Adaptive Control Based on Incremental Hierarchical Sliding Mode for Overhead Crane Systems

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Abstract: Incremental hierarchical sliding mode control (IHSMC) methodology involves $2n - 2$ sign switches of controller parameters for an underactuated system with $n$ subsystems. Too many sign switches trouble parameter tuning. This paper presents an adaptive control design approach based on IHSMC methodology for overhead crane systems with 2 subsystems with only 1 sign switch. The system stability is proven by Barbalat’s lemma and Lasalle’s invariance principle in the sense of Lyapunov theory. Simulation results illustrate the feasibility of the presented method by transport control of overhead crane systems.

Keywords: adaptive control, sliding mode control, hierarchical structure, overhead cranes

1 Introduction

Overhead crane systems are usually employed to move materials horizontally in industries because they can move loads far beyond the normal capability of a human. But their performance may be constrained by the fact that their loads are free to swing with a pendulum-type motion. In practice, operating an overhead crane by manual is hard to resist the pendulum-type motion that is harmful for industry safety. Automation of operation is desirable because high positioning accuracy, small swing angle, and short transportation time are required \cite{1}. As far as transport control of overhead crane systems is concerned, the objective is to transport the loads to the required position as fast and as accurately as possible without free swings. Many control approaches concerning the control problem have been reported in recent years, i.e., fuzzy control \cite{1,4}, adaptive control \cite{3}, feedback linearizing control \cite{8}, wave-based robust control \cite{5}, sliding mode control \cite{2,6,7}, etc. Other reports about this topic can be found in \cite{13} and \cite{14}.

Incremental hierarchical sliding mode control (IHSMC) \cite{2} is a methodology to solve control problems of a class of underactuated systems with $n$ subsystems and 1 control input. For an underactuated system with $n$ subsystems, the incremental sliding surfaces of IHSMC are with $2n - 1$ layers. To ensure the stability of all the surfaces, signs of parameters of the sliding surfaces need to be switched except the first layer (theorem 1 in \cite{2}). But too many sign switches may trouble parameter tuning. In nature, overhead crane systems belong to underactuated systems with 2 subsystems and 1 input. A crane system in \cite{2} was utilized as a benchmark to verify the feasibility of IHSMC. But the signs of two sliding-surface parameters had to be switched for the stability of this system. In this paper, we investigate an adaptive control based on IHSMC for overhead crane systems with only 1 sign switch.

The remainder of this work is organized as follows. Section 2 describes the control design. The stability analysis is presented in section 3. Section 4 shows the simulation results. Conclusion is drawn in section 5 at last.

2 Adaptive Control Design Based on IHSMC for Overhead Crane Systems

2.1 Dynamic Model

Fig. 1 illustrates structure of an overhead crane system. This system consists of the trolley subsystem and the load
subsystem. We assume there is static friction, the rope is inflexible, the rope mass is ignored, and the load is regarded as a material particle. Using Lagrange’s method, its motion equations can be derived as

\[
(m+M)\ddot{x} + mL(\dot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) = f \tag{1}
\]

\[
x \cos \theta + L \dot{\theta} + g \sin \theta = 0 \tag{2}
\]

here \(M\) is trolley mass, \(m\) is load mass, \(L\) is rope length, \(g\) is gravitational acceleration, \(\theta\) is swing angle of the load with respect to the vertical line, \(x\) is trolley position with respect to the origin \(x\), \(f\) is control force applied to the trolley.

\[\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x) + b_1(x) \cdot u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x) + b_2(x) \cdot u
\end{align*} \tag{3}
\]

Here \(x = [x_1, x_2, x_3, x_4]^T\), \(x_1 = x, x_2 = \theta; x_3\) is trolley velocity; \(x_4\) is angular velocity of the load; \(u = f\) is the control input; \(f_i\) and \(b_i\) \((i = 1, 2)\) can be written as

\[
f_1 = \frac{m \cdot L \cdot x_4^2 \cdot \sin x_3 + m \cdot g \cdot \sin x_3 \cdot \cos x_3}{M + m \cdot \sin^2 x_3}
\]

\[
b_1 = \frac{1}{M + m \cdot \sin^2 x_3}
\]

\[
f_2 = \frac{- (m + M) \cdot g \cdot \sin x_3 + m \cdot L \cdot x_4^2 \cdot \sin x_3 \cdot \cos x_3}{(M + m \cdot \sin^2 x_3) \cdot L}
\]

\[
b_2 = \frac{\cos x_3}{(M + m \cdot \sin^2 x_3) \cdot L}
\]

\[\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x) + b_1(x) \cdot u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x) + b_2(x) \cdot u
\end{align*} \tag{3}
\]

2.2 IHSMC-Based Adaptive Control Design

By means of the methodology of IHSMC, the incremental sliding surfaces of this overhead crane system are designed as

\[
s_1 = c_1x_1 + x_2 \\
s_2 = c_2x_3 + s_1 \\
s_3 = c_3x_4 + s_2
\] \tag{4}

where \(c_1\) is a positive constant, \(c_2\) is constant, and \(c_3\) is defined as a time-varying parameter. Due to the derivative relation between \(x_1\) and \(x_2\) in (3), we define \(c_1\) is positive on the aspect of the system stability.

Based on the methodology of equivalent control of variable structure control \([9]\), the SMC law usually includes two parts: switching control and equivalent control. Here we still adopt it and define the total control law \(u\) of the adaptive IHSMC as

\[
u = u_{eq} + u_{sw} \tag{5}
\]

where \(u_{eq}\) is the equivalent control and \(u_{sw}\) is the switching control.

In order to ensure (5) make the last layer sliding surface \(s_3\) asymptotically stable, a Lyapunov function is defined as

\[
V(t) = \frac{1}{2} s_3^2 \tag{6}
\]

Differentiating \(V\) with respect to time \(t\) in (6) yields

\[
\frac{dV}{dt} = \frac{dV}{ds_3} \frac{ds_3}{dt} = s_3 \dot{s}_3 = s_3 \frac{d(c_3x_4 + s_2)}{dt} \tag{7}
\]

Since \(c_3\) is a time-varying parameter, its adaptive law can be derived from (7) in the sense of Lyapunov. So we have

\[
\frac{dV}{dt} = s_3(c_3x_4 + c_3x_4 + s_2) \tag{8}
\]

Substituting (3), (4), and (5) into (8) yields

\[
\frac{dV}{dt} = s_3(c_3x_4 + c_3x_4 + s_2) = s_3(c_3x_4 + c_3x_4 + f_1 + c_1x_2 + (c_3b_2 + b_1)u)
\]

\[
= s_3(c_3x_4 + c_3x_4 + f_1 + c_1x_2 + (c_3b_2 + b_1)(u_{eq} + u_{sw}))
\] \tag{9}

In order to have the stability of the third layer sliding surface, let

\[
\begin{align*}
\dot{c}_3f_2 + c_3x_4 + c_1x_2 + (c_3b_2 + b_1)u_{eq} &= 0 \\
k\dot{x}_3 + \eta \text{sgn}(s_3) + (c_3b_2 + b_1)u_{sw} &= 0 \\
c_3x_4 + f_1 &= 0
\end{align*} \tag{10}
\]

Here \(k\) and \(\eta\) are positive constants, \(\text{sgn}(\cdot)\) is sign function. Substituting (10) into (9), we have

\[
\frac{dV}{dt} = -k\dot{x}_3^2 - \eta |x_3| \leq 0 \tag{11}
\]

which means the sliding motion of the third layer sliding surface \(s_3\) occurs at \(f\). From (10), the equivalent control
law, the switching control law and the adaptive law of $c_3$
of the presented adaptive IHSMC are gotten as

$$\begin{align*}
\dot{u}_q &= \frac{-c_3 f_2 - c_2 x_4 - c_1 x_2}{c_2 b_2 + b_1} \\
\dot{u}_m &= \frac{-k s_3 - \eta sgn(s_3)}{c_2 b_2 + b_1} \\
\dot{s}_3 &= -\frac{f_1 x_4}{||x_4||^2 + \delta}
\end{align*}$$

(12)

Here $\delta$ is a small positive constant to avoid the expression
is singular when $x_4$ is equal to zero.

### 3 Stability Analysis

In this section, we shall prove that only 1 sign switch of the
time controller parameters is able to make this control system possess the asymptotic stability.

**Theorem 3.1.** Consider an overhead crane systems (3)
under the IHSMC-based adaptive control law (12). Then
the third-layer sliding surface $s_3$ and the second-layer sliding surface $s_2$ are asymptotically stable.

**Proof.** Integrating both sides of (11) yields

$$\begin{align*}
V(t) - V(0) &= \int_0^t s_3[-k s_3 - \eta sgn(s_3)] \, dt \\
\dot{V}(t) &= \int_0^t \left( k s_3^2 + \frac{\eta |s_3|}{s_3} \right) \, dt
\end{align*}$$

(13)

So we can obtain

$$V(0) = V(t) + \int_0^t \left( k s_3^2 + \frac{\eta |s_3|}{s_3} \right) \, dt \geq \int_0^t \left( k s_3^2 + \eta |s_3| \right) \, dt$$

(14)

From (14), (15) becomes

$$\lim_{t \to \infty} \int_0^t \left( k s_3^2 + \eta |s_3| \right) \, dt < \infty$$

(15)

From (15), there exist

$$\lim_{t \to \infty} \int_0^t k s_3^2 \, dt < \infty$$

(16)

$$\lim_{t \to \infty} \int_0^t \eta |s_3| \, dt < \infty$$

(17)

We can conclude that

$$s_3 \in \mathcal{L}_2$$

(18)

$$s_3 \in \mathcal{L}_1$$

(19)

From (6) and (14), we can get

$$\frac{1}{2} s_3^2 = V(t) = V(0) - \int_0^t \left( k s_3^2 + \eta |s_3| \right) \, dt < V(0) < \infty$$

(20)

here (20) means

$$s_3 \in \mathcal{L}_\infty$$

(21)

From (11), we have

$$\frac{dV}{dt} = s_3 \frac{ds_3}{dt} = -k s_3^2 - \eta |s_3| < \infty$$

(22)

so we have

$$s_3 \in \mathcal{L}_\infty$$

(23)

(24) can be drawn from (18), (21) and (23) on account of
Barbalat’s lemma [10], i.e., the third layer sliding surface $s_3$ is of asymptotic stability.

$$\lim_{t \to \infty} s_3 = 0$$

(24)

Define a set

$$\mathcal{S}_c = \left\{ s_3 \in \mathbb{R}^2 \mid \frac{dV}{dt} \leq c, c > 0 \right\}$$

(25)

Since $\frac{dV}{dt} \leq 0$ in (11), we know $\mathcal{S}_c$ is positively invariant
and compact. By LaSalle’s principle [11], $s_3$ approaches
the largest invariant set in

$$\mathcal{S} = \left\{ s_3 \in \mathcal{S}_c \mid \frac{dV}{dt} = 0 \right\}$$

(26)

Since the sliding mode of the third-layer sliding surface $s_3$ takes place at $t_f$, this control system does not contain any
discontinuous term in time interval $[t_f, \infty)$, and becomes
an autonomous one. As a result, we have

$$\mathcal{S} = \left\{ s_3 \mid s_3 = 0 \cap s_3 = 0 \right\}$$

(27)

Assume $s_2$ and $x_4$ do not converge to the origin by the
axes $x_4$ and $s_2$ as $t \to \infty$. Then, $s_3$ would converge to a
point of the sliding surface $s_3$ on phase plane by $x_4$ versus
$s_2$ except the origin. This case contradicts the fact that
$\lim_{t \to \infty} s_3 = 0$. So the assumption is false. From proof by
contradiction, we have both $s_2$ and $x_4$ do converge to the
origin rather than other points on the phase plane.
Moreover, we already know $\mathcal{S}$ is attracting, so the largest
invariant set in $\mathcal{S}$ contains no sets other than the
coordinate origin. On account of Lasalle’s invariance principle, we have

$$\lim_{t \to \infty} s_2 = 0$$

(28)

$$\lim_{t \to \infty} x_4 = 0$$

i.e. $s_2$ and $x_4$ are asymptotically stable.

**Theorem 3.2.** Consider an overhead crane systems (3)
under the IHSMC-based adaptive control law (12). Then
the sliding surface $s_1$ is asymptotically stable if (29) is
satisfied.

$$c_2 = \begin{cases} 
  c_2 & \text{if } x_3 \cdot s_1 \geq 0 \\
  -c_2 & \text{if } x_3 \cdot s_1 < 0 
\end{cases}$$

(29)
Proof. Substituting $s_2 = c_2 \cdot x_3 + s_1$ into (28), we have
\[
\lim_{t \to \infty} s_2 = \lim_{t \to \infty} (c_2 \cdot x_3 + s_1) = 0 \tag{30}
\]
Since $\lim_{t \to \infty} x_4 = 0$ and $\dot{x}_3 = x_4$, we have $\lim_{t \to \infty} x_3 = \text{const}$. So (30) becomes
\[
\lim_{t \to \infty} s_1 = -\lim_{t \to \infty} (c_2 \cdot x_3) = \text{const}. \tag{31}
\]
If (29) is satisfied, there exists
\[
\text{sgn}(c_2 \cdot x_3 \cdot s_1) \geq 0 \tag{32}
\]
From (31) and (32), this constant in (31) is zero rather than others. Thus, we can draw
\[
\lim_{t \to \infty} x_3 = 0 \quad \lim_{t \to \infty} s_1 = 0 \tag{33}
\]
(33) means both $x_3$ and $s_1$ are asymptotically stable if (29) is satisfied. □

Comment 1: Substituting $s_2 = x_1$ into $\lim_{t \to \infty} s_1 = 0$ in (33), we have
\[
\lim_{t \to \infty} (c_1 \cdot x_1 + x_1) = 0
\]
This means the two system states $x_1$ and $x_2$ are locally exponentially stable.

Comment 2: As proven in theorems 1 and 2, all the sliding surfaces are asymptotically stable. There also exists tiny difference among them. From (11), the sliding mode of $x_3$ takes place in finite time because of the existence of the discontinuous switching control in (5). But the sliding modes of $x_2$ and $s_1$ are just asymptotically reachable or are reachable in infinite time because their reachability cannot be ensured by (11). Although (29) is discontinuous, it cannot ensure the reachability of $s_1$ and $s_2$ under the Lyapunov function (6). This may inspire one to explore a new Lyapunov function and to deduce a novel switching control law, making all the sliding surfaces possess the reachability.

Comment 3: From our design process, $c_1$ and $c_2$ have each individual function, i.e., $c_2$ switches its sign to get a stable $s_1$, $c_1$ is positive to guarantee $x_1$ and $x_2$ are locally exponentially stable. The function of the adaptive control law of $c_3$ is to make the sliding mode of $s_3$ reachable as soon as possible.

Comment 4: The same incremental sliding surface in [2] was presented. But 2 sign switches of the controller parameters in [2] were needed to guarantee the stability of the sliding surfaces. Although such the method can predigest the system stability analysis [2], it troubles parameter tuning. Whereas, only one sign switch of our controller parameters can ensure the stability of the entire sliding surfaces.

### 4 Simulation Results
In this section, the validity of the IHSMC-based adaptive control is demonstrated by the transport control problem of an overhead crane system. The physical parameters of the overhead crane system are determined as $M = 37.32$ kg, $m = 5$ kg and $L = 1.05$ m. The parameters of the adaptive IHSMC law is selected as $\delta = 0.01$, $c_1 = 3.98$, $c_2 = 0.25$, $k = 1.20$ and $\eta = 0.06$ after trial and error. The initial value of $c_3$ is selected as $c_3^0 = 0.80$. The initial state vector $x^0$ and the desired state vector $x^d$ are $[2, 0, 0, 0]^{T}$ and $[0, 0, 0, 0]^{T}$, respectively.

The simulation results in Fig. 2 demonstrate all the system states and the control input. As proven, all the states are asymptotically stable. From Fig. 2a the system states can achieve the control objective from $x^0$ to $x^d$ at about 4.6 s. Especially, there is no overshoot of the state variable $x_1$, this means the trolley could directly arrive at the desired position with no oscillation. This trait is important for transport control of overhead crane systems in industries. The incremental sliding surfaces, the adaptive process of $c_3$, and the switch process of $c_2$ are displayed in Fig. 3. As proven in theorem 1 and 2, all the surfaces possess the asymptotic stability under the IHSMC-based adaptive control law with only 1 sign switch of the controller parameter $c_2$. Phase plane plots of $s_1$, $s_2$, and $s_3$ are illustrated in Fig. 4. As pointed out in comment 2, only the sliding mode of $s_3$ is reachable in finite time, yet the sliding modes of $s_2$ and $s_1$ is asymptotically reachable.

### 5 Conclusions
This paper has proposed an IHSMC-based adaptive control approach for the transport control problem of overhead crane systems, belonging to underactuated systems extensively used in industries. The system stability is analyzed by Barbalat’s lemma and Lasalle’s invariance principle in the sense of Lyapunov. The presented method with only 1 sign switch of the controller parameters can achieve transport control of overhead crane systems. Simulation results show the feasibility of the presented IHSMC-based adaptive control approach.

### References
Fig. 2: Simulation results. (a). Trolley position $x_1$ and velocity $x_2$, (b). Load angle $x_3$ and angular velocity $x_4$, (c). Control input $u$.

Fig. 3: Simulation results. (a). the third layer sliding surfaces $s_3$ and the second layer sliding surfaces $s_2$, (b). the first layer sliding surface $s_1$, (c). Switch process of $c_3$, (d). Adaptive process of $c_2$.

Fig. 4: Phase plane plots. (a). the third layer sliding surfaces $s_3$, (b). the second layer sliding surfaces $s_2$, (c). the first layer sliding surface $s_1$. 

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