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On Common Fixed Point Results for Set-Valued Mappings in Cone Metric Spaces

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Abstract: In this paper, we establish the existence and uniqueness of common fixed points for a pair of set-valued mappings satisfying a generalized contractive condition in cone metric spaces with normal constant $M = 1$. An example is given to support our results. The presented results improve, unify and generalize many known results in the literature.

Keywords: common fixed point, set-valued mapping, cone metric space.

1 Introduction

Fixed point theory is one of the famous and traditional theories in mathematics and has a broad set of applications. In this theory, contraction is one of the main tools to prove the existence and uniqueness of a fixed point. Banach's contraction principle [5] which gives an answer to the existence and uniqueness of a solution of an operator equation $Tx = x$, is the most widely used fixed point theorem in all of analysis. This principal is constructive in nature and is one of the most useful techniques in the study of nonlinear equations. Different generalizations of the Banach's contraction mapping principle were studied by many authors in metric spaces and Banach spaces, see [6,7,8,15,16,17,19,20,21,22,23,24,25,27,28,31,33] and references given therein. Later, Nadler Jr. [18] has proved multivalued version of the Banach contraction principle which states that each closed bounded valued contraction map on a complete metric space has a fixed point. Many authors have been using the Hausdroff metric to obtain fixed point results for multivalued maps on metric spaces.

Recently, Huang and Zhang [11] generalized the concept of a metric space, replacing the set of real numbers by an ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions. Wardowski [36] introduced the concept of multivalued contractions in cone metric spaces and using the notion of normal cones, obtained fixed point theorems for such mappings. As we know, most of known cones are normal with normal constant M = 1. Further, Rezapour [29] proved two results about common fixed points of multifunctions on cone metric spaces. For a detailed study, see [1,2,3,4,9,10,12,13,14,26,29,30,32,34,35]

Motivated by the above work, in this paper, we obtain a unique common fixed point and for a pair of set-valued mappings satisfying a generalized contractive condition in cone metric spaces with normal constant $M = 1$. An example is given to justify our results. The presented results generalize many known results in cone metric spaces.

2 Preliminaries

In this section, we recall the definition of cone metric spaces and some of their properties.

Definition 2.1. Let E be a real Banach space. A subset P of E is called a cone if the following conditions are satisfied:

(i)P is closed, nonempty and $P \neq \{0\}$;
(ii)$a, b \in \mathbb{R}, a, b \geq 0$ and $x, y \in P$ imply that $ax + by \in P$.
(iii)$P \cap (-P) = \{0\}$.

Given a cone P of E, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall

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write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in intP$.

A cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \text{ implies } \|x\| \leq K\|y\|.$$

The least positive number satisfying the above inequality is called the normal constant of P.

Definition 2.2. Let X be a nonempty set and $d : X \times X \to E$ be a mapping such that the following conditions hold:

(i)$0 \leq d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;
(ii)$d(x,y) = d(y,x)$ for all $x, y \in X$;
(iii)$d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X,d) is called a cone metric space.

Example 2.3. Let $X = R$, $E = R^2$, $P = \{(x,y) \in E : x, y \geq 0\} \subset R^2$ and $d : X \times X \to E$ such that $d(x,y) = (|x-y|, \delta|x-y|)$, where $\delta \geq 0$ is a constant. Then (X,d) is a cone metric space.

Example 2.4. Let $E = C^1_{\mathbb{R}}([0,1])$ with norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$. The cone $P = \{f \in E : f \geq 0\}$ is a non-normal cone.

Definition 2.5. Let (X,d) be a cone metric space. We say that $\{x_n\}$ is;

(i)a Cauchy sequence if for every $c \in E$ with $0 \ll c$, there is N such that for all $m, n > N, d(x_n, x_m) \ll c$;
(ii)a convergent sequence if for every $c \in E$ with $0 \ll c$, there is N such that for all $n > N, d(x_n, x) \ll c$, for some $x \in X$. We denote it by $\lim\limits_{n\to\infty} x_n = x$ or $x_n \to x$.

A cone metric space X is said to be complete if every Cauchy sequence in X is convergent in X. The limit of a convergent sequence is unique provided P is a normal cone with normal constant K (see [11]).

Lemma 2.6. Let (X,d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_m) \to 0$ as $m, n \to \infty$.

Definition 2.7. Let (X,d) be a cone metric space and $B \subseteq X$.

(i)A point b in B is called an interior point of B whenever there exists a point $p, 0 << p$ such that $N(b,p) \subseteq B$, where $N(b,p) = \{y \in X : d(y,b) << p\}$.
(ii)A subset $A \subseteq X$ is called open if each element of A is an interior point of A.

The family $\mathbb{B} = \{N(x,e) : x \in X, 0 << e\}$ is a sub-basis for a topology on X. We denote this cone topology by τ_c is called Hausdroff and first countable.

Lemma 2.8. Let (X,d) be a cone metric space, P be a normal cone with normal constant $M = 1$ and A be a compact set in (X, τ_c). Then, for every $x \in X$ there exists $a_0 \in A$ such that

$$\|d(x,a_0)\| = \inf_{a\in A} \|d(x,a)\|.$$

Lemma 2.9. Let (X,d) be a cone metric space, P be a normal cone with normal constant $M = 1$ and A, B be two compact sets in (X, τ_c). Then,

$$\sup_{x\in B} D(x,A) < \infty,$$

where $D(x,A) = \inf\limits_{a\in A} \|d(x,a)\|$, for each $x \in X$.

Definition 2.10. Let (X,d) be a cone metric space, P be a normal cone with normal constant $M = 1$, H_c denote the set of all compact subsets of (X, τ_c) and $A \in H_c$. Now by Lemma 2.8, we define

$$h_A : H_c(X) \to [0,\infty) \text{ and } d_H : H_c(X) \times H_c(X) \to [0,\infty)$$

by

$$h_A(B) = \sup_{x\in A} D(x,B) < \infty \text{ and } d_H(A,B) = \max\{h_A(B), h_B(A)\},$$

respectively.

Remarks 2.11. Let (X,d) be a cone metric space, P be a normal cone with normal constant $M = 1$. Define $\rho : X \times X \to [0,\infty)$ by $\rho(x,y) = \|d(x,y)\|$. Then, (X,ρ) is a metric space. This implies that for each $A, B \in H_c$ and $x, y \in X$, we have the following relations:

(i)$D \leq \|d(x,y)\| + D(y,A)$,
(ii)$D \leq D(x,B) + h_B(A)$, and
(iii)$D \leq \|d(x,y)\| + D(y,B) + h_B(A)$.

3 Main Results

Theorem 3.1. Let (X,d) be a complete cone metric space with normal constant $M = 1$ and $S, T : X \to H_c(X)$ be two set-valued mappings such that

$$d(Sx,Ty) \leq \alpha \max\{d(x,Sx), d(y,Ty), d(x,y)\} \quad (1)$$

for all $x, y \in X$ where $0 \leq \alpha < 1$. Then S and T have a unique common fixed point in X.

Proof Let $x_0 \in X$be a arbitrary point. Then by Lemma 2.8, there exist $x_1 \in Sx_0$ and $x_2 \in Tx_1$such that

$$D(x_0, Sx_0) = \|d(x_0,x_1)\|$$

and

$$D(x_1, Tx_1) = \|d(x_1,x_2)\|.$$

Likewise, for $n \in \mathbb{N}$, we define a sequence $\{x_n\}$ in X such that

$$x_{2n-1} \in Sx_{2n-2}, x_{2n} \in Tx_{2n-1}.$$

Therefore,

$$D(x_{2n-2}, Sx_{2n-2}) = \|d(x_{2n-2}, x_{2n-1})\|$$

and

$$D(x_{2n-1}, Tx_{2n-1}) = \|d(x_{2n-1}, x_{2n})\|$$

for all $n \in \mathbb{N}$. Thus, for all $n \in \mathbb{N}$, we have the following

$$\begin{aligned}\|d(x_{2n},x_{2n+1})\| &= D(x_{2n},Sx_{2n})\\ &\leq h_{Tx_{2n-1}}(Sx_{2n})\\ &\leq d_H(Tx_{2n-1},Sx_{2n})\\ &\leq \alpha\max\{D(x_{2n},Sx_{2n}),D(x_{2n-1},Tx_{2n-1}),\\ &\quad D(x_{2n},x_{2n-1})\}\\ &= \alpha\max\{\|d(x_{2n},x_{2n+1})\|,\|d(x_{2n-1},x_{2n})\|\}.\end{aligned}$$

Case I:

If $\max\{\|d(x_{2n},x_{2n+1})\|,\|d(x_{2n-1},x_{2n})\|\} = \|d(x_{2n},x_{2n+1})\|$, then

$$\|d(x_{2n},x_{2n+1})\| \leq \alpha\|d(x_{2n},x_{2n+1})\|$$

which implies that

$$\|d(x_{2n},x_{2n+1})\| \to 0 \text{ as } n\to\infty,$$

since $0<\alpha<1$.

Case II:

If $\max\{\|d(x_{2n},x_{2n+1})\|,\|d(x_{2n-1},x_{2n})\|\} = \|d(x_{2n-1},x_{2n})\|$, then

$$\|d(x_{2n},x_{2n+1})\| \leq \alpha\|d(x_{2n-1},x_{2n})\|$$

Proceeding in this way, we obtain

$$\|d(x_{2n},x_{2n+1})\| \leq \alpha^{2n}\|d(x_0,x_1)\|,\ \ n\in\mathbb{N}$$

Also for $n>m$, we have

$$\begin{aligned}\|d(x_n,x_m)\| &\leq \|d(x_n,x_{n-1})\|+\|d(x_{n-1},x_{n-2})\|+\cdots\cdots+\\ &\quad \|d(x_{m+1},x_m,u)\|\\ &\leq (\alpha^{n-1}+\alpha^{n-2}+\cdots\cdots+\alpha^m)\|d(x_1,x_0)\|\\ &\leq \frac{\alpha^m}{1-\alpha}\|d(x_1,x_0)\|\end{aligned}$$

Thus $\|d(x_n,x_m)\|\to 0$, as $n\to\infty$, since $\dfrac{k^m}{1-k}\to 0$, as $n\to\infty$.

Therefore, in both cases, $\{x_n\}$ is a Cauchy sequence in X. Hence there exists a point $z\in X$ such that $x_n\to z$, as $n\to\infty$. Further, by using Remark 2.11, we have

$$\begin{aligned}D(z,Sz) &\leq D(z,Tx_{2n-1})+h_{Tx_{2n-1}}(Sz)\\ &\leq D(z,Tx_{2n-1})+d_H(Tx_{2n-1},Sz)\\ &\leq \|d(z,x_{2n})\|+\\ &\quad \alpha\max\{D(z,Sz),D(x_{2n-1},Tx_{2n-1}),D(z,x_{2n-1})\}\\ &= \|d(z,x_{2n})\|+\\ &\quad \alpha\max\{D(z,Sz),D(x_{2n-1},x_{2n}),D(z,x_{2n-1})\},\end{aligned}$$

now, letting $n\to\infty$, we get $D(z,Sz)=0$. Hence, by Lemma 2.8, $z\in Sz$. Similarly,

$$\begin{aligned}D(z,Tz) &\leq D(z,Sx_{2n})+h_{Sx_{2n}}(Tz)\\ &\leq D(z,Sx_{2n})+d_H(Sx_{2n},Tz)\\ &\leq \|d(z,x_{2n+1})\|+\\ &\quad \alpha\max\{D(z,Tz),D(x_{2n},Sx_{2n}),D(z,x_{2n})\}\\ &= \|d(z,x_{2n+1})\|+\\ &\quad \alpha\max\{D(z,Tz),D(x_{2n},x_{2n+1}),D(z,x_{2n})\},\end{aligned}$$

now, letting $n\to\infty$, we get $D(z,Tz)=0$. Hence, by Lemma 2.8, $z\in Tz$. Therefore, $z\in X$ is a common fixed point of S and T.

Uniqueness:

Let $\tilde{z}$ be another common fixed point of S and T, that is, $S\tilde{z}=T\tilde{z}=\tilde{z}$. Then

$$\begin{aligned}\|d(z,\tilde{z})\| &= \|d(Sz,T\tilde{z})\|\\ &\leq \alpha\max\{\|d(z,Sz)\|,\|d(\tilde{z},T\tilde{z})\|,\|d(z,\tilde{z})\|\}\end{aligned}$$

which implies that $\|d(z,\tilde{z})\|=0$, since $\alpha<1$ for all $u\in X$. Thus z is a unique common fixed point of S and T.

Remarks 3.2. Theorem 3.1 generalizes Theorem 3.1 of [9]. Also, our result establishes the uniqueness of the common fixed point of S and T.

Corollary 3.3. Let (X,d) be a complete cone metric space with normal constant $M=1$ and $S,T:X\to H_c(X)$ be two set-valued mappings such that

$$d(Sx,Ty)\leq ad(x,Sx)+bd(y,Ty)+cd(x,y) \qquad (2)$$

for all $x,y\in X$ and $a,b,c\geq 0$, where $a+b+c<1$. Then S and T have a unique common fixed point in X.

Remarks 3.4. If we take a = b and c = 0 in Corollary 3.3, then we get the Theorem 2.3 of Rezapour [29]. In the case, $S=T$ and $a=b=0$, we obtain Nadler's result [18].

Further, we obtain the following particular results from Corollary 3.3.

Corollary 3.5. Let (X,d) be a complete cone metric space with normal constant $M=1$ and $S,T:X\to H_c(X)$ be two set-valued mappings such that

$$d(Sx,Ty)\leq ad(x,Sx)+bd(y,Ty)$$

for all $x,y\in X$ and $a,b\geq 0$, where $a+b<1$. Then S and T have a unique common fixed point in X.

Corollary 3.6. Let (X,d) be a complete cone metric space with normal constant $M=1$ and $S:X\to H_c(X)$ be a set-valued mapping such that

$$d(Sx,Sy)\leq ad(x,Sx)+bd(y,Sy)+cd(x,y)$$

for all $x,y\in X$ and $a,b,c\geq 0$, where $a+b+c<1$. Then S and T have a unique common fixed point in X.

Corollary 3.7. Let (X,d) be a complete cone metric space with normal constant $M = 1$ and $S : X \to H_c(X)$ be a set-valued mapping such that

$$d(Sx,Sy) \leq ad(x,Sx) + bd(y,Sy)$$

for all $x,y \in X$ and $a,b \geq 0$, where $a+b < 1$. Then S and T have a unique common fixed point in X.

Remarks 3.8. Corollary 3.7 gives the set-valued Kannan type contractive condition [15] in cone metric spaces.

Theorem 3.9. Let (X,d) be a complete cone metric space with normal constant $M = 1$ and $S,T : X \to H_c(X)$ be two set-valued mappings such that

$$d(Sx,Ty) \leq \alpha \max\{d(x,Ty),d(y,Sx),d(x,y)\} \qquad (3)$$

for all $x,y \in X$ where $0 \leq \alpha < 1$. Then S and T have a unique common fixed point in X.

Proof Using the similar argument of the proof of Theorem 3.1, we can show that there exists a Cauchy sequence $\{x_n\}$ in X such that

$$x_{2n-1} \in Sx_{2n-2}, x_{2n} \in Tx_{2n-1}.$$

Therefore,

$$D(x_{2n-2},Sx_{2n-2}) = \|d(x_{2n-2},x_{2n-1})\|$$

and

$$D(x_{2n-1},Tx_{2n-1}) = \|d(x_{2n-1},x_{2n})\|$$

for all $n \in \mathbb{N}$. Thus, we can find a element $\tilde{x} \in X$ such that $x_n \to \tilde{x}$ as $n \to \infty$. Applying Remark 2.11, we obtain

$$\begin{aligned}
D(\tilde{x},S\tilde{x}) &\leq D(\tilde{x},Tx_{2n-1}) + h_{Tx_{2n-1}}(S\tilde{x}) \\
&\leq D(\tilde{x},Tx_{2n-1}) + d_H(Tx_{2n-1},S\tilde{x}) \\
&\leq \|d(\tilde{x},x_{2n})\| + \\
&\quad \alpha \max\{D(\tilde{x},Tx_{2n-1}),D(x_{2n-1},S\tilde{x}),D(\tilde{x},x_{2n-1})\} \\
&= \|d(\tilde{x},x_{2n})\| + \\
&\quad \alpha \max\{D(\tilde{x},x_{2n}),D(x_{2n-1},S\tilde{x}),D(\tilde{x},x_{2n-1})\},
\end{aligned}$$

now, letting $n \to \infty$, we get $D(\tilde{x},S\tilde{x}) = 0$. Hence, by Lemma 2.8, $\tilde{x} \in S\tilde{x}$. Similarly,

$$\begin{aligned}
D(\tilde{x},T\tilde{x}) &\leq D(\tilde{x},Sx_{2n}) + h_{Sx_{2n}}(T\tilde{x}) \\
&\leq D(\tilde{x},Sx_{2n}) + d_H(Sx_{2n},T\tilde{x}) \\
&\leq \|d(\tilde{x},x_{2n+1})\| + \\
&\quad \alpha \max\{D(\tilde{x},Sx_{2n}),D(x_{2n},T\tilde{x}),D(\tilde{x},x_{2n})\} \\
&= \|d(\tilde{x},x_{2n+1})\| + \\
&\quad \alpha \max\{D(\tilde{x},x_{2n+1}),D(x_{2n},T\tilde{x}),D(\tilde{x},x_{2n})\},
\end{aligned}$$

now, letting $n \to \infty$, we get $D(\tilde{x},T\tilde{x}) = 0$. Hence, by Lemma 2.8, $\tilde{x} \in T\tilde{x}$. Therefore, $\tilde{x} \in X$ is a common fixed point of S and T.

Uniqueness:

Let $\tilde{z}$ be another common fixed point of S and T, that is, $S\tilde{z} = T\tilde{z} = \tilde{z}$. Then

$$\begin{aligned}
\|d(\tilde{x},\tilde{z})\| &= \|d(S\tilde{x},T\tilde{z})\| \\
&\leq \alpha \max\{\|d(\tilde{x},T\tilde{z})\|,\|d(\tilde{z},S\tilde{x})\|,\|d(\tilde{x},\tilde{z})\|\}
\end{aligned}$$

which implies that $\|d(\tilde{x},\tilde{z})\| = 0$, since $\alpha < 1$ for all $u \in X$. Thus $\tilde{x}$ is a unique common fixed point of S and T.

Remarks 3.10. Theorem 3.1 generalizes Theorem 3.2 of [9]. Further, it establishes the uniqueness of the common fixed point of S and T.

Corollary 3.11. Let (X,d) be a complete cone metric space with normal constant $M = 1$ and $S,T : X \to H_c(X)$ be two set-valued mappings such that

$$d(Sx,Ty) \leq ad(x,Ty) + bd(y,Sx) + cd(x,y)$$

for all $x,y \in X$ and $a,b,c \geq 0$, where $a+b+c < 1$. Then S and T have a unique common fixed point in X.

Remarks 3.12. If we take a = b and c = 0 in Corollary 3.11, then we get the Theorem 2.4 of Rezapour [29].

The following example supports our results.

Example 3.13. Consider the metric defined in Example 2.3. Now define

$$S,T : X \to X$$

such that

$$Sx = \frac{1}{2} \text{ and } Tx = \frac{x^2}{2}, \ \forall x \in X.$$

$$d(Sx,Ty) = \left(\frac{1}{2}\left|1-y^2\right|, \frac{\delta}{2}\left|1-y^2\right|\right) \qquad (4)$$

$$\max\left\{d(x,Sx),d(y,Ty),d(x,y)\right\} =$$

$$\max\left\{\left(\left|x-\frac{1}{2}\right|, \frac{\delta}{2}\left|x-\frac{1}{2}\right|\right), \left(\left|y-\frac{y^2}{2}\right|, \delta\left|y-\frac{y^2}{2}\right|\right), \left(\left|x-y\right|, \delta\left|x-y\right|\right)\right\}. \qquad (5)$$

From equations (4) and (5), it can be easily seen that all the conditions of Theorems 3.1 and 3.9 are satisfied. Hence, S and T have a unique fixed point 1.

4 Conclusion

We have obtained a unique common fixed point for a pair of set-valued mappings S and T without the property of weakly compatibility in the setting of cone metric spaces by taking the normal constant $M = 1$. We have weakened the contractive nature of already existing maps to achieve the existence and uniqueness of common fixed point that extend and generalize many known results in the literature.

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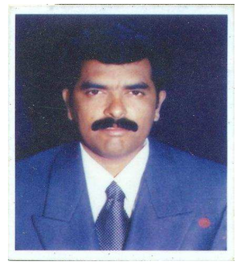

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