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Soft *b - open sets and soft *b - continuous functions

Kh. Abd-Rabou[1,*], *I. K. Halfa* [1,2], *K. Barakat*[1,3]*and M. El sayed*[4]

[1]Department of Mathematics, Faculty of Science, Shaqra University, KSA.
[2]Department of Mathematics and Theoretical Physics, NRC, Atomic Energy Authority, Cairo, Egypt.
[3]Department of Mathematics, Faculty of Science, Helwan University, Egypt.
[4]Department of Mathematics, College of Science and arts, Najran University, KSA.

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Abstract: In this paper, we introduce a new class of open soft sets in soft topological spaces, called soft *b$-$ open sets. Then the relationships among soft $\alpha-$ open, soft semi$-$open, soft pre$-$ open and soft $\beta-$ open are discussed. Also we introduce some types of continuous soft functions based on b*$-$open soft set, namely soft b*$-$continuous and b*$-$ open soft function and we discuss the relations between these types and other weaker forms of continuous soft functions. Finally we study some of their properties.

Keywords: soft set - soft topological space - b**- open soft set - t*- open soft set - B - open soft set - * b- continuous soft function.

1 Introduction

Molodtsov [5] initiated a novel concept of a soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann integration, theory of measurement, and so on. A soft set theory and its applications have shown a great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz [9] introduced the notion of a soft topological space, which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al [6], Aygunoglu , Aygun and Hussain et al [1] continued to study the properties of a soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [3]. He investigated semi-open soft sets in soft topological spaces and studied some properties of it. Arockiarani and Arokialancy defined a β- open soft set and continued to study weak forms of open soft sets in a soft topological space. Later, Akdag and Ozkan [8] defined an α- open (α- closed) soft set.

In the present paper, we introduce some new concepts in soft topolgical spaces such as *b-open soft sets, *b-closed soft sets, *b- soft interior, * b- soft closure, * b-continuous soft functions, *b- open soft functions and *b-closed soft functions. We also study the relationships among continuity [4], α- continuity [8], semi- continuity [7], pre- continuity [8], β- continuity [15] and * b-continuity of soft functions defined on soft topological spaces.

Throughout this paper, the spaces X and Y stand for soft topological spaces with $(X,E,\ \tau)$and $(Y,K,\ \upsilon)$assumed unless otherwise stated. Moreover, throughout this paper, a soft mapping $\tilde{f}: X \to Y$stands for a mapping, where $\tilde{f}: (X,E,\ \tau) \to (Y,K,\upsilon)$, $u: P(X) \to P(Y)$ and $p: E \to K$ are assumed mappings unless otherwise stated.

2 preliminaries

Definition 2.1[5] Let X be an initial universe and E be a set of parameters. Let $P(X)$denote the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F: A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$may be considered as the set of ε-approximate elements of the soft set (F,A).

Definition 2.2[11] A soft set (F,A) over X is called a null soft set, denoted by $\tilde{\Phi}$, if $e \in A$,then $F(e) = \phi$.

* Corresponding author e-mail: k_abdrabo@yahoo.com

Definition 2.3[11] A soft set (F,A) over X is called an absolute soft set, denoted by $\tilde{X}$, if $e \in A$, then $F(e) = X$.

Definition 2.4[9] Let Y be a non- empty subset of X, then $\tilde{Y}$ denotes the soft set (F,E) over X, for which $Y(e) = Y$, for all $e \in E$.

Definition 2.5[11] The union of two soft sets of (F,A) and (G,B) over the common universe X is the soft set (H,C) where $C = A\bigcup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e); & \forall\ e \in A-B. \\ G(e); & \forall\ e \in B-A. \\ F(e)\bigcup G(e); & \forall\ e \in A\bigcap B. \end{cases}$$

We write $(F,A)\tilde{\bigcup}(G,B) = (H,C)$.

Definition 2.6[11] The intersection of two soft sets of (F,A) and (G,B) over a common universe X is the soft set (H,C), where $C = A\bigcap B$ and $H(e) = F(e)\bigcap G(e)$, for all $e \in C$.

Definition 2.7[11] Let (F,A) and (G,B) be two soft sets over a common universe X. Then $(F,A)\tilde{\subset}(G,B)$, if $A \subset B$, and $F(e) \subset G(e)$, for all $e \in A$.

Definition 2.8[9] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if it satisfies the following axioms:

1.$\tilde{\Phi}$, $\tilde{X}$ belong to τ.
2.The union of any number of soft sets in τ belongs to τ.
3.The intersection of any two soft sets in τ belongs to τ.

The triple $(X,E,\ \tau)$ is called a soft topological space. The members of τ are said to be open soft sets over X. A soft set (F,A) over X is said to be soft closed, if its relative complement $(F,A)^c$belongs to τ.

Definition 2.9[10] For a soft set (F,A) over X, the relative complement of (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$, for all $\alpha \in A$.

Definition 2.10[2] A soft set (F,A) in a soft topological space X is called:

1.Soft b-open (sb-open) if $(F,A)\tilde{\subset}(Int(Cl(F,A)))\tilde{\bigcup}(Cl(Int(F,A)))$.
2.Soft b-closed (sb-closed) if$(F,A)\tilde{\supset}(Int(Cl(F,A)))\tilde{\bigcap}(Cl(Int(F,A)))$.

Definition 2.11[2] Let $(X,E,\ \tau)$ be a soft topological space and (F,A) be a soft set. Then

1.[6] The interior of (F,A) is the soft set: $int(F,A) = \tilde{\bigcup}\{(O,A)\tilde{\subset}(F,A) : (O,A)\ is\ soft\ open\ over\ X\}$.
2.[9] The closure of (F,A) is the soft set: $cl(F,A) = \tilde{\bigcap}\{(F,E)\tilde{\supset}(F,A) : (F,E)\ \text{is soft closed over X}\}$.
3.b-closure of a soft set (F,A) over X is denoted by: $\mathrm{b}-cl(F,A) = \tilde{\bigcap}\{(F,E)\tilde{\supset}(F,A) : (F,E)\ \text{is a b}-\text{closed soft set over X}\}$.
4.b-interior of a soft set (F,A) over X is denoted by: $\mathrm{b}-int(F,A) = \tilde{\bigcup}\{(O,A)\tilde{\subset}(F,A) : (O,A)\ \text{is a b}-\text{open soft set over X}\}$.

Clearly b$-Cl(F,A)$ is the smallest b-closed soft set over Xwhich contains (F,A) and b$-Int(F,A)$is the largest b- open soft set over X which is contained in (F,A).

Definition 2.12[2] Let $(X,E,\ \tau)$ be a soft topological space, and (F,A)be a soft set over X. Then
(i)[3] The semi-interior of (F,A) is a soft set: $\mathrm{s}-Int(F,A) = \tilde{\bigcup}\{(O,A)\tilde{\subset}(F,A) : (O,A)\ \text{is soft semi}-\text{open over X}\}$. (ii)[3] The semi- closure of (F,A) is a soft set:
$\mathrm{s}-\mathrm{Cl}(F,A) = \tilde{\bigcap}\{(F,E)\tilde{\supset}(F,A)\}, : (F,E)$ is soft semi-closed over X.
Definition 2.13[13] A subset A of a topological space $(X,\ \tau)$ is said to be:

1.* b- open [14] if $A \subseteq Cl(Int(A))\bigcap Int(Cl(A))$.
2.b**- open [12] if $A \subseteq Int(Cl(Int(A)))\bigcup Cl(Int(Cl(A)))$.
3.**b- open [14] if $A \subseteq Int(Cl(Int(A)))\bigcap Cl(Int(Cl(A)))$.

3 properties of * b-open soft sets and **b-open soft sets

Definition 3.1 A subset (A,E) of a soft topological space $(X,E,\ \tau)$ is said to be:

1.$\tilde{s}$ * b- open if $(A,E)\tilde{\subseteq}Cl(Int(A,E))\tilde{\bigcap}Int(Cl(A,E))$.
2.$\tilde{s}$ b**- open if $(A,E)\tilde{\subseteq}Int(Cl(Int(A,E)))\tilde{\bigcup}Cl(Int(Cl(A,E)))$.
3. $\tilde{s}$ **b- open if $(A,E)\tilde{\subseteq}Int(Cl(Int(A,E)))\tilde{\bigcap}Cl(Int(Cl(A,E)))$.

Definition 3.2 A soft function $f : (X,E,\ \tau_1) \to (Y,E,\ \tau_2)$ is called :

1.$\tilde{s}$ * b- continuous if $f^{-1}(V,E)$ is an $\tilde{s}$ * b-open in $(X,E,\ \tau_1)$, for each open soft set (V,E) of $(Y,E,\ \tau_2)$.
2.$\tilde{s}$ b**- continuous if $f^{-1}(V,E)$ is an $\tilde{s}$ b**-open in $(X,E,\ \tau_1)$, for each open soft set $(V,E)$of $(Y,E,\ \tau_2)$.
3.$\tilde{s}$ t- continuous if $f^{-1}(V,E)$ is a t - $\tilde{s}$ set in $(X,E,\ \tau_1)$, for each open soft set (V,E) of $(Y,E,\ \tau_2)$.
4.$\tilde{s}$ B- continuous if $f^{-1}(V,E)$ is an $\tilde{s}$ B-open in $(X,E,\ \tau_1)$, for each open soft set (V,E) of $(Y,E,\ \tau_2)$.
5.$\tilde{s}$- locally closed continuous if $f^{-1}(V,E)$ is an $\tilde{s}$-locally closed in $(X,E,\ \tau_1)$, for each open soft set (V,E) of $(Y,E,\ \tau_2)$.
6.$\tilde{s}$- locally b -closed continuous if $f^{-1}(V,E)$ is an $\tilde{s}$ - locally b - closed in $(X,E,\ \tau_1)$, for each open soft set (V,E) of $(Y,E,\ \tau_2)$.

7. $\tilde{s}$ - locally * b -closed continuous if $f^{-1}(V,E)$ is an $\tilde{s}$- locally * b- closed in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
8. $\tilde{s}$- locally b** - closed continuous if it is an $\tilde{s}$- locally b**- closed in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
9. $\tilde{s}$- completely continuous if $f^{-1}(V,E)$ is soft regular open in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
10. $\tilde{s}$- D(c, b) continuous if $f^{-1}(V,E)$ is an $\tilde{s}$- D(c, b) – soft set in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
11. $\tilde{s}$- D(c, * b) continuous if $f^{-1}(V,E)$ is an $\tilde{s}$- D(c, * b) - set in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
12. $\tilde{s}$- D(c, b**) continuous if $f^{-1}(V,E)$ is an $\tilde{s}$- D(c, b**) - set in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).

Definition 3.3 A subset (A,E) of a soft topological space (X,E,τ_1) is called:

1. $\tilde{s}$- D(c, * b) if $Int(A,E)$ = * b $Int(A,E)$.
2. $\tilde{s}$- D(c, ** b) if $Int(A,E)$ = ** b $Int(A,E)$.

Result 3.4

1. Every regular open $\tilde{s}$ set is a t - $\tilde{s}$ set.
2. Every regular closed $\tilde{s}$ set is a t* -$\tilde{s}$ set.
3. Every locally closed $\tilde{s}$ set is a B - $\tilde{s}$ set.

Theorem 3.5 Let (A,E)be a soft subset of (X,E,τ_1). Then the following are true:

1. (A,E) is a t- $\tilde{s}$ set iff it is soft semi closed.
2. (A,E) is a t*-$\tilde{s}$ set iff it is soft semi open.

Proof.

1. Let (A,E) be a t - $\tilde{s}$ set . Then
$Int(A,E) = Int(Cl(A,E))$
$\Rightarrow Int(Cl(A,E)) \tilde{\subseteq} (A,E)$
$\Rightarrow (A,E)$ is $\tilde{s}$ semi closed.
Conversly, assume that (A,E) is $\tilde{s}$ semi closed, then
$Int(Cl(A,E)) \tilde{\subseteq} (A,E)$,
$Int(Int(Cl(A,E))) \tilde{\subseteq} Int(A,E)$,
$Int(Cl(A,E)) \tilde{\subseteq} Int(A,E)$.
Since
$Int(A,E) \tilde{\subseteq} Int(Cl(A,E))$,
$Int(A,E) = Int(Cl(A,E)) \Rightarrow (A,E)$ is a t - $\tilde{s}$ set.

2. Let (A,E) be a t*-$\tilde{s}$ set, then $Cl(A,E) = Cl(Int(A,E))$.

Thus$(A,E) \tilde{\subseteq} Cl(Int(A,E)) \Rightarrow (A,E)$ is $\tilde{s}$ semi open.
Conversly, assume that (A,E) is $\tilde{s}$ semi open, then $(A,E) \tilde{\subseteq} Cl(Int(A,E))$, $Cl(A,E) \tilde{\subseteq} Cl(Cl(Int(A,E)))$, $Cl(A,E) \tilde{\subseteq} Cl(Int(A,E))$.
Since
$Cl(Int(A,E)) \tilde{\subseteq} Cl(A,E)$,
$Cl(A,E) = Cl(Int(A,E)) \Rightarrow (A,E)$ is a t*- $\tilde{s}$ set.

Theorem 3.6

1. If (A,E) and (B,E) are t - $\tilde{s}$ sets, then $(A,E)\tilde{\bigcap}(B,E)$ is a t - $\tilde{s}$ set .
2. If (A,E) and (B,E) are t*- $\tilde{s}$ sets, then $(A,E)\tilde{\bigcup}(B,E)$ is a t*- $\tilde{s}$ set .

Proof.

1. Let (A,E) and (B,E) be t - $\tilde{s}$ sets. Then we have
$Int(A,E) = Int(Cl(A,E))$,
$Int(B,E) = Int(Cl(B,E))$.
Since
$Int((A,E)\tilde{\bigcap}(B,E)) \tilde{\subseteq} Int(Cl(A,E)\tilde{\bigcap}(B,E))$
$\tilde{\subseteq} Int(Cl(A,E)\tilde{\bigcap}Cl(B,E))$
$= (Int\ Cl(A,E))\tilde{\bigcap} Int(Cl(B,E))$
$= (Int(A,E))\tilde{\bigcap}(Int(B,E))$
$= Int((A,E)\tilde{\bigcap}(B,E))$,
$Int((A,E)\tilde{\bigcap}(B,E)) \tilde{\subseteq} Int(Cl((A,E)\tilde{\bigcap}(B,E)))$
$\tilde{\subseteq} Int((A,E)\tilde{\bigcap}(B,E))$
$\Rightarrow Int((A,E)\tilde{\bigcap}(B,E)) = Int(Cl((A,E)\tilde{\bigcap}(B,E)))$
$\Rightarrow ((A,E)\tilde{\bigcap}(B,E))$ is a t - $\tilde{s}$ set.

2. Let (A,E) and (B,E) be t*- $\tilde{s}$ sets.
Then we have:
$Cl(A,E) = Cl(Int(A,E))$, and
$Cl(B,E) = Cl(Int(B,E))$.
Since
$Cl((A,E)\tilde{\bigcup}(B,E)) = Cl(A,E)\tilde{\bigcup}Cl(B,E)$
$= Cl(Int(A,E))\tilde{\bigcup} Int(B,E)) \tilde{\subseteq} Cl(Int((A,E)\tilde{\bigcup}(B,E)))$
$= Int((A,E)\tilde{\bigcap}(B,E))$,
$Int((A,E)\tilde{\bigcap}(B,E)) \tilde{\subseteq} Int(Cl((A,E)\tilde{\bigcap}(B,E)))$
$\tilde{\subseteq} Int((A,E)\tilde{\bigcap}(B,E))$
$Cl((A,E)\tilde{\bigcup}(B,E)) \tilde{\subseteq} Cl(Int((A,E)\tilde{\bigcup}(B,E)))$.
Since

$Int((A,E)\tilde{\bigcup}(B,E)) \tilde{\subseteq} ((A,E)\tilde{\bigcup}(B,E))$,
$Cl(Int((A,E)\tilde{\bigcup}(B,E))) \tilde{\subseteq} Cl((A,E)\tilde{\bigcup}(B,E))$, then,

$$Cl(Int((A,E)\tilde{\bigcup}(B,E))) = Cl((A,E)\tilde{\bigcup}(B,E))$$

$\Rightarrow ((A,E)\tilde{\bigcup}(B,E))$ is a t*- $\tilde{s}$ set .

Theorem 3.7 A soft set (A,E) is a t -$\tilde{s}$ set iff its relative complement is a t*- $\tilde{s}$ set .
Proof .
Let (A,E) be a t - $\tilde{s}$ set, then $Int(A,E) = Int(Cl(A,E)) \Leftrightarrow \tilde{X} - Int(A,E)$
$= \tilde{X} - Int(Cl(A,E))$
$\Leftrightarrow Cl(\tilde{X} - (A,E)) = Cl(Int(\tilde{X} - (A,E)))$
$\Leftrightarrow Cl((A,E)^c) = Cl(Int((A,E)^c))$
$\Leftrightarrow (A,E)^C$ is a t*- $\tilde{s}$ set .

Theorem 3.8 For a soft set $(A,E)$of a soft topological space (X,E,τ), the following are equivalent:

1. (A,E) is $\tilde{s}$ open.
2. (A,E) is $\tilde{s}$ pre open and a B - $\tilde{s}$ set.

Proof .

$1 \to 2$: Let (A,E) be $\tilde{s}$ open, then
$(A,E) = Int(A,E) \Rightarrow Int(A,E) \tilde{\subseteq} Int(Cl(A,E))$
$\Rightarrow (A,E) \tilde{\subseteq} Int(Cl(A,E))$
$\Rightarrow (A,E)$ is $\tilde{s}$ pre open.

Let $(U,E) = (A,E) \tilde{\in} (X,E,\tau)$ and $(V,E) = (X,E)$ be a t -$\tilde{s}$ set containing (A,E). Then $(A,E) = (U,E) \tilde{\bigcap} (V,E)$
$\Rightarrow (A,E)$ is a B -$\tilde{s}$ set.

Hence (A,E) is $\tilde{s}$ pre open and a B -$\tilde{s}$ set.

$2 \to 1$: Let (A,E) be $\tilde{s}$ pre open and a B-$\tilde{s}$ set. Since (A,E) is a B-$\tilde{s}$ set, $(A,E) = (U,E) \tilde{\bigcap} (V,E)$, where (U,E) is $\tilde{s}$ open and (V,E) is a t-$\tilde{s}$ set. Thus $Int(V,E) = Int(Cl(V,E))$.
Since (A,E) is $\tilde{s}$ pre open,

$$(A,E) \tilde{\subseteq} Int(Cl(A,E)) = Int(Cl((U,E) \tilde{\bigcap} (V,E))$$
$$\tilde{\subseteq} Int(Cl((U,E)) \tilde{\bigcap} Int(Cl((V,E))$$

$$\Rightarrow ((U,E) \tilde{\bigcap} (V,E)) \tilde{\subset} Int(Cl(U,E) \tilde{\bigcap} Int(V,E)).$$

Consider
$((U,E) \tilde{\bigcap} (V,E)) = ((U,E) \tilde{\bigcap} (V,E)) \tilde{\bigcap} (U,E)$
$\tilde{\subset} [\mathrm{Int}(Cl(U,E) \tilde{\bigcap} \mathrm{Int}(V,E))] \tilde{\bigcap} (U,E)$
$= (U,E) \tilde{\bigcap} \mathrm{Int}(V,E)$
$\Rightarrow ((U,E) \tilde{\bigcap} (V,E)) \tilde{\subset} ((U,E) \tilde{\bigcap} \mathrm{Int}(V,E))$
$\Rightarrow (V,E) \tilde{\subset} \mathrm{Int}(V,E)$
$\Rightarrow (V,E) = \mathrm{Int}(V,E)$
$\Rightarrow (A,E) = ((U,E) \tilde{\bigcap} (V,E)) = ((U,E) \tilde{\bigcap} \mathrm{Int}(V,E))$
$\Rightarrow (A,E)$ is $\tilde{s}$ open.

Theorem 3.9 For a soft set $(A,E)$of a soft topological space (X,E,τ), the following are equivalent:

1.(A,E) is $\tilde{s}$ regular open.
2. (A,E) is $\tilde{s}$ pre open and a t- $\tilde{s}$ set.

Proof .

$1 \to 2$: Let (A,E) be $\tilde{s}$ regular open, then
$(A,E) = Int(Cl(A,E)) \Rightarrow (A,E)$ is $\tilde{s}$ pre open.
Since $(A,E) = Int(Cl(A,E))$,
$Int(A,E) = Int(Cl(A,E)) \Rightarrow (A,E)$ a t -$\tilde{s}$ set.

$2 \to 1$: Let (A,E) be $\tilde{s}$ pre open and a t -$\tilde{s}$ set.
Then $(A,E) = Int(Cl(A,E))$.
Since(A,E) is a t -$\tilde{s}$ set, $Int(Cl(A,E)) \tilde{\subseteq} (A,E)$.
Thus (A,E) is $\tilde{s}$ regular open.

Theorem 3.10 For a soft set (A,E) of a soft topological space (X,E,τ), the following are equivalent:

1.(A,E) is $\tilde{s}$ regular closed.
2. (A,E) is $\tilde{s}$ pre closed and a t* -$\tilde{s}$ set.

Proof .

$1 \to 2$: Let (A,E) be $\tilde{s}$ regular closed, then

$(A,E) = Cl(Int(A,E)) \Rightarrow (A,E)$ is $\tilde{s}$ pre closed.
Since
$(A,E) = Cl(Int(A,E)), Cl(A,E) = Cl(Int(A,E))$ $(A,E)$is a t*-$\tilde{s}$ set.

$2 \to 1$: Let (A,E) be $\tilde{s}$ pre closed and a t*-$\tilde{s}$ set, then $Cl(Int(A,E)) \tilde{\subseteq} (A,E)$. Since (A,E) is a t -$\tilde{s}$ set, $(A,E) \tilde{\subseteq} Cl(Int(A,E))$. Then (A,E) is $\tilde{s}$ regular closed.

Theorem 3.11

1. The intersection of a b –open $\tilde{s}$ set and a *b –open $\tilde{s}$ set is a b –open $\tilde{s}$ set.
2.The intersection of a b** –open $\tilde{s}$ set and a **b –open $\tilde{s}$ set is a b** –open $\tilde{s}$ set.

Proof .

1. Let (A,E) be b- open $\tilde{s}$ set, and (B,E) be *b- open $\tilde{s}$ set, then

$(A,E) \tilde{\subseteq} Cl(Int(A,E)) \tilde{\bigcup} Int(Cl(A,E))$,
$(B,E) \tilde{\subseteq} Cl(Int(B,E)) \tilde{\bigcup} Int(Cl(B,E))$
$\Rightarrow (A,E) \tilde{\bigcap} (B,E) \tilde{\subseteq} [Cl(Int(A,E)) \tilde{\bigcup} Int(Cl(A,E))]$
$\tilde{\bigcap} [Cl(Int(B,E)) \tilde{\bigcup} Int(Cl(B,E))]$
$= Cl(Int(A,E)) \tilde{\bigcap} [Cl(Int(B,E)) \tilde{\bigcap} Int(Cl(B,E)]$
$\tilde{\bigcup} Int(Cl(A,E)) \tilde{\bigcap} [Cl(Int(B,E)) \tilde{\bigcup} Int(Cl(B,E))]$
$\tilde{\subseteq} [Cl(Int(A,E)) \tilde{\bigcap} Cl(Int(B,E))]$
$\tilde{\bigcup} [Int(Cl(A,E)) \tilde{\bigcap} Int(Cl(B,E))]$
$\tilde{\supseteq} Cl(Int(A \bigcap B,E)) \tilde{\bigcup} Int(Cl(A \bigcap B,E))$.

2. Let (A,E) be a b**- open $\tilde{s}$ set, and (B,E) be a **b- open $\tilde{s}$ set. Then
$(A,E) \tilde{\subseteq} Int(Cl(Int(A,E))) \tilde{\bigcup} Cl(Int(Cl(A,E)))$,
$(B,E) \tilde{\subseteq} Int(Cl(Int(B,E))) \tilde{\bigcup} Cl(Int(Cl(B,E)))$,
$\Rightarrow (A,E) \tilde{\bigcap} (B,E) \tilde{\subseteq} [Int(Cl(Int(A,E)))$
$\tilde{\bigcup} Cl(Int(Cl(A,E)))]$
$\tilde{\bigcap} [Int(Cl(Int(B,E))) \tilde{\bigcup} Cl(Int(Cl(B,E)))]$
$= [Int(Cl(Int(A,E))) \tilde{\bigcap} Int(Cl(Int(B,E)))$
$\tilde{\bigcup} Cl(Int(Cl(B,E)))]$
$\tilde{\bigcup} [Cl(Int(Cl(A,E))) \tilde{\bigcap} Int(Cl(Int(B,E)))$
$\tilde{\bigcup} Cl(Int(Cl(B,E)))]$
$\tilde{\subseteq} [Int(Cl(Int(A,E))) \tilde{\bigcap} Int(Cl(Int(B,E)))]$
$\tilde{\bigcup} [Cl(Int(Cl(A,E))) \tilde{\bigcap} Cl(Int(Cl(B,E)))]$
$\tilde{\supset} Int(Cl(Int(A \bigcap B,E))) \tilde{\bigcup} Cl(Int(Cl(A \bigcap B,E)))$.

Theorem 3.12 For a soft set (A,E) of a soft extremally disconnected space (X,E,τ), the following are equivalent:

1.(A,E) is $\tilde{s}$ open.
2. (A,E) is $\tilde{s}$ *b-open and $\tilde{s}$ locally closed.
3. (A,E) is $\tilde{s}$ b-open and $\tilde{s}$ locally closed.

Proof .

Let (A,E) be a soft subset of a soft extremally disconnected space (X,E,τ). Then $Cl(Int(A,E)) = Int(Cl(A,E))$.

$1 \to 2$: Let $(A,E) \tilde{\in} \tau$, *then* $Int(A,E) = (A,E)$.
Now,
$Cl(Int(A,E)) \tilde{\bigcap} Int(Cl(A,E)) = Cl(A,E) \tilde{\supseteq} (A,E)$
$\Rightarrow (A,E)$ is $\tilde{s}$ *b-open.

Clearly $(A,E) = (A,E)\tilde{\bigcap} Cl\,(A,E)$, where $(A,E)\tilde{\in}\tau \Rightarrow Cl\,(A,E)\tilde{\in}\tau' \Rightarrow (A,E)$ is $\tilde{s}$ locally closed.

$2 \to 3$: Let (A,E) be $\tilde{s}$ *b-open and $\tilde{s}$ locally closed. Since every $\tilde{s}$ *b-open is $\tilde{s}$ b-open, (A,E) is $\tilde{s}$ b-open and $\tilde{s}$ locally closed.

$3 \to 1$: From Theorem 2.1 [9].

Theorem 3.13 For a soft set (A,E) of a soft extremally disconnected space (X,E,τ), the following are equivalent:

1.(A,E) is $\tilde{s}$ open.
2. (A,E) is $\tilde{s}$ **b-open and $\tilde{s}$ locally closed.
3. (A,E) is $\tilde{s}$ b**-open and $\tilde{s}$ locally closed.

Proof .

Let (A,E) be a soft set of a soft extremally disconnected space (X,E,τ) . Then $Cl\,(Int(A,E)) = Int\,(Cl\,(A,E))$.

$1 \to 2$: Let (A,E) be $\tilde{s}$ open, then

$$Int\,(Cl\,(Int(A,E))) = Int\,(Cl\,(A,E)),$$
$$Cl\,(Int\,(Cl(A,E))) = Cl\,(Int\,(A,E))$$

$$Int\,(Cl\,(Int(A,E)))\tilde{\bigcap} Cl\,(Int\,(Cl(A,E)))$$
$$= Int\,(Cl\,(A,E))\tilde{\bigcap} Cl\,(Int\,(A,E))$$
$$= Cl(Int(A,E)) = Cl(A,E) \supset (A,E)$$

$\Rightarrow (A,E)$ is $\tilde{s}$ **b-open.

Since $(A,E) = (A,E)\tilde{\bigcap}\, Cl\,(A,E)$, where $(A,E) \in \tau,\ Cl\,(A,E) \in \tau' \Rightarrow (A,E)$ is $\tilde{s}$ locally closed.
$2 \to 3$: Let (A,E) be $\tilde{s}$ **b-open and $\tilde{s}$ locally closed. Since every $\tilde{s}$ **b-open is $\tilde{s}$ b**-open, (A,E) is $\tilde{s}$ b**-open and $\tilde{s}$ locally closed.

$3 \to 1$: Since (X,E,τ) is a soft extremally disconnected space, and (A,E) is $\tilde{s}$ b** -open,
$Int\,(Cl\,(Int(A,E)))\tilde{\bigcup} Cl\,(Int\,(Cl(A,E)))$
$= Int\,(Cl\,(A,E))\tilde{\bigcup} Cl\,(Int\,(A,E))$
$= Cl\,(Int\,(A,E)\tilde{\supseteq}(A,E)$
$\Rightarrow Cl\,(A,E)\tilde{\subseteq} Cl\,(Int\,(A,E))$.
$Int\,(A,E)\tilde{\subseteq} Int\,(Cl\,(A,E)),\ Int\,(A,E)\tilde{\subseteq}(A,E)$
$\Rightarrow Cl\,(Int\,(A,E))\tilde{\subseteq} Cl\,(A,E)$
$\Rightarrow Cl\,(A,E) = Cl\,(Int\,(A,E))$.

By the helpful of $\tilde{s}$ locally closeness nature of (A,E), we get $(A,E)\tilde{\in}\tau$.

4 Applications of *b-continuous and **b-continuous soft functions

Definition 4.1. A soft function $\tilde{f} : (X,E,\tau_1) \to (Y,E,\tau_2)$ is called :

1.$\tilde{s}$ ** b- continuous if $\tilde{f}^{-1}\,(V,E)$ is $\tilde{s}$ ** b-open in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
2.$\tilde{s}$ t*- continuous if $\tilde{f}^{-1}\,(V,E)$ is $\tilde{s}$ t* –set in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
3.$\tilde{s}$ B*- continuous if $\tilde{f}^{-1}\,(V,E)$ is $\tilde{s}$ B*-open in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
4.$\tilde{s}$ locally ** b -closed continuous if $\tilde{f}^{-1}\,(V,E)$ is $\tilde{s}$ locally ** b- closed in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).
5.$\tilde{s}$ D(c, ** b) - continuous if $\tilde{f}^{-1}\,(V,E)$ is D(c, ** b) - $\tilde{s}$ set in (X,E,τ_1), for each open soft set (V,E) of (Y,E,τ_2).

Example 4.2

Let $X = \{a,b,c,d\}$, $E = \{e_1,e_2,e_3,e_4\}$, and
$\tau = \{X_E, \phi_E, \{(e_3,\{c\})\}, \{(e_4,\{d\})\},$
$\{(e_3,\{c\}),(e_4,\{d\})\}\{(e_1,\{a\}),(e_3,\{c\}),(e_4,\{d\})\},$
$\{\{(e_2,\{b\}),(e_3,\{c\}),(e_4,\{d\})\}\},$
where $\exists\, F : E \to P(X)$ such that
$F(e_1) = \{a\}, F(e_2) = \{b\}, F(e_3) = \{c\}, F(e_4) = \{d\}$

1.The collection of **b - open $\tilde{s}$ sets $= \{X_E, \phi_E, \{(e_3,\{c\})\}, \{(e_4,\{d\})\}, \{(e_3,\{c\}),$ $(e_4,\{d\})\}, \{(e_1,\{a\}),(e_3,\{c\}),(e_4,\{d\})\},$ $\{\{(e_2,\{b\}),(e_3,\{c\}),(e_4,\{d\})\}\}$
defines a function $f : X \to X$ such that
$f(a) = b, f(b) = a, f(c) = d, f(d) = c,$
then f is $\tilde{s}$ ** b-continuous.
2.The collection of t *-$\tilde{s}$ sets $= \{X_E, \phi_E, \{(e_2,\{b\})\}, \{(e_3,\{c\})\}, \{(e_4,\{d\})\},$ $\{(e_1,\{a\}),(e_3,\{c\})\}, \{(e_1,\{a\}),(e_4,\{d\})\},$ $\{(e_2,\{b\}),(e_3,\{c\})\}, \{(e_2,\{b\}),(e_4,\{d\})\},$ $\{(e_3,\{c\}),(e_4,\{d\})\}, \{(e_1,\{a\}),(e_2,\{b\}),(e_3,\{c\})\}$ $, \{(e_1,\{a\}),(e_2,\{b\}),,(e_4,d)\},$ $\{(e_2,\{b\}),(e_3,\{c\}),(e_4,\{d\})\}\},$
defines a function $f : X \to X$ such that $f(a) = b, f(b) = d, f(c) = a, f(d) = c,$
then f is t *-$\tilde{s}$ continuous.

3.The collection of B * - $\tilde{s}$ sets $= \{X_E, \phi_E, \{(e_2,\{b\})\}, \{(e_3,\{c\})\}, \{(e_4,\{d\})\}$ $\{(e_1,\{a\}),(e_3,\{c\})\}, \{(e_1,\{a\}),(e_4,\{d\})\}\}$ $\{(e_2,\{b\}),(e_3,\{c\})\}, \{(e_2,\{b\}),(e_4,\{d\})\}$ $\{(e_3,\{c\}),(e_4,\{d\})\}, \{(e_1,\{a\}),(e_2,\{b\}),(e_3,\{c\})\}$ $\{(e_1,\{a\}),(e_2,\{b\}),(e_4,\{d\})\}$ $\{(e_2,\{b\}),(e_3,\{c\}),(e_4,\{d\})\}$ $\{(e_1,\{a\}),(e_3,\{c\}),(e_4,\{d\})\}$
defines a function $f : X \to X$ such that $f(a) = b, f(b) = d, f(c) = a, f(d) = c,$
then f is $\tilde{s}$ B *- continuous.

References

[1] A. Aygunoglu, H. Aygun. Some notes on soft topological spaces. *Neural Comput.Appl.*, Doi, 10.1007/s00521-011-0722-3.
[2] A.Metin, O.Alkan. Soft b- open sets and soft b-continuous functions. *Math Sci.*, 8,124 (2014).
[3] B. Chen. Soft semi- open sets and related properties in soft topological spaces. *Appl.Math.Inf. Sci.*, 7(1), 287-294(2013).
[4] C.Aras, A.Sonmez, H.Cakalli. On soft mappings. *http://arxiv.org/abs/*1305.4545 (submitted).
[5] D. Molodtsov. Soft set theory – First results. *Comput.Math.Appl.*, 37(4-5), 19-31(1999).

[6] I. Zorlutuna, M. Akdag, W. Min, S. Atmaca. Remarks on soft topological spaces. *Ann.Fuzzy Math.Inf.*, 3(2),171-185 (2012).
[7] J. Mahanta, P. Das. On soft topological space via semi open and semi closed soft sets. *http : //arxiv.org/abs/*1203.4133(submitted).
[8] M. Akdag, A. Ozkan. Soft α- open sets and soft α- continuous functions. *Abstr.Anal.Appl. Art.*, ID891341,1-7 (2014).
[9] M. Shabir, M. Naz. On soft topological spaces. *Comput.Math.Appl.*, 61, 1787- 1799 (2011).
[10] M. I. Ali, F. Feng, X. Liu , W. K. Min, M. Shabir. On some new operations in soft set theory. *Comput.Math.Appl* ., 57, 1547- 1553(2009).
[11] P. K. Maji, R. Biswas, A. R. Roy. Soft set theory. *Comput.Math.Appl.*, 45, 555-562(2003).
[12] S. Bharathi, K. Bhuvaneshwari, N. Chandramathi. On locally b**- closed sets. *International Journal of Mathematical Sciences and Applications* ., 1(2) 636-641(2011).
[13] T. Indira, K. Rekha. Application of * b- open sets and **b- open sets in topological spaces. *Pure and Applied Math.*, Vol. 1, (1), 44-56 (2012).
[14] T. Indira and K. Rekha. On locally **b- closed sets, *Proceedings of the Heber International Conference on Applications of Mathematics and Statistics* (HICAMS)(2012).
[15] Y. Yumak, A. Kaymakci. Soft β - open sets and their applications. *http : arxiv.org/abs/*1312.6964(submitted).

Kh. Abdrabou is Associate Professor of Mathematics , Faculty of Science Shaqra University, Al-qawwiya, Kingdom of Saudi Arabia.
He has got MSC degree in Mathematics from Faculty of Science, Zagazig University, Egypt. He has got his Ph.D in Mathematics from Faculty of Science, Tanta University, Egypt. He is research interests are the Function analysis and Topology , Specially, Fixed point theorem and its applications and Soft sets . He has published research articles in reputed international journals of mathematical sciences. He is referee and editor of mathematical journals.

I. K. Halfa is Assistant Professor of Mathematics , He has got Ph. D degree in mathematics for science at Tanta university, Egypt. His work in mathematics and Theoretical Physics Department, Atomic Energy Authority, Cairo, Egypt. His research interests are in the areas of pure mathematics, namely in topology, fuzzy topology, rough set theory, fuzzy and soft metric spaces. Moreover, applications of topology in nuclear physics and radiation pollutions.

K. Barakat received the Ph. D degree in mathematics for science at Helwan university, Egypt. His work in mathematics department, Helwan university, Cairo, Egypt. His research interests are in the areas of pure mathematics, namely in topology, fuzzy topology, intutionistic fuzzy topology, fuzzy and soft metric spaces.

Mohamed EL Sayed received the PhD degree in Mathematics Science at Tanta University, Department of Mathematics, Faculty of science, in 2011. His And my research interests general topology and its applications, soft set theory, soft topology, soft fuzzy topological structures, and its applications and rough sets. Specifically: I work in the areas of generalized continuity, soft near openness, generalized of rough spaces to soft rough spaces