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Intuitionistic Fuzzy Programming Technique to Solve Multi-Objective Transportation Problem

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Abstract: This paper utilizes the fuzzy programming method to explain the multi-objective transportation problem (MOTP), in which products must be transported from one destination to another. Transportation costs and times from origin *i* to destination j were noted. Here, we took into account MOTP with intuitionistic fuzzy numbers and solved the problem both ways. As a result, both the exponential and linear membership functions will continue to provide the best compromise solution. For such a problem, the membership functions are used to solve it. The present facts analysis was conducted using two steps of LINDO statistical software.

Keywords: Multi-objective-transportation problem, membership function, Fuzzy programing Technique.

1 Introduction

Numerous real-world applications for the Transportation Problem, which may be thought of as a special kind of Linear Programming Problem (LPP). TP is one of the strong frame works that ensures effective movement and availability of the products of the raw material. One of the best optimization techniques, it may be used in many real-world human tasks. Early on, it was noticed that selecting the ideal shipment pattern is what is known as a transportation problem. Therefore, it deals with moving various kinds of items from any of *m* sources (i=1,2,3,...m) to any of *n* endpoints (j=1,2,3,...n). The amount x ij that needs to be carried from every origin (i=1, 2, 3...) to every destination (j=1,2,3,...) must be determined in a way that minimises the overall cost. Not every transportation proble m has a single objective instead; several objectives are taken into consideration. The term MOTP mentions to a superior kind of linear programming problem when entirely the objectives are in conflict with one another and the constraints are of the equality kind. Not every transportation problem has a single objective; instead, several objectives are taken into account. The term MOTP denotes to a distinct kind of LPP. when all the objectives are in conflict with one another and the constraints are all of the equality-type. When using the multi-objective fuzzy linear programming method, the objectives are ambiguous. The MOTP is solved with an ideal compromise by using FLPP.

By using an algorithm, Diaz [1, 2] sought to find a solution to the MOTP. For a linear MOTP, Isermann [3] also created a method for recognizing all the non-dominated solutions. Two interactive algorithms were created by Ringuest and Rinks [4] to solve the MOTP. In order to solve a LPP with several goal functions, Zimmermann [5] first used appropriate membership functions. He demonstrated that fuzzy linear programming always produces effective solutions. In order to address the MOTP, Bit et al. [6] used the fuzzy programming technique (FTP) with a linear membership function. Hitchc ock [7] examined and modelled a straightforward transportation problem as a classic linear programming problem. The MOTP's decision-making parameters are initially considered to have fixed values. However, because of numerous uncertain circumstances, including poor road conditions, heavy traffic, fluctuating diesel prices, and many more. and a few other unexpected factors, such as the weather. Because of these factors, standard models are inaccurate. The idea of fuzzy sets was introduced by Zadeh [8] for handling uncertainty. Later, Bellman and Zadeh [9] used it to make decisions about real-world problems. Fuzzy programming was used by Verma et al. [10] to solve MOTP using a few non-linear membership functions. For the solution of the MOTP-with interval cost, source, and destinations parameters, Das et al. [11] developed solution strategies. Fuzzy compromise programming has been

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introduced by Li and Lai [12] as a solution to the multi-objective transportation problem. Wahed [13] started the MOTP's ideal compromise solution, and the outcome was tested for degree of similarity to the ideal solution applying a family of distances. For the vector-maximum linear programming problem, Leberling [14] used a unique sort of nonlinear membership function. Using this kind of nonlinear membership function, he verified that the FLP solution is always effective. Dhingra and H. Moskowitz [15] created additional non-linear (exponential, quadratic, and logarithmic) membership function types and used them to a project problem of optimality. In order to explain a MOTP, Verma, Biswal, and Biswas [16] used the FPT along by a few non-linear (hyperbolic and exponential) membership functions. Recently Triangular intuitionistic fuzzy numbers were recently proposed by Antony et al [17] as a way for solving the transportation problem. Lone et al. fuzzy's programming method [18] for willow wicker cultivation and Intuitionistic fuzzy programming approaches by several authors. Yeola and jahav [19] proposed an approach that is similar to the New Row Maxima Method for solving MOTP applying fuzzy linear membership functions at various costs. In order to track the maximum power point for a PV system, Boualam et al. [20] used fuzzy logic controllers. Usha Rani and Depak [21] conduct a work on FMOLP-based optimal land-allocation for agricultural production planning. Nemat Allah et al. [22] studied the facility location problem, which is a unique type of location problem with point and area destinations. Additionally, they established three fuzzy theorems that demonstrated how the objective function is piecewise continuous, separable, and -convex. Singh and Yadav have done more research in the area of IFTP of type 2 [23]. A solution for actual transportation based on intuitionistic fuzzy programming based on ranking mechanism was proposed by Kumar and Husain [24].

2 Mathematical Models

In this section, we solve MOTP using fuzzy programming using an exponential function and a membership function. The amount to be carried from the source (O_i) to the destination (D_j) is represented by the variable x_{ij}. It is possible to formulate a multi-objective transportation problem quantitatively.

Maximize

$$Z_k = \sum_i^m \sum_j^n c_{ij}\, x_{ij}, \quad k = 1,2,\ldots K. \qquad (1)$$

Subject to

$$\sum_j^n x_{ij} = a_i, i = 1,2,3..m, \qquad (2)$$
$$\sum_i^m x_{ij} = b_j, j = 1,2,3..n. \qquad (3)$$
$$x_{ij} \geq 0, for\ all\ i, j$$

Where the subscript on Z_k and superscript C_{ij}^k, denoted the K^{th} penalties.

Criterion: We assume that

$a_i > 0\ for\ all\ i, b_i > 0, for\ all\ j, C_{ij} > 0, for\ all\ i\ and\ j.$

$\sum_i^m a_i = \sum_j^n b_j$ (Equilibrium condition)

We treat equilibrium condition as iff condition for the existence of a feasible solution to the balanced conditi on linear transportation problem. A transportation problem has exactly mn variables $and\ m + n$ constraints.

3 Fuzzy Algorithms to Solve MOTP

Step 1: Construct the FTP.

Step 2: The following payoff matrix is generated by solving the multi-objective transportation problem k times while focusing on one goal at a time and determining the corresponding values for each objective for each solution

$$\begin{array}{c} \\ X_1 \\ X_2 \\ \vdots \\ X_k \end{array} \begin{array}{c} \begin{array}{cccc} Z_1 & Z_2 & \ldots & Z_K \end{array} \\ \begin{bmatrix} Z_{11} & Z_{12} & \ldots & Z_{1K} \\ Z_{21} & Z_{22} & \ldots & Z_{2K} \\ \vdots & \vdots & & \vdots \\ Z_{K1} & Z_{K2} & \ldots & Z_{KK} \end{bmatrix} \end{array}$$

Where, $X^1, X^2, X^3, X^4 \ldots\ldots\ldots\ldots\ldots\ldots X^K$ are the isolated ideal solutions of the K distinct transpotation problems for K different objective function.

$Z_{ij} = Z_j(X^i)$, $(i = 1,2, \ldots k \quad and\ \ j = 1,2, \ldots k)$ Is the ith row and jth column element of the pay-off table.

Step 3: Set upper and lower bounds for each objective's degree of acceptance and rejection in accordance with the set of solutions in step 2.

Upper and lower bounds for membership functions are provided.

$U_K^\mu = Max(Z_K(X_r))$

$L_K^\mu = Min\,(Z_K(X_r)), \quad 0 \leq r \leq K$

Where, the upper bound U_K^μ and the lower bound for L_K^μ for the K^{th} objective function Z_k,, $k = 1,2, \ldots k$, U_K^μ is the greatest acceptable level of achievement for objective k, L_K^μ is the aspired level of achievement for objective k and $d_k = U_k^\mu - L_k^\mu$, the degradation allowance for objective k.

Step 4. Consider the membership function as following linear functions: -

$$\mu_k\{Z_k(X)\} = \begin{cases} 1, & L_k^\mu \geq Z_k(X) \\ 1 - \dfrac{(U_k^\mu - Z_1(X))}{d_k}, & L_k^\mu \leq Z_k(X) \leq U_k^\mu; \\ 0, & Z_k(X) \geq U_k^\mu \end{cases}$$

$$d_k = U_k^{\mu} - L_k^{\mu}. \quad (4)$$

Step 5. For the initial fuzzy model, LMF is used to find a crisp model.
The crisp model can be made simpler if we apply a LMF.
Minimize α

Such that

$$Z_k(X) - \alpha d_k \le L_k^{\mu},$$
$$\sum_j^n x_{ij} = a_i, \quad i = 1,2,3..m,$$
$$\sum_i^m x_{ij} = b_j, \quad j = 1,2,3..n.$$
$$x_{ij} \ge 0, for\ all\ i\ and\ \alpha \ge 0$$

Statistical LONDO/TORA software can be used to resolve the aforementioned linear programming problem.

Step 6. Use the proper mathematical programming

algorithm to solve the crisp model.

Minimize α

$$C_{ij}^k x_{ij} - \alpha(d_{k)} \le L_k^{\mu} \quad , \quad k = 1,2,\ldots..K,$$

Subject to

$$\sum_j^n x_{ij} = a_i, \quad i = 1,2,3..m,$$
$$\sum_i^m x_{ij} = b_j, \quad j = 1,2,3\ldots\ldots\ldots\ldots$$
$$x_{ij} \ge 0, for\ all\ i,j$$

We use another membership function such as Hyperbolic Tangent function can be formulated as

Minimize α

$$\alpha \ge \frac{1}{2} + \frac{1}{2}\tanh\{\frac{U_k+L_k}{2} + Z_k\}\tau_k,$$

where $\tau_k = \frac{S}{U_k - L_k}$, where S is no. of constraints.

$$\sum_j^n x_{ij} = a_i \quad i = 1,2,3..m,$$
$$\sum_i^m x_{ij} = b_j, \quad j = 1,2,3..n. \quad (5)$$
$$x_{ij} \ge 0, for\ all\ i\ and\ \alpha \ge 0$$

Step 7. With such an exponential membership function for the kth objective function, an intuitionistic-fuzzy optimization for MOLP problem is defined as.

$$\mu_k^e\{Z_k(X)\} = \begin{cases} 1, & L_k^{\mu} \ge Z_k(X) \\ e^{-\frac{1}{2}\left(\frac{Z_k - L_k^{\mu}}{d_k}\right)}, & L_k^{\mu} \le Z_k(X) \le U_k^{\mu}; \\ 0, & Z_k(X) \ge U_k^{\mu} \end{cases}$$

$$d_k = U_k^{\mu} - L_k^{\mu} \quad (6)$$

Where, $k = 1,2,3,\ldots\ldots k$.

4 Data Analysis

This section will go through fuzzy programming techniques for applying the method to solve MOTP. Analyse the overall time and transportation costs. Equations are used to represent the facts on the cost and timing of product supply from source to destination.

$$Maximize\ Z_1 = 1X_{11} + 2X_{12} + 7X_{13}\ 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34}$$

$$Maximize\ Z_2 = 4X_{11} + 4X_{12} + 3X_{13} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34}$$

Such that

$$X_{11} + X_{12} + X_{13} + X_{14} = 18$$
$$X_{21} + X_{22} + X_{23} + X_{24} = 19$$
$$X_{31} + X_{32} + X_{33} + X_{34} = 17$$
$$X_{11} + X_{21} + X_{31} = 11$$
$$X_{12} + X_{22} + X_{32} = 3$$
$$X_{13} + X_{23} + X_{33} = 14$$
$$X_{14} + X_{24} + X_{34} = 16$$
$$X_{ij} \ge 0, \quad i = 1,2,3\ and\ j = 1,2,3.$$

The following is a representation of the problem's optimal compromise solution:

$X^1 = \{X_{13} = 14, X_{14} = 4, X_{22} = 3, X_{24} = 6, X_{31} = 11\ and\ X_{34} = 6\}\ and\ rest\ all\ x_{ij}\ are\ zeros.$

$Z_1(X_1) = 301$

The best solution to the problem is provided by
$X^2 = \{X_{11} = 5, X_{12} = 3, X_{23} = 3, X_{24} = 16, X_{31} = 6, X_{33} = 11,\}\ and\ rest\ all\ x_{ij}\ are\ zeros.$
$Z_2(X_2) = 310$
Similarly $Z_1(X_2) = 176\ and\ Z_2(X_1) = 214$.
Pay -off table is

	Z1	Z2
X1	301	214
X2	176	310

From this we get $U_1^{\mu} = 301$, $U_2^{\mu} = 310$, $L_1^{\mu} = 176, and\ L_2^{\mu}$=214

The membership function is given by

$$\mu_1\{Z_1(X)\} = \begin{cases} 1, & 176 \ge Z_1(X) \\ 1 - \frac{(301 - Z_1(X))}{d_{k1}}, & 176 \le Z_1(X) \le 301 \\ 0, & Z_1(X) \ge 301 \end{cases}$$

$$d_{k1} = 125$$

$$\mu_2\{Z_2(X)\} = \begin{cases} 1, & 214 \ge Z_2(X) \\ 1 - \frac{310 + Z_2(X)}{d_{k2}}, & 214 \le Z_2(X) \le 310 \\ 0, & Z_2(X) \ge 310 \end{cases}$$

$$d_{k2} = 96$$

We find an equivalent crisp model

Minimize α

$Z_1(X) - 125a \le 176$

$Z_2(X) - 96a \le 214$

Solve the crisp model

$1X_{11} + 2X_{12} + 7X_{13} + 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34} - 125\alpha \le 176$

$4X_{11} + 4X_{12} + 3X_{13} + 4X_{14} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34} - 96\alpha \le 214$

$X_{11} + X_{12} + X_{13} + X_{14} = 18$
$X_{21} + X_{22} + X_{23} + X_{24} = 19$
$X_{31} + X_{32} + X_{33} + X_{34} = 17$
$X_{11} + X_{21} + X_{31} = 11$
$X_{12} + X_{22} + X_{32} = 3$
$X_{13} + X_{23} + X_{33} = 14$
$X_{14} + X_{24} + X_{34} = 16$

Using LINDO Software, the best compromise solution to the problem is shown as

$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3$ and rest all X_{ij} are zeros.$\}$

$Z_1^* = 238.496$ $\quad Z_2^* = 261.82$

$\alpha = 0.50$

The fuzzy model can be further expressed as an equivalent crisp model if we apply a different membership function.

Minimize α

$\alpha \ge \frac{1}{2} + \frac{1}{2}\tanh\{\frac{U_k+L_k}{2} + Z_k\}\tau_k,$

where $\tau_k = \frac{7}{U_k - L_k}$.

Which further implies that

$\tau_k Z_k - \tanh^{-1}(2\alpha - 1) \le \frac{U_k+L_k}{2}\tau_k$

Minimize w

$\frac{7}{125}(1X_{11} + 2X_{12} + 7X_{13} + 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34}) - w \le \frac{7}{125}(\frac{477}{2})$

$\frac{7}{96}(4X_{11} + 4X_{12} + 3X_{13} + 4X_{14} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34}) - w \le \frac{7}{96}(\frac{524}{2})$

$X_{11} + X_{12} + X_{13} + X_{14} = 18$
$X_{21} + X_{22} + X_{23} + X_{24} = 19$
$X_{31} + X_{32} + X_{33} + X_{34} = 17$
$X_{11} + X_{21} + X_{31} = 11$
$X_{12} + X_{22} + X_{32} = 3$
$X_{13} + X_{23} + X_{33} = 14$
$X_{14} + X_{24} + X_{34} = 16$

Solving above problem the optimal solution is given by

$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3$ and rest all X_{ij} are zeros.$\}$

$Z_1^* = 238.496$ $\quad Z_2^* = 261.82$

$\alpha = 0.64$, Where $w = \tanh^{-1}(2\alpha - 1)$ and

$w = 0.29$

$$\mu_1^e\{Z_1(X)\} = \begin{cases} 1, & 176 \ge Z_1(X) \\ e^{-\frac{1}{2}\left(\frac{Z_1 - 176}{d_k}\right)} & 176 \le Z_1(X) \le 301 \\ 0, & Z_1(X) \ge 301 \end{cases}$$

$d_k = 125$

$$\mu_2^e\{Z_2(X)\} = \begin{cases} 1, & 214 \ge Z_1(X) \\ e^{-\frac{1}{2}\left(\frac{Z_2 - 214}{d_k}\right)} & 214 \le Z_2(X) \le 310 \\ 0, & Z_2(X) \ge 310 \end{cases}$$

$d_k = 96$

A crisp model that is comparable to the exponential membership functions can be written as

Minimize α

Subject to

$$\alpha \le e^{-\frac{1}{2}\left(\frac{Z_1 - 176}{d_k}\right)} \text{ and } \alpha \le e^{-\frac{1}{2}\left(\frac{Z_2 - 214}{d_k}\right)}$$

The problem is solved by LINDO

$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3$ and rest all X_{ij} are zeros.$\}$

$\alpha = 0.60$

5 Conclusions

In this research, an intuitionistic fuzzy multi-objective transportation problem is solved using three different types of functions. The problem is solved in three steps, the first of which provides the best possible compromise solution. The crisp model becomes linear if we utilise the second type of membership function. Even when we apply a different exponential membership function, the best-case compromise solution remains relatively same. Because in some circumstances there is a minimum quantity received by the destinations related to restricted storage space, the method is very simple and straightforward to address real-life transport problems. A single shipment is not feasible in this circumstance. As a result, goods are transported in stages from the origin to the destinations. The smallest quantity may initially be transported from the origins to the destinations. The method in question is based on fuzzy sets that are intuitionistic.

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