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Reflection of Thermoelastic Waves from Insulated Boundary Fibre-Reinforced Half-Space under Influence of Rotation and Magnetic Field

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Abstract: In this work, an estimation to study the reflection of p-wave, T-wave and SV-wave on the boundary of a fibre-reinforced half-space of homogeneous, isotropic thermoelastic medium under effect of the relaxation times, magnetic field and rotation were taken into our consideration the boundary was stress-free as well as insulated. **GL** model of generalized thermoelasticity which was known as the theory of thermoelasticity with two relaxation times, or the theory of temperature-rate dependent thermoelasticity has been applied to obtain the amplitudes of the reflection coefficients. Lame's potentials were used in the two dimensions oxz that tend to separate the governing equations into three equations that sought in harmonic travelling form. We estimated the equation of the velocities of p-wave, T-wave and SV-wave. The boundary conditions for the mechanical and Maxwell's stresses and the thermal insulated at the boundary are applied to determine the reflection coefficients of the longitudinal p-wave and thermal T-wave as well as the transverse wave SV- and conclude them some special cases. Will arrive at the results of the research proposal consistent with the classic results. The results obtained are calculated numerically by taking an appropriate metal and presented graphically.

Keywords: Magnetic field, Rotation, Reflection, Half Space, p- wave, T- wave, SV- wave, Relaxation Times, Isothermal Boundaries, Thermal insulated, fibre-reinforced.

Nomenclature

$\alpha, \beta, (\mu_L - \mu_T)$	are reinforced anisotropic elastic,
α_1, α_2	are the coefficient of thermal expansion,
δ_{ij}	is Kronecker delta,

$$\gamma = (2\lambda + 3\alpha + 4\mu_T + \beta)\,\alpha_1 + (\lambda + \alpha)\,\alpha_2$$

λ, μ_L, μ_T	are elastic parameters,
μ_e	is the magnetic permeability,
ρ	is the density,
σ_{ij}	are the components of the stress vector,
$\tau_0\,,\ \tau_1$	are the thermal relaxation times,
τ_{ij}	are the Maxwells stress tensor,
ω	is the frequency,
ℓ	is the standard length,
Ω	is the angular velocity,
$\vec{B}$	is the magnetic induction vector,
C_e	is the specific heat per unit mass,
e_{ij}	are the strain components,
$\vec{F}$	is the Lorens body forces, vector,
$\vec{h}$	is the perturbed magnetic field vector,
h_i	is the components of heat flux tensor,
$\vec{H}$	is the magnetic field vector,
H_0	is the constant magnetic field,
$\vec{J}$	is the electric current density vector,
K	is the thermal conductivity,
k	is the wave number,
k_T	is the isothermal,
t	is the time,
T	is the absolute temperature of the medium,
T_0	is the natural temperature of the medium,
u_i	are the components of the displacement vector,
υ	is the phase speed.

1 Introduction

During the second half of twentieth century, nonisothermal problems of the theory of elasticity became increasingly impact. This is due mainly to their many applications in widely diverse fields. First, in the

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nuclear field, the external high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations. Secondly, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. There is also many of uses and applications in various fields, in particular, structures, biology, geology, geophysics, acoustics, physics, plasma, etc. The theory of elasticity with nonuniform heat which was in half-space subjected of thermal shock in this context which known as the theory of uncoupled thermoelasticity and the temperature is governed by a parabolic partial differential equation in temperature term only has been discussed by [8]. [6] introduced the theory of classical thermoelasticity. At present, there are two different theories of the generalized thermoelasticity, the first was developed by [19] who obtained a wave-type heat conduction by postulating a new law of heat conduction to replace the classical Fourier's law. This new law contains the heat flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The second generalization to the coupled theory of thermoelasticity is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity developed by [12]. This theory contains two constants that act as relaxation times and modifies all the equations of the coupled theory not the heat conduction equation only. The two theories both ensure finite speeds of propagation for heat wave. [21] investigated the dynamic problems of thermoelasticity. Theory of thermoelasticity with applications was introduced by [22]. Among the theoretical contributions to the subject are the proofs of uniqueness theorems under different conditions has been introduced by . Dhaliwal and Sherief [17,18]. [7] studied the theory of generalized thermoelasticity with one relaxation time for anisotropic media. In this theory a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is a hyperbolic one and hence automatically eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity. [13,14,15] proposed three new thermoelastic theories based on entropy equality than the usual entropy inequality. The constitutive assumption for the heat flux vector are different in each theory. Thus they obtained three theories which are called thermoelasticity of type I, thermoelasticity of type II and thermoelasticity of type III. When type I theory is linearized we obtain the classical system of thermoelasticity. The type II theory (is a limiting case of type III) does not admit energy dissipation. For many problems involving steep heat gradients and when short time effects are sought this theory is indispensable. Due to the complexity of the partial differential equations of this theory, the work done in this field is unfortunately limited in number. [20] investigated an isotropic linear thermoelasticity with hydrostatic initial stress. A survey article of representative theories in the range of generalized thermoelasticity is due to [16]. Three-dimensional thermal shock problem of generalized thermoelastic half-space was discussed by [11]. [27] studied a three-dimensional thermoelastic problem for a half-space without energy dissipation. Investigation of the dynamic problem concerning the interactions among electromagnetic field, temperature, stress and strain in a thermoelastic solid is immensely important because of its extensive uses in diverse fields. The theory of magneto-thermoelasticity which deals the interactions among strain, temperature and electromagnetic fields has drawn the attention of many researchers because of its extensive uses in diverse fields, such as Geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, optics, etc. Great attention has been devoted to the study of electromagneto-thermoelastic coupled problems based on the generalized thermoelastic theories. The generalized magneto-thermoelasticity in a perfectly conducting medium is investigated by [10]. Some researchers in past have investigated different problem of rotating media. [23] investigated plane waves in generalized thermoelasticity with two relaxation time under the effect of rotation. The effect of rotation on generalized micropolarthermoelasticity for a half-space under five theories was discussed by [24]. [4] discussed the effect of rotation due to various sources at the interface of elastic half space and generalized thermoelastic half space. [5] presented the effect of rotation in a generalized thermoelastic medium with two temperature under the influence of gravity. [26] presented the effect of rotation on 2-d thermal shock problems for a generalized magneto-thermoelasticity half-space under three theories. [3] studied the effect of hydrostatic initial stress and rotation in Green-Naghdi (Type III) thermoelastic half-space with two temperature. Such type of problems in a rotating medium are very important in many dynamical systems. [25] discussed the reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic [9] investigated the reflection of generalized thermoelastic waves from isothermal and insulated boundaries of a half space. [1] pointed out the influence of magnetic field and hydrostatic initial stress on reflection phenomena of P- and SV-waves from a generalized thermoelastic solid half-space. [2] investigated reflection of P and SV waves from stress-free surface elastic half-space under influence of magnetic field and hydrostatic initial stress without energy dissipation. [39] studied propagation of Rayleigh waves in a rotating orthotropic material elastic half-space under initial stress and gravity. [40,41,42,43] studied the fibre-reinforced in elastic media in the beginning. [44]

investigated the propagation, reflection and transmission of magnetoelastic shear waves in a selfreinforced media. [45] and [46] discussed the problem of surface waves in a fibre-reinforced anisotropic elastic media. [47] discussed the reflection of plane waves at the free surface of a fibre-reinforced elastic half-space. [48] discussed the wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. [49] pointed out a model for spherical SH-wave propagation in self-reinforced linearly elastic media. Magnetoelastic surface waves in electrically conducting fibre-reinforced is discussed by [50]. [51] studied the wave motion in an anisotropic fiber-reinforced thermoelastic solid. [52] discussed the problem of wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. [53] studied the effects of anisotropy on reflection coefficients of plane waves in fibre-reinforced thermoelastic solid. [54] investigated a source problem in fibre-reinforced anisotropic generalized thermoelastic solid under acoustic fluid layer. [55] discussed stresses produced in a fibre- reinforced half-space due to a moving load. Recently, [56] investigated LS model of the thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. [57] investigated the reflection of thermoelastic boundary half space with the magnetic field and rotation. [58] investigated the Stoneley waves propagation in magneto-thermoplastic materials. [59] studied the propagation of plane waves of rotating microstretch elastic solid with temperature dependent elastic properties under Green-Naghdi theory. [60] investigated effects of magnetic field and initial stress on plane waves propagation. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in Refs. ([28]-[38]). In this paper, an estimation to study effects of the relaxation times, magnetic field and rotation on the reflection of p-waves and SV-waves on the boundary of a fibre-reinforced half-space of homogeneous, isotropic thermoelastic medium taking into our consideration the boundary is stress-free as well as insulated. GL model of generalized thermoelasticity which is known as the theory of thermoelasticity with two relaxation times, or the theory of temperature-rate dependent thermoelasticity has been applied to obtain the amplitudes of the reflection coefficients. Lame's potentials are used in the two dimensions oxz that tend to separate the governing equations into three equations that sought in harmonic travelling form. We will estimate the equation of the velocities of p-wave, T-wave and SV-wave. The boundary conditions for mechanical and Maxwell's stresses and thermal insulated will be applied to determine the reflection coefficients for p-wave, T-wave and SV-wave. Some new aspects are obtained of the reflection coefficients and displayed graphically and the new conclusions are presented. Effects of relaxation times and magnetic field on the reflection of generalized thermoelastic waves will be noticed and depicted graphically.

2 Basic equations

The governing equations for a fiber-reinforced linearly elastic an isotropic medium with generalized thermoelastic at reference temperature T_0 with respect to direction $\vec{a}$ are

(i) the constitutive equation

$$\sigma_{ij} = \left[\lambda e_{kk} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T\right]\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\right) + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta\left(a_k a_m e_{km} a_i a_j\right), \quad (1)$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad (2)$$

where, e_{ij} are components of strain, $\alpha, \beta, (\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters, λ, μ_L, μ_T are elastic parameters

$$\vec{a} = (a_1, a_2, a_3), \quad a_1^2 + a_2^2 + a_3^2 = 1 \quad (3)$$

If $\vec{a}$ has components that are $(1,0,0)$ so that the preferred direction is the x-axis, simplifies, as given below

$$\sigma_{11} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial u}{\partial x} + (\lambda+\alpha)\frac{\partial w}{\partial z} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T \quad (4)$$

$$\sigma_{33} = (\lambda+\alpha)\frac{\partial u}{\partial x} + (\lambda + 2\mu_T)\frac{\partial w}{\partial z} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T \quad (5)$$

$$\sigma_{13} = \mu_T\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \quad (6)$$

and,

$$\gamma = (2\lambda + 3\alpha + 4\mu_T + \beta)\,\alpha_1 + (\lambda+\alpha)\,\alpha_2 \quad (7)$$

(ii) Maxwell electromagnetic stress τ_{ij} is given by

$$\tau_{ij} = \mu_e[H_i h_j + H_j h_i - H_k h_k \delta_{ij}], \quad (8)$$

(iii) the equation of motion

$$\sigma_{ji,j}+F_i=\rho\left[\ddot{u}+\left(\vec{\Omega}\times\vec{\Omega}\times\vec{u}\right)+\left(2\vec{\Omega}\times\vec{\dot{u}}\right)\right]_i \quad (9)$$

where, $\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)$ is the centripetal acceleration due to the time varying motion only and $2\vec{\Omega}\times\vec{\dot{u}}$ is the Coriolis acceleration. Eq. (9) tends to

$$(\lambda+2\alpha+4\mu_L-2\mu_T+\beta)\,u_{1,11}+(\lambda+\alpha+\mu_T)\,u_{3,13}+\mu_T u_{1,33}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,1}+F_1=\rho\left(\ddot{u}_1-\Omega^2u_1+2\Omega\dot{u}_3\right) \quad (10)$$

$$(\lambda+2\mu_T)\,u_{3,33}+(\lambda+\alpha+\mu_T)\,u_{1,13}+\mu_T u_{3,11}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,3}=\rho\left(\ddot{u}_3-\Omega^2u_3-2\Omega\dot{u}_1\right) \quad (11)$$

where,

$$\vec{F}=\vec{J}\times\vec{B} \quad (12)$$

Consider that the medium is a perfect electric conductor, we take the linearized Maxwell equations governing the electromagnetic field, taking into account absence of the displacement current **(SI)** as the form:

$$\left.\begin{array}{l}\text{curl}\,\vec{h}=\vec{J},\ \text{curl}\,\vec{E}=-\mu_e\,\frac{\partial\vec{h}}{\partial t},\\ \text{div}\,\vec{h}=0,\text{div}\,\vec{E}=0.\end{array}\right\} \quad (13)$$

where,

$$\vec{h}=\text{curl}(\vec{u}\times\vec{H_0}) \quad (14)$$

where we have used,

$$\vec{H}=\vec{H_0}+\vec{h}\,(x,z,t)\,,\vec{H_0}=(0,H_0,0)$$

the constant primary magnetic field $\vec{H_o}$ acting on y direction.

(iv) the equation of heat conduction

$$K\nabla^2T=\rho C_v\left(1+\tau_0\tfrac{\partial}{\partial t}\right)\dot{T}+\gamma T_o\vec{\nabla}\cdot\left(1+\tau_0\delta_{ij}\tfrac{\partial}{\partial t}\right)\vec{\dot{u}} \quad (15)$$

which tends to

$$KT_{,kk}=\rho C_v\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+\gamma T_o\left(1+\tau_0\delta\frac{\partial}{\partial t}\right)\dot{e}_{kk} \quad (16)$$

For **GL** model, the relaxation times τ_o and τ_1 satisfy the inequality $\tau_0\geqslant\tau_1>0$, $\delta=0$ For two-dimensional motion in $(x-z)$ plane, Eqs. (10), (11) and (15) can be written as:

$$(\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_eH^2)\,u_{1,11}+(\lambda+\alpha+\mu_T+\mu_eH^2)\,u_{3,13}+\mu_T u_{1,33}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,1}=\rho\left(\ddot{u}_1-\Omega^2u_1+2\Omega\dot{u}_3\right) \quad (17)$$

$$(\lambda+2\mu_T+\mu_eH^2)\,u_{3,33}+(\lambda+\alpha+\mu_T+\mu_eH^2)\,u_{1,13}+\mu_T u_{3,11}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,3}=\rho\left(\ddot{u}_3-\Omega^2u_3-2\Omega\dot{u}_1\right) \quad (18)$$

$$K\left(T_{,11}+T_{,33}\right)=\rho C_v\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+\gamma T_0\left(\dot{u}_{1,1}+\dot{u}_{3,3}\right) \quad (19)$$

To transform the equations (17)-(19) into non-dimensional form, we take the following dimensionless form

$$x'=\tfrac{x}{\ell},\quad z'=\tfrac{z}{\ell},\quad t'=\tfrac{v}{\ell}t,\quad \tau_0'=\tfrac{v}{\ell}\tau_0,\quad \tau_1'=\tfrac{v}{\ell}\tau_1,\quad \Omega'=\tfrac{\ell}{v}\Omega,\quad T'=\tfrac{T}{T_0},\quad u'=\tfrac{\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_eH^2}{\ell\gamma T_0}u,\quad w'=\tfrac{\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_eH^2}{\ell\gamma T_0}w,\quad \sigma_{ij}'=\tfrac{\sigma_{ij}}{\gamma T_0},\quad \tau_{ij}'=\tfrac{\tau_{ij}}{\gamma T_0} \quad (20)$$

Substituting from equations (20) into equations (17)-(19) and suppressing the primes, we obtain

$$C_2^2\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial z^2}\right)+\left(C_3^2+C_4^2\right)\frac{\partial^2w}{\partial x\partial z}+\left(C_1^2-C_2^2\right)\frac{\partial^2u}{\partial x^2}-C_1^2\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}=\frac{\partial^2u}{\partial t^2}-\Omega^2u+2\Omega\frac{\partial w}{\partial t} \quad (21)$$

$$C_2^2\left(\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial z^2}\right)+\left(C_3^2+C_4^2\right)\frac{\partial^2u}{\partial x\partial z}+C_3^2\frac{\partial^2w}{\partial z^2}-C_1^2\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}=\frac{\partial^2w}{\partial t^2}-\Omega^2w-2\Omega\frac{\partial u}{\partial t} \quad (22)$$

$$C_5^2\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial z^2}\right)-\varepsilon\left(\frac{\partial^2\dot{u}}{\partial x^2}+\frac{\partial^2\dot{w}}{\partial z^2}\right)=\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} \quad (23)$$

Such that

$$C_1^2=\frac{\left(\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_e H^2\right)}{\rho v^2}\quad C_2^2=\frac{\mu_T}{\rho v^2},$$
$$C_3^2=\frac{\left(\lambda+\mu_T+\mu_e H^2\right)}{\rho v^2},\quad C_4^2=\frac{\alpha}{\rho v^2},$$
$$C_5^2=\frac{K}{\rho C_v v\ell},\quad \varepsilon=\frac{\gamma^2 T_0}{\rho C_v\left(\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_e H^2\right)} \tag{24}$$

Let us consider the displacement vector of the form

$$\overrightarrow{u}=\operatorname{grad}\phi+\operatorname{curl}\psi,\operatorname{div}\psi=0 \tag{25}$$

which take the form

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x} \tag{26}$$

and we let the absolute temperature $T=\Theta$. Substituting from Eq. (26) into eqs. (21)-(23)

$$\left[\nabla^2+\frac{1}{C_1^2}\left(C_6^2\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}+\Omega^2\right)\right]\phi-\frac{2\Omega}{C_1^2}\dot{\psi}=\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta \tag{27}$$

$$\left[\nabla^2+\frac{1}{\left(C_2^2+C_3^2\right)}\left(\Omega^2-\left(2C_3^2+C_4^2\right)\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial t^2}\right)\right]\psi+\frac{2\Omega}{\left(C_2^2+C_3^2\right)}\dot{\phi}=0 \tag{28}$$

and from Eq. (23) we get,

$$\left[\nabla^2-\frac{1}{C_5^2}\frac{\partial}{\partial t}-\frac{\tau_0}{C_5^2}\frac{\partial^2}{\partial t^2}\right]\Theta-\frac{\varepsilon}{C_5^2}\nabla^2\dot{\phi}=0 \tag{29}$$

where,

$$C_6^2=C_2^2+C_3^2+C_4^2-C_1^2,\quad C_7^2=C_2^2+C_3^2,\quad C_8^2=2C_3^2+C_4^2 \tag{30}$$

3 Solution of the problem

For the analaytic solution of Eqs. (27)-(29) in the form of the harmonic travelling wave, we suppose the solution takes the form,

$$|\phi,\Theta,\psi|(x,z,t)=|\phi_1,\Theta_1,\psi_1|\exp\left[ik\left(x\sin\theta+z\cos\theta-vt\right)\right] \tag{31}$$

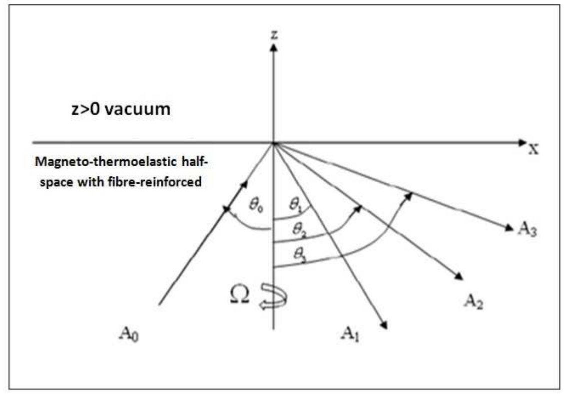

Fig. 1: Schematic of the problem.

where ϕ_1,Θ_1 and ψ_1 are arbitrary constants and the pair $(\sin\theta,\cos\theta)$ denotes the projection of the wave normal onto $xz-$ plane.

Substitute from Eq. (31) into Eqs. (27)-(29) one may obtain,

$$\left[k^2\left(v^2-C_9^2\right)+\Omega^2\right]\phi_1+2ikv\Omega\psi_1-C_1^2\left(1-i\tau_1 kv\right)\Theta_1=0 \tag{32}$$

$$\left[k^2\left(v^2-C_{10}^2\right)+\Omega^2\right]\psi_1-2ikv\Omega\phi_1=0 \tag{33}$$

$$\left[k^2\left(v^2\tau_0-C_5^2\right)+1\right]\Theta_1-\varepsilon ik^3 v\Omega\phi_1=0 \tag{34}$$

where,

$$C_9^2=C_1^2+C_6^2\cos^2\theta,\qquad C_{10}^2=C_7^2-C_8^2\sin^2\theta \tag{35}$$

From Eqs. (32)-(33) we get,

$$L\left(v^2\right)^3+M\left(v^2\right)^2+Nv^2+P=0 \tag{36}$$

where,

$$\begin{aligned}
P&=-\omega^6 C_5^2 C_9^2 C_{10}^2,\\
N&=\varepsilon\tau_1\omega^6\Omega C_1^2 C_{10}^2+\omega^6 C_5^2 C_9^2+\omega^6 C_5^2 C_{10}^2+\tau_0\omega^6 C_9^2 C_{10}^2\\
&\quad+\varepsilon\omega^5\Omega C_1^2 C_{10}^2+\omega^4\Omega^2 C_5^2 C_9^2-2\tau_0\omega^4\Omega^2,\\
M&=-\varepsilon\tau_1\omega^6\Omega C_1^2-\omega^6 C_5^2-\tau_0\omega^6 C_9^2-\tau_0\omega^6 C_{10}^2\\
&\quad-\varepsilon i\omega^5\Omega C_1^2-\varepsilon\omega^4\Omega^3 C_1^2+2\omega^4\Omega^2 C_5^2\\
&\quad-\tau_0\omega^4\Omega^2 C_9^2-\tau_0\omega^4\Omega^2 C_{10}^2+\omega^4 C_9^2+\omega^4 C_{10}^2\\
&\quad+\omega^4\Omega^2 C_5^2 C_{10}^2-\omega^2 C_9^2 C_{10}^2+\omega^2\Omega^2 C_9^2\\
&\quad+\omega^2\Omega^2 C_{10}^2-\varepsilon i\omega^3\Omega^3 C_1^2-\omega^2\Omega^2 C_5^2\\
L&=\tau_0\omega^2-\omega^4+\tau_0\omega^2\Omega^4-\Omega^4+2\omega^2\Omega^2
\end{aligned} \tag{37}$$

then we take into consideration; if the wave normal of the incident wave makes angle θ_0 with the positive

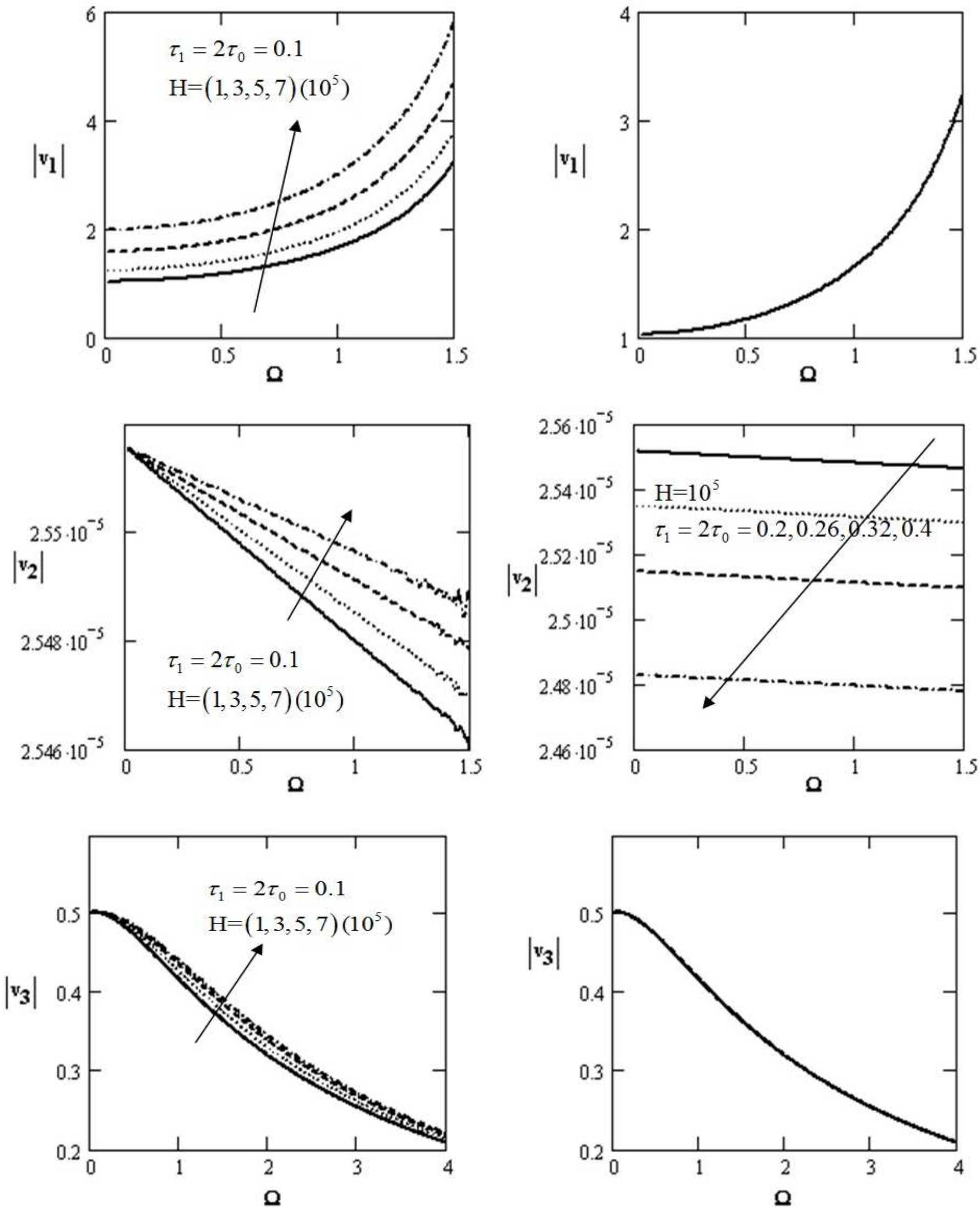

Fig. 2: Variations of the magnetic field and thermal relaxation times on the waves velocities $(|v_1|, |v_2|$ and $|v_3|)$ with respect to the rotation Ω.

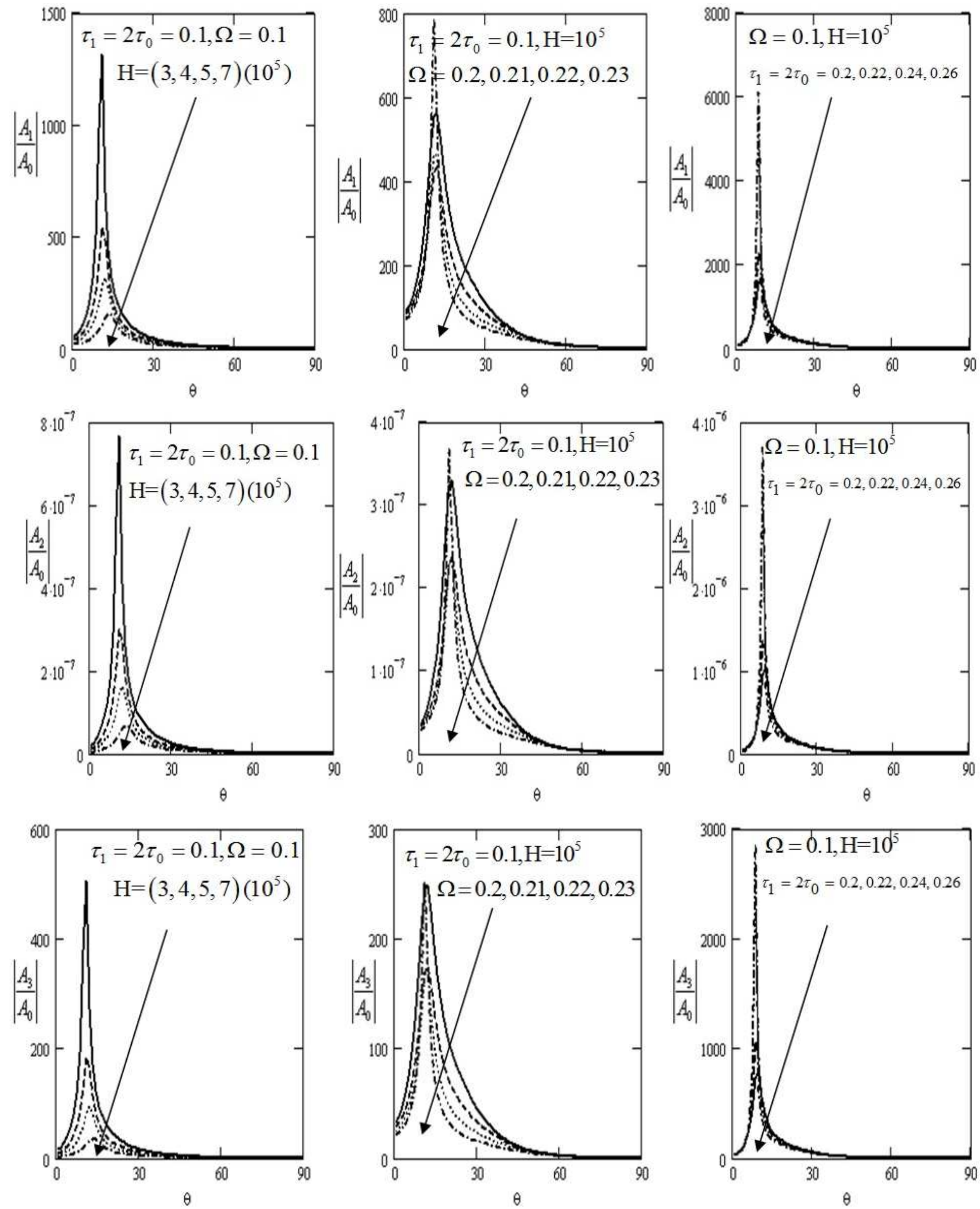

Fig. 3: Variations of the magnetic field, rotation and thermal relaxation times on the waves amplitudes $(|A_1/A_0|, |A_2/A_0|$ *and* $|A_3/A_0|)$ with respect to the angle of incidence θ.

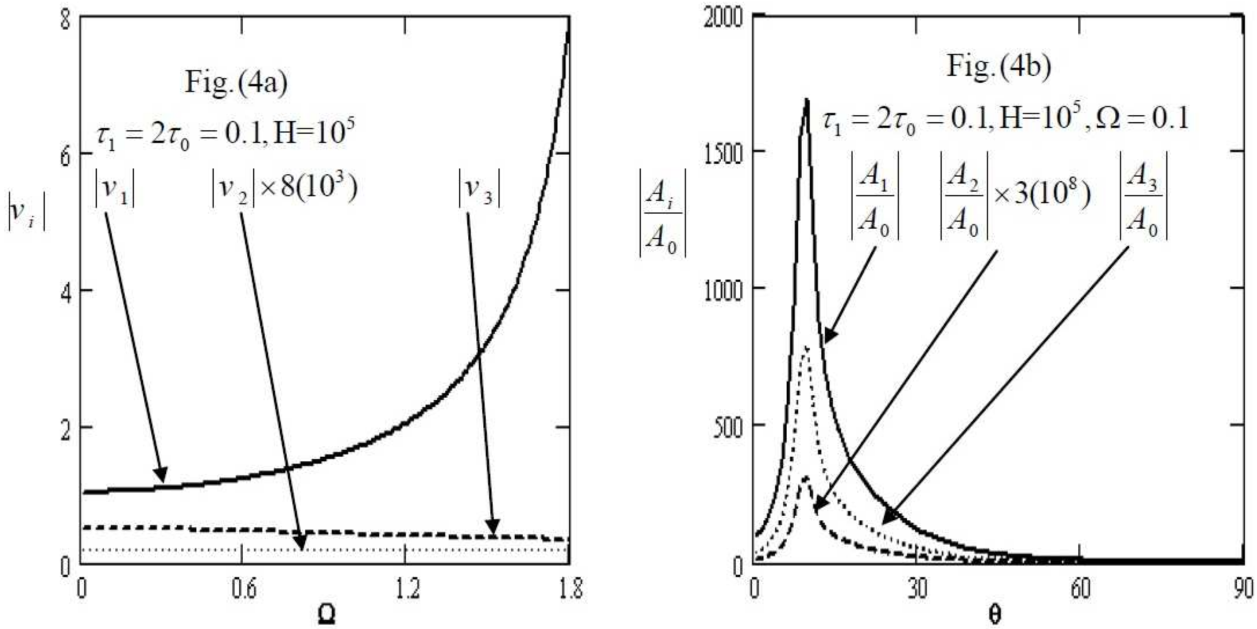

Fig. 4: A comparison between (a) Waves velocities $(|v_i|)$ with respect to Ω (b) Waves amplitudes $(|A_i/A_0|, i = 1,2,3)$ with respect to θ.

direction of z-axis, and those as shown in Fig. 1 of reflected p$-$, $T-$ and SV-waves make $\theta_1, \theta_2, \theta_3$; also with the $z-axis$, the displacement potentials ϕ and ψ and the temperature Θ take the following forms

$$\phi = A_0 \exp\left[ik_0\left(x\sin\theta_0 + z\cos\theta_0 - v_0 t\right)\right] + \sum_{n=1}^{3} A_n \exp\left[ik_n\left(x\sin\theta_n - z\cos\theta_n - v_n t\right)\right] \quad (38)$$

$$\psi = \eta_0 A_0 \exp\left[ik_0\left(x\sin\theta_0 + z\cos\theta_0 - v_0 t\right)\right] + \sum_{n=1}^{3} A_n \eta_n \exp\left[ik_n\left(x\sin\theta_n - z\cos\theta_n - v_n t\right)\right] \quad (39)$$

$$\Theta = \zeta_0 A_0 \exp\left[ik_0\left(x\sin\theta_0 + z\cos\theta_0 - v_0 t\right)\right] + \sum_{n=1}^{3} A_n \zeta_n \exp\left[ik_n\left(x\sin\theta_n - z\cos\theta_n - v_n t\right)\right] \quad (40)$$

From Eqs, (33) and (33) we get

$$\eta_m = \frac{2ik_m v_m \Omega}{k_m^2 v_m^2 - k_m^2 C_{10}^2 + \Omega^2}, \qquad \zeta_m = \frac{\varepsilon 2ik_m^3 v_m \Omega}{k_m^2 v_m^2 \tau_0 - k_m^2 C_5^2 - k_m}, \qquad m = 0,1,2,3 \quad (41)$$

A_0 is the amplitudes of the incident p-wave, and A_1; A_2 and A_3 are the amplitudes of the reflected P_1 , P_2 and P_3 waves, respectively.

$$\sigma_{zz} + \tau_{zz} = 0, \qquad \sigma_{zx} + \tau_{zx} = 0, \qquad \frac{\partial\Theta}{\partial z} = 0, \ at z = 0 \quad (42)$$

For the reflected waves, the wave numbers and the reflected angles may be written as:

$$k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 \quad (43)$$

which take the equivalent form:

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} \quad (44)$$

Substituting from Eqs. (38)-(40) into the boundary conditions in Eq. (42), we obtain a system of three algebraic equations takes the form

$$\Sigma a_{ij} X_j = b_j, \quad (i, j = 1,2,3) \quad (45)$$

where,

$$a_{1j} = k_j^2[\lambda + \mu_e H^2 + 2\mu_T \cos^2\theta_j - \mu_T \eta_j \sin 2\theta_j + \alpha\cos^2\theta_j - \tfrac{1}{2}\alpha\eta_j \sin 2\theta_j] + \left(\lambda + 2\alpha + 4\mu_{\mathrm{L}} - 2\mu_{\mathrm{T}} + \beta + \mu_e H^2\right)\left(1 - i\tau_1 k_j v_j\right)\zeta_j,$$
$$a_{2j} = k_j^2\left[\sin 2\theta_j + \eta_j\left(\cos^2\theta_j - \sin^2\theta_j\right)\right],$$
$$a_{3j} = \zeta_j k_j \cos\theta_j$$

and,

$$X_1=\frac{A_1}{A_0},X_2=\frac{A_2}{A_0},X_3=\frac{A_3}{A_0}. \qquad (46)$$

$$\begin{aligned}b_1&=-k_0^2[\lambda+\mu_eH^2+2\mu_T\cos^2\theta_0-\mu_T\eta_0\sin 2\theta_0\\&+\alpha\cos^2\theta_0-\tfrac{1}{2}\alpha\eta_0\sin 2\theta_0]\\&-\left(\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_eH^2\right)(1-i\tau_1k_0v_0)\zeta_0,\\b_2&=-k_0^2\left[\sin 2\theta_0+\eta_0\left(\cos^2\theta_0-\sin^2\theta_0\right)\right],\\b_3&=-\zeta_0k_0\cos\theta_0.\end{aligned}$$

From the results obtained in Eq. (46), it is concluded that the fibre-reinforced parameters play a significant role on the waves velocities and the ratio of the reflection coefficients, this indicated to the its important applications in diverse filed, especially, in aircraft, geophysics,...etc.

4 Special Case

If the fibre-reinforced is neglected, Eq. (36) tends to:

$$L\left(v^2\right)^3+M\left(v^2\right)^2+Nv^2+P=0 \qquad (47)$$

where

$$\begin{aligned}P&=-C_1^2C_2^2C_3^2\omega^6\\N&=\omega^4[C_1^2C_2^2\omega\left[\omega\left(\tau_0+2\Omega\varepsilon\tau_1\right)+i\left(1+2\,\Omega\varepsilon\right)\right]\\&+C_3^2\left(C_1^2+C_2^2\right)\left(\omega^2+\Omega^2\right)]\\M&=\omega^2[-\left(\omega^2+\Omega^2\right)\left[\left(C_1^2+C_2^2\right)\left(\omega^2\tau_0+i\omega\right)\right.\\&+C_3^2\left(\omega^2+\Omega^2\right)+2i\,\omega\,\Omega\varepsilon C_1^2\left(1-i\,\tau_1\omega\right)]+4\omega^2\,\Omega^2C_3^2]\\L&=\left(\omega^2\tau_0+i\omega\right)\left[\left(\omega^2+\Omega^2\right)^2-4\omega^2\,\Omega^2\right]\end{aligned}$$

where,

$$C_1^2=\frac{\left(\lambda+2\mu+\mu_eH^2\right)}{\rho v^2},\quad C_2^2=\frac{\mu}{\rho v^2},$$

$$C_3^2=\frac{K}{\rho C_v v\ell},\quad \varepsilon=\frac{\gamma^2T_0}{\rho C_v\left(\lambda+2\mu+\mu_eH^2\right)}$$

$$\eta_m=\frac{2ik_mv_m\Omega}{k_m^2v_m^2-k_m^2C_2^2+\Omega^2},$$

$$\zeta_m=\frac{2i\,\varepsilon k_m^3v_m\Omega}{k_m^2v_m^2\tau_0-k_m^2C_3^2+ik_mv_m},$$

$$m=0,1,2,3.$$

Eq. (45) tends to:

$$\Sigma A_{ij}X_j=B_i,\quad (i,\,j=1,2,3)$$

where,

$$\begin{aligned}A_{1j}&=k_j^2[\lambda+\mu_eH^2+2\mu\cos^2\theta_n-\mu\eta_j\sin 2\theta_j]\\&+\zeta_j(\lambda+2\mu+\mu_eH^2)(1-i\tau_1k_jv_j),\\A_{2j}&=k_j^2[\sin 2\theta_j+\eta_j(\cos^2\theta_j-\sin^2\theta_j)]\\A_{3j}&=\zeta_jk_j\cos\theta_j\end{aligned}$$

and,

$$X_1=\frac{A_1}{A_0},\quad X_2=\frac{A_2}{A_0},\quad X_3=\frac{A_3}{A_0}$$

$$\begin{aligned}B_1&=-k_0^2\left[\lambda+\mu_eH^2+2\mu\cos^2\theta_0+\mu\eta_0\sin 2\theta_0\right]\\&-\zeta_0\left(\lambda+2\mu+\mu_eH^2\right)\left(1-i\,\tau_1k_0v_0\right),\\B_2&=k_0^2\left[\sin 2\theta_0+\eta_0\left(\cos^2\theta_0-\sin^2\theta_0\right)\right],\\B_3&=\zeta_0k_0\cos\theta_0.\end{aligned}$$

5 Numerical results and discussion

For computational work, the following material constants at $T_0=300^oC$ are considered a copper material for an elastic solid with generalized thermoelastic solid taking into consideration neglecting the fibre-reinforced property

$$\begin{aligned}&\lambda=8.2\times10^{10}N\,/m^2, && \mu=4.2\times10^{10}N\,/m^2,\\&\rho=8.95\times10^3Kg\,/m^2, && c_v=3.845\times10^2m^2K^{-1}s^{-2},\\&\alpha_t=1.67\times10^{-5}/K, && \omega=10^2.\end{aligned}$$

Fig. 2 displays the variation of the magnitudes of p-wave velocity v_1, T-wave velocity v_2 and SV-wave velocity v_3 with respect to the rotation Ω, where the magnitudes of T-wave velocity and SV-wave velocity decrease with an increasing the rotation but increases with the increased values of the magnetic field H while the magnitudes of p-waves velocity increases with an increasing of Ω and H. Also, it is seen that there is no effect of thermal relaxation time on p-wave velocity and SV-wave velocity, as well the T-wave velocity decreases with increasing the thermal relaxation times.

Fig. 3 shows the variation of the p-wave amplitude $\left|\frac{A_1}{A_0}\right|$, T-wave amplitude $\left|\frac{A_2}{A_0}\right|$ and SV-wave amplitude $\left|\frac{A_3}{A_0}\right|$ with respect to the angle of incident θ, the waves amplitudes increase arriving their maximum values nearly at $12^o<\theta<17^o$, and decrease with an increasing of the angle of incident vanishing even when $\theta=90^o$, while it decrease with an increase in the magnetic field H and rotation Ω but the thermal relaxation times τ_0 and τ_1 affects increasing on the waves amplitudes nearly at $0^o<\theta<12^o$, after that it decrease with an increasing of the thermal relaxation times.

Physically, it is clear that the amplitudes ratios of the waves arrive to their maximum values with small values of the angle of incidence, and tends to zero as the waves incidence perpendicular to its primary incidence, this indicate to the significant role on the waves reflection.

Fig. 4 plots a comparison between the magnitudes of the waves velocities with respect to the rotation Ω (Fig. 4a) and the magnitudes of the reflection coefficients with respect to the angle of incidence θ From Fig. 4a, it is obvious that $|v_1| > |v_3| > |v_2|$, also, it is seen that $\left|\frac{A_1}{A_0}\right| > \left|\frac{A_3}{A_0}\right| > \left|\frac{A_2}{A_0}\right|$.

6 Conclusion

The main conclusions due to the influences of the fibre-reinforced, magnetic field and thermal relaxation times on the reflection of p-, T-, and SV-waves, can be pointed as follow:

(i) The reflection coefficients are affected strongly by the angle of incidence θ fibre-reinforced, magnetic field and thermal relaxation times.

(ii) The magnetic field affected strongly on the absolute values of all reflection coefficients unless $|B_1/A_o|$ for p-wave and $|A_1/B_o|$ for SV-wave at a stress-free thermally insulated.

(iii) The thermal relaxation times affected strongly on all values of the reflection coefficients, it is seen that the angle of incidence θ affects very strong on all values of the reflection coefficients and $0^o < \theta < 12^o$, displays a critical value for the reflection coefficients in all waves.

(iv) It is concluded that, $|v_1| > |v_3| > |v_2|$, also, it appears that $\left|\frac{A_1}{A_0}\right| > \left|\frac{A_3}{A_0}\right| > \left|\frac{A_2}{A_0}\right|$.

(v) Finally, it is too clear that all operators are affected on the amplitude ratios, which have a good influence on Aircraft, Seismic waves, Earthquakes, Geophysics, Volcanoes, Plasma, Geometrical Geology, Nuclear fields, Geology and etc.

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