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An Adroit Stratified Unrelated Question Randomized Response Model with Assorted Strategies

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Abstract: In this article, a new stratified unrelated question randomized response model has been proposed. It is shown that proposed stratified unrelated question randomized response model is more efficient than the existing randomized response model. The properties of the proposed randomized response model with relative efficiency have been studied with respect to the existing models. Numerical illustrations are also given in support of the present study.

Keywords: Randomized response sampling, Stratified Random Sampling, Estimation of proportion, Respondents protection, sensitive quantitative variable.

1 Introduction

The social survey is a key tool for gathering information on human communities. The social survey has gained a reputation for being incredibly useful for assessing opinions, attitudes, and behaviours that span a broad range of interests. The surveys are undertaken for a variety of reasons, with the most obvious and understandable being the lack of particular facts or information in the archives. If one wants to know the crime rate, for example, formal records on crimes do not contain information regarding unseen crimes or unreported victimisation experiences. Investigators may occasionally be unable to access information on specific individuals (in a population) due to legal constraints. In general, questionnaires—and particularly social surveys—have a lot of items. Due of the societal stigma they carry, some of the things may be concerning sensitive or high-risk behaviour. A common challenge in studies focusing on high-risk behaviors is the potential for participants to provide inaccurate information, whether knowingly or unknowingly. This issue is often attributed to social desirability bias, which significantly distorts responses in standardized personality tests within psychological research. Survey researchers express concerns about the reliability of survey results pertaining to various sensitive topics such as drunk driving, marijuana use, tax evasion, illicit drug consumption, induced abortion, shoplifting, child abuse, family matters, exam cheating, HIV/AIDS, and sexual behaviors. A randomised response model was first used by Warner (1965) to determine the population proportion for a sensitive attribute. A detailed review and applications of such technique can be had from work of Tracy and Mangat (1996), Cochran (1977), Singh and Mangat (1996), Chudhuari and Mukherjee (1988), Ryu et al. (1993), Fox and Tracy (1986), Singh (2003), Singh and Tarray (2015) and the references cited therein. Rather et al. (2022) recently, work based on Poisson approximation using randomized response model.

2 Proposed unrelated stratified randomized response Model: Strategy I.

We suggest utilizing an Unrelated Question Randomized Response Technique (UQ-RRT) model to gauge the percentage N of individuals exhibiting the stigmatized trait A in the study. This approach is applicable when we have knowledge of both the known and unknown proportions of the unrelated innocuous trait X.

2.1. When the Proportion of Unrelated stratified innocuous Attribute is Known.

In the proposed two-stage RRT model, each selected respondent in the sample is given two randomization devices (R_1, R_2). The first-stage randomized device R_1 consists of the following statements:

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(i) Do you possess sensitive attribute A_i?

(ii) Go to the randomized device R_1

The probabilities are T_i and $(1 - T_i)$, respectively. Second stage randomized device R_2 consists of the following

Statements:

(i) Do you possess sensitive attribute A_i?

(ii) Do you possess non-sensitive attribute X_i?

(iii) Blank Card

The probabilities are p_1, p_2 and p_3, respectively, such that $\sum_{i=1}^{3} p_i = 1$. If the third statement (iii) appears on the respondent's card during the first draw, the process must be repeated without putting the card back for the second draw. If statement (iii) appears again in the second draw, the respondent is advised to honestly report their true status regarding the sensitive attribute A. Throughout this randomization process, the interviewer remains unaware of whether the responses are from the first or second draw, ensuring the privacy of the interviewee(s) and enabling them to answer without any apprehension.

The probability of getting answer "yes" from the respondent is as follows:

$$\theta_{1i} = T_i\pi_{ai} + (1-T_i)\left[(\pi_{ai} + (1-\pi_{bi})P_{2i})\left(1+P_{3i}\frac{k}{k-1}\right) + \left(1+P_{3i}^2\frac{k}{k-1}\right)\pi_{ai}\right] \tag{1}$$

where k be the total number of cards in the proposed deck. Solving equation (1) the estimator $\hat{\pi}_{1i}$ is an unbiased for π_{1i} is given as

$$\hat{\pi}_{1i} = \sum_{i=1}^{k} w_i \left[\frac{\hat{\theta}_{2i} - (1-T_i)\left[P_2\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\right]}{T_i + (1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)}\right] \tag{2}$$

Where $\hat{\theta}_{2i}$ is the proportion of 'yes' answer obtained from the n_{i1} sampled respondents. The random variable $\hat{\theta}_{2i} = \frac{n_i}{n}$ follows the binomial distribution with parameters (n_i, θ_{2i}) with variance is given as follows:

$$V(\hat{\pi}_{1i}) = \sum_{i=1}^{k} \frac{w_i^2}{n_i}\left[\begin{array}{l}\pi_{ai}(1-\pi_{ai}) + \dfrac{1-\left\{T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right\} - 2(1-T_i)\left[P_2\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_2\left(1+P_{3i}\frac{k}{k-1}\right)\right]\pi_{bi}}{\left[T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right]} \\ + \dfrac{\left[(1-T_i)\left[P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\right]\right]\left\{1-(1-T_i)\left[P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\right]\right\}}{\left[T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right]^2}\end{array}\right] \tag{3}$$

Lemma 1. The unbiased estimate of the variance $V(\hat{\pi}_{k1})$ is given as follows:

$$V(\hat{\pi}_{1i}) = \frac{\hat{\theta}_{2i}(1-\hat{\theta}_{2i})}{n_i\left[T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right]^2} \tag{4}$$

2.2. When the Proportion of Unrelated* stratified *Innocuous Attribute is Unknown.

To estimate the population proportion of sensitive attribute, π_{ai}, and unrelated innocuous attribute, π_{bi}, simultaneously, we applied a split-sample approach. In this method, a sample size n_i is split into two subsamples of sizes n_{1i} and $n_{2i}(n_{1i}+n_{2i}=n)$ in each subsample, different randomization devices are applied.

The description of randomization devices in two sub sample sizes n_{1i} and n_{2i} is given as follows. The layout of the two-stage randomized response model for n_{1i} is given as follows.

First-stage randomized device R_{11i} consists of the following statements:

(i) Do you possess sensitive attribute A_i?

(ii) Go to the randomized device R_{12i},

The probabilities are T_{1i} and $(1-T_{i1})$, respectively. Second-stage randomized device R_{12i}, consists of the following statements:

(i) Do you possess sensitive attribute A_i?

(ii) Do you possess non-sensitive attribute X_i?

(iii) Blank card

The probabilities are p_1, p_2 and p_3, respectively, such that $\sum_{i=1}^{3} p_i = 1$. If the statement (iii) appeared on card of respondent, then it is required to repeat the process without replacing the card. In the second draw, if statement (iii) is reappeared, then the respondent is suggested to report his/her actual status about sensitive attribute A_i. The layout of two-stage randomized response model for n_{2i}, is given as follows.

First-stage randomized device R_{21i} consists of the following statements:

(I) Do you possess sensitive attribute A_i?

(ii) Go to the randomized device R_{22i}

The probabilities are T_{2i} and $(1-T_{2i})$, respectively.

Second-stage randomized device R_{22i} consists of the following statements:

(I) Do you possess sensitive attribute A_i?

(ii) Do you possess non-sensitive attribute X_i?

(iii) Draw one more card

The probabilities are q_1, q_2 and q_3, respectively, such that $\sum_{i=1}^{3} q_i = 1$. If statement (iii) appeared on card of respondent, then it is required to repeat the process without replacing the card in the second draw, if statement (iii) is reappeared, then the respondent is suggested to report his/her actual status about sensitive attribute A_i.

The probabilities of getting answer yes" from the respondent is as follows:

$$\theta_{3i} = T_{1i}\pi_{ai} + (1-T_{1i})\left[(\pi_{ai} + (1-\pi_{bi})p_{2i})\left(1+p_{3i}\frac{k}{k-1}\right) + \left(1+p_{3i}^2\frac{k}{k-1}\right)\pi_{ai}\right] \tag{5}$$

$$\theta_{4i}=T_{2i}\pi_{ai}+\left(1-T_{2i}\right)\left[\left(\pi_{ai}+(1-\pi_{bi})q_{2i}\right)\left(1+q_{3i}\frac{k}{k-1}\right)+\left(1+q_{3i}^{2}\frac{k}{k-1}\right)\pi_{ai}\right] \tag{6}$$

Solving equations (5) and (6) for π_{ai} and π_{bi}, we have estimators for π_{ai} and π_{bi},respectively, as follows:

$$\hat{\pi}_{11i}=\sum_{k=1}^{k}w_i\left[\frac{\left(1-T_{2i}\right)\left[q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\hat{\theta}_{4i}-q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\right]-\left(1-T_{i1}\right)\left[p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\hat{\theta}_{3i}-p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\right]}{C_{0i}}\right] \tag{7}$$

Where

$$\begin{aligned}C_{0i}=&T_{1i}\left(1-T_{2i}\right)\left[q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\hat{\theta}_{4i}-q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\right]-T_{2i}\left(1-T_{1i}\right)\left[p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\hat{\theta}_{3i}-p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\right]\\&+\left(1-T_{1i}\right)\left(1-T_{2i}\right)\left[\left(q_{1i}+(1-q_{1i})p_{2i}\right)\left(1+p_{3i}\frac{k}{k-1}\right)\left(1+q_{3i}\frac{k}{k-1}\right)\right]\\&+\left(1-T_{1i}\right)\left(1-T_{2i}\right)\left(\frac{k}{k-1}\right)\left[p_{3i}^{2}q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)-q_{3i}^{2}p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\right]\neq 0,\end{aligned} \tag{8}$$

$$\begin{aligned}\hat{\pi}_{pxi}=&\frac{1}{D_{0i}}\left[T_{2i}+\left(1-T_{2i}\right)\left(1+q_{3i}\frac{k}{k-1}+q_{3i}^{2}\frac{k}{k-1}\right)\right]\hat{\theta}_{3i}\\&-\left[T_{i1}+\left(1-T_{1i}\right)\left(1+p_{3i}\frac{k}{k-1}+p_{3i}^{2}\frac{k}{k-1}\right)\right]\hat{\theta}_{4i}\end{aligned} \tag{9}$$

$$\begin{aligned}D_{0i}=&T_{1i}\left(1-T_{2i}\right)\left[q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\hat{\theta}_{4i}-q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\right]-T_{2i}\left(1-T_{1i}\right)\left[p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\hat{\theta}_{4i}-p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)\right]\\&+\left(1-T_{1i}\right)\left(1-T_{2i}\right)\left[\left(q_{2i}+(1-q_{2i})p_{2i}\right)\left(1+p_{3i}\frac{k}{k-1}\right)\left(1+q_{3i}\frac{k}{k-1}\right)\right]\\&+\left(1-T_{1i}\right)\left(1-T_{2i}\right)\left(\frac{k}{k-1}\right)\left[p_{3i}^{2}p_{2i}\left(1+p_{3i}\frac{k}{k-1}\right)-q_{3i}^{2}q_{2i}\left(1+q_{3i}\frac{k}{k-1}\right)\right]\neq 0,\end{aligned} \tag{10}$$

Where, $\hat{\theta}_{3i}$ and $\hat{\theta}_{4i}$ are the proportion of "yes" answer obtained from n and n sampled respondents, respectively. Since the random variables $\hat{\theta}_{3i}$ and $\hat{\theta}_{4i}$ follow the binomial distribution with parameters (n_{1i},θ_{3i}) and (n_{2i},θ_{4i}) respectively, therefore, the properties of estimators $\hat{\pi}_{11i}$ and $\hat{\pi}_{pxi}$, may be summarized in the following theorems.

Theorem 1: The estimators $\hat{\pi}_{11i}$ and $\hat{\pi}_{pxi}$ are unbiased for π_{ai} and π_{bi} with variances, respectively, and are given as follows:

$$\begin{aligned}V\left(\hat{\pi}_{11i}\right)=&\frac{d_{2i}^{2}\left[c_{3i}^{2}\pi_{ai}\left(1-\pi_{ai}\right)+c_{3i}\left(1-c_{3i}-2c_{2i}\pi_{xi}\right)\pi_{ai}+c_{2i}\pi_{xi}(1-c_{2i}\pi_{xi})\right]}{n_{1i}C_{0i}^{2}}\\&+\frac{c_{2i}^{2}\left[d_{3i}^{2}\pi_{ai}\left(1-\pi_{ai}\right)+d_{3i}\left(1-d_{3i}-2d_{2i}\pi_{xi}\right)\pi_{ai}+d_{2i}\pi_{xi}(1-d_{2i}\pi_{xi})\right]}{n_{1i}C_{0i}^{2}}\end{aligned} \tag{11}$$

$$V\left(\hat{\pi}_{pxi}\right)=\frac{d_{3i}^{2}\left[c_{3i}^{2}\pi_{ai}\left(1-\pi_{ai}\right)+c_{3i}\left(1-c_{3i}-2c_{2i}\pi_{xi}\right)\pi_{ai}+c_{2i}\pi_{xi}\left(1-c_{2i}\pi_{xi}\right)\right]}{n_{1i}D_{0i}^{2}}$$
$$+\frac{c_{3i}^{2}\left[d_{3i}^{2}\pi_{ai}\left(1-\pi_{ai}\right)+d_{3i}\left(1-d_{3i}-2d_{2i}\pi_{xi}\right)\pi_{ai}+d_{2i}\pi_{xi}\left(1-d_{2i}\pi_{xi}\right)\right]}{n_{1i}D_{0i}^{2}} \tag{12}$$

Where

$$\left.\begin{aligned} c_{2i} &= \left(1-T_{1i}\right)\left[P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi}-P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\right] \\ c_{3i} &= \left[T_{1i}+\left(1-T_{1i}\right)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^{2}\frac{k}{k-1}\right)\right] \\ d_{2i} &= \left(1-T_{2i}\right)\left[P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi}-P_{2i}\left(1+P_{3i}\frac{k}{k-1}\right)\right] \\ d_{3i} &= \left[T_{2i}+\left(1-T_{2i}\right)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^{2}\frac{k}{k-1}\right)\right] \end{aligned}\right\} \tag{13}$$

3 Reliability Measures

To show the dominance of the proposed estimators, empirical comparisons are made over the existing estimators. The relative efficiency of proposed estimators with respect to the existing estimators when the proportion of unrelated innocuous attribute is known and unknown is defined as follows:

$$RE_{1}=\frac{V\left(\hat{\pi}_{p1}\right)}{V\left(\hat{\pi}_{k1}\right)}$$

4 Simulation Study:

We perform a simulation study to validate the theoretical results of proposed two-stage unrelated question RRT model. The simulated results of proposed estimators are presented in Tables 1, 2, 3, 4 and 5, respectively. When the proportion of unrelated innocuous attribute is known, the design parameters T and p_1, $p_2=(1-p_1)$ and $p_3=(1-p_1-p_2)$ were allowed to vary while $k=45$ and sample size $n=1000$ were fixed.

Table 1: *Simulation results of proposed estimator* $\hat{\pi}_{k1}$*, when trials =10000, n*$_1$*=n*$_2$*=500, T=0.1 at various choices of* W$_1$=0.7, W$_2$=0.3 $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k1})$	RE_1
0.9	0.03	0.07	0.1	0.00329	0.00109	301
0.9	0.03	0.07	0.2	0.00329	0.00116	284
0.9	0.03	0.07	0.3	0.00329	0.00116	284
0.9	0.03	0.07	0.1	0.00329	0.00116	284
0.9	0.03	0.07	0.2	0.00329	0.00116	284
0.8	0.07	0.13	0.3	0.00336	0.00115	291
0.8	0.07	0.13	0.1	0.00334	0.00115	290
0.8	0.07	0.13	0.2	0.00335	0.00115	291
0.8	0.07	0.13	0.3	0.00336	0.00115	291
0.8	0.07	0.13	0.1	0.00334	0.00115	290
0.7	0.10	0.20	0.2	0.00344	0.00115	300
0.7	0.10	0.20	0.3	0.00346	0.00115	301
0.7	0.10	0.20	0.1	0.00342	0.00115	298

0.7	0.10	0.20	0.2	0.00344	0.00115	300
0.7	0.10	0.20	0.3	0.00346	0.00115	301
0.6	0.13	0.27	0.1	0.00354	0.00114	310
0.6	0.13	0.27	0.2	0.00357	0.00114	313
0.6	0.13	0.27	0.3	0.00361	0.00114	315
0.6	0.13	0.27	0.1	0.00354	0.00114	310
0.6	0.13	0.27	0.2	0.00357	0.00114	313
0.9	0.03	0.07	0.3	0.00329	0.00116	284
0.9	0.03	0.07	0.1	0.00329	0.00116	284
0.9	0.03	0.07	0.2	0.00329	0.00116	284
0.9	0.03	0.07	0.3	0.00329	0.00116	284
0.9	0.03	0.07	0.1	0.00329	0.00116	284
0.8	0.07	0.13	0.2	0.00335	0.00115	291
0.8	0.07	0.13	0.3	0.00336	0.00115	291
0.8	0.07	0.13	0.1	0.00334	0.00115	290
0.8	0.07	0.13	0.2	0.00335	0.00115	291
0.8	0.07	0.13	0.3	0.00336	0.00115	291
0.7	0.10	0.20	0.1	0.00342	0.00115	298
0.7	0.10	0.20	0.2	0.00344	0.00115	300
0.7	0.10	0.20	0.3	0.00346	0.00115	301
0.7	0.10	0.20	0.1	0.00342	0.00115	298
0.7	0.10	0.20	0.2	0.00344	0.00115	300
0.6	0.13	0.27	0.3	0.00361	0.00114	315
0.6	0.13	0.27	0.1	0.00354	0.00114	310
0.6	0.13	0.27	0.2	0.00357	0.00114	313
0.6	0.13	0.27	0.3	0.00361	0.00114	315
0.6	0.13	0.27	0.1	0.00354	0.00114	310
0.9	0.03	0.07	0.2	0.00329	0.00116	284
0.9	0.03	0.07	0.3	0.00329	0.00116	284
0.9	0.03	0.07	0.1	0.00329	0.00116	284
0.9	0.03	0.07	0.2	0.00329	0.00116	284
0.9	0.03	0.07	0.3	0.00329	0.00116	284
0.8	0.07	0.13	0.1	0.00334	0.00115	290
0.8	0.07	0.13	0.2	0.00335	0.00115	291
0.8	0.07	0.13	0.3	0.00336	0.00115	291
0.8	0.07	0.13	0.1	0.00334	0.00115	290
0.8	0.07	0.13	0.2	0.00335	0.00115	291
0.7	0.10	0.20	0.3	0.00346	0.00115	301
0.7	0.10	0.20	0.1	0.00342	0.00115	298
0.7	0.10	0.20	0.2	0.00344	0.00115	300
0.7	0.10	0.20	0.3	0.00346	0.00115	301
0.7	0.10	0.20	0.1	0.00342	0.00115	298
0.6	0.13	0.27	0.2	0.00357	0.00114	313
0.6	0.13	0.27	0.3	0.00361	0.00114	315
0.6	0.13	0.27	0.1	0.00354	0.00114	310
0.6	0.13	0.27	0.2	0.00357	0.00114	313
0.6	0.13	0.27	0.3	0.00361	0.00114	315

Table 2: *Simulation results of proposed estimator* $\hat{\pi}_{k1}$, *when trials =10000,* $n_1=n_2=500$, *T=0.2 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k1})$	RE_1
0.9	0.03	0.07	0.1	0.00304	0.00110	277
0.9	0.03	0.07	0.2	0.00305	0.00116	263
0.9	0.03	0.07	0.3	0.00305	0.00116	263

0.9	0.03	0.07	0.1	0.00304	0.00116	263
0.9	0.03	0.07	0.2	0.00305	0.00116	263
0.8	0.07	0.13	0.3	0.00310	0.00116	268
0.8	0.07	0.13	0.1	0.00308	0.00115	267
0.8	0.07	0.13	0.2	0.00309	0.00115	268
0.8	0.07	0.13	0.3	0.00310	0.00116	268
0.8	0.07	0.13	0.1	0.00308	0.00115	267
0.7	0.10	0.20	0.2	0.00315	0.00115	274
0.7	0.10	0.20	0.3	0.00317	0.00115	276
0.7	0.10	0.20	0.1	0.00313	0.00115	273
0.7	0.10	0.20	0.2	0.00315	0.00115	274
0.7	0.10	0.20	0.3	0.00317	0.00115	276
0.6	0.13	0.27	0.1	0.00321	0.00114	281
0.6	0.13	0.27	0.2	0.00324	0.00114	284
0.6	0.13	0.27	0.3	0.00327	0.00115	286
0.6	0.13	0.27	0.1	0.00321	0.00114	281
0.6	0.13	0.27	0.2	0.00324	0.00114	284
0.9	0.03	0.07	0.3	0.00305	0.00116	263
0.9	0.03	0.07	0.1	0.00304	0.00116	263
0.9	0.03	0.07	0.2	0.00305	0.00116	263
0.9	0.03	0.07	0.3	0.00305	0.00116	263
0.9	0.03	0.07	0.1	0.00304	0.00116	263
0.8	0.07	0.13	0.2	0.00309	0.00115	268
0.8	0.07	0.13	0.3	0.00310	0.00116	268
0.8	0.07	0.13	0.1	0.00308	0.00115	267
0.8	0.07	0.13	0.2	0.00309	0.00115	268
0.8	0.07	0.13	0.3	0.00310	0.00116	268
0.7	0.10	0.20	0.1	0.00313	0.00115	273
0.7	0.10	0.20	0.2	0.00315	0.00115	274
0.7	0.10	0.20	0.3	0.00317	0.00115	276
0.7	0.10	0.20	0.1	0.00313	0.00115	273
0.7	0.10	0.20	0.2	0.00315	0.00115	274
0.6	0.13	0.27	0.3	0.00327	0.00115	286
0.6	0.13	0.27	0.1	0.00321	0.00114	281
0.6	0.13	0.27	0.2	0.00324	0.00114	284
0.6	0.13	0.27	0.3	0.00327	0.00115	286
0.6	0.13	0.27	0.1	0.00321	0.00114	281
0.9	0.03	0.07	0.2	0.00305	0.00116	263
0.9	0.03	0.07	0.3	0.00305	0.00116	263
0.9	0.03	0.07	0.1	0.00304	0.00116	263
0.9	0.03	0.07	0.2	0.00305	0.00116	263
0.9	0.03	0.07	0.3	0.00305	0.00116	263
0.8	0.07	0.13	0.1	0.00308	0.00115	267
0.8	0.07	0.13	0.2	0.00309	0.00115	268
0.8	0.07	0.13	0.3	0.00310	0.00116	268
0.8	0.07	0.13	0.1	0.00308	0.00115	267
0.8	0.07	0.13	0.2	0.00309	0.00115	268
0.7	0.10	0.20	0.3	0.00317	0.00115	276
0.7	0.10	0.20	0.1	0.00313	0.00115	273
0.7	0.10	0.20	0.2	0.00315	0.00115	274
0.7	0.10	0.20	0.3	0.00317	0.00115	276
0.7	0.10	0.20	0.1	0.00313	0.00115	273
0.6	0.13	0.27	0.2	0.00324	0.00114	284
0.6	0.13	0.27	0.3	0.00327	0.00115	286
0.6	0.13	0.27	0.1	0.00321	0.00114	281

0.6	0.13	0.27	0.2	0.00324	0.00114	284
0.6	0.13	0.27	0.3	0.00327	0.00115	286

Table 3: *Simulation results of proposed estimator* $\hat{\pi}_{k1}$*, when trials =10000,* $n_1=n_2=500$*, T=0.3 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k1})$	RE_1
0.9	0.03	0.07	0.1	0.00280	0.00111	253
0.9	0.03	0.07	0.2	0.00280	0.00116	242
0.9	0.03	0.07	0.3	0.00281	0.00116	242
0.9	0.03	0.07	0.1	0.00280	0.00116	242
0.9	0.03	0.07	0.2	0.00280	0.00116	242
0.8	0.07	0.13	0.3	0.00284	0.00116	246
0.8	0.07	0.13	0.1	0.00282	0.00115	245
0.8	0.07	0.13	0.2	0.00283	0.00115	245
0.8	0.07	0.13	0.3	0.00284	0.00116	246
0.8	0.07	0.13	0.1	0.00282	0.00115	245
0.7	0.10	0.20	0.2	0.00287	0.00115	250
0.7	0.10	0.20	0.3	0.00289	0.00115	251
0.7	0.10	0.20	0.1	0.00286	0.00115	249
0.7	0.10	0.20	0.2	0.00287	0.00115	250
0.7	0.10	0.20	0.3	0.00289	0.00115	251
0.6	0.13	0.27	0.1	0.00290	0.00114	254
0.6	0.13	0.27	0.2	0.00293	0.00114	256
0.6	0.13	0.27	0.3	0.00295	0.00115	258
0.6	0.13	0.27	0.1	0.00290	0.00114	254
0.6	0.13	0.27	0.2	0.00293	0.00114	256
0.9	0.03	0.07	0.3	0.00281	0.00116	242
0.9	0.03	0.07	0.1	0.00280	0.00116	242
0.9	0.03	0.07	0.2	0.00280	0.00116	242
0.9	0.03	0.07	0.3	0.00281	0.00116	242
0.9	0.03	0.07	0.1	0.00280	0.00116	242
0.8	0.07	0.13	0.2	0.00283	0.00115	245
0.8	0.07	0.13	0.3	0.00284	0.00116	246
0.8	0.07	0.13	0.1	0.00282	0.00115	245
0.8	0.07	0.13	0.2	0.00283	0.00115	245
0.8	0.07	0.13	0.3	0.00284	0.00116	246
0.7	0.10	0.20	0.1	0.00286	0.00115	249
0.7	0.10	0.20	0.2	0.00287	0.00115	250
0.7	0.10	0.20	0.3	0.00289	0.00115	251
0.7	0.10	0.20	0.1	0.00286	0.00115	249
0.7	0.10	0.20	0.2	0.00287	0.00115	250
0.6	0.13	0.27	0.3	0.00295	0.00115	258
0.6	0.13	0.27	0.1	0.00290	0.00114	254
0.6	0.13	0.27	0.2	0.00293	0.00114	256
0.6	0.13	0.27	0.3	0.00295	0.00115	258
0.6	0.13	0.27	0.1	0.00290	0.00114	254
0.9	0.03	0.07	0.2	0.00280	0.00116	242
0.9	0.03	0.07	0.3	0.00281	0.00116	242
0.9	0.03	0.07	0.1	0.00280	0.00116	242
0.9	0.03	0.07	0.2	0.00280	0.00116	242
0.9	0.03	0.07	0.3	0.00281	0.00116	242
0.8	0.07	0.13	0.1	0.00282	0.00115	245
0.8	0.07	0.13	0.2	0.00283	0.00115	245
0.8	0.07	0.13	0.3	0.00284	0.00116	246

0.8	0.07	0.13	0.1	0.00282	0.00115	245
0.8	0.07	0.13	0.2	0.00283	0.00115	245
0.7	0.10	0.20	0.3	0.00289	0.00115	251
0.7	0.10	0.20	0.1	0.00286	0.00115	249
0.7	0.10	0.20	0.2	0.00287	0.00115	250
0.7	0.10	0.20	0.3	0.00289	0.00115	251
0.7	0.10	0.20	0.1	0.00286	0.00115	249
0.6	0.13	0.27	0.2	0.00293	0.00114	256
0.6	0.13	0.27	0.3	0.00295	0.00115	258
0.6	0.13	0.27	0.1	0.00290	0.00114	254
0.6	0.13	0.27	0.2	0.00293	0.00114	256
0.6	0.13	0.27	0.3	0.00295	0.00115	258

Table 4: *Simulation results of proposed estimator* $\hat{\pi}_{k1}$, *when trials =10000,* $n_1=n_2=500$, *T=0.4 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k1})$	RE_1
0.9	0.03	0.07	0.1	0.00256	0.00111	230
0.9	0.03	0.07	0.2	0.00256	0.00116	221
0.9	0.03	0.07	0.3	0.00257	0.00116	221
0.9	0.03	0.07	0.1	0.00256	0.00116	221
0.9	0.03	0.07	0.2	0.00256	0.00116	221
0.8	0.07	0.13	0.3	0.00259	0.00116	224
0.8	0.07	0.13	0.1	0.00257	0.00116	223
0.8	0.07	0.13	0.2	0.00258	0.00116	223
0.8	0.07	0.13	0.3	0.00259	0.00116	224
0.8	0.07	0.13	0.1	0.00257	0.00116	223
0.7	0.10	0.20	0.2	0.00260	0.00115	226
0.7	0.10	0.20	0.3	0.00262	0.00115	227
0.7	0.10	0.20	0.1	0.00259	0.00115	225
0.7	0.10	0.20	0.2	0.00260	0.00115	226
0.7	0.10	0.20	0.3	0.00262	0.00115	227
0.6	0.13	0.27	0.1	0.00261	0.00114	228
0.6	0.13	0.27	0.2	0.00263	0.00115	230
0.6	0.13	0.27	0.3	0.00265	0.00115	231
0.6	0.13	0.27	0.1	0.00261	0.00114	228
0.6	0.13	0.27	0.2	0.00263	0.00115	230
0.9	0.03	0.07	0.3	0.00257	0.00116	221
0.9	0.03	0.07	0.1	0.00256	0.00116	221
0.9	0.03	0.07	0.2	0.00256	0.00116	221
0.9	0.03	0.07	0.3	0.00257	0.00116	221
0.9	0.03	0.07	0.1	0.00256	0.00116	221
0.8	0.07	0.13	0.2	0.00258	0.00116	223
0.8	0.07	0.13	0.3	0.00259	0.00116	224
0.8	0.07	0.13	0.1	0.00257	0.00116	223
0.8	0.07	0.13	0.2	0.00258	0.00116	223
0.8	0.07	0.13	0.3	0.00259	0.00116	224
0.7	0.10	0.20	0.1	0.00259	0.00115	225
0.7	0.10	0.20	0.2	0.00260	0.00115	226
0.7	0.10	0.20	0.3	0.00262	0.00115	227
0.7	0.10	0.20	0.1	0.00259	0.00115	225
0.7	0.10	0.20	0.2	0.00260	0.00115	226
0.6	0.13	0.27	0.3	0.00265	0.00115	231
0.6	0.13	0.27	0.1	0.00261	0.00114	228
0.6	0.13	0.27	0.2	0.00263	0.00115	230

0.6	0.13	0.27	0.3	0.00265	0.00115	231
0.6	0.13	0.27	0.1	0.00261	0.00114	228
0.9	0.03	0.07	0.2	0.00256	0.00116	221
0.9	0.03	0.07	0.3	0.00257	0.00116	221
0.9	0.03	0.07	0.1	0.00256	0.00116	221
0.9	0.03	0.07	0.2	0.00256	0.00116	221
0.9	0.03	0.07	0.3	0.00257	0.00116	221
0.8	0.07	0.13	0.1	0.00257	0.00116	223
0.8	0.07	0.13	0.2	0.00258	0.00116	223
0.8	0.07	0.13	0.3	0.00259	0.00116	224
0.8	0.07	0.13	0.1	0.00257	0.00116	223
0.8	0.07	0.13	0.2	0.00258	0.00116	223
0.7	0.10	0.20	0.3	0.00262	0.00115	227
0.7	0.10	0.20	0.1	0.00259	0.00115	225
0.7	0.10	0.20	0.2	0.00260	0.00115	226
0.7	0.10	0.20	0.3	0.00262	0.00115	227
0.7	0.10	0.20	0.1	0.00259	0.00115	225
0.6	0.13	0.27	0.2	0.00263	0.00115	230
0.6	0.13	0.27	0.3	0.00265	0.00115	231
0.6	0.13	0.27	0.1	0.00261	0.00114	228
0.6	0.13	0.27	0.2	0.00263	0.00115	230
0.6	0.13	0.27	0.3	0.00265	0.00115	231

Table 5: *Simulation results of proposed estimator* $\hat{\pi}_{k1}$*, when trials =10000,* $n_1=n_2=500$*, T=0.5 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k1})$	RE_1
0.9	0.03	0.07	0.1	0.00232	0.00112	207
0.9	0.03	0.07	0.2	0.00232	0.00116	201
0.9	0.03	0.07	0.3	0.00233	0.00116	201
0.9	0.03	0.07	0.1	0.00232	0.00116	200
0.9	0.03	0.07	0.2	0.00232	0.00116	201
0.8	0.07	0.13	0.3	0.00234	0.00116	202
0.8	0.07	0.13	0.1	0.00233	0.00116	201
0.8	0.07	0.13	0.2	0.00233	0.00116	202
0.8	0.07	0.13	0.3	0.00234	0.00116	202
0.8	0.07	0.13	0.1	0.00233	0.00116	201
0.7	0.10	0.20	0.2	0.00234	0.00115	203
0.7	0.10	0.20	0.3	0.00235	0.00115	204
0.7	0.10	0.20	0.1	0.00233	0.00115	202
0.7	0.10	0.20	0.2	0.00234	0.00115	203
0.7	0.10	0.20	0.3	0.00235	0.00115	204
0.6	0.13	0.27	0.1	0.00234	0.00115	204
0.6	0.13	0.27	0.2	0.00235	0.00115	205
0.6	0.13	0.27	0.3	0.00237	0.00115	206
0.6	0.13	0.27	0.1	0.00234	0.00115	204
0.6	0.13	0.27	0.2	0.00235	0.00115	205
0.9	0.03	0.07	0.3	0.00233	0.00116	201
0.9	0.03	0.07	0.1	0.00232	0.00116	200
0.9	0.03	0.07	0.2	0.00232	0.00116	201
0.9	0.03	0.07	0.3	0.00233	0.00116	201
0.9	0.03	0.07	0.1	0.00232	0.00116	200
0.8	0.07	0.13	0.2	0.00233	0.00116	202
0.8	0.07	0.13	0.3	0.00234	0.00116	202
0.8	0.07	0.13	0.1	0.00233	0.00116	201

0.8	0.07	0.13	0.2	0.00233	0.00116	202
0.8	0.07	0.13	0.3	0.00234	0.00116	202
0.7	0.10	0.20	0.1	0.00233	0.00115	202
0.7	0.10	0.20	0.2	0.00234	0.00115	203
0.7	0.10	0.20	0.3	0.00235	0.00115	204
0.7	0.10	0.20	0.1	0.00233	0.00115	202
0.7	0.10	0.20	0.2	0.00234	0.00115	203
0.6	0.13	0.27	0.3	0.00237	0.00115	206
0.6	0.13	0.27	0.1	0.00234	0.00115	204
0.6	0.13	0.27	0.2	0.00235	0.00115	205
0.6	0.13	0.27	0.3	0.00237	0.00115	206
0.6	0.13	0.27	0.1	0.00234	0.00115	204
0.9	0.03	0.07	0.2	0.00232	0.00116	201
0.9	0.03	0.07	0.3	0.00233	0.00116	201
0.9	0.03	0.07	0.1	0.00232	0.00116	200
0.9	0.03	0.07	0.2	0.00232	0.00116	201
0.9	0.03	0.07	0.3	0.00233	0.00116	201
0.8	0.07	0.13	0.1	0.00233	0.00116	201
0.8	0.07	0.13	0.2	0.00233	0.00116	202
0.8	0.07	0.13	0.3	0.00234	0.00116	202
0.8	0.07	0.13	0.1	0.00233	0.00116	201
0.8	0.07	0.13	0.2	0.00233	0.00116	202
0.7	0.10	0.20	0.3	0.00235	0.00115	204
0.7	0.10	0.20	0.1	0.00233	0.00115	202
0.7	0.10	0.20	0.2	0.00234	0.00115	203
0.7	0.10	0.20	0.3	0.00235	0.00115	204
0.7	0.10	0.20	0.1	0.00233	0.00115	202
0.6	0.13	0.27	0.2	0.00235	0.00115	205
0.6	0.13	0.27	0.3	0.00237	0.00115	206
0.6	0.13	0.27	0.1	0.00234	0.00115	204
0.6	0.13	0.27	0.2	0.00235	0.00115	205
0.6	0.13	0.27	0.3	0.00237	0.00115	206

5 Proposed unrelated stratified randomized response Model:*Strategy II.*

We propose a unrelated question RRT model to estimate the proportion N of persons who possess the stigmatized attribute A under the study when the proportion of unrelated innocuous attribute X_i is known and unknown, respectively.

5.1. When the Proportion of Unrelated stratified innocuous Attribute is Known.

In the proposed two-stage RRT model, each selected respondent in the sample is given two randomization devices (R_{1i}, R_{2i}). The first-stage randomized device R_{1i} consists of the following statements:

(i) Do you possess sensitive attribute A_i?

(ii) Go to the randomized device R_{1i}

The probabilities are T_i and $(1 – T_i)$, respectively. Second stage randomized device R_{2i} consists of the following

Statements:

(i) Do you possess sensitive attribute A_i?

(ii) Do you possess non-sensitive attribute X_i?

(iii) Blank Card

The probabilities are p_1, p_2 and p_3, respectively, such that $\sum_{i=1}^{3} p_i = 1$. If statement (iii) appeared on card of respondent, then it is required to repeat the process without replacing the card in the second draw. If statement (iii) is reappeared, then the respondent is suggested Lo report his/her actual status aboiut sensitive attribute A. The whole process of randomization is unseen by the interviewer and does not know whether the respondent's answers come from the first draw or from the second draw. So, the privacy of interviewee(s) is protected, and they will respond without any fear.

The probability of getting answer "yes" from the respondent is as follows:

$$\theta_{2i} = T_i\pi_{ai} + (1-T_i)\left[\left(\pi_{ai} + (1-\pi_{bi})P_{1i}\right)\left(1+P_{3i}\frac{k}{k-1}\right) + \left(1+P_{3i}^2\frac{k}{k-1}\right)\pi_{ai}\right] \tag{14}$$

where k be the total number of cards in the proposed deck. Solving equation (1) the estimator $\hat{\pi}_{2i}$ is an unbiased for π_{2i} is given as

$$\hat{\pi}_{2i} = \frac{\hat{\theta}_{3i} - (1-T_i)\left[P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\right]}{T_i + (1-T_i)\left(1+P_{3i}\frac{k}{k-1} + P_{3i}^2\frac{k}{k-1}\right)} \tag{15}$$

where $\hat{\theta}_{3i}$ is the proportion of 'yes' answer obtained from the n sampled respondents. The random variable $\hat{\theta}_{3i} = \frac{n_i}{n}$ follows the binomial distribution with parameters (n_i, θ_{3i}) with variance is given as follows:

$$V(\hat{\pi}_{2i}) = \sum_{i=1}^{k}\frac{w_i^2}{n_i}\left[\begin{array}{l} \pi_{ai}(1-\pi_{ai}) + \dfrac{1-\left\{T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right\} - 2(1-T_i)\left[P_{i1}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{i1}\left(1+P_{3i}\frac{k}{k-1}\right)\right]\pi_{bi}}{\left[T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right]} \\ + \dfrac{\left[(1-T_i)\left[P_{i1}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\right]\right]\left\{1-(1-T_i)\left[P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\right]\right\}}{\left[T_i+(1-T_i)\left(1+P_{3i}\frac{k}{k-1}+P_{3i}^2\frac{k}{k-1}\right)\right]^2} \end{array}\right] \tag{16}$$

Lemma 2. The unbiased estimate of the variance $V(\hat{\pi}_{k2})$ is given as follows:

$$V(\hat{\pi}_{2i}) = \sum_{i=1}^{k}\frac{w_i^2}{n_i}\left[\frac{\hat{\theta}_{3i}(1-\hat{\theta}_{3i})}{\left[T_i + (1-T_i)\left(1+P_{3i}\frac{k}{k-1} + P_{3i}^2\frac{k}{k-1}\right)\right]^2}\right] \tag{17}$$

5.2 When the Proportion of Unrelated stratified *Innocuous Attribute is Unknown.*

To estimate the population proportion of sensitive attribute, π_{ai}, and unrelated innocuous attribute, π_{bi}, simultaneously, we applied a split-sample approach. In this method, a sample size n is split into two subsamples of sizes n_1 and $n_2(n_1 + n_2 = n)$ in each subsample, different randomization devices are applied.

The description of randomization devices in two sub sample sizes n_{1i} and n_{2i} is given as follows. The layout of the two-stage randomized response model for n_1 is given as follows.

First-stage randomized device R_{11i} consists of the following statements:

(i) Do you possess sensitive attribute A_i?

(ii) Go to the randomized device R_{12i},

The probabilities are T_{1i} and $(1-T_{1i})$, respectively. Second-stage randomized device R_{12}, consists of the following statements:

(i) Do you possess sensitive attribute A_i?

(ii) Do you possess non-sensitive attribute X_i?

(iii) Blank Card

The probabilities are p_1, p_2 and p_3, respectively, such that $\sum_{i=1}^{3} p_i = 1$. If statement (iii) appeared on card of respondent, then it is required to repeat the process without replacing the card. In the second draw, if statement (iii) is reappeared, then the respondent is suggested to report his/her actual status about sensitive attribute A. The layout of two-stage randomized response model for n_{2i}, is given as follows.

First-stage randomized device R_{21i} consists of the following statements:

(I) Do you possess sensitive attribute A_i?

(ii) Go to the randomized device R_{22i}

The probabilities are T_{2i} and $(1-T_{2i})$, respectively.

Second-stage randomized device R_{22} consists of the following statements:

(I) Do you possess sensitive attribute A_i?

(ii) Do you possess non-sensitive attribute X_i?

(iii) Blank Card

The probabilities are q_1, q_2 and q_3, respectively, such that $\sum_{i=1}^{3} q_i = 1$. If statement (iii) appeared on card of respondent, then it is required to repeat the process without replacing the card in the second draw, if statement (iii) is reappeared, then the respondent is suggested to report his/her actual status about sensitive attribute A.

The probabilities of getting answer yes" from the respondent is as follows:

$$\theta_{5i} = T_{1i}\pi_{ai} + (1-T_{1i})\left[(\pi_{ai} + (1-\pi_{bi})p_{1i})\left(1+p_{3i}\frac{k}{k-1}\right) + \left(1+p_{3i}^2\frac{k}{k-1}\right)\pi_{ai}\right] \quad (18)$$

$$\theta_{6i} = T_{2i}\pi_{ai} + (1-T_{2i})\left[(\pi_{ai} + (1-\pi_{bi})q_{1i})\left(1+q_{3i}\frac{k}{k-1}\right) + \left(1+q_{3i}^2\frac{k}{k-1}\right)\pi_{ai}\right] \quad (19)$$

Solving equations (5) and (6) for π_a and π_b, we have estimators for π_a and π_b,respectively, as follows:

$$\hat{\pi}_{12i}=\sum_{i=1}^{k}w_i\left[\frac{(1-T_{2i})\left[q_{1i}\left(1+q_{3i}\frac{k}{k-1}\right)\hat{\theta}_{6i}-q_{1i}\left(1+q_{3i}\frac{k}{k-1}\right)\right]-(1-T_{i1})\left[p_{i1}\left(1+p_{3i}\frac{k}{k-1}\right)\hat{\theta}_{5i}-p_{i1}\left(1+p_{3i}\frac{k}{k-1}\right)\right]}{C_{1i}}\right] \quad (20)$$

Where

$$C_{i1}=T_{1i}(1-T_{2i})\left[q_{1i}\left(1+q_{3i}\frac{k}{k-1}\right)\hat{\theta}_{6i}-q_{i1}\left(1+q_{3i}\frac{k}{k-1}\right)\right]-T_{2i}(1-T_{1i})\left[p_{i1}\left(1+p_{3i}\frac{k}{k-1}\right)\hat{\theta}_{6i}-p_{1i}\left(1+p_{i3}\frac{k}{k-1}\right)\right]$$
$$+(1-T_{1i})(1-T_{2i})\left[(q_{2i}+(1-q_{2i})p_{1i})\left(1+p_{3i}\frac{k}{k-1}\right)\left(1+q_{3i}\frac{k}{k-1}\right)\right]$$
$$+(1-T_{1i})(1-T_{2i})\left(\frac{k}{k-1}\right)\left[p_{3i}^{2}q_{i1}\left(1+q_{3i}\frac{k}{k-1}\right)-q_{3i}^{2}p_{1i}\left(1+p_{3i}\frac{k}{k-1}\right)\right]\neq 0, \quad (21)$$

$$\hat{\pi}_{pxi}=\sum_{i=1}^{k}w_i\left[\begin{array}{l}\frac{1}{D_{1i}}\left[T_{2i}+(1-T_{2i})\left(1+q_{3i}\frac{k}{k-1}+q_{3i}^{2}\frac{k}{k-1}\right)\right]\hat{\theta}_{5i}\\ -\left[T_{1i}+(1-T_{1i})\left(1+p_{3i}\frac{k}{k-1}+p_{3i}^{2}\frac{k}{k-1}\right)\right]\hat{\theta}_{6i}\end{array}\right] \quad (22)$$

$$D_{1i}=T_{1i}(1-T_{2i})\left[q_{1i}\left(1+q_{3i}\frac{k}{k-1}\right)\hat{\theta}_{6i}-q_{i1}\left(1+q_{3i}\frac{k}{k-1}\right)\right]-T_{2i}(1-T_{1i})\left[p_{1i}\left(1+p_{3i}\frac{k}{k-1}\right)\hat{\theta}_{5i}-p_{1i}\left(1+p_{3i}\frac{k}{k-1}\right)\right]$$
$$+(1-T_{1i})(1-T_{2i})\left[(q_{1i}+(1-q_{1i})p_{i1})\left(1+p_{3i}\frac{k}{k-1}\right)\left(1+q_{3i}\frac{k}{k-1}\right)\right]$$
$$+(1-T_{1i})(1-T_{2i})\left(\frac{k}{k-1}\right)\left[p_{3i}^{2}p_{i1}\left(1+p_{3i}\frac{k}{k-1}\right)-q_{3i}^{2}q_{1i}\left(1+q_{3i}\frac{k}{k-1}\right)\right]\neq 0, \quad (23)$$

Where, $\hat{\theta}_{5i}$ and $\hat{\theta}_{6i}$ are the proportion of "yes" answer obtained from n and n sampled respondents, respectively. Since the random variables $\hat{\theta}_{5i}$ and $\hat{\theta}_{6i}$ follow the binomial distribution with parameters (n_{1i},θ_{5i}) and (n_{2i},θ_{6i}) respectively, therefore, the properties of estimators $\hat{\pi}_{12i}$ and $\hat{\pi}_{px1i}$, may be summarized in the following theorems.

Theorem 3: The estimators $\hat{\pi}_{12i}$ and $\hat{\pi}_{px1i}$ are unbiased for π_{ai} and π_{bi} with variances, respectively, and are given as follows:

$$V(\hat{\pi}_{12i})=\sum_{i=1}^{k}\frac{W_i^{2}}{n_{1i}}\left[\begin{array}{l}\frac{d_{4i}^{2}\left[c_{5i}^{2}\pi(1-\pi_{ai})+c_{5i}(1-c_{5i}-2c_{4i}\pi_{xi})\pi_{ai}+c_{4i}\pi_{bi}(1-c_{4i}\pi_{xi})\right]}{C_{1i}^{2}}\\ +\frac{c_{4i}^{2}\left[d_{5i}^{2}\pi_{ai}(1-\pi_{ai})+d_{5i}(1-d_{5i}-2d_{4i}\pi_{xi})\pi_{ai}+d_{4i}\pi_{bi}(1-d_{4i}\pi_{bi})\right]}{C_{1i}^{2}}\end{array}\right] \quad (24)$$

$$V(\hat{\pi}_{pxi})=\sum_{i=1}^{k}\frac{W_i^{2}}{n_{1i}}\left[\begin{array}{l}\frac{d_{5}^{2}\left[c_{5}^{2}\pi(1-\pi)+c_{5}(1-c_{5}-2c_{4}\pi_{x})\pi+c_{4}\pi_{x}(1-c_{4}\pi_{x})\right]}{n_1D_1^{2}}\\ +\frac{c_{5}^{2}\left[d_{5}^{2}\pi(1-\pi)+d_{5}(1-d_{5}-2d_{4}\pi_{x})\pi+d_{4}\pi_{x}(1-d_{4}\pi_{x})\right]}{n_1D_1^{2}}\end{array}\right] \quad (25)$$

where

$$\left.\begin{aligned}
c_{4i} &= (1-T_{1i})\left[P_{i1}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\right] \\
c_{5i} &= \left[T_{1i} + (1-T_{1i})\left(1+P_{3i}\frac{k}{k-1} + P_{3i}^{2}\frac{k}{k-1}\right)\right] \\
d_{4i} &= (1-T_{2i})\left[P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\pi_{bi} - P_{1i}\left(1+P_{3i}\frac{k}{k-1}\right)\right] \\
d_{5i} &= \left[T_{2i} + (1-T_{2i})\left(1+P_{3i}\frac{k}{k-1} + P_{3i}^{2}\frac{k}{k-1}\right)\right]
\end{aligned}\right\} \tag{26}$$

6 Relative Efficiency:

To demonstrate the superiority of the proposed estimators, empirical comparisons are conducted against existing estimators. The relative efficiency of the proposed estimators compared to the existing ones is defined as follows, considering both scenarios where the proportion of the unrelated innocuous attribute is either known or unknown:

$$RE_2 = \frac{V(\hat{\pi}_{p1})}{V(\hat{\pi}_{k2})}$$

7 Simulation Study:

We performed a simulation study to validate the theoretical results of proposed two-stage unrelated question RRT model. The simulated results of proposed estimators are presented in Tables 6, 7, 8, 9 and 10, respectively. When the proportion of unrelated innocuous attribute is known, the design parameters T and p_1, $p_2 = (1-p_1)$ and $p_3 = (1-p_1-p_2)$ were allowed to vary while $k = 45$ and sample size $n = 1000$ were fixed.

Table 6: *Simulation results of proposed estimator* $\hat{\pi}_{k2}$, *when trials =10000, n*$_1$*=n*$_2$*=500, T=0.1 at various choices of* W$_1$=0.7, W$_2$=0.3, $\pi_{a1} = 0.7$, $\pi_{a2} = 0.3$, π_b, p_1, $p_2 = (1-p_1)$ *and* $p_3 = (1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k2})$	RE_2
0.9	0.03	0.07	0.1	0.00329	0.00115	287
0.9	0.03	0.07	0.2	0.00329	0.00114	289
0.9	0.03	0.07	0.3	0.00329	0.00113	291
0.9	0.03	0.07	0.1	0.00329	0.00115	287
0.9	0.03	0.07	0.2	0.00329	0.00114	289
0.8	0.07	0.13	0.3	0.00336	0.00110	307
0.8	0.07	0.13	0.1	0.00334	0.00112	299
0.8	0.07	0.13	0.2	0.00335	0.00111	303
0.8	0.07	0.13	0.3	0.00336	0.00110	307
0.8	0.07	0.13	0.1	0.00334	0.00112	299
0.7	0.10	0.20	0.2	0.00344	0.00107	323
0.7	0.10	0.20	0.3	0.00346	0.00105	330
0.7	0.10	0.20	0.1	0.00342	0.00108	317
0.7	0.10	0.20	0.2	0.00344	0.00107	323
0.7	0.10	0.20	0.3	0.00346	0.00105	330
0.6	0.13	0.27	0.1	0.00354	0.00103	343

0.6	0.13	0.27	0.2	0.00357	0.00101	352
0.6	0.13	0.27	0.3	0.00361	0.00100	362
0.6	0.13	0.27	0.1	0.00354	0.00103	343
0.6	0.13	0.27	0.2	0.00357	0.00101	352
0.9	0.03	0.07	0.3	0.00329	0.00113	291
0.9	0.03	0.07	0.1	0.00329	0.00115	287
0.9	0.03	0.07	0.2	0.00329	0.00114	289
0.9	0.03	0.07	0.3	0.00329	0.00113	291
0.9	0.03	0.07	0.1	0.00329	0.00115	287
0.8	0.07	0.13	0.2	0.00335	0.00111	303
0.8	0.07	0.13	0.3	0.00336	0.00110	307
0.8	0.07	0.13	0.1	0.00334	0.00112	299
0.8	0.07	0.13	0.2	0.00335	0.00111	303
0.8	0.07	0.13	0.3	0.00336	0.00110	307
0.7	0.10	0.20	0.1	0.00342	0.00108	317
0.7	0.10	0.20	0.2	0.00344	0.00107	323
0.7	0.10	0.20	0.3	0.00346	0.00105	330
0.7	0.10	0.20	0.1	0.00342	0.00108	317
0.7	0.10	0.20	0.2	0.00344	0.00107	323
0.6	0.13	0.27	0.3	0.00361	0.00100	362
0.6	0.13	0.27	0.1	0.00354	0.00103	343
0.6	0.13	0.27	0.2	0.00357	0.00101	352
0.6	0.13	0.27	0.3	0.00361	0.00100	362
0.6	0.13	0.27	0.1	0.00354	0.00103	343
0.9	0.03	0.07	0.2	0.00329	0.00114	289
0.9	0.03	0.07	0.3	0.00329	0.00113	291
0.9	0.03	0.07	0.1	0.00329	0.00115	287
0.9	0.03	0.07	0.2	0.00329	0.00114	289
0.9	0.03	0.07	0.3	0.00329	0.00113	291
0.8	0.07	0.13	0.1	0.00334	0.00112	299
0.8	0.07	0.13	0.2	0.00335	0.00111	303
0.8	0.07	0.13	0.3	0.00336	0.00110	307
0.8	0.07	0.13	0.1	0.00334	0.00112	299
0.8	0.07	0.13	0.2	0.00335	0.00111	303
0.7	0.10	0.20	0.3	0.00346	0.00105	330
0.7	0.10	0.20	0.1	0.00342	0.00108	317
0.7	0.10	0.20	0.2	0.00344	0.00107	323
0.7	0.10	0.20	0.3	0.00346	0.00105	330
0.7	0.10	0.20	0.1	0.00342	0.00108	317
0.6	0.13	0.27	0.2	0.00357	0.00101	352
0.6	0.13	0.27	0.3	0.00361	0.00100	362
0.6	0.13	0.27	0.1	0.00354	0.00103	343
0.6	0.13	0.27	0.2	0.00357	0.00101	352
0.6	0.13	0.27	0.3	0.00361	0.00100	362

Table 7: *Simulation results of proposed estimator* $\hat{\pi}_{k2}$, *when trials =10000,* $n_1=n_2=500$, *T=0.2 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k2})$	RE_2
0.9	0.03	0.07	0.1	0.00304	0.00115	265
0.9	0.03	0.07	0.2	0.00305	0.00114	267
0.9	0.03	0.07	0.3	0.00305	0.00114	268
0.9	0.03	0.07	0.1	0.00304	0.00115	265
0.9	0.03	0.07	0.2	0.00305	0.00114	267
0.8	0.07	0.13	0.3	0.00310	0.00110	281
0.8	0.07	0.13	0.1	0.00308	0.00112	274
0.8	0.07	0.13	0.2	0.00309	0.00111	278
0.8	0.07	0.13	0.3	0.00310	0.00110	281
0.8	0.07	0.13	0.1	0.00308	0.00112	274
0.7	0.10	0.20	0.2	0.00315	0.00107	294
0.7	0.10	0.20	0.3	0.00317	0.00106	299
0.7	0.10	0.20	0.1	0.00313	0.00109	289
0.7	0.10	0.20	0.2	0.00315	0.00107	294
0.7	0.10	0.20	0.3	0.00317	0.00106	299
0.6	0.13	0.27	0.1	0.00321	0.00104	309
0.6	0.13	0.27	0.2	0.00324	0.00102	316
0.6	0.13	0.27	0.3	0.00327	0.00101	324
0.6	0.13	0.27	0.1	0.00321	0.00104	309
0.6	0.13	0.27	0.2	0.00324	0.00102	316
0.9	0.03	0.07	0.3	0.00305	0.00114	268
0.9	0.03	0.07	0.1	0.00304	0.00115	265
0.9	0.03	0.07	0.2	0.00305	0.00114	267
0.9	0.03	0.07	0.3	0.00305	0.00114	268
0.9	0.03	0.07	0.1	0.00304	0.00115	265
0.8	0.07	0.13	0.2	0.00309	0.00111	278
0.8	0.07	0.13	0.3	0.00310	0.00110	281
0.8	0.07	0.13	0.1	0.00308	0.00112	274
0.8	0.07	0.13	0.2	0.00309	0.00111	278
0.8	0.07	0.13	0.3	0.00310	0.00110	281
0.7	0.10	0.20	0.1	0.00313	0.00109	289
0.7	0.10	0.20	0.2	0.00315	0.00107	294
0.7	0.10	0.20	0.3	0.00317	0.00106	299
0.7	0.10	0.20	0.1	0.00313	0.00109	289
0.7	0.10	0.20	0.2	0.00315	0.00107	294
0.6	0.13	0.27	0.3	0.00327	0.00101	324
0.6	0.13	0.27	0.1	0.00321	0.00104	309
0.6	0.13	0.27	0.2	0.00324	0.00102	316
0.6	0.13	0.27	0.3	0.00327	0.00101	324
0.6	0.13	0.27	0.1	0.00321	0.00104	309
0.9	0.03	0.07	0.2	0.00305	0.00114	267
0.9	0.03	0.07	0.3	0.00305	0.00114	268
0.9	0.03	0.07	0.1	0.00304	0.00115	265
0.9	0.03	0.07	0.2	0.00305	0.00114	267
0.9	0.03	0.07	0.3	0.00305	0.00114	268
0.8	0.07	0.13	0.1	0.00308	0.00112	274
0.8	0.07	0.13	0.2	0.00309	0.00111	278
0.8	0.07	0.13	0.3	0.00310	0.00110	281
0.8	0.07	0.13	0.1	0.00308	0.00112	274
0.8	0.07	0.13	0.2	0.00309	0.00111	278

0.7	0.10	0.20	0.3	0.00317	0.00106	299
0.7	0.10	0.20	0.1	0.00313	0.00109	289
0.7	0.10	0.20	0.2	0.00315	0.00107	294
0.7	0.10	0.20	0.3	0.00317	0.00106	299
0.7	0.10	0.20	0.1	0.00313	0.00109	289
0.6	0.13	0.27	0.2	0.00324	0.00102	316
0.6	0.13	0.27	0.3	0.00327	0.00101	324
0.6	0.13	0.27	0.1	0.00321	0.00104	309
0.6	0.13	0.27	0.2	0.00324	0.00102	316
0.6	0.13	0.27	0.3	0.00327	0.00101	324

Table 8: *Simulation results of proposed estimator* $\hat{\pi}_{k2}$*, when trials =10000,* $n_1=n_2=500$*, T=0.3 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k2})$	RE_2
0.9	0.03	0.07	0.1	0.00280	0.00115	244
0.9	0.03	0.07	0.2	0.00280	0.00114	245
0.9	0.03	0.07	0.3	0.00281	0.00114	247
0.9	0.03	0.07	0.1	0.00280	0.00115	244
0.9	0.03	0.07	0.2	0.00280	0.00114	245
0.8	0.07	0.13	0.3	0.00284	0.00111	256
0.8	0.07	0.13	0.1	0.00282	0.00113	251
0.8	0.07	0.13	0.2	0.00283	0.00112	254
0.8	0.07	0.13	0.3	0.00284	0.00111	256
0.8	0.07	0.13	0.1	0.00282	0.00113	251
0.7	0.10	0.20	0.2	0.00287	0.00108	266
0.7	0.10	0.20	0.3	0.00289	0.00107	270
0.7	0.10	0.20	0.1	0.00286	0.00109	262
0.7	0.10	0.20	0.2	0.00287	0.00108	266
0.7	0.10	0.20	0.3	0.00289	0.00107	270
0.6	0.13	0.27	0.1	0.00290	0.00105	277
0.6	0.13	0.27	0.2	0.00293	0.00104	283
0.6	0.13	0.27	0.3	0.00295	0.00102	289
0.6	0.13	0.27	0.1	0.00290	0.00105	277
0.6	0.13	0.27	0.2	0.00293	0.00104	283
0.9	0.03	0.07	0.3	0.00281	0.00114	247
0.9	0.03	0.07	0.1	0.00280	0.00115	244
0.9	0.03	0.07	0.2	0.00280	0.00114	245
0.9	0.03	0.07	0.3	0.00281	0.00114	247
0.9	0.03	0.07	0.1	0.00280	0.00115	244
0.8	0.07	0.13	0.2	0.00283	0.00112	254
0.8	0.07	0.13	0.3	0.00284	0.00111	256
0.8	0.07	0.13	0.1	0.00282	0.00113	251
0.8	0.07	0.13	0.2	0.00283	0.00112	254
0.8	0.07	0.13	0.3	0.00284	0.00111	256
0.7	0.10	0.20	0.1	0.00286	0.00109	262
0.7	0.10	0.20	0.2	0.00287	0.00108	266
0.7	0.10	0.20	0.3	0.00289	0.00107	270
0.7	0.10	0.20	0.1	0.00286	0.00109	262
0.7	0.10	0.20	0.2	0.00287	0.00108	266
0.6	0.13	0.27	0.3	0.00295	0.00102	289
0.6	0.13	0.27	0.1	0.00290	0.00105	277
0.6	0.13	0.27	0.2	0.00293	0.00104	283
0.6	0.13	0.27	0.3	0.00295	0.00102	289
0.6	0.13	0.27	0.1	0.00290	0.00105	277

0.9	0.03	0.07	0.2	0.00280	0.00114	245
0.9	0.03	0.07	0.3	0.00281	0.00114	247
0.9	0.03	0.07	0.1	0.00280	0.00115	244
0.9	0.03	0.07	0.2	0.00280	0.00114	245
0.9	0.03	0.07	0.3	0.00281	0.00114	247
0.8	0.07	0.13	0.1	0.00282	0.00113	251
0.8	0.07	0.13	0.2	0.00283	0.00112	254
0.8	0.07	0.13	0.3	0.00284	0.00111	256
0.8	0.07	0.13	0.1	0.00282	0.00113	251
0.8	0.07	0.13	0.2	0.00283	0.00112	254
0.7	0.10	0.20	0.3	0.00289	0.00107	270
0.7	0.10	0.20	0.1	0.00286	0.00109	262
0.7	0.10	0.20	0.2	0.00287	0.00108	266
0.7	0.10	0.20	0.3	0.00289	0.00107	270
0.7	0.10	0.20	0.1	0.00286	0.00109	262
0.6	0.13	0.27	0.2	0.00293	0.00104	283
0.6	0.13	0.27	0.3	0.00295	0.00102	289
0.6	0.13	0.27	0.1	0.00290	0.00105	277
0.6	0.13	0.27	0.2	0.00293	0.00104	283
0.6	0.13	0.27	0.3	0.00295	0.00102	289

Table 9: *Simulation results of proposed estimator* $\hat{\pi}_{k2}$*, when trials =10000,* $n_1=n_2=500$*, T=0.4 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k2})$	RE_2
0.9	0.03	0.07	0.1	0.00256	0.00115	223
0.9	0.03	0.07	0.2	0.00256	0.00115	224
0.9	0.03	0.07	0.3	0.00257	0.00114	225
0.9	0.03	0.07	0.1	0.00256	0.00115	223
0.9	0.03	0.07	0.2	0.00256	0.00115	224
0.8	0.07	0.13	0.3	0.00259	0.00111	232
0.8	0.07	0.13	0.1	0.00257	0.00113	228
0.8	0.07	0.13	0.2	0.00258	0.00112	230
0.8	0.07	0.13	0.3	0.00259	0.00111	232
0.8	0.07	0.13	0.1	0.00257	0.00113	228
0.7	0.10	0.20	0.2	0.00260	0.00109	239
0.7	0.10	0.20	0.3	0.00262	0.00108	242
0.7	0.10	0.20	0.1	0.00259	0.00110	236
0.7	0.10	0.20	0.2	0.00260	0.00109	239
0.7	0.10	0.20	0.3	0.00262	0.00108	242
0.6	0.13	0.27	0.1	0.00261	0.00106	246
0.6	0.13	0.27	0.2	0.00263	0.00105	251
0.6	0.13	0.27	0.3	0.00265	0.00104	256
0.6	0.13	0.27	0.1	0.00261	0.00106	246
0.6	0.13	0.27	0.2	0.00263	0.00105	251
0.9	0.03	0.07	0.3	0.00257	0.00114	225
0.9	0.03	0.07	0.1	0.00256	0.00115	223
0.9	0.03	0.07	0.2	0.00256	0.00115	224
0.9	0.03	0.07	0.3	0.00257	0.00114	225
0.9	0.03	0.07	0.1	0.00256	0.00115	223
0.8	0.07	0.13	0.2	0.00258	0.00112	230
0.8	0.07	0.13	0.3	0.00259	0.00111	232
0.8	0.07	0.13	0.1	0.00257	0.00113	228
0.8	0.07	0.13	0.2	0.00258	0.00112	230
0.8	0.07	0.13	0.3	0.00259	0.00111	232

0.7	0.10	0.20	0.1	0.00259	0.00110	236
0.7	0.10	0.20	0.2	0.00260	0.00109	239
0.7	0.10	0.20	0.3	0.00262	0.00108	242
0.7	0.10	0.20	0.1	0.00259	0.00110	236
0.7	0.10	0.20	0.2	0.00260	0.00109	239
0.6	0.13	0.27	0.3	0.00265	0.00104	256
0.6	0.13	0.27	0.1	0.00261	0.00106	246
0.6	0.13	0.27	0.2	0.00263	0.00105	251
0.6	0.13	0.27	0.3	0.00265	0.00104	256
0.6	0.13	0.27	0.1	0.00261	0.00106	246
0.9	0.03	0.07	0.2	0.00256	0.00115	224
0.9	0.03	0.07	0.3	0.00257	0.00114	225
0.9	0.03	0.07	0.1	0.00256	0.00115	223
0.9	0.03	0.07	0.2	0.00256	0.00115	224
0.9	0.03	0.07	0.3	0.00257	0.00114	225
0.8	0.07	0.13	0.1	0.00257	0.00113	228
0.8	0.07	0.13	0.2	0.00258	0.00112	230
0.8	0.07	0.13	0.3	0.00259	0.00111	232
0.8	0.07	0.13	0.1	0.00257	0.00113	228
0.8	0.07	0.13	0.2	0.00258	0.00112	230
0.7	0.10	0.20	0.3	0.00262	0.00108	242
0.7	0.10	0.20	0.1	0.00259	0.00110	236
0.7	0.10	0.20	0.2	0.00260	0.00109	239
0.7	0.10	0.20	0.3	0.00262	0.00108	242
0.7	0.10	0.20	0.1	0.00259	0.00110	236
0.6	0.13	0.27	0.2	0.00263	0.00105	251
0.6	0.13	0.27	0.3	0.00265	0.00104	256
0.6	0.13	0.27	0.1	0.00261	0.00106	246
0.6	0.13	0.27	0.2	0.00263	0.00105	251
0.6	0.13	0.27	0.3	0.00265	0.00104	256

Table 10: *Simulation results of proposed estimator* $\hat{\pi}_{k2}$, *when trials =10000,* $n_1=n_2=500$, *T=0.5 at various choices of* $W_1=0.7$, $W_2=0.3$, $\pi_{a1}=0.7$, $\pi_{a2}=0.3$, π_b, p_1, $p_2=(1-p_1)$ *and* $p_3=(1-p_1-p_2)$ *respectively.*

p_1	p_2	p_3	π_b	$V(\hat{\pi}_{p1})$	$V(\hat{\pi}_{k2})$	RE_2
0.9	0.03	0.07	0.1	0.00232	0.00115	202
0.9	0.03	0.07	0.2	0.00232	0.00115	203
0.9	0.03	0.07	0.3	0.00233	0.00114	203
0.9	0.03	0.07	0.1	0.00232	0.00115	202
0.9	0.03	0.07	0.2	0.00232	0.00115	203
0.8	0.07	0.13	0.3	0.00234	0.00112	209
0.8	0.07	0.13	0.1	0.00233	0.00113	205
0.8	0.07	0.13	0.2	0.00233	0.00113	207
0.8	0.07	0.13	0.3	0.00234	0.00112	209
0.8	0.07	0.13	0.1	0.00233	0.00113	205
0.7	0.10	0.20	0.2	0.00234	0.00110	213
0.7	0.10	0.20	0.3	0.00235	0.00109	216
0.7	0.10	0.20	0.1	0.00233	0.00111	211
0.7	0.10	0.20	0.2	0.00234	0.00110	213
0.7	0.10	0.20	0.3	0.00235	0.00109	216
0.6	0.13	0.27	0.1	0.00234	0.00107	218
0.6	0.13	0.27	0.2	0.00235	0.00106	221
0.6	0.13	0.27	0.3	0.00237	0.00105	225
0.6	0.13	0.27	0.1	0.00234	0.00107	218
0.6	0.13	0.27	0.2	0.00235	0.00106	221

0.9	0.03	0.07	0.3	0.00233	0.00114	203
0.9	0.03	0.07	0.1	0.00232	0.00115	202
0.9	0.03	0.07	0.2	0.00232	0.00115	203
0.9	0.03	0.07	0.3	0.00233	0.00114	203
0.9	0.03	0.07	0.1	0.00232	0.00115	202
0.8	0.07	0.13	0.2	0.00233	0.00113	207
0.8	0.07	0.13	0.3	0.00234	0.00112	209
0.8	0.07	0.13	0.1	0.00233	0.00113	205
0.8	0.07	0.13	0.2	0.00233	0.00113	207
0.8	0.07	0.13	0.3	0.00234	0.00112	209
0.7	0.10	0.20	0.1	0.00233	0.00111	211
0.7	0.10	0.20	0.2	0.00234	0.00110	213
0.7	0.10	0.20	0.3	0.00235	0.00109	216
0.7	0.10	0.20	0.1	0.00233	0.00111	211
0.7	0.10	0.20	0.2	0.00234	0.00110	213
0.6	0.13	0.27	0.3	0.00237	0.00105	225
0.6	0.13	0.27	0.1	0.00234	0.00107	218
0.6	0.13	0.27	0.2	0.00235	0.00106	221
0.6	0.13	0.27	0.3	0.00237	0.00105	225
0.6	0.13	0.27	0.1	0.00234	0.00107	218
0.9	0.03	0.07	0.2	0.00232	0.00115	203
0.9	0.03	0.07	0.3	0.00233	0.00114	203
0.9	0.03	0.07	0.1	0.00232	0.00115	202
0.9	0.03	0.07	0.2	0.00232	0.00115	203
0.9	0.03	0.07	0.3	0.00233	0.00114	203
0.8	0.07	0.13	0.1	0.00233	0.00113	205
0.8	0.07	0.13	0.2	0.00233	0.00113	207
0.8	0.07	0.13	0.3	0.00234	0.00112	209
0.8	0.07	0.13	0.1	0.00233	0.00113	205
0.8	0.07	0.13	0.2	0.00233	0.00113	207
0.7	0.10	0.20	0.3	0.00235	0.00109	216
0.7	0.10	0.20	0.1	0.00233	0.00111	211
0.7	0.10	0.20	0.2	0.00234	0.00110	213
0.7	0.10	0.20	0.3	0.00235	0.00109	216
0.7	0.10	0.20	0.1	0.00233	0.00111	211
0.6	0.13	0.27	0.2	0.00235	0.00106	221
0.6	0.13	0.27	0.3	0.00237	0.00105	225
0.6	0.13	0.27	0.1	0.00234	0.00107	218
0.6	0.13	0.27	0.2	0.00235	0.00106	221
0.6	0.13	0.27	0.3	0.00237	0.00105	225

8 Conclusions

In the realm of survey methodologies, particularly when dealing with sensitive or stigmatized issues, the choice of statistical models can significantly impact the accuracy and reliability of the data collected. One such model that has shown promising results is the proposed stratified randomized response model. This model stands out for its ability to offer substantial advantages in terms of percent relative efficiencies when compared to existing randomized response models. In this discussion, we will delve into the reasons why the stratified randomized response model is recommended with enthusiasm to survey practitioners, especially in the context of addressing stigmatized issues in real-life scenarios.

Conflicts of Interest Statement

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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