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The Effects of Fractional Derivatives of Bio-Heat Model in Living Tissues using Analytical-Numerical Method

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Abstract: In this article, the fractional order bioheat model of heat transfer is used to offer a new interpretation to investigate the thermal damage in living tissues under laser irradiations. The effects of thermal relaxation time and the fractional order parameter on the temperature of living tissues and the thermal damages are studied. The basic equation is formulated in the domain of Laplace transform to get the analytical solution. The numerical solution is obtained by using the implicit finite difference method (IFDM). The efficiency and accuracy of this method have been verified using numerical example. Considering the thermophysical properties of living tissue, the effects of fractional time parameter and the thermal relaxation time on the temperature distributions in living tissues were analyzed and presented graphically.

Keywords: Fractional derivative; finite difference method; living tissue; thermal damage; Laplace transforms.

1 Introduction

A human body behaves under various environmental conditions of humidity, air temperature and wind speed. Recently studies development show that the thermal transfer problems in living tissue becomes one of complicated issues but there are several discussions a finding in this field. In the modernistic medicine, the methods of heat treatments have been used like laser surgery [1], laser tissues soldering [2] and hyperthermia [3]. Therefore, a study on the treatment of tumors is urgently needed to save human lives around the world. Many techniques are adopted to treat the tumors, among which laser treatments are considered one of the most prospective technologies.

A thorough understanding of mechanical properties and thermal behaviors of living tissue are beneficial in improving treatments efficiencies and preventing thermal damages through laser tissues interactions. Pioneering work dates back to Pennes [4] to analyze the relationships between arterials blood and tissues temperature, in which a thermal transfer equations are presented and have been widely used to describe the thermal behaviors of living tissue in the treatments. Recently, fractional calculus have found very successful application to solve real-world problems, such as heat transfer, fluid flow and viscoelasticity. Several published researches have proven that mathematical theories with fractional derivative can be used to describe the process of heat transfer in different media. Ezzat *et. al.* [5,6], have presented the fractional thermal waves models overtime for the Pennes bioheat equations to study thermal behaviors in biological tissue caused by instantaneous surface heating Patra *et. al.* [7] applied the finite difference method to investigate the computational models on thermoelasticity analysis under the magnetic field in a rotating cylinder.

Mukhopadhyay and Kumar [8] studied the generalized thermoelastic problem of hollo cylinder by using the finite difference method. Abd-Alla *et. al.* [9] applied the finite difference method to study the thermoelastic interaction in annular cylinders. Abd-Alla *et. al.* [10] have used by the implicit finite-difference technique to investigate the solution of the transient coupled thermoelasticity of a hollo fins. Abd-Alla *et. al.* [11] discussed the impacts of magnetic field in an isotropic non-homogeneous cylinder. Zhao *et. al.* [12] investigated a two level finite difference method for one dimensional Pennes' bioheat model. Singh *et. al.* [13] studied the solution of fractional bioheat equation by finite difference approach and homotopy perturbation method. Patil *et. al.* [14] used the finite difference scheme

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based analysis of bioheat transfers in human breast cyst. Damor *et. al.* [15] studied the numerical solutions of a fractional bio-heat model with constant and sinusoidal heat flux condition on skin tissues Hobiny and Abbas[16] have studied the analysis of thermal damages in skin tissues subject to moving heating source. Finite-decomposition method is used to get the solutions of the heating transfer problems in tissue during hyperthermia as in Gupta *et. al.* [17]. The Homotopy perturbation method are presented to studied the numerical simulation for heating transfer in tissue during thermal therapy as in Gupta *et. al.* [18]. Esneault and Dillenseger [19] investigated the temperature increment over time in hypothermia by the finite difference method. Analytical solution is very interesting because of its lower outlay and exact evaluations than experimental and numerical computations. By using the finite element scheme, Diaz *et. al.* [20] studied the solution of thermo-diffusion model in the tissues to investigate the resulting of thermal damages. Saeed and Abbas [21] applied the finite element approach to study the nonlinear dual-phase lag bioheat model in spherical tissue. When a real phenomenon regards to heat transfers in finite mediums are investigated, the linear/nonlinear models of heating transfer has been expanded and their numerical or analytical solutions have been solved by many authors [22,23,24,25,26,27,28,29,30,31,32]. The present work is devoted to study the effects of fractional time parameter and the thermal relaxation time on the temperature distribution and the thermal damages of biological tissues by the bioheat model in living tissues. The numerical results can be used as a substantiation division for living tissue interaction such as continuous scanning laser interaction.

2 Mathematical model

In this work, we consider a semi-infinite living tissue with the thickness of $L = 0.3cm$. Based on Cattaneo [33] and Ezzat *et. al.* [5], the fractional bioheat formulations in living tissues can be given as

$$k\nabla^2 T = \left(1+\frac{\tau_0^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c\frac{\partial T}{\partial t}+\omega_b\rho_b c_b(T-T_b)-Q_m-Q_{ext}\right), 0<\alpha\le 1, \tag{1}$$

Taking into account the definition can be expressed by

$$\frac{\partial^\alpha h(r,t)}{\partial t^\alpha} = \begin{cases} h(r,t)-h(r,0), & \alpha\to 0,\\ I^{\alpha-1}\frac{\partial h(r,t)}{\partial t}, & 0<\alpha<1,\\ \frac{\partial h(r,t)}{\partial t}, & \alpha=1 \end{cases} \tag{2}$$

$$I^\nu h(r,t) = \int_0^t \frac{(t-s)^\nu}{\Gamma(\nu)} h(r,s)ds, \quad \nu > 0 \tag{3}$$

$$\lim_{\nu\to 1}\frac{\partial^\nu h(r,t)}{\partial t^\nu} = \frac{\partial h(r,t)}{\partial t}. \tag{4}$$

The full spectrum of local thermal conduction is described by standard thermal conduction to ballistic thermal conduction as shown in equation (2). The different values of fractional parameter $0<\alpha\le 1$ cover two types of conductivity, $\alpha = 1$ for normal conductivity while $0<\alpha<1$ for low conductivity, $\omega_b$is the rate of blood perfusions, t is the time, T is the tissues temperature, c is the specific heat of tissues, k is the thermal conductive of tissues, ρ is the tissues mass density, ρ_b is the blood mass density, T_b is the blood temperature, c_b is the blood specific heat, τ_0 is the thermal relaxation time, Q_m is the metabolic heating generations in living tissue and Q_{ext} refer to the heat generated per unit volume of tissue. Gardner *et. al.* [34] supposed the laser heat sources as follow

$$Q_{ext}(x,t) = I_0\mu_a[U(t)-U(t-\tau_p)][C_1 e^{-\frac{k_1}{\delta}x}-C_2 e^{-\frac{k_2}{\delta}x}], \tag{5}$$

where μ_a is the absorption coefficient, $U(t)$ is the step function, δ is penetration depth, τ_p is the laser exposure time, I_0 is the laser intensity and C_1, C_2, k_1 and k_2 are the functions of diffuse reflectance R_d and they are mentioned as in [34]. The penetrations depth can be given by [34]:

$$\delta = \frac{1}{\sqrt{3(\mu_s(1-g)+\mu_a)\mu_a}}, \tag{6}$$

where g is the anisotropy factor and μ_s is the scattering coefficient. Now, the boundary conditions and initial condition are expressed as

$$T(0,t)=0,\ T(L,t)=0, \tag{7}$$

$$T(x,0)=T_b,\ \frac{\partial T(x,0)}{\partial t}=0 \tag{8}$$

For appropriateness, the dimensionless forms are defined as

$$\begin{aligned} T' &= \frac{T-T_0}{T_0},\ T_b' = \frac{T_b-T_0}{T_0},\\ (t',\tau_0',\tau_b') &= \frac{k}{\rho c L^2}(t,\tau_0,\tau_b)\\ (k_1',k_2') &= \frac{L}{\delta}(k_1,k_2),\ R_b = \frac{\omega_b\rho_b c_b L^2}{k},\\ R_m &= \frac{L^2 Q_m}{kT_0},\ R_r = \frac{L^2 I_0\mu_a}{kT_0}. \end{aligned} \tag{9}$$

In the dimensionless forms of parameters in (9), the governing formulation (1) with the boundary (7) and the initial conditions (8) can be written by (for appropriateness, the dash has been neglected)

$$\frac{\partial^2 T}{\partial x^2} = \left(1+\frac{\tau_0^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\partial T}{\partial t}-R_b(T_b-T)-R_m-R_r f(x,t)\right), \tag{10}$$

$$T(0,t)=0,\ T(L,t)=0, \tag{11}$$

$$T(x,0) = T_{b,} \frac{\partial T(x,0)}{\partial t} = 0 \quad (12)$$

where

$$f(x,t) = [U(t) - U(t-\tau_p)][C_1 e^{-k_1 x} - C_2 e^{-k_2 x}].$$

3 Analytical method

Applying the Laplace transforms for equations (10)-(12), which are defined by the formula

$$\overline{M}(x,s) = L[M(x,t)] = \int_0^\infty M(x,t) e^{-st} dt, \ \ s > 0, \quad (13)$$

$$\frac{\partial^2 \overline{T}}{\partial x^2} - f_1 \overline{T} = -f_2 - f_3 e^{-k_1 x} - f_4 e^{-k_2 x}, \quad (14)$$

$$\frac{\partial \overline{T}(0,t)}{\partial x} = 0, \ \frac{\partial \overline{T}(L,t)}{\partial x} = 0, \quad (15)$$

where s is the parameter of Laplace transforms, $f_1 = 1 + \frac{s^\alpha \tau_0^\alpha}{\Gamma(\alpha+1)}(s+R_b)$, $f_2 = \frac{1}{s}(R_b T_b + R_m)$, $f_3 = \frac{R_r C_1}{s}(1-e^{-s\tau_b})$, $f_4 = \frac{-R_r C_2}{s}(1-e^{-s\tau_b})$

The exact solution of equation (14) can be given as

$$\overline{T}(x,s) = \frac{f_2}{f_1} + A_1 e^{\sqrt{f_1}x} + A_2 e^{-\sqrt{f_1}x} + \frac{f_3}{f_1 - k_1^2} e^{-k_1 x} + \frac{f_4}{f_1 - k_1^2} e^{-k_2 x}. \quad (16)$$

where $A_1$and A_2 are constants. For finally solution of temperature distribution, a numerically reversal method was adopted depending on Stehfest[35]. In this method, the inverse $M(x,t)$ of the Laplace transform $\overline{M}(x,s)$ is approximated by the relation

$$M(x,t) = \frac{\ln 2}{t} \sum_{j=1}^{M} V_j \overline{M}(\chi, j\frac{\ln 2}{t}), \quad (17)$$

whereV_j is given by the following equation:

$$V_j = (-1)^{\frac{n}{2}+1} \sum_{k=\frac{i+1}{2}}^{\min(i,\frac{n}{2})} \frac{k^{\frac{n}{2}+1}(2k)!}{(\frac{n}{2}-k)!k!(i-k)!(2k-1)!} \quad (18)$$

Numerical scheme

The linear partial differential governing equation are obtained. For the numerical solution, the implicit finite differences methods (IFDM) are used. The domain of solution $0 \le x \le x_f$, $0 \le t \le t_f$, are replaced by grids described by the set of nodes points (x_m, t_s), in which $x_m = mh, m = 0,1,2,\ldots.,M$ and$t_s = sk, s = 0,1,2,\ldots.,S$. Therefore,$k = \frac{t_f}{s}, h = \frac{x_f}{M}$ are expressed as the times steps and mess width respectively. For the time derivative and the space derivative, the derivative is replaced the central difference. Thus, the approximations of finite difference scheme for the system of partial differential equations with respect to the independent variables:

$$\frac{\partial f}{\partial t} = \frac{f_m^{s+1} - f_m^{s-1}}{2k} + o(k^2), \frac{\partial^2 f}{\partial t^2} = \frac{f_m^{s+1} - 2f_m^s + f_m^{s-1}}{2k} + o(k^2), \quad (19)$$

$$\frac{\partial f}{\partial x} = \frac{f_{m+1}^{s+1} - f_{m-1}^{s+1}}{2h} + o(h^2), \frac{\partial^2 f}{\partial x^2} = \frac{f_{m+1}^{s+1} - 2f_m^{s+1} + f_{m-1}^{s+1}}{2k} + o(h^2) \quad (20)$$

$$\frac{\partial^\alpha f}{\partial t^\alpha} = \frac{k^{-\alpha}}{\Gamma(2-\alpha)} \sum_{i=0}^{s-1} b_i^{1-\alpha}[f_m^{s-i} - f_m^{s-i-1}], 0 < \alpha \le 1 \quad (21)$$

$$\frac{\partial^\gamma f}{\partial t^\gamma} = \frac{k^{-\gamma}}{\Gamma(3-\gamma)} \sum_{i=0}^{s-1} b_i^{2-\gamma}[f_m^{s-i} - 2f_m^{s-i-1} + f_m^{s-i-2}], 1 < \gamma \le 2 \quad (22)$$

where $b_i^{1-\alpha} = (i+1)^{1-\alpha} - (i)^{1-\alpha}, \gamma = \alpha + 1$
The equation (10) is then replaced by the implicit finite difference equations by

$$\begin{aligned} &\frac{T_{m+1}^{s+1} - 2T_m^{s+1} + T_{m-1}^{s+1}}{h^2} \\ &- \frac{k^{-\alpha}}{\Gamma(2-\alpha)} \sum_{i=0}^{s-1} b_i^{1-\alpha}[f_m^{s-i} - f_m^{s-i-1}] - T_m^{s+1} \\ &+ R_m + R_r f - \frac{\tau_0^\alpha}{\Gamma(\alpha+1)} (\frac{k^{-\gamma}}{\Gamma(3-\gamma)} \\ &\times \sum_{i=0}^{s-1} b_i^{2-\gamma}[f_m^{s-i} - 2f_m^{s-i-1} + f_m^{s-i-2}] \\ &+ R_b \frac{k^{-\alpha}}{\Gamma(2-\alpha)} \sum_{i=0}^{s-1} b_i^{1-\alpha}[f_m^{s-i} - f_m^{s-i-1}]) = 0 \end{aligned} \quad (23)$$

4 Evaluation of thermal damages

Burn assessment is one of the most important attributes of skin tissue bioengineering science. To quantify thermal damage, the method improved by Moritz and Henriques [36,37] can be used:

$$\Omega = \int_0^t B e^{-\frac{E_a}{RT}} dt, \quad (24)$$

where $B = 3.1 \times 10\hat{}98 s^{-1}$ is the frequency factor,$E_a = 6.28 \times 10\hat{}5 J/mol$ is the activation energy and$R = 8.313 J/mol{\cdot}K$ is the constant of universal gas.

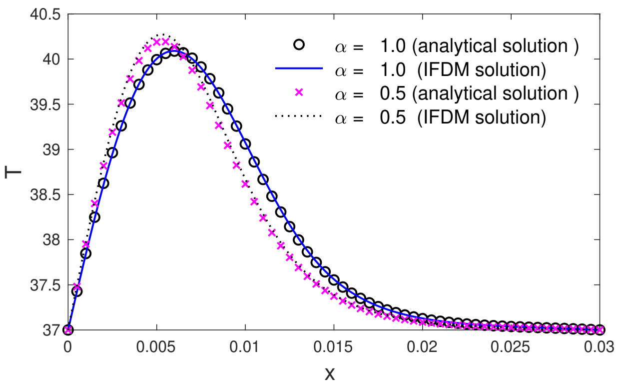

Fig. 1: The variation of temperature via the distance with and without the time fractional derivative.

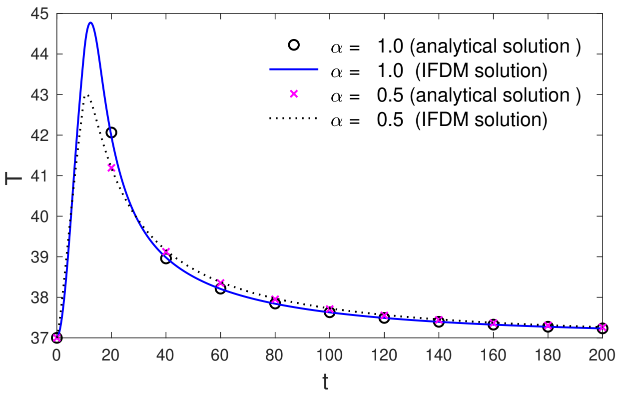

Fig. 2: The variation of temperature through the time with and without the time fractional derivative.

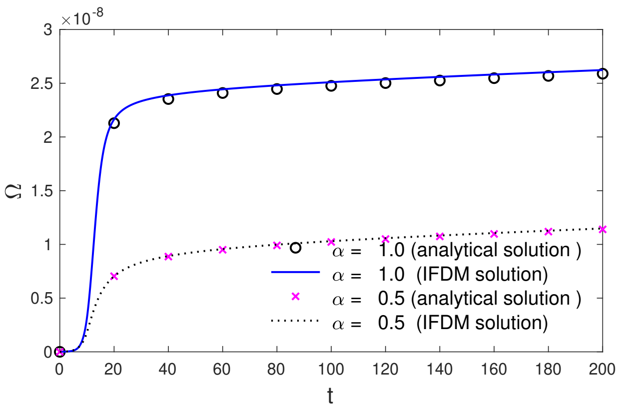

Fig. 3: The variation of thermal damages through the time with and without the time fractional derivative.

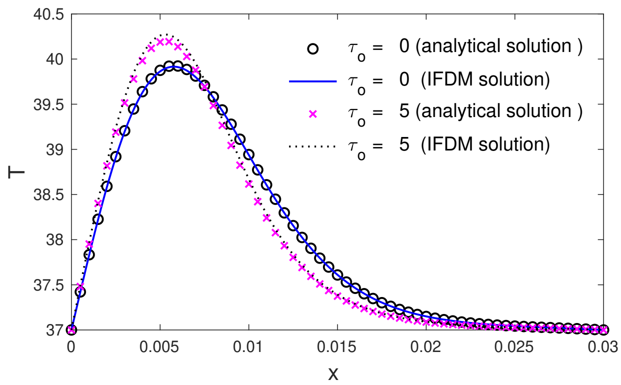

Fig. 4: The variation of temperature via the distance with and without the thermal relaxation time.

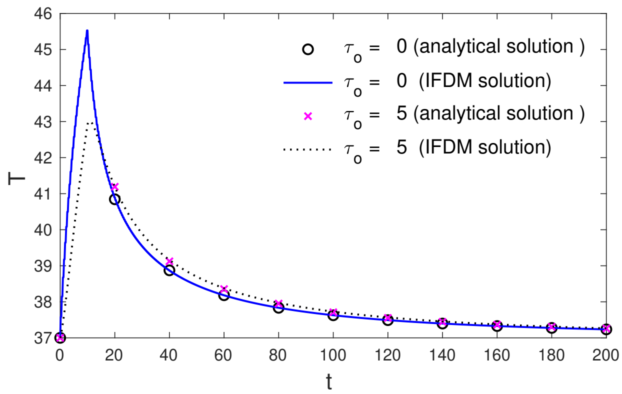

Fig. 5: The variation of temperature through the time with and without the thermal relaxation time.

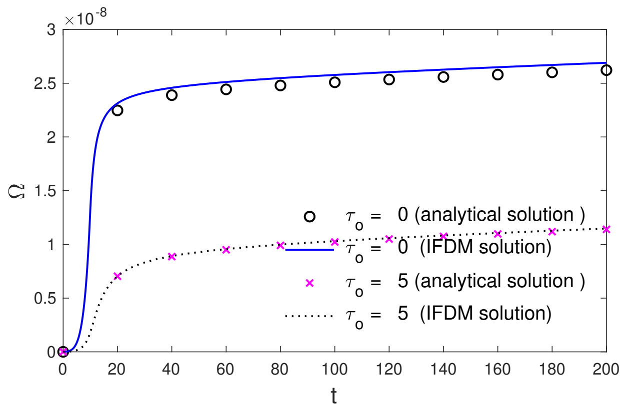

Fig. 6: The variation of thermal damages through the time with and without the thermal relaxation time.

5 Results and Discussion

Several simulations were carried out to test the performance of the proposed nonlinear thermal model based on the hyperbolic bio-heat transfer. For numerical calculations, examples of thermal property values for skin tissues have been written [38]

ρ_b=$1060(kg)(m^{-3}), c_b = 3860(J)(kg^{-1})(k^{-1}),$
$\omega_b = 1.87\times10^{-3}(s^{-1}), T_b = 37°\text{C}, \tau_p = 10(s),$
$Q_m = 1.19\times10^3(W)(m^{-3}),$
$I_o = 122\times10^3(W)(m^{-2}), L = 0.03(m), g = 0.9,$
$\rho = 1000(kg)(m^{-3}), c = 4187(J)(kg^{-1})(k^{-1}),$
$k = 0.628(W)(m^{-1})(k^{-1}), \tau_o = 5(s),$
$T_o = 37°\text{C}, \mu_s = 12000(m^{-1}), \mu_a = 40(m^{-1}).$

This mathematical model which is based on hyperbolic Pennes bioheat transfer has been founded with the interface and suitable boundary conditions. The perfusion, metabolic and conducting heat resource terms have been used in the formulations[39,40,41,42]. A slab of tissues is $3cm$ thick and its reference temperature equal to normal temperature i.e. $T_b = T_o = 37°C$. For the aim of studying the impacts of fractional parameter α and the thermal relaxation time τ_o on the temperature and the resulting of thermal damage. the numerical outcomes have been presented as in figures 1-6. Figure 1 shows the variations of temperature with respect to the distance x at$t = 80$ second when the thermal relaxation time$\tau_o = 5s$ and the laser exposure$\tau_p = 10s$ with $(\alpha = 0.5)$ and without $(\alpha = 1)$ fractional time derivative. It is clear from the graph that the temperature starts from maximum values, after that it reduces continuously to the normal temperature $T_b = 37°C$. Figure 2 exabits the time histories of the temperature with $(\alpha = 0.5)$ and without$(\alpha = 1)$ fractional time derivative. It is notices that the temperature begins from the normal temperature T_b and rises with the time till maxmum values after that reduces to the normal temperature again. Figure 3 show the resulting of thermal damages through the time t. It is observed that the time histories of the thermal damage have been obtained with$(\alpha = 0.5)$ and without$(\alpha = 1)$ fractional time derivative. As expect, the fractional time derivative has major effect on the distributions on the temperature and the thermal damages. Figures 4, 5 and 6 show the effect of thermal relaxation time $\tau_o$with fractional bioheat model $(\alpha = 0.5)$ on the the temperature distributions and the resulting of thermal damage. As expect, thermal relaxation time τ_o has great effects on the variations of temperature and the resulting of thermal damage. Otherwise, Figures 1-6 illustrate the solutions obtained numerically by the implicit finite differences methods (IFDM) superimposed on the solution obtained analytically. The precision of the Implicit Finite Differences Methods (IFDM) formulations was validated by comparing the numerical solution and the analytical solution for the field quantities

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References

[1] J. Zhou, J. Chen, and Y. Zhang, Dual-phase lag effects on thermal damage to biological tissues caused by laser irradiation. *Computers in Biology and Medicine*,**39**,pp. 286-293 (2009).
[2] I. Gabay, et al.,Temperature-controlled two-wavelength laser soldering of tissues. *Lasers in surgery and medicine*, **43**,pp. 907-913 (2011).
[3] S. Mahjoob,. and K. Vafai, Analytical characterization of heat transport through biological media incorporating hyperthermia treatment. *International Journal of Heat and Mass Transfe*r, **52**, pp.1608-1618 (2009).
[4] H. Pennes, Analysis of tissue and arterial blood temperatures in the resting human forearm. *Journal of applied physiology*,**2**,pp. 93-122 (1948).
[5] M.A. Ezzat, et al., Fractional modelling of Pennes' bioheat transfer equation. *Heat and Mass Transfer*,. **50**, pp 907-914 (2014).
[6] M.A. Ezzat, A.A. El-bary, and N.S. Al-sowayan, Tissue responses to fractional transient heating with sinusoidal heat flux condition on skin surface. *Animal Science Journal*,**87**,pp 1304-1311 (2016).
[7] S. Patra, G. Shit, and B. Das, Computational model on magnetothermoelastic analysis of a rotating cylinder using finite difference method. *Waves in Random and Complex Media*, :pp. 1-18 (2020).
[8] S. Mukhopadhyay, and R. Kumar, Solution of a problem of generalized thermoelasticity of an annular cylinder with variable material properties by finite difference method. *Computational Methods in Science and Technology*,**15**,pp. 169-176 (2009).
[9] A.M. Abd-Alla, S.M. Abo-Dahab, and A.A. Kilany, Finite difference technique to solve a problem of generalized thermoelasticity on an annular cylinder under the effect of rotation. *Numerical Methods for Partial Differential Equations*.
[10] A. Abd-Alla, et al., An implicit finite-difference method for solving the transient coupled thermoelasticity of an annular fin. *Appl. Math. Inf. Sci*, **1**,pp. 79-93 (2007).
[11] A. Abd-Alla, A. El-Naggar, and M. Fahmy, Magneto-thermoelastic problem in non-homogeneous isotropic cylinder. *Heat and Mass transfer*,**39**,pp. 625-629 (2003).

[12] J.J. Zhao, et al., A two level finite difference scheme for one dimensional Pennes' bioheat equation. *Applied Mathematics and Computation*,. **171,**pp. 320-331 (2005).
[13] J. Singh, P.K. Gupta, and K. Rai, Solution of fractional bioheat equations by finite difference method and HPM. *Mathematical and Computer Modelling*, **54**,pp. 2316-2325 (2011).
[14] H.M. Patil,.and R. Maniyeri, Finite difference method based analysis of bio-heat transfer in human breast cyst. *Thermal Science and Engineering Progress*,**10**,pp. 42-47 (2019).
[15] R. Damor, S. Kumar, and A. Shukla, Numerical solution of fractional bioheat equation with constant and sinusoidal heat flux condition on skin tissue. *American Journal of Mathematical Analysis*, **1**,pp 20-24 (2013).
[16] A.D. Hobiny, and I.A. Abbas, Theoretical analysis of thermal damages in skin tissue induced by intense moving heat source. *International Journal of Heat and Mass Transfer*,**124**,pp. 1011-1014 (2018).
[17] P.K. Gupta, et al., Solution of the heat transfer problem in tissues during hyperthermia by finite difference–decomposition method. *Applied Mathematics and Computation*,**219,p**p 6882-6892 (2013).
[18] P.K. Gupta, J. Singh, and K. Rai, Numerical simulation for heat transfer in tissues during thermal therapy. *Journal of Thermal Biology*,**35,**pp. 295-301 (2010).
[19] J. Dillenseger, and S. Esneault, Fast FFT-based bioheat transfer equation computation. *Computers in Biology and Medicine*, **40,**pp. 119-123 (2010).
[20] S.H. Diaz, J.S. Nelson, and B.J. Wong, Rate process analysis of thermal damage in cartilage. *Physics in medicine and biology*, **48,**pp. 19 (2002).
[21] T. Saeed, and I. Abbas, Finite element analyses of nonlinear DPL bioheat model in spherical tissues using experimental data. *Mechanics Based Design of Structures and Machines*, p. 1-11 (2020).
[22] A.M. Zenkour, and I.A. Abbas, A generalized thermoelasticity problem of an annular cylinder with temperature-dependent density and material properties. *International Journal of Mechanical Sciences*, **84**,pp. 54-60 (2014).
[23] I.A. Abbas, The effects of relaxation times and a moving heat source on a two-temperature generalized thermoelastic thin slim strip. *Canadian Journal of Physics*,**93**,pp. 585-590 (2014).
[24] I.A. Abbas, and H.M. Youssef, Finite element analysis of two-temperature generalized magneto-thermoelasticity. *Archive of Applied Mechanics*, **79**,pp. 917-925 (2009).
[25] S. Mondal, P. Pal, and M. Kanoria, Transient response in a thermoelastic half-space solid due to a laser pulse under three theories with memory-dependent derivative. *Acta Mechanica*, **230**,pp. 179-199 (2019).
[26] T. Saeed, I. Abbas, and M. Marin, A gl model on thermo-elastic interaction in a poroelastic material using finite element method. *Symmetr*y, **12**,pp. 488 (2020).
[27] M. Marin, An evolutionary equation in thermoelasticity of dipolar bodies. *Journal of Mathematical Physics*,**40**,pp. 1391-1399 (1999).
[28] A. El-Naggar, et al., On the initial stress, magnetic field, voids and rotation effects on plane waves in generalized thermoelasticity. *Journal of Computational and Theoretical Nanoscience*, **10**,pp. 1408-1417 (2013).
[29] G. Palani, and I. Abbas, Free convection MHD flow with thermal radiation from an impulsively-started vertical plate. *Nonlinear Analysis: Modelling and Control*,**14**,pp. 73-84 (2009).
[30] A.M. Zenkour, and I.A. Abbas, Magneto-thermoelastic response of an infinite functionally graded cylinder using the finite element method. *Journal of Vibration and Control*,**20**,pp. 1907-1919 (2014).
[31] A.D. Hobiny, and I.A. Abbas, A study on photothermal waves in an unbounded semiconductor medium with cylindrical cavity. *Mechanics of Time-Dependent Materials*, **21**,pp. 61-72 (2017).
[32] I.A. Abbas,.and M. Marin, Analytical solutions of a two-dimensional generalized thermoelastic diffusions problem due to laser pulse. *Iranian Journal of Science and Technology*, *Transactions of Mechanical Engineering*, **42**,pp. 57-71 (2018).
[33] C. Cattaneo, A form of heat conduction equation which eliminates the paradox of instantaneous propagation. *Compte Rendus*, **247**,pp. 431-433 (1958).
[34] C.M. Gardner, S.L. Jacques, and A. Welch, Light transport in tissue: Accurate expressions for one-dimensional fluence rate and escape function based upon Monte Carlo simulation. Lasers in Surgery and Medicine: *The Official Journal of the American Society for Laser Medicine and Surgery*, **18,**pp. 129-138 (1996).
[35] H. Stehfest, Algorithm 368: Numerical inversion of Laplace transforms [D5]. *Communications of the ACM*, **13,**pp. 47-49 (1970).
[36] J. Henriques, and A. Moritz, Studies of thermal injury: I. The conduction of heat to and through skin and the temperatures attained therein. A theoretical and an experimental investigation. *The American journal of pathology*, **23**,pp. 530 (1947).
[37] A.R. Moritz, and F. Henriques, Studies of thermal injury: II. The relative importance of time and surface temperature in the causation of cutaneous burns. *The American journal of pathology*, **23**,pp. 695 (1947).
[38] H. Askarizadeh, and H. Ahmadikia, Analytical analysis of the dual-phase-lag model of bioheat transfer equation during transient heating of skin tissue. *Heat and Mass Transfer*, **50,p**p. 1673-1684 (2014).
[39] D. Vernardou,A. Kazas,M. Apostolopoulou,N. E. Katsarakis,E. Koudoumas,Hydrothermal Growth of MnO2 at 95 oC as an Anode Material,*Int. J. of Thin Film Science and Technology* **5**, pp. 121-127 (2016).
[40] M. S Al-Qrinawi,T. M El-Agez,M. S Abdel-Latif,S. A Taya,Capacitance-voltage measurements of hetero-layer OLEDs treated by an electric field and thermal

annealing,*Int. J. of Thin Film Science and Technology* **10**, pp. 217-226 (2021).

[41] A. A Elhadary,A. El-Zein,M. Talaat,G. El-Aragi,A. El-Amawy,Studying The Effect of The Dielectric Barrier Discharge Non- thermal Plasma on Colon Cancer Cell line,*Int. J. of Thin Film Science and Technology* **10**, pp. 161-168 (2021).

[42] M. Shapaan,DC Conductivity,Thermal Stability and Crystallization Kinetics of the Semiconducting 30P2O5 (50-x)V2O5 xB2O3 20Fe2O3 Oxide Glasses,*Int. J. of Thin Film Science and Technology* **5**, pp. 143-153 (2016).