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An Approximate Study of Fisher's Equation by Using a Semi-Analytical Iterative Method

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Abstract: In this paper, a new fractional analytical iterative technique was used to get analytical solutions of the nonlinear Fisher's equation with time-fractional order. The novelty of the study comes from the application of the Caputo fractional operator to classical equations, which results in very accurate solutions via well-known series solutions. It also doesn't require any presumptions for nonlinear terms. The numerical results for numerous instances of the equations are displayed in tables and graphs. The method can drastically reduce the number of analytical steps while also being efficient and convenient for solving nonlinear fractional equations.

Keywords: Fractional calculus, Fisher's equation, Temimi-Ansari method (TAM), semi-analytical iterative method, numerical results.

1 Introduction

In recent years, there has been a lot of interest in fractional calculus implementations [1-6]. Much progress has been made in the theory and applications of fractional differential equations in the recent years (FDEs). A variety of observable phenomena are frequently described using linear and nonlinear FDEs devices like dynamic processes in acoustics, electromagnetics, electrochemistry, engineering, viscoelasticity, physics, and materials science [7-10]. Analytical and approximate solutions are completely presented to provide more insight and comprehension of the events illustrated by nonlinear FDEs. However, various academics have disputed over analytical and approximate solutions to the nonlinear Fisher equation using a fraction of the famous methodologies for more clarification [11-18]. FDEs have the benefit of having a nonlocal characteristic that reveals the novel features of these challenges. Finding an exact solution to these kinds of issues, though, is quite difficult. As a result, the majority of fractional differential equations may be approximated using some analytical techniques, such as perturbation methods [19-24]. It is obvious that analytical approaches have problems with nonlinearity in differential equations, although the approximate solutions have acceptable criteria or domains. We recognize that most nonlinear differential equations are difficult to solve; hence, various strategies for solving such equations have been investigated in-depth and narratively. The Temimi-Ansari technique (TAM) [25-27], which was introduced by Temimi and Ansari in 2011 for solving linear and nonlinear ordinary and partial differential equations, is one of the approaches that fiercely fight in this field.

The purpose of this study is to easily expand the usage of the TAM with Caputo's operator to argue and construct approximate and analytical solutions to the fractional nonlinear Fisher's equation.
As a nonlinear model for a physical system, the Fisher equation including linear diffusion and nonlinear growth takes the non dimensional form:

$$u_t = u_{xx} + \gamma\left(1 - u^{\beta}\right)(u - \xi). \tag{1}$$

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Fisher presented the previous equation as a gene selection model where the positive constant γ represents the reaction factor and u indicating population density. The previous equation is about populations, which can be tumor cells, humans, or fish. Fisher's equation describes the wave of beneficial genes advancing [28-29]. This equation piques the interest of scholars due to its numerous implementations in various sectors of science and engineering. The same equation appears in the Brownian motion process, autocatalytic chemical reactions, neurophysiology, flame propagation, and nuclear reactor theory.
The following are the findings of the study: The introduction is in the first section. We utilize some of the fractional calculus requisite concepts that will be utilized in this work in the second section. The fundamental notion of the fractional-order TAM for predicting fractional differential equation solutions is presented in the third section. The suggested approach was used to solve the nonlinear Fisher's equation in the fourth section. Lastly, in section five, the findings are provided.

2 Preliminaries

Many definitions of fractional calculus have been developed over the past hundreds of years. These definitions include the fractional Riemann-Leuvel (R-L) derivative, the fractional Caputo derivative, the Caputo-Fabrizio (C-F) partial integral, and the fractional AB derivative, among others.

Definition 1

For a function $u(t) \in C_\upsilon, \upsilon \geq -1$, the Riemann-Liouville fractional integral operator is defined as [1]

$$\mathfrak{I}_0^\alpha u(t) = \begin{cases} \frac{1}{\Gamma(\alpha+1)} \int\limits_0^t u(t)\,(dt)^\alpha = \frac{1}{\Gamma(\alpha)} \int\limits_0^t (t-\varepsilon)^{\alpha-1} u(\varepsilon)\,d\varepsilon, & t,\alpha > 0, \\ u(t), & \alpha = 0. \end{cases} \tag{2}$$

Definition 2

The Caputo fractional differential operator of order $\alpha > 0$ is defined as [2]

$$\mathfrak{I}^{n-\alpha} D^n u(t) = D_*^\alpha u(t) = \begin{cases} \frac{d^n}{dt^n} u(t), & \alpha = n \in \mathbb{N}, \\ \frac{1}{\Gamma(n-\alpha)} \int\limits_\varepsilon^t (t-\varepsilon)^{n-\alpha-1} u^{(n)}(\varepsilon)\,d\varepsilon, & n-1 < \alpha \leq n \in \mathbb{N}. \end{cases} \tag{3}$$

3 Construction of fractional TAM

To characterize the essential concept of the suggested technique, we consider the generic non-homogeneous fractional differential equation (FPDE) as [25-27]

$$\Upsilon(u(x,t)) + \Phi(u(x,t)) = p(x,t),\; n-1 < \alpha \leq n, \tag{4}$$

with the boundary conditions

$$\mathscr{B}\left(u, \frac{\partial u}{\partial x}\right) = 0, \tag{5}$$

where the Caputo fractional operator of $u(x,t)$ is indicated by $\Upsilon = D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$, Φ is indicate to the general differential operators, $u(x,t)$ is representing the nameless function, the independent variable is denoted by x, the dependent variable is denoted by t, the recognized continuous functions are represented by $p(x,t)$ and the boundary operator is indicated by $\mathscr{B}$. Υ is the main requirement here and it is the general fractional differential operator, but we can take several linear expressions and lay them as needed along with the nonlinear expressions.
The suggested methodology begins with obtaining the initial condition as a result of eliminating the nonlinear part as:

$$D_t^{\alpha} u_0(x,t) = p(x,t), \quad \mathscr{B}\left(u_0, \frac{\partial u_0}{\partial x}\right) = 0. \tag{6}$$

To create the next iteration of the solution, we solve the following equation

$$D_t^{\alpha} u_1(x,t) + \Phi(u_0(x,t)) = p(x,t), \quad \mathscr{B}\left(u_1, \frac{\partial u_1}{\partial x}\right) = 0 \tag{7}$$

As a result, we have a simple iterative stride $u_{n+1}(x,t)$ which is the adequate approach to a linear and nonlinear set of problems

$$D_t^{\alpha} u_{n+1}(x,t) + \Phi(u_n(x,t)) = p(x,t), \quad \mathscr{B}\left(u_{n+1}, \frac{\partial u_{n+1}}{\partial x}\right) = 0 \tag{8}$$

In this approach, it is very important to note that either $u_{n+1}(x,t)$ is solving for problem (4) separately. The iterative approach is easy to apply and each iteration is better than the previous iteration. The iterative approach is easy to apply and each iteration is closer to the exact solution than the previous iteration. Continuing with this method, an ideal approximate solution corresponding to the exact solution can be obtained. In this way, the solution of Eq. (4) Displayed as

$$u(x,t) = \lim_{n\to\infty} u_n(x,t). \tag{9}$$

4 Applications by TAM

Several physical and practical implementations sophisticated with FDEs cannot be solved with accuracy. As a result, solving the problem often just requires a numerical estimate of a solution. The method described here may be utilized to sink the solution of TAM in order to acquire such rough approximations. This department examines three examples to explain the viability, usefulness, and notability of our approach to existing approaches in solving the nonlinear Fisher's equation.

Application 1

The time-fractional order of the following nonlinear Fisher's equation is taken into consideration as [30]

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\partial^2 u(x,t)}{\partial x^2} + u^2(x,t)(1-u(x,t)), \quad 0<\alpha\le 1,\ x\in\mathbb{I},\ t>0, \tag{10}$$

subject to the initial condition

$$u(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{11}$$

By first rewriting the problem using the semi-analytical iterative approach (TAM),

$$\Upsilon(u(x,t)) = D_t^{\alpha} u(x,t) = \frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}}, \quad \Phi(u(x,t)) = \frac{\partial^2 u(x,t)}{\partial x^2} + u^2(x,t)(1-u(x,t)), \quad p(x,t) = 0. \tag{12}$$

The initial problem that has to be resolved is

$$\Upsilon(u_0(x,t)) = 0, \qquad u_0(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{13}$$

Using a basic treatment, the following is the solution to Eq. (13):

$$I^{\alpha}\left(D_t^{\alpha} u_0(x,t)\right) = 0, \qquad u_0(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{14}$$

Hence, given the fundamental characteristics of definition (2), we get the primary iteration as

$$u_0(x,t) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{15}$$

One may compute the next iteration as

$$\Upsilon\left(u_1(x,t)\right) + \Phi\left(u_0(x,t)\right) + w(x,t) = 0, \qquad u_1(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{16}$$

So, by integrating both sides of Eq. (16) and using the fundamental characteristics of definition (1), we arrive to

$$I^{\alpha}\left(D_t^{\alpha}u_1(x,t)\right) = I^{\alpha}\left(\frac{\partial^2 u_0(x,t)}{\partial x^2} + u_0^2(x,t)\left(1-u_0(x,t)\right)\right), \qquad u_1(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{17}$$

Then, we acquire the next iteration as

$$u_1(x,t) = u_0(x,t) + \frac{t^{\alpha}}{4\Gamma(\alpha+1)\left(1+\cosh\left(\frac{x}{\sqrt{2}}\right)\right)}. \tag{18}$$

Calculating the third iteration is as follows:

$$\Upsilon\left(u_2(x,t)\right) + \Phi\left(u_1(x,t)\right) + w(x,t) = 0, \qquad u_2(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{19}$$

Hence, by integrating both sides of Eq. (19) and using the fundamental features of definition (1), we have

$$I^{\alpha}\left(D_t^{\alpha}u_2(x,t)\right) = I^{\alpha}\left(\frac{\partial^2 u_1(x,t)}{\partial x^2} + u_1^2(x,t)\left(1-u_1(x,t)\right)\right), \qquad u_2(x,0) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}. \tag{20}$$

Then we acquire the next iteration as

$$\begin{aligned} u_2(x,t) = u_1(x,t) + \frac{e^{\frac{x}{\sqrt{2}}}t^{2\alpha}}{8\left(e^{\frac{x}{\sqrt{2}}}+1\right)^6}\Bigg(&\frac{2e^{\frac{x}{\sqrt{2}}}\left(-e^{\frac{x}{\sqrt{2}}}+e^{\sqrt{2}x}-2\right)\Gamma(2\alpha+1)t^{\alpha}}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} \\ &+\frac{2\left(e^{\frac{x}{\sqrt{2}}}-1\right)\left(e^{\frac{x}{\sqrt{2}}}+1\right)^3}{\Gamma(2\alpha+1)} - \frac{e^{\sqrt{2}x}\Gamma(3\alpha+1)t^{2\alpha}}{\Gamma(\alpha+1)^3\Gamma(4\alpha+1)}\Bigg). \end{aligned} \tag{21}$$

Each iteration of the $u_n(x,t)$ indicates an approximation of the solution to Eq. (9), and when the quantity of iterations increases, the approximate solution becomes closer to the exact solution, according to Eq. (9). By continuing using this strategy, we are able to create the following series template for analytical solutions as

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) \simeq u_2(x,t), \tag{22}$$

which contains the exact solution as [30]

$$u(x,t) = \frac{1}{1+e^{\frac{x-tv}{\sqrt{2}}}}. \tag{23}$$

Application 2

The time-fractional order of the following nonlinear Fisher's equation is taken into consideration as [30]

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}=\frac{\partial^{2}u(x,t)}{\partial x^{2}}+au(x,t)\left(1-u(x,t)\right),\quad 0<\alpha\leq 1, \tag{24}$$

subject to the initial condition

$$u(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{25}$$

By first rewriting the problem using the semi-analytical iterative approach (TAM),

$$\Upsilon\left(u(x,t)\right)=D_{t}^{\alpha}u(x,t)=\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}},\quad \Phi\left(u(x,t)\right)=\frac{\partial^{2}u(x,t)}{\partial x^{2}}+au(x,t)\left(1-u(x,t)\right),\quad p(x,t)=0. \tag{26}$$

The initial problem that has to be resolved is

$$\Upsilon\left(u_{0}(x,t)\right)=0,\qquad u_{0}(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{27}$$

Using a basic treatment, the following is the solution to Eq. (13):

$$I^{\alpha}\left(D_{t}^{\alpha}u_{0}(x,t)\right)=0,\qquad u_{0}(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{28}$$

Hence, given the fundamental characteristics of definition (2), we get the primary iteration as

$$u_{0}(x,t)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{29}$$

One may compute the next iteration as

$$\Upsilon\left(u_{1}(x,t)\right)+\Phi\left(u_{0}(x,t)\right)+w(x,t)=0,\qquad u_{1}(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{30}$$

So, by integrating both sides of Eq. (16) and using the fundamental characteristics of definition (1), we arrive to

$$I^{\alpha}\left(D_{t}^{\alpha}u_{1}(x,t)\right)=I^{\alpha}\left(\frac{\partial^{2}u_{0}(x,t)}{\partial x^{2}}+au_{0}(x,t)\left(1-u_{0}(x,t)\right)\right),\qquad u_{1}(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{31}$$

Then, we acquire the next iteration as

$$u_{1}(x,t)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}+\frac{5ae^{\sqrt{\frac{a}{6}}x}t^{\alpha}}{3\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{3}\Gamma(\alpha+1)}. \tag{32}$$

Calculating the third iteration is as follows:

$$\Upsilon\left(u_{2}(x,t)\right)+\Phi\left(u_{1}(x,t)\right)+w(x,t)=0,\qquad u_{2}(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{33}$$

Hence, by integrating both sides of Eq. (19) and using the fundamental features of definition (1), we have

$$I^{\alpha}\left(D_{t}^{\alpha}u_{2}(x,t)\right)=I^{\alpha}\left(\frac{\partial^{2}u_{1}(x,t)}{\partial x^{2}}+au_{1}(x,t)\left(1-u_{1}(x,t)\right)\right),\qquad u_{2}(x,0)=\frac{1}{\left(1+e^{\sqrt{\frac{a}{6}}x}\right)^{2}}. \tag{34}$$

Then we acquire the next iteration as

$$u_2(x,t) = u_1(x,t) + \frac{25a^2 e^{\frac{\sqrt{a}x}{\sqrt{6}}} t^{2\alpha}}{18\left(e^{\frac{\sqrt{a}x}{\sqrt{6}}}+1\right)^6} \left(\frac{\left(e^{\frac{\sqrt{a}x}{\sqrt{6}}}+1\right)^2 \left(2e^{\frac{\sqrt{a}x}{\sqrt{6}}}-1\right)}{\Gamma(2\alpha+1)} - \frac{2^{2\alpha+1} a e^{\frac{\sqrt{a}x}{\sqrt{6}}} \Gamma\left(\alpha+\frac{1}{2}\right) t^{\alpha}}{\sqrt{\pi}\Gamma(\alpha+1)\Gamma(3\alpha+1)} \right). \tag{35}$$

Each iteration of the $u_n(x,t)$ indicates an approximation of the solution to Eq. (9), and when the quantity of iterations increases, the approximate solution becomes closer to the exact solution, according to Eq. (9). By continuing using this strategy, we are able to create the following series template for analytical solutions as

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) \simeq u_2(x,t), \tag{36}$$

which contains the exact solution as [30],

$$u(x,t) = \frac{1}{\left[1+\exp\left(\sqrt{\frac{a}{6}}x - \frac{5a}{6}t\right)\right]^2}. \tag{37}$$

Application 3

The time-fractional order of the following nonlinear Fisher's equation is taken into consideration as [30]

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\partial^2 u(x,t)}{\partial x^2} + u(x,t)\left(1-u(x,t)\right)\left(u(x,t)-a\right), \quad 0<\alpha\leq 1. \tag{38}$$

subject to the initial condition

$$u(x,0) = \frac{1+a}{2} + \frac{1-a}{2}\tanh\left(\frac{(1-a)x}{2\sqrt{2}}\right). \tag{39}$$

We obtain the following analytical solutions by applying the same fundamental idea of the TAM of fractional order

$$u_0(x,t) = \frac{1+a}{2} + \frac{1-a}{2}\tanh\left(\frac{(1-a)x}{2\sqrt{2}}\right), \tag{40}$$

$$u_1(x,t) = u_0(x,t) + \frac{(a-1)^2(a+1)t^{\alpha}\operatorname{sech}^2\left(\frac{(a-1)x}{2\sqrt{2}}\right)}{8\Gamma(\alpha+1)}, \tag{41}$$

$$\begin{aligned} u_2(x,t) = u_1(x,t) - \frac{(a-1)^3(a+1)^2 t^{2\alpha}\operatorname{sech}^2\left(\frac{(a-1)x}{2\sqrt{2}}\right)}{2048} \Bigg\{ & \frac{3(a-1)^3(a+1)\Gamma(3\alpha)t^{2\alpha}\operatorname{sech}^4\left(\frac{(a-1)x}{2\sqrt{2}}\right)}{\Gamma(4\alpha)\Gamma(\alpha+1)^3} \\ & + \frac{16(a-1)\Gamma(2\alpha+1)t^{\alpha}\left(3(a-1)\tanh\left(\frac{(a-1)x}{2\sqrt{2}}\right)+a+1\right)\operatorname{sech}^2\left(\frac{(a-1)x}{2\sqrt{2}}\right)}{\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} + \frac{128\tanh\left(\frac{(a-1)x}{2\sqrt{2}}\right)}{\Gamma(2\alpha+1)} \Bigg\}, \cdots \end{aligned} \tag{42}$$

Each iteration of the $u_n(x,t)$ indicates an approximation of the solution to Eq. (9), and when the quantity of iterations increases, the approximate solution becomes closer to the exact solution, according to Eq. (9). By continuing using this strategy, we are able to create the following series template for analytical solutions as

$$u(x,t) = \lim_{m\to\infty} u_m(x,t) \simeq u_2(x,t), \tag{43}$$

which contains the exact solution as [30]

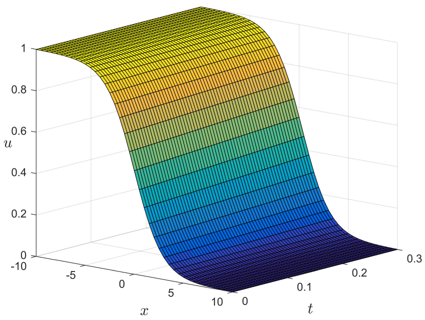
$$u(x,t) = \frac{a+1}{2} + \frac{1-a}{2}\tanh\left(\frac{(1-a)x}{2\sqrt{2}} + \frac{1-a^2}{4}t\right). \tag{44}$$

Table 1: Comparison of approximate solutions acquired by TAM with exact solution at $t = 0.1$.

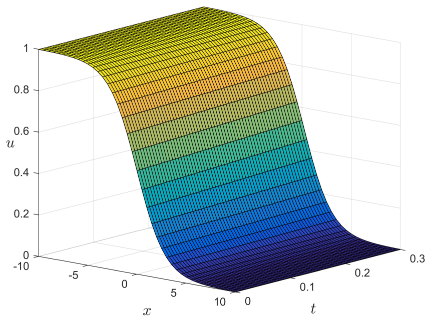
x	Application 1 ($v = 1$)			Application 2 ($a = 1$)			Application 3 ($a = 0.5$)		
	u_{Ex}	u_{TAM}	E_u	u_{Ex}	u_{TAM}	E_u	u_{Ex}	u_{TAM}	E_u
0	0.501768	0.501250	5.177E-4	0.271255	0.271255	1.008E-7	0.754687	0.754687	2.652E-9
1	0.331804	0.331345	4.590E-4	0.175962	0.175962	8.131E-8	0.798268	0.798268	1.286E-7
2	0.196685	0.196358	3.270E-4	0.105301	0.105301	1.161E-7	0.839001	0.839001	2.649E-7
3	0.107720	0.107521	1.989E-4	0.058593	0.058593	1.717E-7	0.874958	0.874957	3.598E-7
4	0.056181	0.056071	1.097E-4	0.030672	0.030672	1.863E-7	0.905131	0.905131	3.928E-7
5	0.028513	0.028455	5.731E-5	0.015300	0.015300	1.536E-7	0.929394	0.929394	3.733E-7
6	0.014265	0.014236	2.909E-5	0.007359	0.007359	1.038E-7	0.948245	0.948245	3.231E-7
7	0.007084	0.007070	1.455E-5	0.003448	0.003448	6.118E-8	0.962502	0.962502	2.623E-7
8	0.003505	0.003498	7.228E-6	0.001585	0.001585	3.281E-8	0.973068	0.973068	2.037E-7
9	0.001731	0.001728	3.576E-6	0.000720	0.000720	1.650E-8	0.980780	0.980780	1.534E-7
10	0.000854	0.000852	1.766E-6	0.000324	0.000324	7.956E-9	0.986348	0.986348	1.131E-7

5 Conclusion

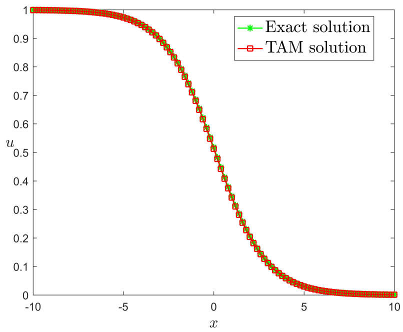
The primary contribution of this work is that it confirms the successful use of TAM with fractional time derivatives to get analytical solutions to various nonlinear fractional differential equations. According to the results, a small number of approximate expressions produce outcomes with a high level of dependability, and the approximate solution error reduces rapidly as the number of these expressions increases. It also consumes fewer CPU and computational resources than previous techniques. TAM's accuracy and dependability have been proved by watching the efficiency of this technology. The outcomes show that this method is genuinely effective for solving a class of nonlinear FPD equations with fractional operators. The proposed method is efficient and effective for solving different sorts of fractional equations and systems.



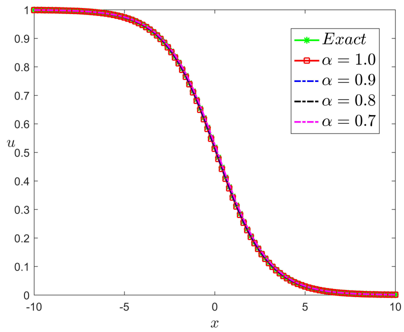
(a) Exact solution of $u(x,t)$.



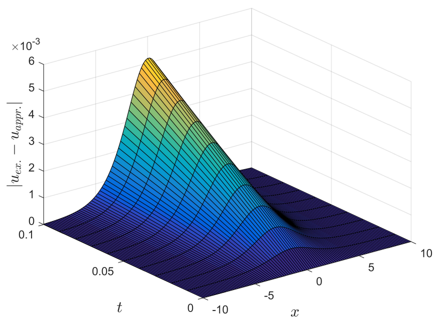
(b) TAM solution of $u(x,t)$.



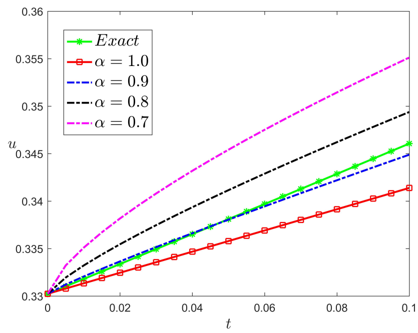
(c) Graph of the TAM solution with the exact solution.



(d) Graphical simulation for α at $t = 0.1$.



(e) The absolute error between exact and TAM solutions.



(f) Graphical simulation for α at $x = 1$.

Fig. 1: The behavior of approximate solutions obtained by TAM for application (1) at $v = 1$.

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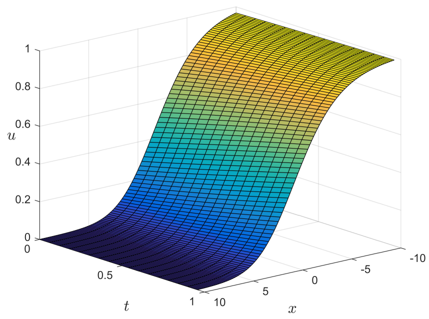
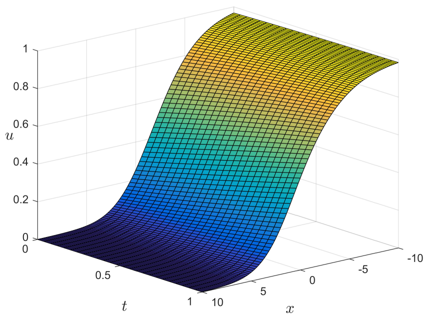
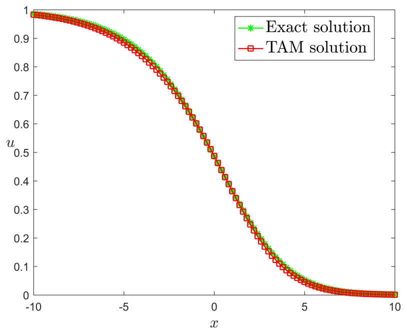


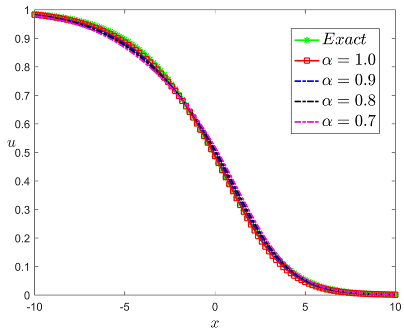
(a) Exact solution of $u(x,t)$.



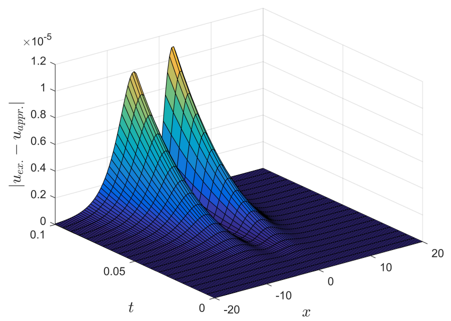
(b) TAM solution of $u(x,t)$.



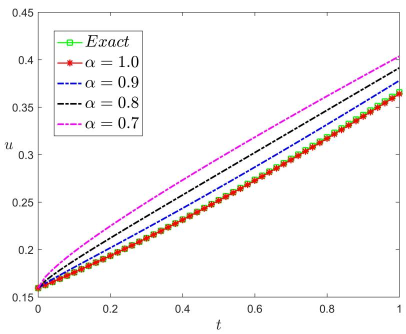
(c) Graph of the TAM solution with the exact solution.



(d) Graphical simulation for α at $t = 1$.



(e) The absolute error between exact and TAM solutions.



(f) Graphical simulation for α at $x = 1$.

Fig. 2: The behavior of approximate solutions obtained by TAM for application (2) at $a = 1$.

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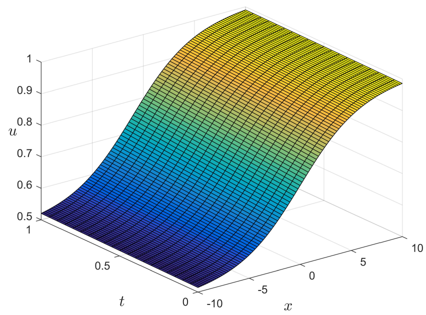
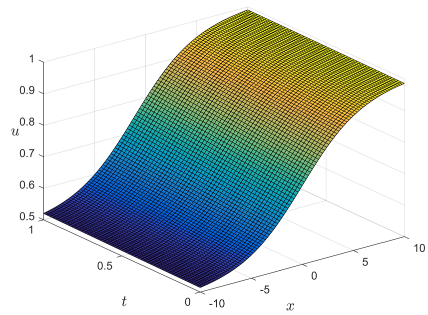
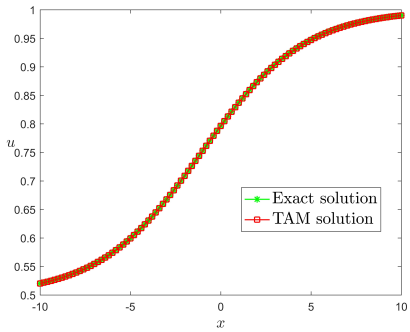


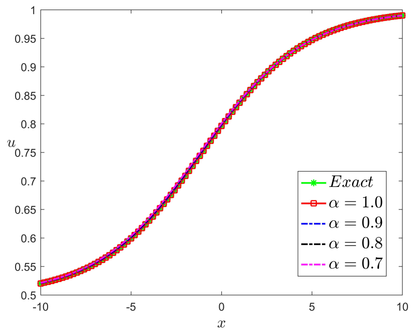
(a) Exact solution of $u(x,t)$.



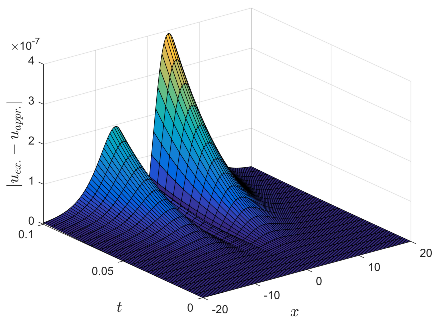
(b) TAM solution of $u(x,t)$.



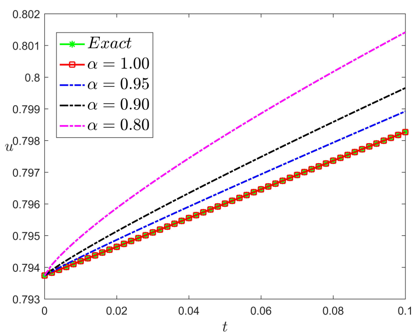
(c) Graph of the TAM solution with the exact solution.



(d) Graphical simulation for α at $t = 1$.



(e) The absolute error between exact and TAM solutions.



(f) Graphical simulation for α at $x = 1$.

Fig. 3: The behavior of approximate solutions obtained by TAM for application (3) at $a = 0.5$.

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