Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/110203

Mathematical Model for Unsteady Remediation of River Pollution by Aeration

Mohamed Kh. Hadhouda[1,*] *and Zeinab S. Hassan* [2]

[1]Elgazeera High Institute for Engineering and Technology, Basic Sciences Department, Cairo, Egypt
[2]Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt

Received: 22 Jun. 2021, Revised: 12 Aug. 2021, Accepted: 28 Sep. 2021
Published online: 1 Mar. 2022

Abstract: The governing coupled pair of nonlinear unsteady-state equations describing the concentration of the pollutant $P(x,t)$ and the dissolved oxygen concentration $X(x,t)$ in a river are solved numerically in one dimension x. The initial values of both $P(x,t)$ and $X(x,t)$ are assumed to be decreasing exponential functions in x. The results are presented and discussed graphically. We have proved mathematically the fact that the high concentration of pollutant can be reduced by releasing adequate discharges from barrage in a river. The different effects of parameters controlling the flow on both $P(x,t)$ and $X(x,t)$ are studied along the river at different time t.

Keywords: pollution in a river, aeration, dissolved oxygen, finite difference method

1 Introduction

Globally, pollution is drastically increasing in water bodies, negatively influencing the surrounding areas species and grounds. Hence, mathematical models have been developed to control and reduce contamination. Such models originated in 1920s, specifically Streeter and Phelp's model [1] which presented the significance of the quantity of dissolved oxygen in rivers. The dissolved oxygen concentration is the most essential factor because it is essential for aquatic life, without it, a river will be an aquatic desert devoid of fish, plants, and insects. As a result, the concentration of dissolved oxygen is investigated in depth in this study. Despite the fact that the Nile is Egypt's lifeblood, it is sadly contaminated by a variety of chemical and biological toxins, as well as agricultural waste.

In summer 2020 and 2021, a very big quantity of fresh water came to the Lake Naser and by releasing this fresh water to the Nile River the high polluted regions were remediated. Herein lies the significance of this research; it will help us understand how to predict the size of a pollutant concentration at a reasonable time, as well as how to control pollutants by increasing dissolved oxygen concentration. Huang et al. [2] developed an analytical model for one-dimensional solute transport in heterogeneous porous media with scale-dependent dispersion. They supposed that the dispersion coefficient is proportional to pore water velocity in a linear form. Later, Pimpunchat et al. [3] developed a basic mathematical model for river pollution and investigated the impact of aeration on pollutant reduction. In reality, their model is made up of coupled reaction-diffusion-advection equations for both pollutant and dissolved oxygen concentrations, with the steady-state case considered in one spatial dimension for simplified cases. Kumar et al. [4] studied solute dispersion in a semi-infinite porous medium with a source/trough effect. Hussain et al. [5] investigated a mathematical model that allows for the prediction of contaminant concentration levels in rivers. The pollutant and dissolved oxygen concentrations are described by a pair of coupled reaction diffusion-advection equations. They looked at the steady-state case in a one-dimensional space with zero dispersion. Dimian et al. [6] studied the effect of an additional pollutant along a river on the pollutant concentration as defined by the one-dimensional advection diffusion equation, and discovered that the pollutant concentration increases as time passes at any cross-section. Svetislav and Alexander [7] investigated a one-dimensional advection-diffusion equation with variable coefficients in semi-infinite media, and the resulting equations were solved numerically using the explicit finite difference method (EFDM). Dimian et al.

* Corresponding author e-mail: drmohamed.khaieralla@gi.edu.eg

© 2022 NSP
Natural Sciences Publishing Cor.

[8] studied the impact of the El-Rahawy Drain on the water quality of the Rosetta Branch of the Nile River in Egypt, determining the best zone from which water can be taken for drinking or irrigation. Wadi et al. [9] obtained solution for the one-dimensional advection-dispersion equation of pollutant concentration and divided the river into two sections. Ibrahim et al. [10] studied pollution remediation in a river using unsteady aeration with arbitrary initial and boundary conditions. Alexander and Svetislav [11] investigated the explicit finite-difference solution of two-dimensional solute transport in homogeneous porous media with periodic flow, adsorption retardation, periodic seepage velocity and a dispersion coefficient proportional to this velocity. Pawarisa and Nopparat [12] studied the numerical simulation of a one-dimensional water-quality model in a stream using Saulyev technique with quadratic interpolation initial-boundary conditions. Yadav and Kumar [13] investigated analytical solutions for the two-dimensional advection dispersion equation in a semi-infinite heterogeneous porous medium with a uniform nature pulse type input point source for conservative solute transport. In general form, the dispersion coefficient and velocity are both spatially and temporally dependent and the retardation factor is considered in degenerate form. Hadhouda and Abdelwahid [14] studied the numerical solution for contamination in a river and its remediation by two-dimensional unsteady aeration, assuming that the initial values of both P and X are zero. Manitchoen and Pimpunchat [15] obtained analytical and numerical solutions of pollutant concentration with uniformly and exponentially increasing forms of sources, they found that the concentration increases as the rate of pollutant addition along the river and the arbitrary constant of exponential pollution source term increase. These models are used to simulate the spatial and temporal distribution of various water quality variables in the research field.

However, the objective of this study is to develop a general applicable solution of the coupled partial differential equations describing the flow of the pollutant. The initial values of both $P^*(x^*,t^*)$ and $X^*(x^*,t^*)$ are assumed to be decreasing exponential functions in x^*. These equations which represent the diffusion of both the pollutant concentration $P^*(x^*,t^*)$ and dissolved oxygen concentration $X^*(x^*,t^*)$ are solved by using explicit finite- difference method. Our results generalize the earlier solutions obtained by Manitcharoen and Pimpunchat [15] and lbrahim et al. [10], which form a subset of our solutions for the limited case when $t^* \to \infty$ and the half-saturated oxygen demand concentration for pollutant decay $k^* = 0$. Where the superscript $(^*)$ means that the physical quantity is in the dimensional form.

2 Mathematical formulation of the problem

We assume the mathematical model for unsteady flow in the river as being one dimensional characterized by a single distance $x^*(m)$ measured from the origin $x^* = 0$. We consider the diffusion is accompanied by forced convection and so the pollutant concentration $P^*(x^*,t^*)$ and dissolved oxygen concentration $X^*(x^*,t^*)$ satisfy diffusion-advection equations. The governing coupled pair of nonlinear partial differential equations are given by [15], [16] and [17].

$$\frac{\partial (A^*P^*)}{\partial t^*} = D_P^* \frac{\partial^2 (A^*P^*)}{\partial x^{*2}} - \frac{\partial (v^*A^*P^*)}{\partial x^*} - k_1^* \frac{X^*}{X^*+k^*} A^*P^* + q^*, \quad (0 \le x^* \le L^*, t^* \ge 0), \tag{1}$$

$$\frac{\partial (A^*X^*)}{\partial t^*} = D_X^* \frac{\partial^2 (A^*X^*)}{\partial x^{*2}} - \frac{\partial (v^*A^*X^*)}{\partial x^*} - k_2^* \frac{X^*}{X^*+k^*} A^*P^* + \alpha^*(S^* - X^*), \quad (0 \le x^* \le L^*, t^* \ge 0), \tag{2}$$

where A^* is the cross-section area (m^2), D_p^* is the dispersion coefficient of pollutant in the x^*-direction $(m^2\, day^{-1})$, v^* is the water velocity in the x^*-direction $(m\, day^{-1})$, k_1^* is the degradation rate coefficient for pollutant (day^{-1}), q^* is the added pollutant rate along the river $(\text{kg m}^{-1}\, day^{-1})$, L^* is the length of river(m), D_X^* is the dispersion coefficient of dissolved oxygen in the x^*-direction $(m^2\, day^{-1})$, k_2^* is the de-aeration rate coefficient for dissolved oxygen (day^{-1}), α^* is the mass transfer of oxygen from air to water $(m^2\, day^{-1})$ and S^* is the saturated oxygen concentration (kg m^{-3}). The last term in equation (1), represents the addition of pollutant at a rate q^*, while the third term, in the right hand side, represents its removal by aeration. The rate of depletion of pollutant concentration P^*, due to the biochemical reaction with dissolved oxygen concentration X^* has been described using "Michaelis Menten" term $k_1^* \frac{X^*A^*}{X^*+k^*} P^*$ [17]. Two different case studies are taken into account, where:
First case: $P^*(x^*,0) \neq 0$, $X^*(x^*,0) \neq 0$, $P^*(0,t^*) \neq 0$, $X^*(0,t^*) \neq 0$.
Second case: $P^*(x^*,0) \neq 0$ and $X^*(x^*,0) = 0$, while $P^*(0,t^*) = 0$ and $X^*(0,t^*) \neq 0$.

3 Case study one

The initial and boundary conditions associated with equations (1) and (2), for this case are:

$$\begin{cases} P^*(x^*,0) = P_{in}^* \, e^{\frac{-x^*}{\lambda_1^*}}, & 0 \le x^* \le L^*, \\ X^*(x^*,0) = X_{in}^* \, e^{\frac{-x^*}{\lambda_2^*}}, & 0 \le x^* \le L^*, \end{cases} \tag{3}$$

© 2022 NSP
Natural Sciences Publishing Cor.

$$\begin{cases} P^*(0,t^*) = P_1^*, & t^* > 0, \\ X^*(0,t^*) = X_1^*, & t^* > 0, \end{cases} \tag{4}$$

$$\begin{cases} \frac{\partial P^*(L^*,t^*)}{\partial x^*} = 0, & t^* \geq 0, \\ \frac{\partial X^*(L^*,t^*)}{\partial x^*} = 0, & t^* \geq 0, \end{cases} \tag{5}$$

where P_{in}^* and X_{in}^* are the initial pollutant and dissolved oxygen concentrations respectively. λ_1^* and λ_2^* are the initial pollutant and dissolved oxygen concentrations decay length respectively. P_1^* and X_1^* are the pollutant and dissolved oxygen concentrations at the origin, which are taken constants.

The following non-dimensional variables are used in the governing equations, initial and boundary conditions.

$$\begin{cases} x = \frac{x^*}{L^*},\ t = k_1^* t^*,\ P = \frac{P^*}{P_{in}^*}, \\ X = \frac{X^*}{X_{in}^*},\ P_1 = \frac{P_1^*}{P_{in}^*} \text{ and } X_1 = \frac{X_1^*}{X_{in}^*}, \end{cases} \tag{6}$$

where x,t,P,X,P_1 and X_1 are the dimensionless distance, time, pollutant and dissolved oxygen concentrations, the pollutant and dissolved oxygen concentrations at the origin respectively. Equation (6), transforms equations (1-5) into the non-dimensional form:

$$\frac{\partial P}{\partial t} = D_P \frac{\partial^2 P}{\partial x^2} - v\frac{\partial P}{\partial x} - \frac{X}{X+k}P + q, \quad (0 \leq x \leq 1,\ t \geq 0), \tag{7}$$

$$\frac{\partial X}{\partial t} = D_X \frac{\partial^2 X}{\partial x^2} - v\frac{\partial X}{\partial x} - \frac{k_3 X}{X+k}P + \alpha(S-X), \quad (0 \leq x \leq 1,\ t \geq 0), \tag{8}$$

$$P(x,0) = e^{\frac{-x}{\lambda_1}}, \quad X(x,0) = e^{\frac{-x}{\lambda_2}}, \quad 0 \leq x \leq 1, \tag{9}$$

$$P(0,t) = P_1, \quad X(0,t) = X_1, \quad t > 0, \tag{10}$$

$$\frac{\partial P(1,t)}{\partial x} = 0, \quad \frac{\partial X(1,t)}{\partial x} = 0, \quad t \geq 0, \tag{11}$$

where $D_P, v, k, q, D_X, k_3, \alpha, S, \lambda_1$ and λ_2 are dimensionless parameters which are given by:

$$\begin{cases} D_P = \frac{D_P^*}{L^{*2} k_1^*},\ v = \frac{v^*}{L^* k_1^*},\ k = \frac{k^*}{X_{in}^*}, \\ q = \frac{q^*}{A^* k_1^* P_{in}^*},\ D_X = \frac{D_X^*}{L^{*2} k_1^*},\ k_3 = \frac{k_2^* P_{in}^*}{k_1^* X_{in}^*}, \\ \alpha = \frac{\alpha^*}{A^* k_1^*},\ S = \frac{S^*}{X_{in}^*},\ \lambda_1 = \frac{\lambda_1^*}{L^*} \text{ and } \lambda_2 = \frac{\lambda_2^*}{L^*}. \end{cases} \tag{12}$$

4 Case study two

Assume that the river's water is polluted and the aeration is removed at the initial time $(t = 0)$ along the river. Assume also that at the origin $(x = 0)$, at any time $(t > 0)$, the source of pollution is removed while the source of aeration exists. Then the initial and boundary conditions associated with equations (7 and 8) in the dimensionless form are:

$$P(x,0) = P_2, \quad X(x,0) = 0, \quad 0 \leq x \leq 1, \tag{13}$$

$$P(0,t) = 0, \quad X(0,t) = X_2, \quad t > 0, \tag{14}$$

$$\frac{\partial P(1,t)}{\partial x} = 0, \quad \frac{\partial X(1,t)}{\partial x} = 0, \quad t \geq 0, \tag{15}$$

where P_2 is the value of initial pollution concentration and X_2 is the value of dissolved oxygen concentration at the origin.

5 Numerical solution

The explicit finite-difference method (EFDM) is applied to solve equations (7) and (8) associated with the initial and boundary conditions (9-11). The central difference scheme was used for $\frac{\partial^2 P}{\partial x^2}, \frac{\partial^2 X}{\partial x^2}, \frac{\partial P}{\partial x}$ and $\frac{\partial X}{\partial x}$. The forward difference scheme was used for $\frac{\partial P}{\partial t}$ and $\frac{\partial X}{\partial t}$. With these substitutions, equations (7)and (8) can be written as:

$$P_{i,j+1} = (r_1 + r_2) P_{i-1,j} + \left(1 - 2r_1 - \frac{X_{i,j}\,\Delta t}{X_{i,j}+k}\right) P_{i,j} + (r_1 - r_2) P_{i+1,j} + \Delta t q, \tag{16}$$

$$X_{i,j+1} = (r_3 + r_2) X_{i-1,j} + \left(1 - 2r_3 - k_3\,\Delta t \frac{P_{i,j}}{X_{i,j}+k} - \alpha \Delta t\right) X_{i,j} + (r_3 - r_2) X_{i+1,j} + \alpha\, S\, \Delta t, \tag{17}$$

where i and j refer to the discrete step lengths Δx and Δt for the coordinate x and time t, respectively, and $r_1 = \frac{D_p\,\Delta t}{(\Delta x)^2}$, $r_2 = \frac{v\,\Delta t}{2\Delta x}$, $r_3 = \frac{D_x\,\Delta t}{(\Delta x)^2}$. The initial and boundary conditions (9-11) can be written in the finite difference form as:

$$P(i,0) = e^{\frac{-x_i}{\lambda_1}}, \quad X(i,0) = e^{\frac{-x_i}{\lambda_2}}, \tag{18}$$

$$P(0,j) = P_1, \quad X(0,j) = X_1, \tag{19}$$

$$\begin{cases} P(N,j) = P(N-1,j), & x = 1,\ t \geq 0, \\ X(N,j) = X(N-1,j), & x = 1,\ t \geq 0, \end{cases} \tag{20}$$

where $t[j] = j\Delta t$, $x[i] = i\Delta x$ and $N = \frac{1}{\Delta x}$ is the grid dimension in the x direction and 1 is the distance in the direction x at which $\frac{\partial P}{\partial x} \to 0$ and $\frac{\partial X}{\partial x} \to 0$.

© 2022 NSP
Natural Sciences Publishing Cor.

6 Results and discussions

The coupled equations (16) and (17), with the initial and the boundary conditions (18-20) are solved numerically by using EFDM. We take in our model the domain of dimensionless longitudinal distance in the region $0 \leq x \leq 1$ and the dimensionless time $0 \leq t \leq 1.5$. In the numerical calculations, the step length $\Delta x = 0.1$ and the step of time $\Delta t = 0.002$ have been used to achieve the stability of the finite difference scheme. P_1 and X_1 are taken equal to 0.2 and 1.9, respectively, Pimpunchat et al. [3]. We have presented the variations of P and X along the river for several values of the time t and the parameters $D_p, v, k, q, D_x, k_3, \alpha$ and S. The parameters D_p, v, k, q and D_x are taken to be equal $1, k_3 = 0.5, \lambda_1 = \lambda_2 = 1.02, \alpha$ and S are taken to be equal 2 and $t = 0.5$, Pimpunchat et al. [3]. Figures (1-8) illustrate the variations of P and X with one of the parameters of the flow keeping the other parameters constants.

Figure (1) shows the variation of both P and X with t along the river for $t = 0.1, 0.2, 0.5$ and 1. From figure (1) it is clear that: (i) For any fixed value of x, the values of P decrease and the values of X increase as the time t increases. (ii) At any time, the decrease of the values of X is associated with the increase of the values of P along the river. (iii) The numerical study shows that the steady state case, i. e. the state for which $t \to \infty$ achieved at $t \approx 1.2$. For values of $t \geq 1.2$, the increase in the values of t has no effect on both P and X. This result agrees with that obtained by pimpunchat et al. [3].

Figure (2) shows the variation of P and X with D_p along the river for $D_P = 0.1, 0.5, 1$ and 2 . From figure (2) it is clear that: for any fixed value of x, as D_p increases, the values of P decrease and the values of X increase. As expected, the effect of D_P on P is dominant, while the effect of D_p on X is very small. Numerical studies show that, at $t \leq 0.04$, as D_P increases the values of P increase and as D_x increases the values of X increase.

Figure (3) shows the variation of both P and X with v along the river for $v = 0.1, 1, 5$ and 10. From figure (3) it is clear that: (i) At any constant value of x, as v increases P decreases, while X increases, this result agrees with that obtained by Guoyuan, [18]. (ii) Numerical studies show that for small values of $t (t \leq 0.04)$, v has an opposite effect only on P, i.e., as v increases P increases.

Figure (4) shows the variation of P and X with k along the river for $k = 0.01, 0.5, 1$ and 2 . From figure (4) it is clear that: (i) As x increases P increases and X decreases, for a fixed value of k. (ii) At any fixed value of x, as k increases P and X increase. In general, the effect of k on P is dominant while its effect on X is small. Our results agree with that obtained by Pimpunchat et al. [3].

Figure (5) shows the variation of P and X with q along the river for $q = 0.01, 0.5, 1$ and 1.5. From figure (5) it is clear that: (i) At any fixed value of x along the river, as q increases the values of P increase and the values of X decrease, our results agree with that obtained by Maitchoen and Pimpunchat [15] and Ibrahim et al.[10]. As expected the effect of q on P is dominant, while its effect of X is very small.

Figure (6) shows the variation of P and X with k_3 along the river for $k_3 = 0.1, 1, 5$ and 10 . From figure (6) it is clear that: (i) As k_3 increases the value of X decreases and the values of P increase along the river. (ii) In general, the effect of k_3 on X is dominant while the effect on P is very small.

Figure (7) shows the variation of P and X with α along the river for $\alpha = 0.01, 0.5, 1$ and 2 . From figure (7) it is clear that: as α increases the values of X increase while the values of P rarely decrease. Our results agree with that obtained by Wadi et al. [9].

Figure (8) shows the variation of P with dissolved oxygen concentration at the origin X_1 along the river, for $X_1 = 0.1, 2$ and 10 .From figure (8) it is clear that, at a fixed value of x, as X_1 increases the values of P decrease.

Equations (7) and (8), associated with the initial and boundary conditions (13-15) are solved numerically and illustrated in figure (9) and table (1) for the common input data $t = 0.5, P_2 = 0.2, X_2 = 1.9, D_P = 1, k = 1, q = 1, D_X = 1, k_3 = 0.5, \alpha = 2$ and $S = 2$. Let the cross section area of the river at $x^* = 0$ be A^*, then the flux of the water (the volume of water crossing A^* every day) will be $Q^* = A^* v^*$. By using equation (12), the flux of water in the dimensionless form is given by: $Q = \frac{Q^*}{A^* L^* k_1^*} = \frac{A^* v^*}{A^* L^* k_1^*} = v$. Consequently, increasing the values of Q means increasing the values of v. Figure (9) shows the variation of P with flow velocity for the values $0.1 \leq v \leq 15$. From figure (9), it is clear that: as v increases the value of P decreases along the river, hence figure (9) emphasize the fact that the zone of clean water measured form $x = 0$ in the direction of the flow increases as the quantity of the clean water Q entering the cross section A increases.

Table 1: The variations of both $P(x,t)$ and $X(x,t)$ with Q at the middle of the river $(x = 0.5)$.

Q	$P(x,t)$	$X(x,t)$
0.1	0.2666	1.7653
1	0.2155	1.8292
5	0.0917	1.9129
7	0.0683	1.9115
10	0.0488	1.9088
12	0.0409	1.9075
15	0.0328	1.9061

© 2022 NSP
Natural Sciences Publishing Cor.

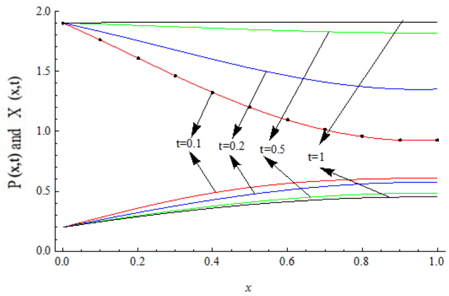

Fig. 1: The variation of both P and X with t along the river for $D_P = 1, v = 1, , k = 1, q = 1, D_X = 1, k_3 = 0.5$ $\alpha = 2, S = 2$ and $t = 0.1, 0.2, 0.5$ and 1.

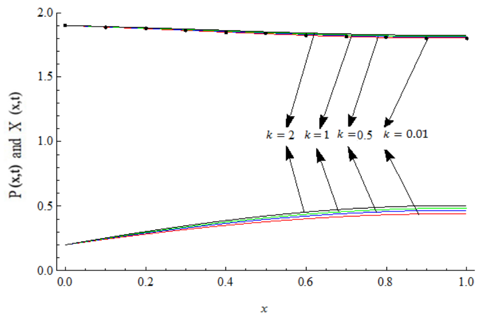

Fig. 4: The variation of P and X with k along the river for $t = 0.5, D_P = 1, v = 1, q = 1, D_X = 1$ $k_3 = 0.5,$ $\alpha = 2, S = 2$ and $k = 0.01, 0.5,$ 1 and 2.

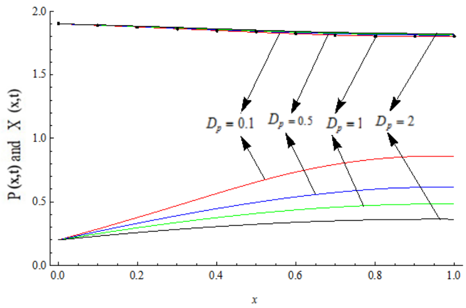

Fig. 2: The variation of P and X with Dp along the river for $t = 0.5, v = 1, \mathrm{k} = 1, q = 1, D_X = 1$ $k_3 = 0.5, \alpha = 2, S = 2$ and $D_P = 0.1, 0.5, 1$ and 2.

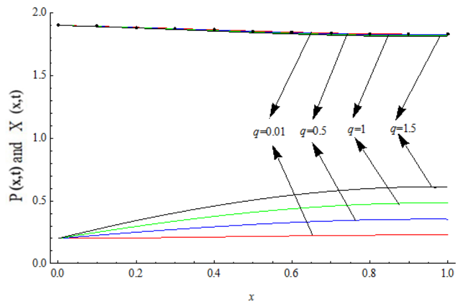

Fig. 5: The variation of P and X with q along the river for $t = 0.5, D_P = 1, v = 1, k = 1, D_X = 1, k_3 = 0.5, \alpha = 2, S = 2$ and $q = 0.01, 0.5, 1$ and 1.5.

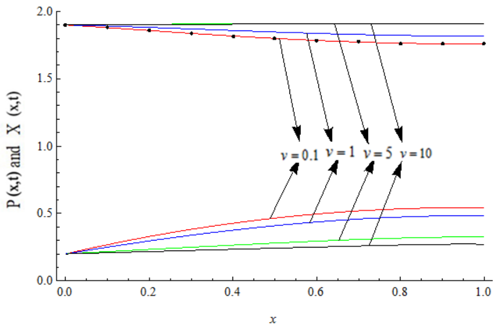

Fig. 3: The variation of both P and X with v along the river for $t = 0.5, D_P = 1, k = 1, q = 1,$ $D_X = 1$ $k_3 = 0.5, \alpha = 2, S = 2$ and $v = 0.1, 1, 5$ and 10.

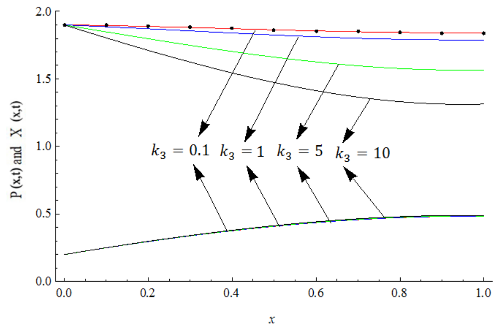

Fig. 6: The variation of both P and X with k_3 along the river for $t = 0.5, D_P = 1, v = 1, k = 1, q = 1,$ $D_X = 1, \alpha = 2, S = 2$ and $k_3 = 0.1, 1, 5$ and 10.

7 Conclusions

We have presented a mathematical model for river pollution and investigated the effect of aeration on the degradation of pollution for all the parameters controlling the flow. We consider the flow one-dimensional along the river. The governing coupled pair of nonlinear unsteady-state equations for river pollutant and dissolved oxygen concentrations are non-dimensionalized by using appropriate transformations. The resulting equations are solved numerically by using explicit finite difference method (EFDM) and the results are plotted. It is found that, as t increases P decreases and X increases. At any constant value of x as k increases values of P increase. As

© 2022 NSP
Natural Sciences Publishing Cor.

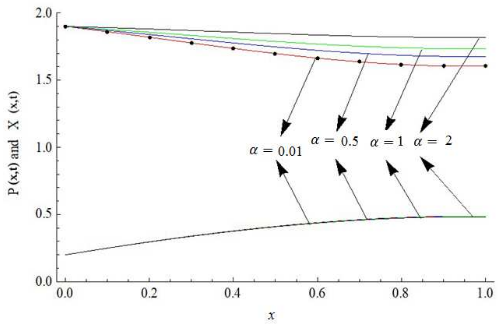

Fig. 7: The variation of both P and X with α along the river for $t = 0.5, D_P = 1, v = 1, k = 1, q = 1,\ D_X = 1, k_3 = 0.5, S = 2$ and $\alpha = 0.01, 0.5, 1$ and 2.

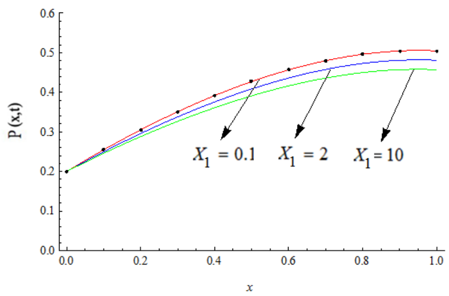

Fig. 8: The variation of P with X_1 along the river for $t = 0.5, D_P = 1, v = 1, k = 1, q = 1, \quad D_X = 1, k_3 = 0.5, \alpha = 2, S = 2$ and $X_1 = 0.1, 2$ and 10.

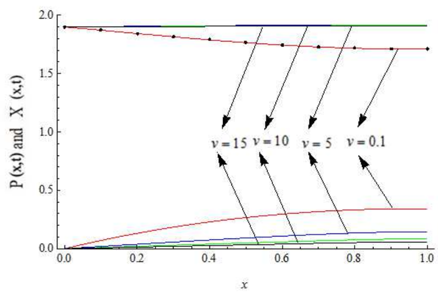

Fig. 9: The variation of P and X with v along the river for $t = 0.5, D_P = 1, k = 1, q = 1, \quad D_X = 1\ k_3 = 0.5, \alpha = 2, S = 2$ and $v = 0.1, 5, 10$ and 15.

v increases, values of P decrease and values of X increase. It is also observed that, as α increases values of X increase. Our results generalize the earlier solutions obtained by Manitcharoen and Pimpunchat [15] and Ibrahim et al. [10].

Acknowledgements

This project was supported financially by the Academy of Scientific Research and Technology (ASRT), Egypt, Grant No (6654), (ASRT) is the second affiliation of this paper.We would like to thank Assistant Lecturer/ Ahmed Saleh Hanafy as he helped us in this research with his constructive scientific discussions with us, Dr. Mahmoud Amin and the many colleagues who contributed in different ways to this work.

Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

References

[1] H.W. Streeter, E.B. Phelps, A study of the pollution and nutrial purification of the Ohio river, U.S. Public Health Service Bulletin, U.S.A., 146, (1925).
[2] K. Huang, M.T.V. Genuchten, R. Zhang, Exact solutions for one-dimensional transport with asymptotic scale-dependent dispersion, Appl. Math. Modelling,20,298-308(1996) .
[3] B. Pimpunchat, W.L. Sweatman, W. Triampo, G.C. Wake, A. Parshotam, Modelling river pollution and removal by aeration, MODSIM 2007, International Congress on Modelling and Simulation. Land, Water and Environmental Management: Integrated Systems for Sustainability, 2431-2437,(2007).
[4] R. Kumar, A. Chatterjee, M. K.Singh, V. P. Singh, Study of solute dispersion with source/sink impact in semi- infinite porous medium, Pollution,6,87-98(2020).
[5] S.A. Hussain, W.G. Atshan, Z.M. Najam, Mathematical model for the concentration of pollution and dissolved oxygen in the diwaniya river (Iraq), American J. Sci. Rese.,78,33-37(2012).
[6] M. F. Dimain, A. S. Wadi, F.N. Ibrahim, The effect of added pollutant along a river on the pollutant concentration described by one-dimensional advection diffusion equation, International Journal of Engineering Science and Technology,5,1662 –1671(2013).
[7] S. Svetislav, D. Alexander, Numerical solution for temporally and spatially dependent solute dispersion of pulse type input concentration in semi-infinite media, Int. J. Heat and Mass transfer,60,291-295(2013).
[8] M.F. Dimian, A.S. Wadi, F.N. Ibrahim, Impact of El- rahawy drain on the water quality of rosetta branch of the Nile River, Egypt, International Journal of Environmental Science and Engineering,5, 17-25(2014) .
[9] A.S. Wadi, M.F. Dimian, F.N. Ibrahim, Analytical solutions for one-dimensional advection–dispersion equation of the pollutant concentration, J. Earth Syst. Sci.,123, 1317-1324(2014).
[10] F.N. Ibrahim, M.F. Dimain, A.S. Wadi, Remediation of pollution in a river by unsteady aeration with arbitrary initial and boundary conditions, J. hydro.,525, 793-797(2015).

© 2022 NSP
Natural Sciences Publishing Cor.

[11] D. Alexander, S. Svetislav, Explicit finite- difference solution of two-dimensional solute transport with periodic flow in homogenous porous media, J. Hydrol. Hydromech.,65, 426-432(2017) .
[12] S. Pawarisa, P. Nopparat, Numerical simulation of a one-dimensional water-quality model in a stream using saulyev technique with quadratic interpolation initial-boundary conditions, Abstract and Applied Analysis,2018, 1-7(2018).
[13] R. R. Yadav, L. K. Kumar, Solute transport for pulse type input point source along temporally and spatially dependent flow, Pollution,5, 53-70(2019) .
[14] M. Kh.Hadhouda, T. Z.Abdelwahid, On the numerical solution for pollution in a river and its remediation by two-dimensional unsteady aeration. international journal of applied engineering research, 13, 15824-15829(2018).
[15] N. Manitcharoen, B. Pimpunchat, Analytical and numerical solutions of pollution concentration with uniformly and exponentially increasing forms of sources, J. of Applied Mathematics, 2020, 1-9(2020).
[16] S.C. Chapra, Surface Water quality Modelling, McGraw-Hill., New York, 345, (1997).
[17] B. Pimpunchat, W. L. Sweatman, W. Triampo, G.C. Wake, A. Parshotam, A mathematical model for pollution in a river and its remediation by aeration, App. Math. Letters,22, 304–308(2009) .
[18] Li. Guoyuan, Stream temperature and dissolved oxygen modelling in the flint river basin, thesis requirements for the degree doctor of Philosophy, Georgia (Chapter 1), 2006.

© 2022 NSP
Natural Sciences Publishing Cor.