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Application in multi-attribute decision-making using possibility single-valued neutrosophic hypersoft graphs

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Abstract: Experts frequently employ three-dimensional arguments to steer decisions in neutrosophic environments. In explicit cases, parameters are grouped into sub-classes, and the degree of possibility is utilized to appraise the acceptability of professional opinions for potential outcomes. A multi-attribute decision-making (MADM) process is the most fitting approach that entails these types of settings. It is indispensable to make sure that the attributes are pertinent and non-discriminatory so that the decision-making process remains transparent and fair. In this context, the possibility single-valued neutrosophic hypersoft set (pSVNHSS) is a new hybrid model designed to address the limitations of the possibility intuitionistic fuzzy set and soft set regarding indeterminacy levels and multi-argument approximation functions, respectively. This paper introduces the concept of pSVNHSS and integrates it with graph theory to develop a novel framework called the possibility single-valued neutrosophic hypersoft graph (psvNHSG) for data management based on pSVNHSS information. First, it reviews basic concepts and set-theoretical operations of psvNHSG using examples and illustrations. Furthermore, it confers its products, compositions, and related theorems. By combining pSVNHSS, the derived psvNHSG, the psvNHSG-based incidence matrix, and the score function, an ample MADM algorithm is suggested for selecting an assistant manager in an organization. The adaptability of this new structure is evaluated by comparing it with other existing models.

Keywords: Neutrosophic Set; Hypersoft set; Optimization; Neutrosophic Hypersoft Graph; Decision Making.

1 Introduction

Graph theory is the study of graphs and involves examining properties such as connectivity, paths, cycles, coloring, embedding, and algorithms. It aids in analyzing the structure and behavior of graphs and solving problems like finding the shortest paths, optimizing resource allocation, and identifying key nodes. However, uncertainties can arise depending on the context, and the application of the study can introduce various uncertainties. In graph theory, different sources of uncertainty can be identified. Data uncertainty, which can arise from incomplete or noisy data, can affect the structure and properties of a graph, affecting the accuracy of analyses or predictions. Model uncertainty, which is related to the choice of a graph model to represent a real-world system, can also affect the properties and predictions of the graph. Algorithmic uncertainty, which can be influenced by input data uncertainties, algorithm choice, and parameters, can affect the performance and accuracy of graph algorithms. Dynamic uncertainty, which arises from changes in the underlying system,

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limitations of available data, and the modeling approach used to represent the system, can also affect the graph structure and properties. These uncertainties can significantly impact the accuracy of graph theory applications. Overall, uncertainties in graph theory are an important consideration in many applications, and it is important to be aware of these uncertainties and to take them into account when analyzing or modeling real-world systems using graphs. To cope with such uncertainties, the single-valued neutrosophic graph (SVNG) idea was developed by Broumi et al. [1] in response to the intuitionistic fuzzy graph's (IFG) [2] inadequacy for indeterminacy grade. The single-valued neutrosophic set (SVNS) [3] and conventional graph theory are joined by the SVNG. As a continuation of their research, Broumi et al. [4] examined the isolation and homogeneity of SVNG in more detail. Naz et al. [5] covered specific SVNG operations with remarkable graphical representations. By utilizing the idea of SVNGs, Akram et al. [6] used an algorithmic strategy to tackle a decision-making problem. Numerous researchers made significant contributions to the creation of SVNS and its use in numerous academic disciplines. Molodtsov [7] created the soft set (SS) as a parametrization tool. Thumbakara [8] presented soft graphs. Shah et al. [9] introduced the notions of neutrosophic soft graphs by integrating neutrosophic soft sets [10] with graph theory. By assigning a possibility degree to each approximate element of the corresponding structures, Alkhazaleh et al. [11], Bashir et al. [12], and Karaaslan [13], Husain et al. [14], Noori et al. [15] respectively, characterized possibility fuzzy soft set (pFSS), possibility intuitionistic fuzzy soft set (pIFSS), and possibility neutrosophic soft set (pNSS).
It is required to group the attributes into their corresponding sub-attributive values in the form of sets in a variety of real-world settings. Smarandache [16] proposed the notion of the hypersoft set (HSS) to solve the inadequacy of SSs and to deal with circumstances with multi-argument approximate functions because the present concept of SSs is insufficient and incompatible with such scenarios. The authors like Musa and Asaad [17] and Asaad and Musa [18] developed topological structures in HSS environments. Debnath [19] discussed decision-making applications in intuitionistic HSS environments with interval-type settings. Using hybrids of HSS and possibility grade settings, Rahman et al. [20,21,22] and Zhao et al. [23] presented decision-making applications. Sajid et al. [24] evaluated suppliers in the health care industry using cosine similarity measures of single-valued neutrosophic cubic hypersoft sets. Rahman et al. [25] and Saeed et al. [26] introduced the notions of picture fuzzy hypersoft graphs and their properties. They also discussed their applications in recruitment process and micro-enterprise supermarket investment risk assessment, respectively. Saeed et al. [27] introduced the notions of neutrosophic hypersoft graph (NHSG) and discussed their examples and applications. Recently Smarandache [28,29,30] introduced some new types of HSSs and discussed their examples and applications. Similarly, the researchers like Abdullah et al. [31], and Rahman et al. [32] have made useful contributions in the field of hypersoft sets.
Recently, Rahman et al. [33] introduced 36 novel hybrid set models by integrating the notions of several fuzzy HSS extensions with fuzzy parameterization and possibility degree setting. They explained each theoretical context with illustrative examples. Surya and Vimala [34] discussed the pattern recognition using the similarity measures formulations of complex non-linear Diophantine fuzzy HSS. Subramanian et al. [35] discussed the medical diagnosis using the integrated approach of fuzzy HSS and weight-based support vector machine. Hussein et al. [36] investigated several properties of possibility interval valued neutrosophic hypersoft matrices and formulated the notions for the correlation coefficient. They employed the proposed notions in human resource management for the recruitment process. Al-Hagery and Abdalla Musa [37] explored the properties of possibility neutrosophic HSS to enhance the network security using a cyber-attack detection method based on the proposed model. The following problems cannot be addressed collectively by any algebraic model, according to careful observation and analysis of previous research works:

1. Sometimes the situations happen when decision-makers appraise diverse alternatives but cannot articulate their findings with inclusive certainty. Instead, they use single-valued neutrosophic information to signify their considerations through degrees of truth, indeterminacy, and falsity. This approach permits them to handle ambiguity, hesitation, and incomplete information that frequently happen in real-world decision-making, particularly when the selection of alternatives depends on complex or uncertain parameters.
2. Mostly, the circumstances happen when the issue entails multiple correlated parameters that manipulate the evaluation process, making it indispensable to consider a multi-argument function. This function permits the approximation of sub-parametric disjoint sets, where each subset corresponds to distinct yet related parameter combinations. This approach assists detain the complex relationships and dependencies among parameters, leading to a more precise and inclusive depiction of uncertain or overlapping information in decision-making or modeling processes.
3. When professionals present their findings as approximations of alternatives, these assessments are frequently uncertain or imprecise. To decide how acceptable each alternative is, the evaluations must be quantified through a possibility degree, which measures the degree to which an alternative may be

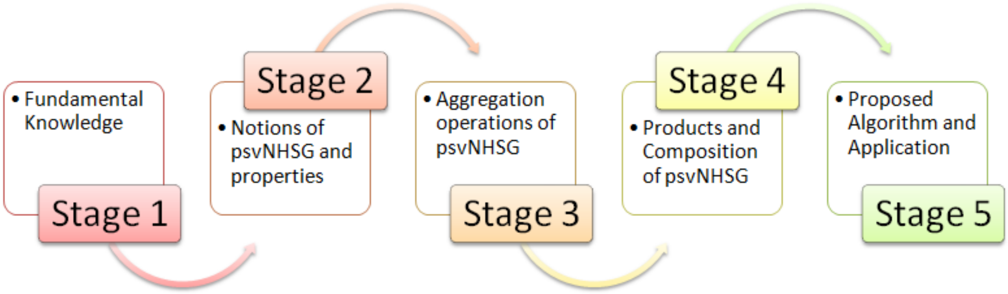

Fig. 1: Methodological stages

regarded as suitable. This process assists rank or compares alternatives based on their degree of acceptability derived from expert opinions.

Since there is no suitable set-theoretic or graph model that can handle all of the aforementioned situations, therefore, the goal of this study is to provide a novel framework called possibility single-valued neutrosophic hypersoft graph (psvNHSG), which is capable of handling all such issues collectively. It can easily address the first issue through its possibility single-valued neutrosophic settings. Similarly, it can address the second and third issues with its hypersoft settings. It can tackle the fourth one by possibility grade settings.The proposed model offers greater adaptability by addressing the issues present in current models for managing uncertainties. It assigns a degree of possibility to each approximate element in its multi-argument approximation to effectively handle the ambiguous behavior of each element.
The section-wise layout of the remaining paper is as follows: Some essential terms are reviewed in Section 2.1 to help readers understand the main results. Section 2.2 introduces the concepts of psvNHSG and its properties. Section 2.3 examines some aggregation operations of psvNHSG. Specific products and compositions of psvNHSG are discussed in Section 2.4 with graphical illustrations and examples. A decision-making framework is developed in Section 2.5 with an algorithm utilizing psvNHSG aggregations. Section 2.6 provides a comparison analysis and discussion of the results. Finally, the overall study is summarized in Section 3, including a brief overview of future scope and limitations.

2 Methodology

This section presents the main methodology of the proposed study. It begins with a review of fundamental definitions and then examines the main proposed concepts. The Figure 1 presents the different stage involved in the proposed methodology.

2.1 Elementary Knowledge

To facilitate readers' better understanding of the main proposed concepts, this section aims to recall some essential basic definitions from the published literature.

Definition 1.[3]
A SVNS $\mathcal{R}$ defined as $\mathcal{R} = \{(\hat{u}, <\mathcal{A}_{\mathcal{R}}(\hat{u}), \mathcal{B}_{\mathcal{R}}(\hat{u}), \mathcal{C}_{\mathcal{R}}(\hat{u})>) | \hat{u} \in \mathcal{U}\}$ such that $\mathcal{A}_{\mathcal{R}}(\hat{u}), \mathcal{B}_{\mathcal{R}}(\hat{u}), \mathcal{C}_{\mathcal{R}}(\hat{u}) : \mathcal{U} \to [0,1]$, where $\mathcal{A}_{\mathcal{R}}(\hat{u}), \mathcal{B}_{\mathcal{R}}(\hat{u})$ and $\mathcal{C}_{\mathcal{R}}(\hat{u})$ represent the grades of membership, indeterminacy and non-membership of $\hat{u} \in \mathcal{U}$ subject to the condition that $0 \leq \mathcal{A}_{\mathcal{R}}(\hat{u}) + \mathcal{B}_{\mathcal{R}}(\hat{u}) + \mathcal{C}_{\mathcal{R}}(\hat{u}) \leq 3$.

Definition 2.[7]
A SS over $\mathcal{U}$ is a pair $(\mathcal{F}_{\mathcal{S}}, \mathscr{E})$, which is defined by an approximate mapping $\mathcal{F}_{\mathcal{S}} : \mathscr{E} \to \mathbb{P}(\mathcal{U})$ such that $\mathcal{F}_{\mathcal{S}}(\hat{h}) \subseteq \mathbb{P}(\mathcal{U})$ where $\mathscr{E}$ is a collection of parameters and

Definition 3.[10]
A pair $(\mathcal{M}_{\mathcal{NS}}, \mathcal{Z})$ is called a NSS over $\mathcal{U}$, where $\mathcal{M}_{\mathcal{FS}} : \mathcal{Z} \to \mathcal{N}(\mathcal{U})$ where $\mathcal{Z} \subseteq \mathscr{E}$ and $\mathcal{N}(\mathcal{U})$ is the collection of neutrosophic subsets over $\mathcal{U}$.

Definition 4.[16]

A HSS over $\mathcal{U}$ is a set of objects $(\mathcal{W},\mathcal{H})$, such that $\mathcal{H} = \bigsqcup_{i=1}^{n} \mathcal{H}^i$, and $\mathcal{H}^i$ are non overlapping sets consisting of sub-parametric values of parameters $\hat{h}^i, i = 1,2,3,...,n, \hat{h}^i \neq \hat{h}^j, i \neq j$ respectively and $\mathcal{W} : \mathcal{H} \to \mathbb{P}(\mathcal{U})$. Any HSS is claimed to be NHSS when $\mathbb{P}(\mathcal{U})$ is substituted by $\mathbb{N}(\mathcal{U})$ (a family consisting of neutrosophic subsets over $\mathcal{U}$).

2.2 Possibility Single-Valued Neutrosophic Hypersoft Graphs (psvNHSG)

The basic notions of psvNHSG are characterized with the description of graph-based presentations.
Contrasting to available neutrosophic and hypersoft graph frameworks, the suggested structure psvNHSG concurrently incorporates multi-parameter, multi-sub-attribute structures along with possibility-based truth, indeterminacy, and falsity components, enabling a more communicative illustration of uncertainty. This arrangement permits more improved decision-making in environments where both hierarchical parameters and degrees of possibility are indispensable, which is not addressed in conventional neutrosophic or hypersoft graph-based approaches. The manuscript has been updated to explicitly emphasize these discerning aspects and their benefits over existing models. Now $\mathscr{A} = (\mathscr{V},\mathscr{E})$ will represent as simple graph where $\mathscr{V}$ is a set consisting of vertices and $\mathscr{E}$ is consisting of edges, the $\mathbb{E}$ is consisting of parameters and the disjoint sets $\mathscr{Q}^i$ consisting of sub-parametric values with respect to distinct parameters $\hat{e}_i, i = 1,2,...,n$ of $\mathbb{E}$. Also $\mathscr{Q} = \mathscr{Q}^1 \subseteq \mathscr{Q}^2 \subseteq \mathscr{Q}^3 \subseteq ... \subseteq \mathscr{Q}^n$.

Definition 5.*A possibility single-valued neutrosophic set (pSVNS) $\mathbb{P}_N$ over $\mathcal{U}$ is stated as $\mathbb{P}_N = \{(\hat{u}, < \mathcal{A}_{\mathcal{R}}(\hat{u}), \mathcal{B}_{\mathcal{R}}(\hat{u}), \mathcal{C}_{\mathcal{R}}(\hat{u}) >, \Delta(\hat{u})) | \hat{u} \in \mathcal{U}\}$ where $\mathcal{A}_{\mathcal{R}}(\hat{u})$, $\mathcal{B}_{\mathcal{R}}(\hat{u})$, $\mathcal{C}_{\mathcal{R}}(\hat{u})$ are uncertain components of SVNS and $\Delta : \mathcal{U} \to [0,1]$ with $\Delta(\hat{u})$ is the possibility degree of $\hat{u}$ to $\mathbb{P}_N$. The collection of all pSVNSs over $\mathcal{U}$ is represented by $\Omega_{\mathbb{P}\mathbb{N}}(\mathcal{U})$.*

The following definition is an extension and motivation of the concept presented by Saeed et al. [27].

Definition 6.*A psvNHSG is a 4-tuple $\mathscr{A} = (\mathscr{A},\mathscr{Q},Æ,\Xi)$ where $Æ : \mathscr{Q} \to \mathbb{P}_N(\mathscr{V}), \Xi : \mathscr{Q} \to \mathbb{P}_N(\mathscr{V} \subseteq \mathscr{V})$ given by $Æ(\sigma) = Æ_\sigma = \left\{ (\hat{v}, < \mathscr{T}_{Æ_\sigma}(\hat{v}), \mathscr{I}_{Æ_\sigma}(\hat{v}), \mathscr{F}_{Æ_\sigma}(\hat{v}) >, \mu(\hat{v})), \hat{v} \in \mathscr{V} \right\}$ and*

$$\Xi(\sigma) = \Xi_\sigma = \left\{ \begin{array}{l} \left((\hat{v}_1,\hat{v}_2), \left\langle \begin{array}{l} \mathscr{T}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2), \\ \mathscr{I}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2), \\ \mathscr{F}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2) \end{array} \right\rangle, \right. \\ \left. \mu_{Æ_\sigma}(\hat{v}_1,\hat{v}_2)\right), (\hat{v}_1,\hat{v}_2) \in \mathscr{V} \subseteq \mathscr{V} \end{array} \right\}$$

are pSVNSs over $\mathscr{V}$ and $\mathscr{V} \subseteq \mathscr{V}$ with $\mathscr{T}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2) \leq \min\{\mathscr{T}_{Æ_\sigma}(\hat{v}_1), \mathscr{T}_{Æ_\sigma}(\hat{v}_2)\}$, $\mathscr{I}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2) \leq \min\{\mathscr{I}_{Æ_\sigma}(\hat{v}_1), \mathscr{I}_{Æ_\sigma}(\hat{v}_2)\}$, $\mathscr{F}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2) \geq \max\{\mathscr{F}_{Æ_\sigma}(\hat{v}_1), \mathscr{F}_{Æ_\sigma}(\hat{v}_2)\}$, $\mu_{Æ_\sigma}(\hat{v}_1,\hat{v}_2) \leq \min\{\mu_{Æ_\sigma}(\hat{v}_1), \mu_{Æ_\sigma}(\hat{v}_2)\}$, $(\hat{v}_1,\hat{v}_2) \in (\mathscr{V} \subseteq \mathscr{V})$ and $\sigma \in \mathscr{Q}$. Note: The collection of all psvNHSGs is represented by $\Omega_{psvNHSG}$.

*Example 1.*Let $\mathscr{A} = (\mathscr{V},\mathscr{E})$ be a simple graph with $\mathscr{V} = \{\hat{v}_1,\hat{v}_2,\hat{v}_3\}$ and $\mathscr{Q}_1 = \{\hat{q}_{11},\hat{q}_{12}\}$, $\mathscr{Q}_2 = \{\hat{q}_{21},\hat{q}_{22}\}$ and $\mathscr{Q}_3 = \{\hat{q}_{31}\}$ such that $\mathscr{Q} = \mathscr{Q}_1 \subseteq \mathscr{Q}_2 \subseteq \mathscr{Q}_3 = \{\sigma_1,\sigma_2,\sigma_3,\sigma_4\}$ and $\mathscr{T}_{\Xi_\sigma}(\hat{v}_i,\hat{v}_j) = 0, \mathscr{I}_{\Xi_\sigma}(\hat{v}_i,\hat{v}_j) = 0, \mathscr{F}_{\Xi_\sigma}(\hat{v}_i,\hat{v}_j) = 1, \mu_{\Xi_\sigma}(\hat{v}_i,\hat{v}_j) = 0 \ \ (\hat{v}_i,\hat{v}_j) \in \mathscr{V} \subseteq \mathscr{V} \setminus \{(\hat{v}_1,\hat{v}_2),(\hat{v}_2,\hat{v}_3),(\hat{v}_1,\hat{v}_3)\}$. The Table 1 and Figure 2 presents its numerical tabular-form and graph-based presentation respectively.

Definition 7.*A psvNHSG $\mathscr{G} = (\mathscr{A},\mathscr{Q},Æ^1,\Xi^1)$ is called a psvNHS-subgraph of $\mathscr{A} = (\mathscr{A},\mathscr{Q},Æ,\Xi)$ if*

1. $\mathscr{Q} \subseteq \mathscr{Q}$

2. $Æ^1_\sigma \subseteq Æ$ implies $\mathscr{T}_{Æ^1_\sigma}(\hat{v}) \leq \mathscr{T}_{Æ_\sigma}(\hat{v}), \mathscr{I}_{Æ^1_\sigma}(\hat{v}) \leq \mathscr{I}_{Æ_\sigma}(\hat{v}), \mathscr{F}_{Æ^1_\sigma}(\hat{v}) \geq \mathscr{F}_{Æ_\sigma}(\hat{v}), \mu_{Æ^1_\sigma}(\hat{v}) \leq \mu_{Æ_\sigma}(\hat{v})$

3. $\Xi^1_\sigma \subseteq \Xi$ implies $\mathscr{T}_{\Xi^1_\sigma}(\hat{v}) \leq \mathscr{T}_{\Xi_\sigma}(\hat{v}), \mathscr{I}_{\Xi^1_\sigma}(\hat{v}) \leq \mathscr{I}_{\Xi_\sigma}(\hat{v}), \mathscr{F}_{\Xi^1_\sigma}(\hat{v}) \geq \mathscr{F}_{\Xi_\sigma}(\hat{v}), \mu_{\Xi^1_\sigma}(\hat{v}) \leq \mu_{\Xi_\sigma}(\hat{v})$

$\sigma \in \mathscr{Q}$.

*Example 2.*Repeating the Example 1 with $\mathscr{Q}_1 = \{\alpha_{11},\alpha_{12}\}$, $\mathscr{Q}_2 = \{\alpha_{21}\}$ and $\mathscr{Q}_3 = \{\alpha_{31}\}$, $\mathscr{Q} = \mathscr{Q}_1 \subseteq \mathscr{Q}_2 \subseteq \mathscr{Q}_3 = \{\sigma_1,\sigma_2,\sigma_3\}$, it gives a new psvNHSG $\mathscr{A} = (\mathscr{A},\mathscr{Q},Æ^1,\Xi^1)$ which is psvNHS-subgraph of psvNHSG given in Example 1. Its tabular-form and graph-based presentation are provided in Table 2 and Figure **??** respectively.

Definition 8.*A psvNHS-subgraph $(\mathscr{A},\mathscr{Q},Æ^1,\Xi^1)$ is called a psvNHS-spanning subgraph of psvNHSG $(\mathscr{A},\mathscr{Q},Æ,\Xi)$ when $Æ^1_\sigma(\hat{v}) = Æ_\sigma(\hat{v}) \ \ \hat{v} \in \mathscr{V}, \sigma \in \mathscr{Q}$.1*

Definition 9.*A psvNHS-subgraph $(\mathscr{A},\mathscr{Q},Æ^1,\Xi^1)$ is called a strong psvNHS-subgraph (SSVNHS-subgraph) of psvNHSG $(\mathscr{A},\mathscr{Q},Æ,\Xi)$ when $\Xi_\sigma(\hat{v}_1,\hat{v}_2) = Æ_\sigma(\hat{v}_1) \Im Æ_\sigma(\hat{v}_2)$ for $\hat{v}_1,\hat{v}_2 \in \mathscr{V}$ and $\sigma \in \mathscr{Q}$.*

Table 1: Numerical Computation of Example 1 with (a) $\mathbb{P}_N(\sigma_1)$, (b) $\mathbb{P}_N(\sigma_2)$, (c) $\mathbb{P}_N(\sigma_3)$ and (d) $\mathbb{P}_N(\sigma_4)$

Æ	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$	$\hat{\vartheta}_3$
σ_1	(0.2,0.4,0.7,0.2)	(0.3,0.6,0.3,0.3)	(0,0,1,0)
σ_2	(0.1,0.4,0.3,0.2)	(0.3,0.4,0.5,0.1)	(0,0,1,0)
σ_3	(0.1,0.5,0.6,0.3)	(0.3,0.3,0.8,0.2)	(0.3,0.2,0.5,0.1)
σ_4	(0.4,0.2,0.6,0.3)	(0.3,0.6,0.5,0.4)	(0.4,0.3,0.6,0.2)
Ξ	$(\hat{\vartheta}_1,\hat{\vartheta}_2)$	$(\hat{\vartheta}_2,\hat{\vartheta}_3)$	$(\hat{\vartheta}_1,\hat{\vartheta}_3)$
σ_1	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)
σ_2	(0.1,0.3,0.2,0.4)	(0,0,1,0)	(0,0,1,0)
σ_3	(0.1,0.5,0.4,0.3)	(0.2,0.4,0.3,0.1)	(0,0,1,0)
σ_4	(0.2,0.3,0.4,0.6)	(0.2,0.5,0.3,0.4)	(0.4,0.2,0.7,0.5)

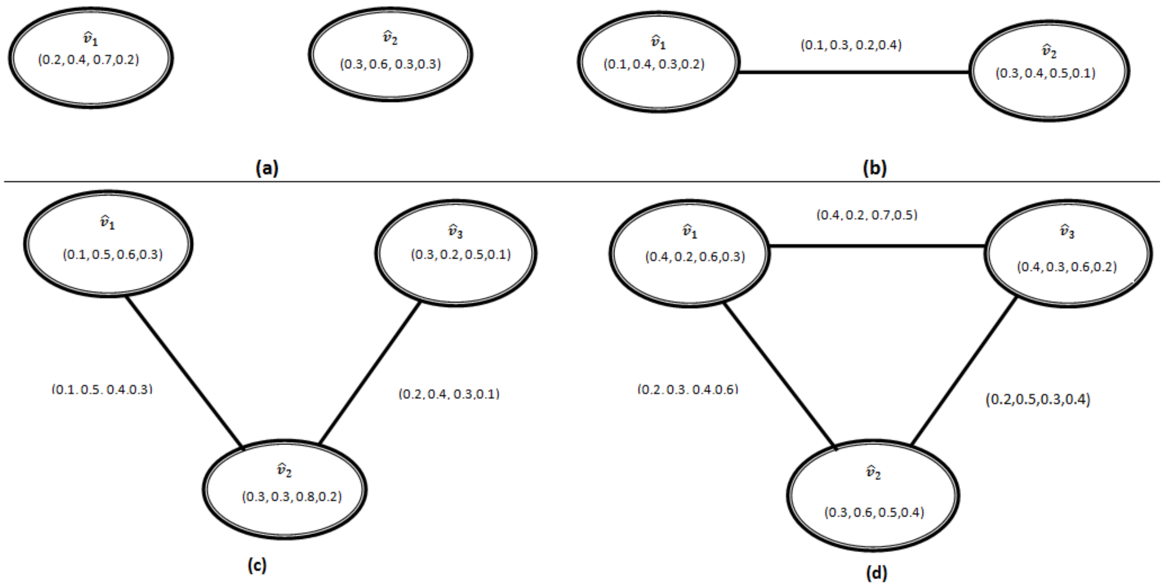

Fig. 2: Geometrical Interpretation of Table 1

Table 2: Tabular-form of Example 2 with (a) $\mathbb{P}_N(\sigma_1)$, (b) $\mathbb{P}_N(\sigma_2)$ and (c) $\mathbb{P}_N(\sigma_3)$

Æ	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$	$\hat{\vartheta}_3$
σ_1	(0.1,0.3,0.8,0.1)	(0.2,0.3,0.4,0.2)	(0,0,1,0)
σ_2	(0.1,0.2,0.4,0.1)	(0.2,0.3,0.8,0.1)	(0,0,1,0)
σ_3	(0.1,0.4,0.7,0.2)	(0.2,0.2,0.9,0.1)	(0.2,0.1,0.6,0.1)
Ξ	$(\hat{\vartheta}_1,\hat{\vartheta}_2)$	$(\hat{\vartheta}_2,\hat{\vartheta}_3)$	$(\hat{\vartheta}_1,\hat{\vartheta}_3)$
σ_1	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)
σ_2	(0.1,0.2,0.3,0.1)	(0,0,1,0)	(0,0,1,0)
σ_3	(0.1,0.4,0.5,0.2)	(0.1,0.3,0.4,0.1)	(0,0,1,0)

2.3 Aggregation Operations of psvNHSG

Some aggregation operations of psvNHSG are investigated and illustrated by graph representations.

Definition 10. *The union of two psvNHSGs* $\mathscr{A}_1 = (\mathscr{A}_1, \mathscr{Q}^1, Æ^1, \Xi^1)$, $\mathscr{A}_2 = (\mathscr{A}_2, \mathscr{Q}^2, Æ^2, \Xi^2)$, *denoted by* $\mathscr{A}_1 \Re \mathscr{A}_2$, *is a psvNHSG* $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ *such that* $\mathscr{Q} = \mathscr{Q}^1 \Re \mathscr{Q}^2$. *In this graph,*
$\mathscr{T}_{Æ_\sigma}(\hat{\vartheta}) =$

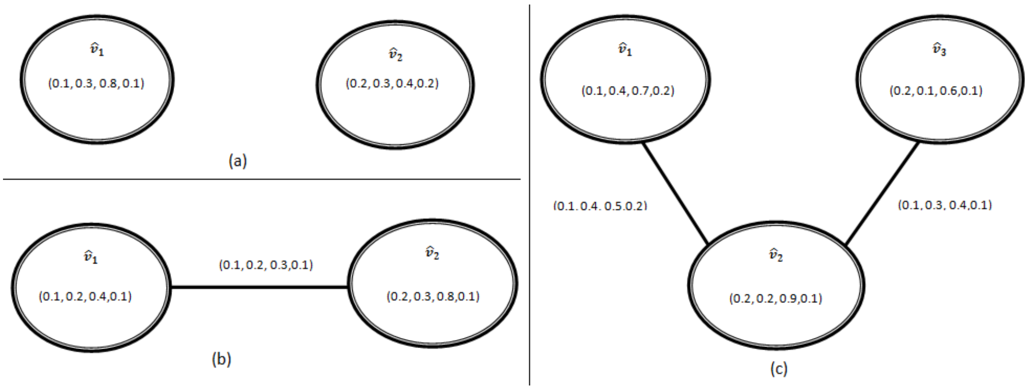

Fig. 3: Graph-based presentation of Table 2

$$
\begin{cases}
\mathscr{T}_{\text{Æ}_\sigma^1}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2 \end{cases} \\
\mathscr{T}_{\text{Æ}_\sigma^2}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \Im \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1 \end{cases} \\
\max\left\{\mathscr{T}_{\text{Æ}_\sigma^1}(\hat{v}), \mathscr{T}_{\text{Æ}_\sigma^2}(\hat{v})\right\}\left\{if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\right. \\
0, otherwise
\end{cases} \quad \&
$$

$$
\mathscr{I}_{\text{Æ}_\sigma}(\hat{v}) = \begin{cases}
\mathscr{I}_{\text{Æ}_\sigma^1}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2 \end{cases} \\
\mathscr{I}_{\text{Æ}_\sigma^2}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \Im \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1 \end{cases} \\
\max\left\{\mathscr{I}_{\text{Æ}_\sigma^1}(\hat{v}), \mathscr{I}_{\text{Æ}_\sigma^2}(\hat{v})\right\}\left\{if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\right. \\
0, otherwise
\end{cases} \quad \&
$$

$$
\mathscr{F}_{\text{Æ}_\sigma}(\hat{v}) =
$$

$$
\begin{cases}
\mathscr{F}_{\text{Æ}_\sigma^1}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2 \end{cases} \\
\mathscr{F}_{\text{Æ}_\sigma^2}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \Im \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1 \end{cases} \\
\min\left\{\mathscr{F}_{\text{Æ}_\sigma^1}(\hat{v}), \mathscr{F}_{\text{Æ}_\sigma^2}(\hat{v})\right\}\left\{if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\right. \\
0, otherwise
\end{cases} \quad \&
$$

$$
\mu_{\text{Æ}_\sigma}(\hat{v}) =
$$

$$
\begin{cases}
\mu_{\text{Æ}_\sigma^1}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \circ \mathscr{V}_2 \end{cases} \\
\mu_{\text{Æ}_\sigma^2}(\hat{v}) \begin{cases} if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \,\&\, \hat{v} \in \mathscr{V}_2 \Im \mathscr{V}_1\, or \\ if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_2 \circ \mathscr{V}_1 \end{cases} \\
\max\left\{\mu_{\text{Æ}_\sigma^1}(\hat{v}), \mu_{\text{Æ}_\sigma^2}(\hat{v})\right\}\left\{if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \,\&\, \hat{v} \in \mathscr{V}_1 \Im \mathscr{V}_2\right. \\
0, otherwise
\end{cases} .
$$

Also the neutrosophic components for Ξ are given as follows:

Theorem 1. *If $\mathscr{A}_1, \mathscr{A}_2 \in \Omega_{NHSG}$ then $\mathscr{A}_1 \Re \mathscr{A}_2 \in \Omega_{NHSG}$.*

Proof. Consider two psvNHSGs $\mathscr{A}_1 = \left(\mathscr{A}_1, \mathscr{Q}^1, \text{Æ}^1, \Xi^1\right)$ and $\mathscr{A}_2 = \left(\mathscr{A}_2, \mathscr{Q}^2, \text{Æ}^2, \Xi^2\right)$. Let $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, \text{Æ}, \Xi)$ be the union of psvNHSGs $\mathscr{A}_1$ and $\mathscr{A}_2$ where $\mathscr{Q} = \mathscr{Q}^1 \Re \mathscr{Q}^2$.
Now let $\sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2$ and $(\hat{v}_1, \hat{v}_2) \in (\mathscr{V}_1 \subseteq \mathscr{V}_1) \circ (\mathscr{V}_2 \subseteq \mathscr{V}_2)$, then

$$\mathscr{T}_{\Xi_\sigma}(\hat{\upsilon}_1)=\begin{cases}\mathscr{T}_{\Xi^1_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\end{cases}\\ \mathscr{T}_{\Xi^2_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\Im(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\end{cases}\\ \max\left\{\mathscr{T}_{\Xi^1_\sigma}(\hat{\upsilon}_1),\mathscr{T}_{\Xi^2_\sigma}(\hat{\upsilon}_1)\right\}\left\{if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\right.\\ 0,\,otherwise\end{cases}\quad\&$$

$$\mathscr{I}_{\Xi_\sigma}(\hat{\upsilon}_1)=\begin{cases}\mathscr{I}_{\Xi^1_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\end{cases}\\ \mathscr{I}_{\Xi^2_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\Im(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\end{cases}\\ \max\left\{\mathscr{I}_{\Xi^1_\sigma}(\hat{\upsilon}_1),\mathscr{I}_{\Xi^2_\sigma}(\hat{\upsilon}_1)\right\}\left\{if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\right.\\ 0,\,otherwise\end{cases}$$

$$\mathscr{F}_{\Xi_\sigma}(\hat{\upsilon}_1)=\begin{cases}\mathscr{F}_{\Xi^1_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\end{cases}\\ \mathscr{F}_{\Xi^2_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\Im(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\end{cases}\\ \min\left\{\mathscr{F}_{\Xi^1_\sigma}(\hat{\upsilon}_1),\mathscr{F}_{\Xi^2_\sigma}(\hat{\upsilon}_1)\right\}\left\{if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\right.\\ 0,\,otherwise\end{cases}\quad\&$$

$$\mu_{\Xi_\sigma}(\hat{\upsilon}_1)=\begin{cases}\mu_{\Xi^1_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)\end{cases}\\ \mu_{\Xi^2_\sigma}(\hat{\upsilon}_1)\begin{cases} if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\Im(\mathscr{V}_1\subseteq\mathscr{V}_1)\;or\\ if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_2\subseteq\mathscr{V}_2)\circ(\mathscr{V}_1\subseteq\mathscr{V}_1)\end{cases}\\ \max\left\{\mu_{\Xi^1_\sigma}(\hat{\upsilon}_1),\mu_{\Xi^2_\sigma}(\hat{\upsilon}_1)\right\}\left\{if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\,\&\,(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)\right.\\ 0,\,otherwise\end{cases}.$$

$\mathscr{T}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)=\mathscr{T}_{\Xi^1_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\leq\min\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{\upsilon}_1),\mathscr{T}_{Æ^1_\sigma}(\hat{\upsilon}_2)\right\}$
$=\min\left\{\mathscr{T}_{Æ_\sigma}(\hat{\upsilon}_1),\mathscr{T}_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$ so
$\mathscr{T}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\leq\min\left\{\mathscr{T}_{Æ_\sigma}(\hat{\upsilon}_1),\mathscr{T}_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$.
Also
$\mathscr{I}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)=\mathscr{I}_{\Xi^1_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\leq\min\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{\upsilon}_1),\mathscr{I}_{Æ^1_\sigma}(\hat{\upsilon}_2)\right\}$
$=\min\left\{\mathscr{I}_{Æ_\sigma}(\hat{\upsilon}_1),\mathscr{I}_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$ so
$\mathscr{I}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\leq\min\left\{\mathscr{I}_{Æ_\sigma}(\hat{\upsilon}_1),\mathscr{I}_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$.
Now
$\mathscr{F}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)=\mathscr{F}_{\Xi^1_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\geq\max\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{\upsilon}_1),\mathscr{F}_{Æ^1_\sigma}(\hat{\upsilon}_2)\right\}$
$=\max\left\{\mathscr{F}_{Æ_\sigma}(\hat{\upsilon}_1),\mathscr{F}_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$ so
$\mathscr{F}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\geq\max\left\{\mathscr{F}_{Æ_\sigma}(\hat{\upsilon}_1),\mathscr{F}_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$.
Similar results are obtained when $\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2$ and
$(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)$
or $\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2$ and $(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\circ(\mathscr{V}_2\subseteq\mathscr{V}_2)$.
Now if $\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2$ and $(\hat{\upsilon}_1,\hat{\upsilon}_2)\in(\mathscr{V}_1\subseteq\mathscr{V}_1)\Im(\mathscr{V}_2\subseteq\mathscr{V}_2)$ then
$\mathscr{T}_{\Xi_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)=\max\left\{\mathscr{T}_{\Xi^1_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2),\mathscr{T}_{\Xi^2_\sigma}(\hat{\upsilon}_1,\hat{\upsilon}_2)\right\}$
$\leq\max\left\{\begin{matrix}\min\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{\upsilon}_1),\mathscr{T}_{Æ^1_\sigma}(\hat{\upsilon}_2)\right\},\\ \min\left\{\mathscr{T}_{Æ^2_\sigma}(\hat{\upsilon}_1),\mathscr{T}_{Æ^2_\sigma}(\hat{\upsilon}_2)\right\}\end{matrix}\right\}$

Table 3: Tabular form of Example 3

Æ	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$	$\hat{\vartheta}_3$
σ_1	(0.2,0.3,0.4,0.2)	(0.3,0.6,0.8,0.3)	(0.3,0.4,0.5,0.3)
σ_2	(0.2,0.4,0.8,0.2)	(0.2,0.3,0.4,0.2)	(0.5,0.7,0.8,0.5)
σ_3	(0.6,0.7,0.8,0.6)	(0.4,0.5,0.7,0.4)	(0.7,0.9,0.9,0.7)
Ξ	$(\hat{\vartheta}_1,\hat{\vartheta}_2)$	$(\hat{\vartheta}_2,\hat{\vartheta}_3)$	$(\hat{\vartheta}_1,\hat{\vartheta}_3)$
σ_1	(0.2,0.3,0.6,0.2)	(0.2,0.4,0.9,0.2)	(0.2,0.3,0.8,0.2)
σ_2	(0.2,0.3,0.9,0.2)	(0.2,0.2,0.9,0.2)	(0.2,0.3,0.8,0.2)
σ_3	(0,0,1,0)	(0.3,0.4,0.9,0.3)	(0.2,0.4,0.9,0.2)

$\leq \min\left\{\begin{array}{l}\max\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{\vartheta}_1),\mathscr{T}_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \max\left\{\mathscr{T}_{Æ^2_\sigma}(\hat{\vartheta}_1),\mathscr{T}_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$=\min\left\{\mathscr{T}_{Æ_\sigma}(\hat{\vartheta}_1),\mathscr{T}_{Æ_\sigma}(\hat{\vartheta}_2)\right\}.$
$\mathscr{I}_{\Xi_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2)=\max\left\{\mathscr{I}_{\Xi^1_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2),\mathscr{I}_{\Xi^2_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2)\right\}$
$\leq \max\left\{\begin{array}{l}\min\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{\vartheta}_1),\mathscr{I}_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \min\left\{\mathscr{I}_{Æ^2_\sigma}(\hat{\vartheta}_1),\mathscr{I}_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$\leq \min\left\{\begin{array}{l}\max\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{\vartheta}_1),\mathscr{I}_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \max\left\{\mathscr{I}_{Æ^2_\sigma}(\hat{\vartheta}_1),\mathscr{I}_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$=\min\left\{\mathscr{I}_{Æ_\sigma}(\hat{\vartheta}_1),\mathscr{I}_{Æ_\sigma}(\hat{\vartheta}_2)\right\}.$
$\mathscr{F}_{\Xi_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2)=\min\left\{\mathscr{F}_{\Xi^1_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2),\mathscr{F}_{\Xi^2_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2)\right\}$
$\geq \min\left\{\begin{array}{l}\max\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{\vartheta}_1),\mathscr{F}_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \max\left\{\mathscr{F}_{Æ^2_\sigma}(\hat{\vartheta}_1),\mathscr{F}_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$\geq \max\left\{\begin{array}{l}\min\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{\vartheta}_1),\mathscr{F}_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \min\left\{\mathscr{F}_{Æ^2_\sigma}(\hat{\vartheta}_1),\mathscr{F}_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$=\max\left\{\mathscr{F}_{Æ_\sigma}(\hat{\vartheta}_1),\mathscr{F}_{Æ_\sigma}(\hat{\vartheta}_2)\right\}.$
$\mu_{\Xi_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2)=\max\left\{\mu_{\Xi^1_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2),\mu_{\Xi^2_\sigma}(\hat{\vartheta}_1,\hat{\vartheta}_2)\right\}$
$\leq \max\left\{\begin{array}{l}\min\left\{\mu_{Æ^1_\sigma}(\hat{\vartheta}_1),\mu_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \min\left\{\mu_{Æ^2_\sigma}(\hat{\vartheta}_1),\mu_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$\leq \min\left\{\begin{array}{l}\max\left\{\mu_{Æ^1_\sigma}(\hat{\vartheta}_1),\mu_{Æ^1_\sigma}(\hat{\vartheta}_2)\right\},\\ \max\left\{\mu_{Æ^2_\sigma}(\hat{\vartheta}_1),\mu_{Æ^2_\sigma}(\hat{\vartheta}_2)\right\}\end{array}\right\}$
$=\min\left\{\mu_{Æ_\sigma}(\hat{\vartheta}_1),\mu_{Æ_\sigma}(\hat{\vartheta}_2)\right\}$. Hence the union $\mathscr{A}=\mathscr{A}_1\Re\mathscr{A}_2$ is psvNHSGs.

*Example 3.*Let $\mathscr{A}_1=(\mathscr{A}_1,\mathscr{Q}^1,Æ^1,\Xi^1)$ be a psvNHSG where $\mathscr{A}_1=(\mathscr{V}_1,\mathscr{E}_1)$ with $\mathscr{V}_1=\{\hat{\vartheta}_1,\hat{\vartheta}_2,\hat{\vartheta}_3\}$ and $\mathscr{Q}_1=\{\alpha_{11}\}$, $\mathscr{Q}_2=\{\alpha_{21}\}$ and $\mathscr{Q}_3=\{\alpha_{31},\alpha_{32},\alpha_{33}\}$ such that $\mathscr{Q}^1=\mathscr{Q}_1\subseteq\mathscr{Q}_2\subseteq\mathscr{Q}_3=\{\sigma_1,\sigma_2,\sigma_3\}$ and $\mathscr{T}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0,\mathscr{I}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0,\mathscr{F}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=1,\mu_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0$ $(\hat{\vartheta}_i,\hat{\vartheta}_j)\in\mathscr{V}_1\subseteq\mathscr{V}_1\backslash\{(\hat{\vartheta}_1,\hat{\vartheta}_2),(\hat{\vartheta}_2,\hat{\vartheta}_3),(\hat{\vartheta}_1,\hat{\vartheta}_3)\}$. Its tabulation is given in Table 3. Also let $\mathscr{A}_2=\left(\mathscr{A}_2,\mathscr{Q}^2,Æ^2,\Xi^2\right)$ be a psvNHSG where $\mathscr{A}_2=(\mathscr{V}_2,\mathscr{E}_2)$ with $\mathscr{V}_2=\{\hat{\vartheta}_3,\hat{\vartheta}_4,\hat{\vartheta}_5\}$ and $\mathscr{Q}_3=\{\alpha_{31},\alpha_{32}\}$, $\mathscr{Q}_4=\{\alpha_{41}\}$ such that $\mathscr{Q}_5=\{\alpha_{51}\}$. $\mathscr{Q}^2=\mathscr{Q}_3\subseteq\mathscr{Q}_4\subseteq\mathscr{Q}_5=\{\sigma_2,\sigma_4\}$ and $\mathscr{T}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0,\mathscr{I}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0,\mathscr{F}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=1,\mu_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0$ $(\hat{\vartheta}_i,\hat{\vartheta}_j)\in\mathscr{V}_2\subseteq\mathscr{V}_2\backslash\{(\hat{\vartheta}_3,\hat{\vartheta}_4),(\hat{\vartheta}_4,\hat{\vartheta}_5),(\hat{\vartheta}_3,\hat{\vartheta}_5)\}$. Its tabulation is given in Table 4. Now Let $\mathscr{A}=\mathscr{A}_1\Re\mathscr{A}_2$ with $\mathscr{Q}=\mathscr{Q}^1\Re\mathscr{Q}^2$ and $\mathscr{T}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0,\mathscr{I}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0,\mathscr{F}_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=1,\mu_{\Xi_\sigma}(\hat{\vartheta}_i,\hat{\vartheta}_j)=0$ $(\hat{\vartheta}_i,\hat{\vartheta}_j)\in\mathscr{V}\subseteq\mathscr{V}\backslash\{(\hat{\vartheta}_1,\hat{\vartheta}_2),(\hat{\vartheta}_1,\hat{\vartheta}_3),(\hat{\vartheta}_2,\hat{\vartheta}_3),(\hat{\vartheta}_3,\hat{\vartheta}_4),(\hat{\vartheta}_3,\hat{\vartheta}_5),(\hat{\vartheta}_4,\hat{\vartheta}_5)\}$. Its tabulation is given in Table 5.

Definition 11.*The intersection of two psvNHSGs $\mathscr{G}_1=\left(\mathscr{G}_1,\mathscr{Q}^1,Æ^1,\Xi^1\right)$, $\mathscr{G}_2=\left(\mathscr{G}_2,\mathscr{Q}^2,Æ^2,\Xi^2\right)$, denoted by $\mathscr{G}_1\Im\mathscr{G}_2$, is a psvNHSG $\mathscr{G}=(\mathscr{G},\mathscr{Q},Æ,\Xi)$ such that $\mathscr{Q}=\mathscr{Q}^1\Im\mathscr{Q}^2,\mathscr{V}=\mathscr{V}_1\Im\mathscr{V}_2$. The uncertain parts in this graph, for Æ are as follows:*

$$\mathscr{T}_{Æ_\sigma}=\left\{\begin{array}{c}\mathscr{T}^1_{Æ_\sigma}(\hat{\vartheta})\,if\,\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\\ \mathscr{T}^2_{Æ_\sigma}(\hat{\vartheta})\,if\,\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\\ \min\left\{\mathscr{T}^1_{Æ_\sigma}(\hat{\vartheta}),\mathscr{T}^2_{Æ_\sigma}(\hat{\vartheta})\right\}\,if\,\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\end{array}\right.,$$

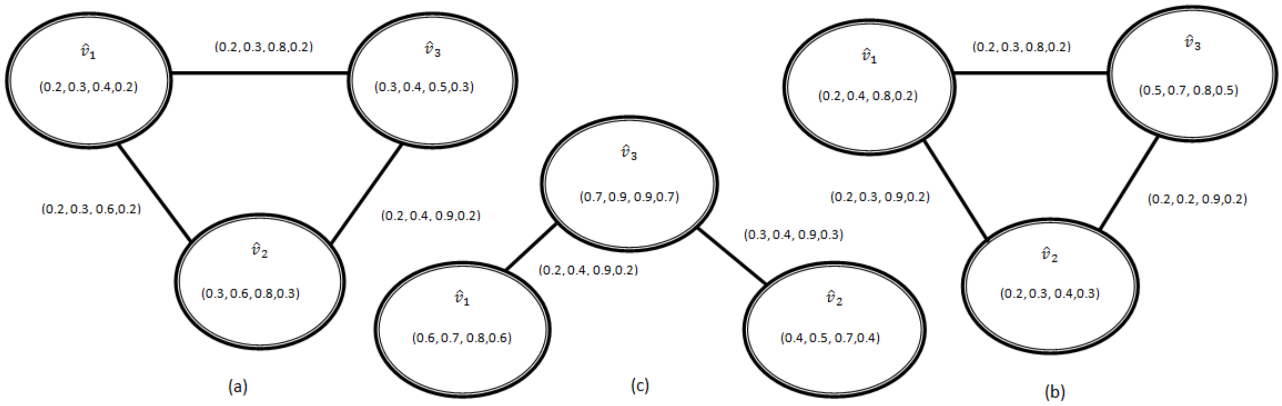

Fig. 4: Graph-based presentation of Table 3

Table 4: Tabular form of psvNHSG $\mathscr{A}_2 = (\mathscr{A}_2, \mathscr{Q}^2, Æ^2, \Xi^2)$ according to Example 3

Æ	$\hat{v}_3$	$\hat{v}_4$	$\hat{v}_5$
σ_2	(0.3,0.4,0.5,0.3)	(0.2,0.3,0.5,0.2)	(0.5,0.7,0.8,0.5)
σ_4	(0.6,0.8,0.9,0.6)	(0.4,0.7,0.9,0.4)	(0.4,0.5,0.6,0.4)
Ξ	$(\hat{v}_3, \hat{v}_4)$	$(\hat{v}_4, \hat{v}_5)$	$(\hat{v}_3, \hat{v}_5)$
σ_2	(0.2,0.3,0.9,0.2)	(0.3,0.4,0.9,0.3)	(0,0,1,0)
σ_4	(0.2,0.2,0.9,0.2)	(0.3,0.3,0.9,0.3)	(0.3,0.4,0.9,0.3)

Fig. 5: Graph-based presentation of Table 4

Table 5: Tabular form of $\mathscr{A} = \mathscr{A}_1 \Re \mathscr{A}_2$

Æ	$\hat{v}_1$	$\hat{v}_2$	$\hat{v}_3$	$\hat{v}_4$	$\hat{v}_5$	
σ_1	(0.2,0.3,0.4,0.2)	(0.3,0.4,0.5,0.3)	(0.3,0.6,0.8,0.3)	(0,0,1,0)	(0,0,1,0)	
σ_2	(0.2,0.4,0.8,0.2)	(0.2,0.3,0.4,0.2)	(0.3,0.5,0.5,0.3)	(0.2,0.3,0.4,0.2)	(0.5,0.7,0.8,0.5)	
σ_3	(0.6,0.7,0.8,0.6)	(0.4,0.5,0.7,0.4)	(0.7,0.9,0.9,0.7)	(0,0,1,0)	(0,0,1,0)	
σ_4	(0,0,1,0)	(0,0,1,0)	(0.6,0.8,0.9,0.6)	(0.4,0.7,0.9,0.4)	(0.4,0.5,0.6,0.4)	
Ξ	$(\hat{v}_1, \hat{v}_2)$	$(\hat{v}_1, \hat{v}_3)$	$(\hat{v}_2, \hat{v}_3)$	$(\hat{v}_3, \hat{v}_4)$	$(\hat{v}_3, \hat{v}_5)$	$(\hat{v}_4, \hat{v}_5)$
σ_1	(0.2,0.3,0.8,0.2)	(0.2,0.3,0.9,0.2)	(0.2,0.4,0.9,0.2)	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)
σ_2	(0.2,0.3,0.8,0.2)	(0.2,0.3,0.9,0.2)	(0.2,0.2,0.9,0.2)	(0.2,0.3,0.9,0.2)	(0.3,0.4,0.9,0.3)	(0,0,1,0)
σ_3	(0.2,0.4,0.9,0.2)	(0,0,1,0)	(0.3,0.4,0.9,0.3)	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)
σ_4	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)	(0.2,0.2,0.9,0.2)	(0.3,0.3,0.9,0.3)	(0.3,0.4,0.9,0.3)

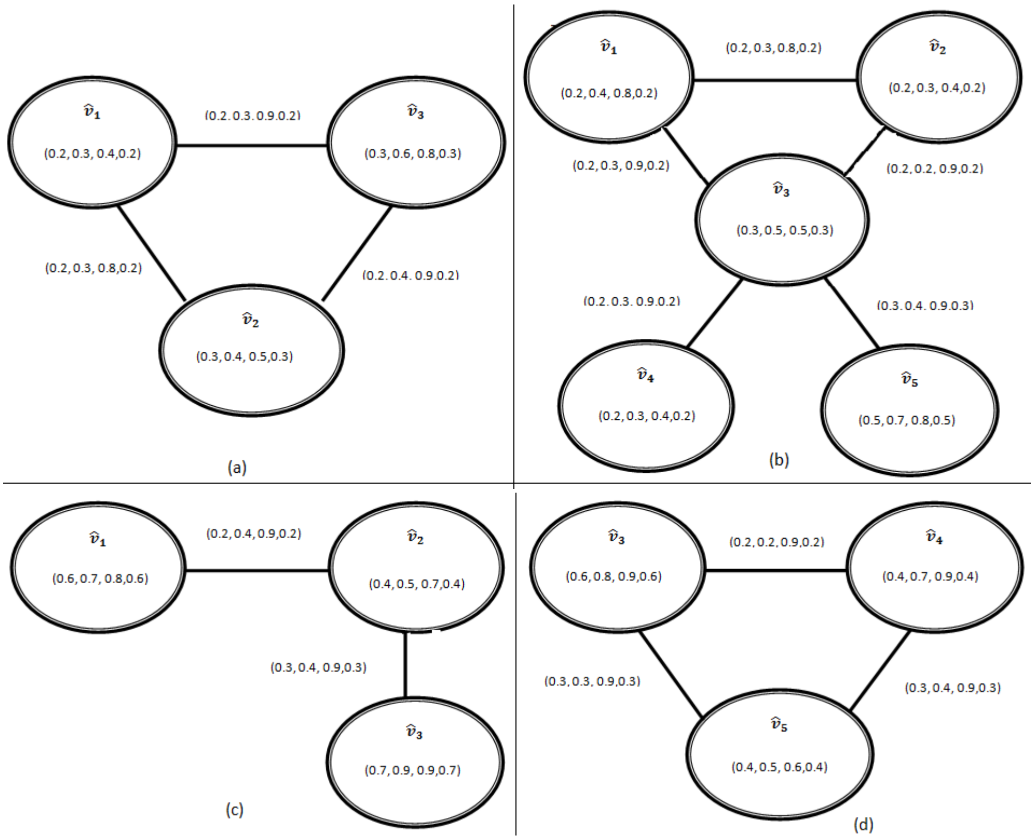

Fig. 6: Graph-based presentation of Table 5

$$\mathscr{I}_{\text{Æ}_\sigma} = \begin{cases} \mathscr{I}^1_{\text{Æ}_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{I}^2_{\text{Æ}_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \min\left\{\mathscr{I}^1_{\text{Æ}_\sigma}(\hat{v}), \mathscr{I}^2_{\text{Æ}_\sigma}(\hat{v})\right\}\, if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$

$$\mathscr{F}_{\text{Æ}_\sigma} = \begin{cases} \mathscr{F}^1_{\text{Æ}_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{F}^2_{\text{Æ}_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \max\left\{\mathscr{F}^1_{\text{Æ}_\sigma}(\hat{v}), \mathscr{F}^2_{\text{Æ}_\sigma}(\hat{v})\right\}\, if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$

$$\mu_{\text{Æ}_\sigma} = \begin{cases} \mu^1_{\text{Æ}_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mu^2_{\text{Æ}_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \min\left\{\mu^1_{\text{Æ}_\sigma}(\hat{v}), \mu^2_{\text{Æ}_\sigma}(\hat{v})\right\}\, if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases}.$$

The uncertain parts for Ξ *are given as follows:*

$$\mathscr{T}_{\Xi_\sigma} = \begin{cases} \mathscr{T}^1_{\Xi_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{T}^2_{\Xi_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \min\left\{\mathscr{T}^1_{\Xi_\sigma}(\hat{v}), \mathscr{T}^2_{\Xi_\sigma}(\hat{v})\right\}\, if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$

$$\mathscr{I}_{\Xi_\sigma} = \begin{cases} \mathscr{I}^1_{\Xi_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{I}^2_{\Xi_\sigma}(\hat{v})\, if\, \sigma \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \min\left\{\mathscr{I}^1_{\Xi_\sigma}(\hat{v}), \mathscr{I}^2_{\Xi_\sigma}(\hat{v})\right\}\, if\, \sigma \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$

$$\mathscr{F}_{\Xi_\sigma}=\begin{cases}\mathscr{F}^1_{\Xi_\sigma}(\hat{v})\, if\, \sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\\ \mathscr{F}^2_{\Xi_\sigma}(\hat{v})\, if\, \sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\\ \max\left\{\mathscr{F}^1_{\Xi_\sigma}(\hat{v}),\mathscr{F}^2_{\Xi_\sigma}(\hat{v})\right\} if\, \sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\end{cases},$$

$$\mu_{\Xi_\sigma}=\begin{cases}\mu^1_{\Xi_\sigma}(\hat{v})\, if\, \sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2\\ \mu^2_{\Xi_\sigma}(\hat{v})\, if\, \sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1\\ \min\left\{\mu^1_{\Xi_\sigma}(\hat{v}),\mu^2_{\Xi_\sigma}(\hat{v})\right\} if\, \sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2\end{cases}.$$

Theorem 2. *If $\mathscr{A}_1,\mathscr{A}_2\in\Omega_{NHSG}$ then $\mathscr{A}_1\Im\mathscr{A}_2\in\Omega_{NHSG}$.*

Proof. Consider two psvNHSGs $\mathscr{A}_1=\left(\mathscr{A}_1,\mathscr{Q}^1,Æ^1,\Xi^1\right)$ and $\mathscr{A}_2=\left(\mathscr{A}_2,\mathscr{Q}^2,Æ^2,\Xi^2\right)$ as defined in Definition 6. Let $\mathscr{A}=(\mathscr{A},\mathscr{Q},Æ,\Xi)$ be the intersection of psvNHSGs $\mathscr{A}_1$ and $\mathscr{A}_2$ where $\mathscr{Q}=\mathscr{Q}^1\Re\mathscr{Q}^2$ and $\mathscr{V}=\mathscr{V}_1\Im\mathscr{V}_2$. Let $\sigma\in\mathscr{Q}^1\circ\mathscr{Q}^2$ then

$\mathscr{T}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\mathscr{T}_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2)\le$
$\min\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{T}_{Æ^1_\sigma}(\hat{v}_2)\right\}=\min\left\{\mathscr{T}_{Æ_\sigma}(\hat{v}_1),\mathscr{T}_{Æ_\sigma}(\hat{v}_2)\right\}$ so
$\mathscr{T}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)\le\min\left\{\mathscr{T}_{Æ_\sigma}(\hat{v}_1),\mathscr{T}_{Æ_\sigma}(\hat{v}_2)\right\}$
& $\mathscr{I}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\mathscr{I}_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2)\le$
$\min\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{I}_{Æ^1_\sigma}(\hat{v}_2)\right\}=\min\left\{\mathscr{I}_{Æ_\sigma}(\hat{v}_1),\mathscr{I}_{Æ_\sigma}(\hat{v}_2)\right\}$ so
$\mathscr{I}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)\le\min\left\{\mathscr{I}_{Æ_\sigma}(\hat{v}_1),\mathscr{I}_{Æ_\sigma}(\hat{v}_2)\right\}$ &
$\mathscr{F}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\mathscr{F}_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2)\ge$
$\max\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{F}_{Æ^1_\sigma}(\hat{v}_2)\right\}$
$=\max\left\{\mathscr{F}_{Æ_\sigma}(\hat{v}_1),\mathscr{F}_{Æ_\sigma}(\hat{v}_2)\right\}$
so $\mathscr{F}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)\ge\max\left\{\mathscr{F}_{Æ_\sigma}(\hat{v}_1),\mathscr{F}_{Æ_\sigma}(\hat{v}_2)\right\}$
& $\mu_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\mu_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2)$
$\le\min\left\{\mu_{Æ^1_\sigma}(\hat{v}_1),\mu_{Æ^1_\sigma}(\hat{v}_2)\right\}=\min\left\{\mu_{Æ_\sigma}(\hat{v}_1),\mu_{Æ_\sigma}(\hat{v}_2)\right\}$ so
$\mu_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)\le\min\left\{\mu_{Æ_\sigma}(\hat{v}_1),\mu_{Æ_\sigma}(\hat{v}_2)\right\}$
Similar results are obtained when $\sigma\in\mathscr{Q}^2\circ\mathscr{Q}^1$
Now if $\sigma\in\mathscr{Q}^1\Im\mathscr{Q}^2$ then

$$\mathscr{T}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\min\left\{\mathscr{T}_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2),\mathscr{T}_{\Xi^2_\sigma}(\hat{v}_1,\hat{v}_2)\right\}$$
$$\le\min\left\{\begin{array}{c}\min\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{T}_{Æ^1_\sigma}(\hat{v}_2)\right\},\\ \min\left\{\mathscr{T}_{Æ^2_\sigma}(\hat{v}_1),\mathscr{T}_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$
$$\le\min\left\{\begin{array}{c}\min\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{T}_{Æ^2_\sigma}(\hat{v}_2)\right\},\\ \min\left\{\mathscr{T}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{T}_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$
$$=\min\left\{\mathscr{T}_{Æ_\sigma}(\hat{v}_1),\mathscr{T}_{Æ_\sigma}(\hat{v}_2)\right\}$$
$$\mathscr{I}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\min\left\{\mathscr{I}_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2),\mathscr{I}_{\Xi^2_\sigma}(\hat{v}_1,\hat{v}_2)\right\}$$
$$\le\min\left\{\begin{array}{c}\min\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{I}_{Æ^1_\sigma}(\hat{v}_2)\right\},\\ \min\left\{\mathscr{I}_{Æ^2_\sigma}(\hat{v}_1),\mathscr{I}_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$
$$\le\min\left\{\begin{array}{c}\min\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{I}_{Æ^2_\sigma}(\hat{v}_2)\right\},\\ \min\left\{\mathscr{I}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{I}_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$
$$=\min\left\{\mathscr{I}_{Æ_\sigma}(\hat{v}_1),\mathscr{I}_{Æ_\sigma}(\hat{v}_2)\right\}$$
$$\mathscr{F}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\max\left\{\mathscr{F}_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2),\mathscr{F}_{\Xi^2_\sigma}(\hat{v}_1,\hat{v}_2)\right\}$$
$$\ge\max\left\{\begin{array}{c}\max\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{F}_{Æ^1_\sigma}(\hat{v}_2)\right\},\\ \max\left\{\mathscr{F}_{Æ^2_\sigma}(\hat{v}_1),\mathscr{F}_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$
$$\ge\max\left\{\begin{array}{c}\max\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{F}_{Æ^2_\sigma}(\hat{v}_2)\right\},\\ \max\left\{\mathscr{F}_{Æ^1_\sigma}(\hat{v}_1),\mathscr{F}_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$
$$=\max\left\{\mathscr{F}_{Æ_\sigma}(\hat{v}_1),\mathscr{F}_{Æ_\sigma}(\hat{v}_2)\right\}$$
$$\mu_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2)=\min\left\{\mu_{\Xi^1_\sigma}(\hat{v}_1,\hat{v}_2),\mu_{\Xi^2_\sigma}(\hat{v}_1,\hat{v}_2)\right\}$$
$$\le\min\left\{\begin{array}{c}\min\left\{\mu_{Æ^1_\sigma}(\hat{v}_1),\mu_{Æ^1_\sigma}(\hat{v}_2)\right\},\\ \min\left\{\mu_{Æ^2_\sigma}(\hat{v}_1),\mu_{Æ^2_\sigma}(\hat{v}_2)\right\}\end{array}\right\}$$

Table 6: Tabular form of psvNHSG $\mathscr{A}_1 = (\mathscr{A}_1, \mathscr{Q}^1, Æ^1, \Xi^1)$ for Example 4

Æ	$\hat{\upsilon}_1$	$\hat{\upsilon}_2$	$\hat{\upsilon}_3$
σ_1	(0.2,0.3,0.4,0.2)	(0.3,0.5,0.6,0.3)	(0.2,0.6,0.8,0.2)
σ_2	(0.3,0.4,0.8,0.3)	(0.5,0.7,0.8,0.5)	(0.4,0.5,0.7,0.4)
Ξ	$(\hat{\upsilon}_1,\hat{\upsilon}_2)$	$(\hat{\upsilon}_2,\hat{\upsilon}_3)$	$(\hat{\upsilon}_1,\hat{\upsilon}_3)$
σ_1	(0.2,0.2,0.7,0.2)	(0.2,0.4,0.9,0.2)	(0.2,0.2,0.9,0.2)
σ_2	(0.3,0.4,0.8,0.3)	(0.4,0.5,0.9,0.4)	(0.3,0.4,0.8,0.3)

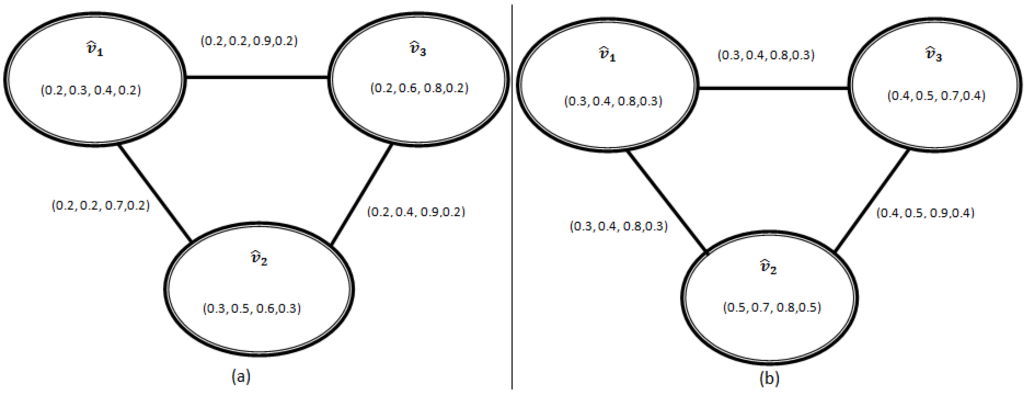

Fig. 7: Graph-based presentation of Table 6

Table 7: Tabular form of psvNHSG $\mathscr{A}_2 = (\mathscr{A}_2, \mathscr{Q}^2, Æ^2, \Xi^2)$ for Example 4

Æ	$\hat{\upsilon}_2$	$\hat{\upsilon}_3$	$\hat{\upsilon}_4$
σ_2	(0.4,0.6,0.7,0.4)	(0.5,0.6,0.9,0.5)	(0.3,0.5,0.7,0.3)
σ_3	(0.3,0.5,0.6,0.3)	(0.2,0.6,0.8,0.2)	(0.2,0.3,0.7,0.2)
Ξ	$(\hat{\upsilon}_2,\hat{\upsilon}_3)$	$(\hat{\upsilon}_3,\hat{\upsilon}_4)$	$(\hat{\upsilon}_2,\hat{\upsilon}_4)$
σ_2	(0.2,0.2,0.7,0.2)	(0.2,0.4,0.9,0.2)	(0.2,0.2,0.9,0.2)
σ_3	(0.3,0.4,0.8,0.3)	(0.4,0.5,0.9,0.4)	(0.3,0.4,0.8,0.3)

$$\leq \min\left\{\begin{array}{c}\min\left\{\mu_{Æ^1_\sigma}(\hat{\upsilon}_1), \mu_{Æ^2_\sigma}(\hat{\upsilon}_2)\right\},\\ \min\left\{\mu_{Æ^1_\sigma}(\hat{\upsilon}_1), \mu_{Æ^2_\sigma}(\hat{\upsilon}_2)\right\}\end{array}\right\}$$
$$= \min\left\{\mu_{Æ_\sigma}(\hat{\upsilon}_1), \mu_{Æ_\sigma}(\hat{\upsilon}_2)\right\}$$
Hence the intersection $\mathscr{A} = \mathscr{A}_1 \Im \mathscr{A}_2$ is psvNHSGs.

*Example 4.*Let $\mathscr{A}_1 = (\mathscr{A}_1, \mathscr{Q}^1, Æ^1, \Xi^1)$ be a psvNHSG where $\mathscr{A}_1 = (\mathscr{V}_1, \mathscr{E}_1)$ with $\mathscr{V}_1 = \{\hat{\upsilon}_1, \hat{\upsilon}_2, \hat{\upsilon}_3\}$ and $\mathscr{Q}_1, \mathscr{Q}_2, \mathscr{Q}_3$ are sub-parametric non-overlapping sets w.r.t. distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathscr{Q}_1 = \{\alpha_{11}\}$, $\mathscr{Q}_2 = \{\alpha_{21}\}$ and $\mathscr{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$. $\mathscr{Q}^1 = \mathscr{Q}_1 \subseteq \mathscr{Q}_2 \subseteq \mathscr{Q}_3 = \{\sigma_1, \sigma_2\}$ and $\mathscr{T}_{\Xi_\sigma}(\hat{\upsilon}_i, \hat{\upsilon}_j) = 0, \mathscr{I}_{\Xi_\sigma}(\hat{\upsilon}_i, \hat{\upsilon}_j) = 0, \mathscr{F}_{\Xi_\sigma}(\hat{\upsilon}_i, \hat{\upsilon}_j) = 1$ $(\hat{\upsilon}_i, \hat{\upsilon}_j) \in \mathscr{V}_1 \subseteq \mathscr{V}_1 \backslash \{(\hat{\upsilon}_1, \hat{\upsilon}_2), (\hat{\upsilon}_2, \hat{\upsilon}_3), (\hat{\upsilon}_1, \hat{\upsilon}_3)\}$. The Table 6 and Figure 7 elaborate its tabular form and graph-based presentation respectively.
Also let $\mathscr{A}_2 = (\mathscr{A}_2, \mathscr{Q}^2, Æ^2, \Xi^2)$ be a psvNHSG where $\mathscr{A}_2 = (\mathscr{V}_2, \mathscr{E}_2)$ with $\mathscr{V}_2 = \{\hat{\upsilon}_2, \hat{\upsilon}_3, \hat{\upsilon}_4\}$ and $\mathscr{Q}_2, \mathscr{Q}_3, \mathscr{Q}_4$ are sub-parametric non-overlapping sets w.r.t. distinct attributes $\alpha_2, \alpha_3, \alpha_4$ where $\mathscr{Q}_2 = \{\alpha_{21}\}$, $\mathscr{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$, $\mathscr{Q}_4 = \{\alpha_{41}\}$. $\mathscr{Q}^2 = \mathscr{Q}_2 \subseteq \mathscr{Q}_3 \subseteq \mathscr{Q}_4 = \{\sigma_2, \sigma_3\}$ and $\mathcal{T}_{\Xi_\sigma}(\hat{\upsilon}_i, \hat{\upsilon}_j) = 0, \mathscr{I}_{\Xi_\sigma}(\hat{\upsilon}_i, \hat{\upsilon}_j) = 0, \mathscr{F}_{\Xi_\sigma}(\hat{\upsilon}_i, \hat{\upsilon}_j) = 1$ $(\hat{\upsilon}_i, \hat{\upsilon}_j) \in \mathscr{V}_2 \subseteq \mathscr{V}_2 \backslash \{(\hat{\upsilon}_2, \hat{\upsilon}_3), (\hat{\upsilon}_3, \hat{\upsilon}_4), (\hat{\upsilon}_2, \hat{\upsilon}_4)\}$. Its tabular form and graph-based presentation are provided in Table 7 and Figure 8 respectively. Consider $\mathscr{A} = \mathscr{A}_1 \Im \mathscr{A}_2$ with $\mathscr{Q} = \mathscr{Q}^1 \Im \mathscr{Q}^2$. Its tabular form and graph-based presentation are stated in Table 8 and Figure 9 respectively.

Definition 12.*The compliment* $\overline{\mathfrak{A}} = (\overline{\mathscr{A}}, \overline{\mathscr{Q}}, \overline{\mathbb{F}}, \overline{\mathbb{G}})$ *of SSVNHS-subgraph* $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ *with* $\Xi_\sigma(\hat{\upsilon}_1, \hat{\upsilon}_2) = Æ_\sigma(\hat{\upsilon}_1) \Im Æ_\sigma(\hat{\upsilon}_2)$ *where*

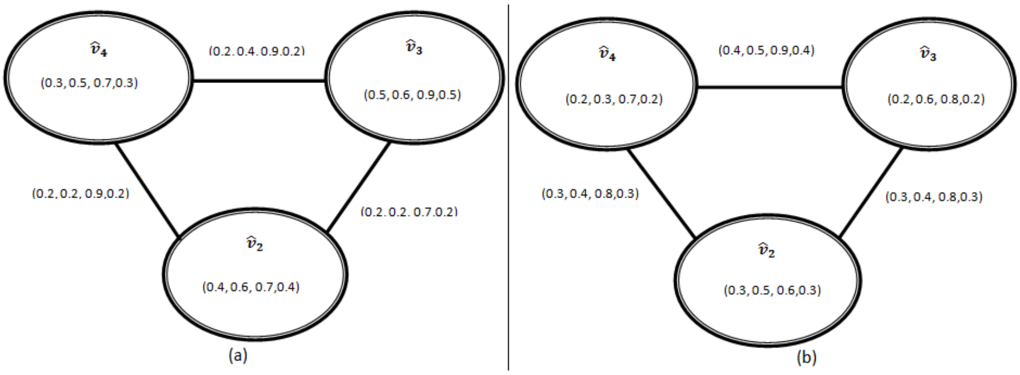

Fig. 8: Graph presentation of Table 7

Table 8: Tabular form of psvNHSG $\mathscr{A} = \mathscr{A}_1 \Im \mathscr{A}_2$

Æ	$\hat{v}_2$	$\hat{v}_3$
σ_1	$(0.3, 0.5, 0.6, 0.3)$	$(0.2, 0.6, 0.8, 0.2)$
σ_2	$(0.4, 0.6, 0.8, 0.4)$	$(0.4, 0.5, 0.9, 0.4)$
σ_3	$(0.3, 0.5, 0.6, 0.3)$	$(0.2, 0.6, 0.8, 0.2)$
Ξ	$(\hat{v}_2, \hat{v}_3)$	
σ_1	$(0.2, 0.4, 0.9, 0.2)$	
σ_2	$(0.3, 0.5, 0.9, 0.3)$	
σ_3	$(0.2, 0.5, 0.9, 0.2)$	

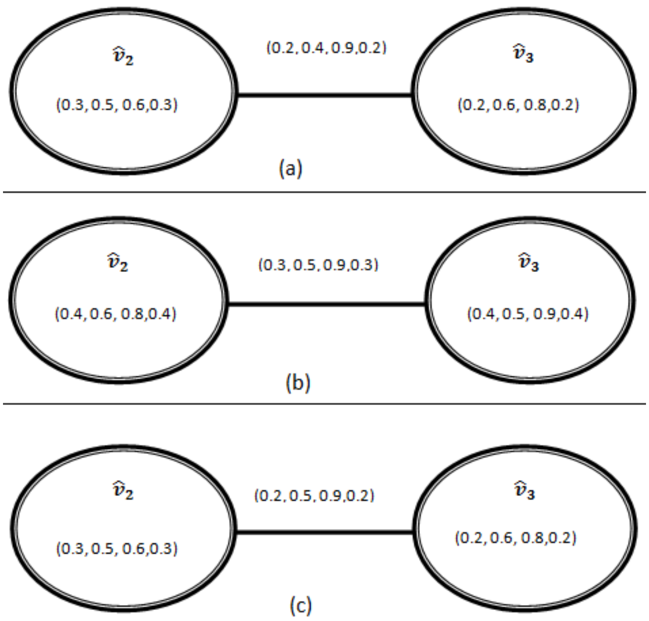

Fig. 9: Graph-based presentation of Table 8

1. $\overline{\mathscr{Q}} = \mathscr{Q}$
2. $\overline{\mathscr{T}_{Æ_\sigma}(\hat{v})} = \mathscr{T}_{Æ_\sigma}(\hat{v}), \overline{\mathscr{I}_{Æ_\sigma}(\hat{v})} = \mathscr{I}_{Æ_\sigma}(\hat{v}), \overline{\mathscr{F}_{Æ_\sigma}(\hat{v})} = \mathscr{F}_{Æ_\sigma}(\hat{v}), \overline{\mu_{Æ_\sigma}(\hat{v})} = \mu_{Æ_\sigma}(\hat{v}) \ \ \hat{v} \in \mathscr{V}$
3. $\overline{\mathscr{T}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2)} =$
$\begin{cases} \min\{\mathscr{T}_{Æ_\sigma}(\hat{v}_1),\mathscr{T}_{Æ_\sigma}(\hat{v}_2)\} \, if \, \mathscr{T}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2) = 0 \\ 0 \quad otherwise \end{cases}$,
$\overline{\mathscr{I}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2)} =$
$\begin{cases} \min\{\mathscr{I}_{Æ_\sigma}(\hat{v}_1),\mathscr{I}_{Æ_\sigma}(\hat{v}_2)\} \, if \, \mathscr{I}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2) = 0 \\ 0 \quad otherwise \end{cases}$,
$\overline{\mathscr{F}_{Æ_\sigma}(\hat{v}_1,\hat{v}_2)} =$
$\begin{cases} \max\{\mathscr{F}_{Æ_\sigma}(\hat{v}_1),\mathscr{F}_{Æ_\sigma}(\hat{v}_2)\} \, if \, \mathscr{F}_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2) = 0 \\ 0 \quad otherwise \end{cases}$,
$\overline{\mu_{Æ_\sigma}(\hat{v}_1,\hat{v}_2)} =$
$\begin{cases} \min\{\mu_{Æ_\sigma}(\hat{v}_1),\mu_{Æ_\sigma}(\hat{v}_2)\} \, if \, \mu_{\Xi_\sigma}(\hat{v}_1,\hat{v}_2) = 0 \\ 0 \quad otherwise \end{cases}$.

2.4 Composition and Products of psvNHSG

Definition 13. *For two psvNHSGs $\mathscr{A}^1 = (\mathscr{A}^1,\mathscr{Q}^1,Æ^1,\Xi^1)$ and $\mathscr{A}^2 = (\mathscr{A}^2,\mathscr{Q}^2,Æ^2,\Xi^2)$ w.r.t. $\mathscr{A}^1 = (\mathscr{V}_1,\mathscr{E}_1)$ and $\mathscr{A}^2 = (\mathscr{V}_2,\mathscr{E}_2)$. Let $\mathscr{A} = \mathscr{A}^1 \subseteq_{\mathbb{P}} \mathscr{A}^2$ where $\mathscr{A} = (Æ,\Xi,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ and $(Æ = Æ^1 \subseteq Æ^2, \Xi = \Xi^1 \subseteq \Xi^2)$ is pSVNHSS over $\mathscr{V} = \mathscr{V}_1 \subseteq \mathscr{V}_2$, $\Xi = (\Xi^1 \subseteq \Xi^2, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ is pSVNHSS over $\mathscr{E} = \{((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) | \hat{\varrho} \in \mathscr{V}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\}$ $\Re\{((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})) | \hat{\varsigma} \in \mathscr{V}_2, (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1\}$ and $\Xi = (Æ,\Xi,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ are psvNHSGs where as*

1. $\mathscr{T}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{Æ^2(t)}(\hat{\varsigma})$,
$\mathscr{I}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{Æ^2(t)}(\hat{\varsigma})$,
$\mathscr{F}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{F}_{Æ^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{Æ^2(t)}(\hat{\varsigma})$,
$\mu_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mu_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mu_{Æ^2(t)}(\hat{\varsigma}) \ \ (\hat{\varrho},\hat{\varsigma}) \in \mathscr{V}, (s,t) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$.
2. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) =$
$\mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2)$,
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) =$
$\mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2)$,
$\mathscr{F}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) =$
$\mathscr{F}_{Æ^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2)$,
$\mu_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) =$
$\mu_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2), \ \hat{\varrho} \in \mathscr{V}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2$.
3. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})) =$
$\mathscr{T}_{Æ^2(\zeta)}(\hat{\varsigma}) \prime \mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2)$,
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})) =$
$\mathscr{I}_{Æ^2(\zeta)}(\hat{\varsigma}) \prime \mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2)$,
$\mathscr{F}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})) =$
$\mathscr{F}_{Æ^2(\zeta)}(\hat{\varsigma}) \prime \mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2)$,
$\mu_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})) =$
$\mu_{Æ^2(\zeta)}(\hat{\varsigma}) \prime \mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2), \ \hat{\varsigma} \in \mathscr{V}_2, (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1$

$(\gamma,\zeta) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$, $\mathbb{W}(\gamma,\zeta) = \mathbb{W}_1(\gamma) \subseteq \mathbb{W}_2(\zeta)$ *are psvNHSGs of $\mathscr{A}$.*

Example 5. Let $\mathscr{A}^1 = (\mathscr{V}_1,\mathscr{E}_1)$ be a simple graph with $\mathscr{V}_1 = \{\hat{\varrho}_1,\hat{\varrho}_2,\hat{\varrho}_3\}$ and $\mathscr{E}_1 = \{\hat{\varrho}_1\hat{\varrho}_2,\hat{\varrho}_1\hat{\varrho}_3,\hat{\varrho}_2\hat{\varrho}_3\}$ and $\mathscr{Q}_1,\mathscr{Q}_2,\mathscr{Q}_3$ are sub-parametric non-overlapping sets w.r.t. distinct attributes $\alpha_1,\alpha_2,\alpha_3$ where $\mathscr{Q}_1 = \{\alpha_{11}\}$, $\mathscr{Q}_2 = \{\alpha_{21},\alpha_{22}\}$ and $\mathscr{Q}_3 = \{\alpha_{31}\}$. $\mathscr{Q}^1 = \mathscr{Q}_1 \subseteq \mathscr{Q}_2 \subseteq \mathscr{Q}_3 = \{\hat{\omega}_1,\hat{\omega}_2\}$. $\mathscr{A}^1 = \{(\mathbb{W}_1,\mathscr{Q}^1)\} = \{(\mathbb{W}_1(\hat{\omega}_1)),(\mathbb{W}_1(\hat{\omega}_2))\}$ is psvNHSG which is stated in Table 9. Let $\mathscr{A}^2 = (\mathscr{V}_2,\mathscr{E}_2)$ be a simple graph with $\mathscr{V}_2 = \{\hat{\varsigma}_1,\hat{\varsigma}_2,\hat{\varsigma}_3,\hat{\varsigma}_4\}$, $\mathscr{E}_2 = \{\hat{\varsigma}_1\hat{\varsigma}_2,\hat{\varsigma}_1\hat{\varsigma}_3,\hat{\varsigma}_1\hat{\varsigma}_4,\hat{\varsigma}_3\hat{\varsigma}_4\}$ and $\mathscr{Q}_1,\mathscr{Q}_2,\mathscr{Q}_3$ are sub-parametric non-overlapping sets w.r.t. distinct attributes

Table 9: Tabular form of psvNHSG $\mathscr{A}^1 = (\mathscr{A}, \mathscr{Q}^1, Æ^1, \Xi^1)$ Example 5 demonstrated in Fig. 10

Æ	$\hat{\varrho}_1$	$\hat{\varrho}_2$	$\hat{\varrho}_3$
$\hat{\omega}_1$	(0.3,0.5,0.7,0.3)	(0.5,0.6,0.8,0.5)	(0.5,0.6,0.8,0.5)
$\hat{\omega}_2$	(0.4,0.6,0.8,0.4)	(0.5,0.6,0.7,0.5)	(0.6,0.5,0.4,0.6)
Ξ	$(\hat{\varrho}_1,\hat{\varrho}_2)$	$(\hat{\varrho}_1,\hat{\varrho}_3)$	$(\hat{\varrho}_2,\hat{\varrho}_3)$
$\hat{\omega}_1$	(0.3,0.4,0.5,0.3)	(0.2,0.3,0.6,0.2)	(0.3,0.4,0.5,0.3)
$\hat{\omega}_2$	(0.3,0.5,0.6,0.3)	(0.3,0.4,0.5,0.3)	(0,0,1,0)

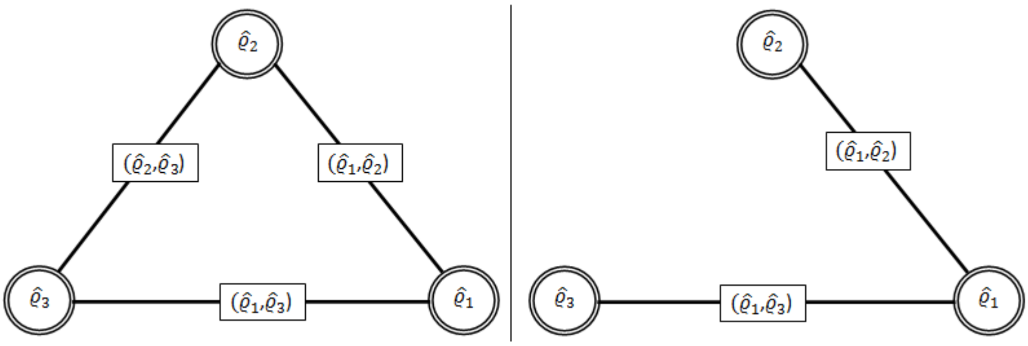

Fig. 10: Graph-based presentation of Table 9 with (a): $\mathbb{W}(\hat{\omega}_1)$, (b): $\mathbb{W}(\hat{\omega}_2)$

Table 10: psvNHSG $\mathscr{A}^2 = (\mathscr{A}, \mathscr{Q}^2, Æ^2, \Xi^2)$ Example 5 demonstrated in Fig.11

Æ	$\hat{\varsigma}_1$	$\hat{\varsigma}_2$	$\hat{\varsigma}_3$	$\hat{\varsigma}_4$		
$\hat{\omega}_3$	(0.5,0.6,0.4,0.5)	(0.4,0.5,0.2,0.4)	(0.4,0.6,0.9,0.4)	(0.6,0.4,0.5,0.6)		
$\hat{\omega}_4$	(0.5,0.6,0.9,0.5)	(0.7,0.4,0.8,0.7)	(0.5,0.5,0.6,0.5)	(0.8,0.3,0.7,0.8)		
Ξ	$(\hat{\varsigma}_1,\hat{\varsigma}_2)$	$(\hat{\varsigma}_1,\hat{\varsigma}_3)$	$(\hat{\varsigma}_1,\hat{\varsigma}_4)$	$(\hat{\varsigma}_2,\hat{\varsigma}_3)$	$(\hat{\varsigma}_2,\hat{\varsigma}_4)$	$(\hat{\varsigma}_3,\hat{\varsigma}_4)$
$\hat{\omega}_3$	(0.3,0.4,0.4,0.3)	(0.3,0.4,0.6,0.3)	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)	(0.3,0.3,0.6,0.3)
$\hat{\omega}_4$	(0.4,0.5,0.7,0.4)	(0.3,0.4,0.6,0.3)	(0.4,0.3,0.6,0.4)	(0,0,1,0)	(0,0,1,0)	(0,0,1,0)

$\alpha_1,\alpha_2,\alpha_3$ where $\mathscr{Q}_1 = \{\alpha_{11}\}$, $\mathscr{Q}_2 = \{\alpha_{21},\alpha_{22}\}$ and $\mathscr{Q}_3 = \{\alpha_{31}\}$. $\mathscr{Q}^2 = \mathscr{Q}_1 \subseteq \mathscr{Q}_2 \subseteq \mathscr{Q}_3 = \{\hat{\omega}_3,\hat{\omega}_4\}$. $\mathscr{A}^2 = \{(\mathbb{W}_2,\mathscr{Q}^2)\} = \{(\mathbb{W}_2(\hat{\omega}_3)),(\mathbb{W}_2(\hat{\omega}_4))\}$ is psvNHSG which is depicted in Table 10. $\mathscr{A} = \mathscr{A}^1 \subseteq_{\mathbb{P}} \mathscr{A}^2 = (\mathbb{W}, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ where
$\mathscr{Q}^1 \subseteq \mathscr{Q}^2 = \{(\hat{\omega}_1,\hat{\omega}_3),(\hat{\omega}_2,\hat{\omega}_3),(\hat{\omega}_1,\hat{\omega}_4),(\hat{\omega}_2,\hat{\omega}_4)\}$. Here $\mathbb{W}(\hat{\omega}_1,\hat{\omega}_3) = \mathbb{W}_1(\hat{\omega}_1) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_3)$, $\mathbb{W}(\hat{\omega}_2,\hat{\omega}_3) = \mathbb{W}_1(\hat{\omega}_2) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_3)$, $\mathbb{W}(\hat{\omega}_1,\hat{\omega}_4) = \mathbb{W}_1(\hat{\omega}_1) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_4)$ and $\mathbb{W}(\hat{\omega}_2,\hat{\omega}_4) = \mathbb{W}_1(\hat{\omega}_2) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_4)$ for convenience we will write $(\hat{\varrho}_p,\hat{\varsigma}_q) = \hat{\partial}_{pq}$ for $p = 1,2,3$ and $q = 1,2,3,4$ also $\mathscr{T}_{\Xi_{\hat{\omega}}}(\hat{\partial}_i,\hat{\partial}_j) = 0, \mathscr{I}_{\Xi_{\hat{\omega}}}(\hat{\partial}_i,\hat{\partial}_j) = 0, \mathscr{F}_{\Xi_{\hat{\omega}}}(\hat{\partial}_i,\hat{\partial}_j) = 1$

$$(\hat{\partial}_{pq}) \in \mathscr{V} \subseteq \mathscr{V} \setminus \left\{ \begin{array}{l} (\hat{\partial}_{11},\hat{\partial}_{12}),(\hat{\partial}_{11},\hat{\partial}_{13}),(\hat{\partial}_{11},\hat{\partial}_{21}),(\hat{\partial}_{11},\hat{\partial}_{31}),(\hat{\partial}_{12},\hat{\partial}_{22}), \\ (\hat{\partial}_{12},\hat{\partial}_{32}),(\hat{\partial}_{13},\hat{\partial}_{23}),(\hat{\partial}_{13},\hat{\partial}_{33}),(\hat{\partial}_{13},\hat{\partial}_{14}),(\hat{\partial}_{14},\hat{\partial}_{24}), \\ (\hat{\partial}_{14},\hat{\partial}_{34}),(\hat{\partial}_{21},\hat{\partial}_{22}),(\hat{\partial}_{21},\hat{\partial}_{23}),(\hat{\partial}_{21},\hat{\partial}_{31}),(\hat{\partial}_{22},\hat{\partial}_{32}), \\ (\hat{\partial}_{23},\hat{\partial}_{24}),(\hat{\partial}_{23},\hat{\partial}_{33}),(\hat{\partial}_{24},\hat{\partial}_{34}),(\hat{\partial}_{31},\hat{\partial}_{32}),(\hat{\partial}_{31},\hat{\partial}_{33}), \\ (\hat{\partial}_{33},\hat{\partial}_{34}) \end{array} \right\}$$

psvNHSG of $\mathbb{W}(\hat{\omega}_1,\hat{\omega}_3) = \mathbb{W}_1(\hat{\omega}_1) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_3)$, is given in Table 11.

Theorem 3. *The cartesian product of two psvNHSGs is psvNHSG.*

Definition 14. *For two psvNHSGs $\mathscr{A}^1 = (\mathscr{A}^1, \mathscr{Q}^1, Æ^1, \Xi^1)$ and $\mathscr{A}^2 = (\mathscr{A}^2, \mathscr{Q}^2, Æ^2, \Xi^2)$ w.r.t. $\mathscr{A}^1 = (\mathscr{V}_1, \mathscr{E}_2)$ and $\mathscr{A}^2 = (\mathscr{V}_2, \mathscr{E}_2)$. Let $\mathscr{A} = \mathscr{A}^1 \parallel_{\mathbb{P}} \mathscr{A}^2$ be cross product $\mathscr{A}^1$ and $\mathscr{A}^2$ where $\mathscr{A} = (Æ, \Xi, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ is pSVNHSS over*

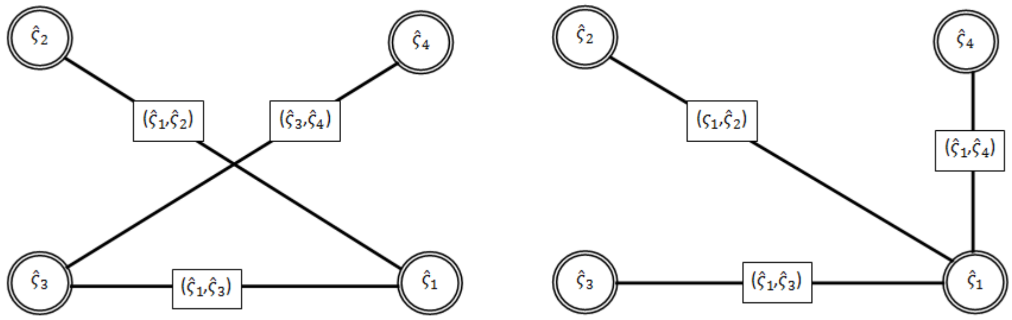

Fig. 11: Graph-based presentation of Table 3 with (c): $\mathbb{W}(\hat{\omega}_3)$, (d): $\mathbb{W}(\hat{\omega}_4)$

Table 11: psvNHSG $\mathbb{W}(\hat{\omega}_1,\hat{\omega}_3) = \mathbb{W}_1(\hat{\omega}_1) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_3)$ of $\mathscr{A} = \mathscr{A}^1 \subseteq_{\mathbb{P}} \mathscr{A}^2$ Example 5 demonstrated in Fig. 12

Æ	$\hat{\partial}_{11}$	$\hat{\partial}_{12}$	$\hat{\partial}_{13}$	$\hat{\partial}_{14}$	$\hat{\partial}_{21}$	$\hat{\partial}_{22}$
$(\hat{\omega}_1,\hat{\omega}_3)$	(0.3,0.5,0.7,0.3)	(0.3,0.5,0.7,0.3)	(0.3,0.5,0.9,0.3)	(0.3,0.4,0.7,0.3)	(0.5,0.6,0.8,0.5)	(0.4,0.5,0.8,0.4)
Æ	$\hat{\partial}_{23}$	$\hat{\partial}_{24}$	$\hat{\partial}_{31}$	$\hat{\partial}_{32}$	$\hat{\partial}_{33}$	$\hat{\partial}_{34}$
$(\hat{\omega}_1,\hat{\omega}_3)$	(0.4,0.6,0.9,0.4)	(0.5,0.4,0.8,0.5)	(0.5,0.6,0.8,0.5)	(0.4,0.5,0.8,0.4)	(0.4,0.6,0.9,0.4)	(0.5,0.4,0.8,0.5)
Ξ	$(\hat{\partial}_{11},\hat{\partial}_{12})$	$(\hat{\partial}_{11},\hat{\partial}_{13})$	$(\hat{\partial}_{11},\hat{\partial}_{21})$	$(\hat{\partial}_{11},\hat{\partial}_{31})$	$(\hat{\partial}_{12},\hat{\partial}_{22})$	$(\hat{\partial}_{12},\hat{\partial}_{32})$
$(\hat{\omega}_1,\hat{\omega}_3)$	(0.3,0.4,0.7,0.3)	(0.3,0.4,0.7,0.3)	(0.3,0.4,0.5,0.3)	(0.5,0.6,0.8,0.5)	(0.3,0.4,0.5,0.3)	(0.2,0.3,0.6,0.2)
Ξ	$(\hat{\partial}_{13},\hat{\partial}_{33})$	$(\hat{\partial}_{13},\hat{\partial}_{14})$	$(\hat{\partial}_{14},\hat{\partial}_{24})$	$(\hat{\partial}_{14},\hat{\partial}_{34})$	$(\hat{\partial}_{21},\hat{\partial}_{22})$	$(\hat{\partial}_{21},\hat{\partial}_{23})$
$(\hat{\omega}_1,\hat{\omega}_3)$	(0.2,0.3,0.9,0.2)	(0.3,0.3,0.7,0.3)	(0.3,0.3,0.6,0.3)	(0.2,0.3,0.6,0.2)	(0.3,0.4,0.8,0.3)	(0.3,0.4,0.8,0.3)
Ξ	$(\hat{\partial}_{22},\hat{\partial}_{32})$	$(\hat{\partial}_{23},\hat{\partial}_{24})$	$(\hat{\partial}_{23},\hat{\partial}_{33})$	$(\hat{\partial}_{24},\hat{\partial}_{34})$	$(\hat{\partial}_{31},\hat{\partial}_{32})$	$(\hat{\partial}_{31},\hat{\partial}_{33})$
$(\hat{\omega}_1,\hat{\omega}_3)$	(0.3,0.4,0.5,0.3)	(0.3,0.3,0.8,0.3)	(0.3,0.4,0.9,0.3)	(0.3,0.4,0.5,0.3)	(0.3,0.4,0.8,0.3)	(0.3,0.4,0.8,0.3)
Ξ	$(\hat{\partial}_{13},\hat{\partial}_{23})$	$(\hat{\partial}_{21},\hat{\partial}_{31})$	$(\hat{\partial}_{33},\hat{\partial}_{34})$			
$(\hat{\omega}_1,\hat{\omega}_3)$	(0.3,0.4,0.9,0.3)	(0.3,0.4,0.5,0.3)	(0.3,0.3,0.8,0.3)			

$\mathscr{V} = \mathscr{V}_1 \subseteq \mathscr{V}_2$, $\Xi = (\Xi^1 \parallel_{\mathbb{P}} \Xi^2, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ is pSVNHSS over $\mathscr{E} = \{((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) | (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\}$ and $\Xi = (\Xi^1 \parallel_{\mathbb{P}} \Xi^2, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ are psvNHSGs where as

1. $\mathscr{T}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \, ı \, \mathscr{T}_{Æ^2(t)}(\hat{\varsigma})$,
$\mathscr{I}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \, ı \, \mathscr{I}_{Æ^2(t)}(\hat{\varsigma})$,
$\mathscr{F}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{F}_{Æ^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{Æ^2(t)}(\hat{\varsigma})$,
$\mu_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mu_{Æ^1(\gamma)}(\hat{\varrho}) \, ı \, \mu_{Æ^2(t)}(\hat{\varsigma}) \quad (\hat{\varrho},\hat{\varsigma}) \in \mathscr{V}, (s,t) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$.
2. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \, ı \, \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2)$,
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \, ı \, \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2)$,
$\mathscr{F}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2)$,
$\mu_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \, ı \, \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2), \ \hat{\varrho}_1,\hat{\varrho}_2 \in \mathscr{E}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2$

$(\hat{\varrho},\hat{\varsigma}) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$, $\mathbb{W}(\gamma,\zeta) = \mathbb{W}_1(\gamma) \parallel_{\mathbb{P}} \mathbb{W}_2(\zeta)$ are psvNHSGs of $\mathscr{A}$.

Theorem 4.*The cross product of two psvNHSGs is psvNHSG.*

Definition 15.*For two psvNHSGs $\mathscr{A}^1 = (\mathscr{A}^1, \mathscr{Q}^1, Æ^1, \Xi^1)$ and $\mathscr{A}^2 = (\mathscr{A}^2, \mathscr{Q}^2, Æ^2, \Xi^2)$ w.r.t. $\mathscr{A}^1 = (\mathscr{V}_1, \mathscr{E}_2)$ and $\mathscr{A}^2 = (\mathscr{V}_2, \mathscr{E}_2)$. Let $\mathscr{A} = \mathscr{A}^1 |_{\mathbb{P}} \mathscr{A}^2$ be lexicographic product $\mathscr{A}^1$ and $\mathscr{A}^2$ where $\mathscr{A} = (Æ, \Xi, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ is pSVNHSS over $\mathscr{V} = \mathscr{V}_1 \subseteq \mathscr{V}_2$, $\Xi = (\Xi^1 |_{\mathbb{P}} \Xi^2, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ is pSVNHSS over*

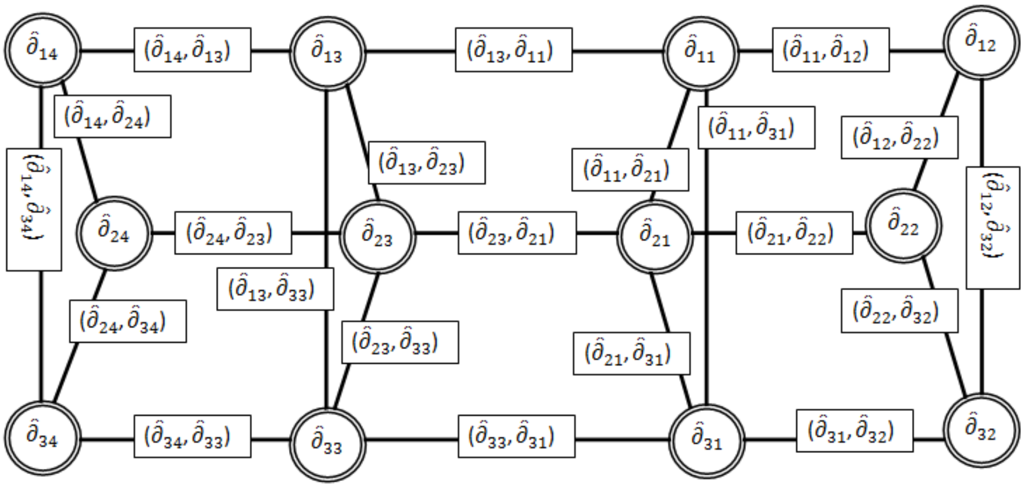

Fig. 12: The graph-based presentation of $\mathbb{W}(\hat{\omega}_1,\hat{\omega}_3) = \mathbb{W}_1(\hat{\omega}_1) \subseteq_{\mathbb{P}} \mathbb{W}_2(\hat{\omega}_3)$

$\mathscr{E} = \{((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2))|\hat{\varrho} \in \mathscr{V}_1,(\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\} \Re \{((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2))|(\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1,(\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\}$ *and* $\Xi = (\Xi^1 \,|_{\mathbb{P}}\, \Xi^2, \mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *are psvNHSGs where as*

1. $\mathscr{T}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{Æ^2(t)}(\hat{\varsigma}),$
$\mathscr{I}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{Æ^2(t)}(\hat{\varsigma}),$
$\mathscr{F}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{F}_{Æ^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{Æ^2(t)}(\hat{\varsigma}),$
$\mu_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mu_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mu_{Æ^2(t)}(\hat{\varsigma}) \quad (\hat{\varrho},\hat{\varsigma}) \in \mathscr{V},(s,t) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2.$
2. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) = \mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) = \mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{F}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) = \mathscr{F}_{Æ^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mu_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) = \mu_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2), \ \hat{\varrho} \in \mathscr{V}_1,(\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2.$
3. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{F}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mu_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) = \mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2), \ (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1,(\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2.$

Here $(\hat{\varrho},\hat{\varsigma}) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$, $\mathbb{W}(\gamma,\zeta) = \mathbb{W}_1(\gamma) \,|_{\mathbb{P}}\, \mathbb{W}_2(\zeta)$ *are psvNHSGs of* $\mathscr{A}$.

Theorem 5. *The lexicographical product of two psvNHSGs is psvNHSG.*

Definition 16. *For two psvNHSGs* $\mathscr{A}^1 = (\mathscr{A}^1,\mathscr{Q}^1,Æ^1,\Xi^1)$ *and* $\mathscr{A}^2 = (\mathscr{A}^2,\mathscr{Q}^2,Æ^2,\Xi^2)$ *w.r.t.* $\mathscr{A}^1 = (\mathscr{V}_1,\mathscr{E}_2)$ *and* $\mathscr{A}^2 = (\mathscr{V}_2,\mathscr{E}_2)$. *Let* $\mathscr{A} = \mathscr{A}^1 \rangle_{\mathbb{P}} \mathscr{A}^2$ *be strong product of* $\mathscr{A}^1$ *and* $\mathscr{A}^2$ *where* $\mathscr{A} = (Æ,\Xi,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *is pSVNHSS over* $\mathscr{V} = \mathscr{V}_1 \subseteq \mathscr{V}_2$, $\Xi = (\Xi^1 \rangle_{\mathbb{P}} \Xi^2,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *is pSVNHSS over* $\mathscr{E} = \{((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2))|\hat{\varrho} \in \mathscr{V}_1,(\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\} \Re \{((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma}))|(\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1,\hat{\varsigma} \in \mathscr{V}_2\}$
$\Re \{((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2))|(\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1,(\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\}$ *and* $\Xi = (\Xi^1 \rangle_{\mathbb{P}} \Xi^2,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *are psvNHSGs where as*

1. $\mathscr{T}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{Æ^2(t)}(\hat{\varsigma}),$
$\mathscr{I}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{Æ^2(t)}(\hat{\varsigma}),$
$\mathscr{F}_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{F}_{Æ^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{Æ^2(t)}(\hat{\varsigma}),$
$\mu_{Æ(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mu_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mu_{Æ^2(t)}(\hat{\varsigma}) \quad (\hat{\varrho},\hat{\varsigma}) \in \mathscr{V},(s,t) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2.$
2. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) =$
$\mathscr{T}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) =$
$\mathscr{I}_{Æ^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$

$\mathscr{F}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)\right) =$
$\mathscr{F}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mu_{\Xi(\gamma,\zeta)}\left((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)\right) =$
$\mu_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),\ \hat{\varrho} \in \mathscr{V}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2.$

3. $\mathscr{T}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{T}_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{I}_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),$
$\mathscr{F}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \infty \mathscr{F}_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),$
$\mu_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mu_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),\ (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1, \hat{\varsigma} \in \mathscr{V}_2.$

4. $\mathscr{T}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)\right) =$
$\mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)\right) =$
$\mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{F}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)\right) =$
$\mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mu_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)\right) =$
$\mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),\ (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2.$

Here $(\hat{\varrho},\hat{\varsigma}) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$, $\mathbb{W}(\gamma,\zeta) = \mathbb{W}_1(\gamma) \rangle_{\mathbb{P}} \mathbb{W}_2(\zeta)$ *are psvNHSGs of* $\mathscr{A}$.

Theorem 6. *The strong product of two psvNHSGs is psvNHSG.*

Definition 17. *For two psvNHSGs* $\mathscr{A}^1 = \left(\mathscr{A}^1,\mathscr{Q}^1,\text{Æ}^1,\Xi^1\right)$ *and* $\mathscr{A}^2 = \left(\mathscr{A}^2,\mathscr{Q}^2,\text{Æ}^2,\Xi^2\right)$ *w.r.t.* $\mathscr{A}^1 = (\mathscr{V}_1,\mathscr{E}_2)$ *and* $\mathscr{A}^2 = (\mathscr{V}_2,\mathscr{E}_2)$. *Let* $\mathscr{A} = \mathscr{A}^1[\mathscr{A}^2]$ *be composition of* $\mathscr{A}^1$ *and* $\mathscr{A}^2$ *where* $\mathscr{A} = (\text{Æ},\Xi,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *is pSVNHSS over* $\mathscr{V} = \mathscr{V}_1 \subseteq \mathscr{V}_2$, $\Xi = (\Xi^1 \subseteq \Xi^2,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *is pSVNHSS over* $\mathscr{E} = \{((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)) | \hat{\varrho} \in \mathscr{V}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2\} \Re \{((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})) | (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1, \hat{\varsigma} \in \mathscr{V}_2\}$
$\Re \{((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) | (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1, \hat{\varsigma}_1 \neq \hat{\varsigma}_2\}$ *and* $\Xi = (\Xi^1 \subseteq \Xi^2,\mathscr{Q}^1 \subseteq \mathscr{Q}^2)$ *are psvNHSGs where as*

1. $\mathscr{T}_{\text{Æ}(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{T}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{\text{Æ}^2(t)}(\hat{\varsigma}),$
$\mathscr{I}_{\text{Æ}(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{I}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{\text{Æ}^2(t)}(\hat{\varsigma}),$
$\mathscr{F}_{\text{Æ}(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mathscr{F}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{\text{Æ}^2(t)}(\hat{\varsigma}),$
$\mu_{\text{Æ}(\gamma,\zeta)}(\hat{\varrho},\hat{\varsigma}) = \mu_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mu_{\text{Æ}^2(t)}(\hat{\varsigma})\ \ (\hat{\varrho},\hat{\varsigma}) \in \mathscr{V}, (s,t) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2.$

2. $\mathscr{T}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)\right) =$
$\mathscr{T}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mathscr{T}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)\right) =$
$\mathscr{I}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mathscr{I}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mathscr{F}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)\right) =$
$\mathscr{F}_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \infty \mathscr{F}_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),$
$\mu_{\Xi(\gamma,\zeta)}\left((\hat{\varrho},\hat{\varsigma}_1),(\hat{\varrho},\hat{\varsigma}_2)\right) =$
$\mu_{\text{Æ}^1(\gamma)}(\hat{\varrho}) \prime \mu_{\Xi^2(\zeta)}(\hat{\varsigma}_1,\hat{\varsigma}_2),\ \hat{\varrho} \in \mathscr{V}_1, (\hat{\varsigma}_1,\hat{\varsigma}_2) \in \mathscr{E}_2.$

3. $\mathscr{T}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{T}_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),$
$\mathscr{I}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mathscr{I}_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),$
$\mathscr{F}_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \infty \mathscr{F}_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),$
$\mu_{\Xi(\gamma,\zeta)}\left((\hat{\varrho}_1,\hat{\varsigma}),(\hat{\varrho}_2,\hat{\varsigma})\right) =$
$\mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \prime \mu_{\text{Æ}^2(\zeta)}(\hat{\varsigma}),\ (\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1, \hat{\varsigma} \in \mathscr{V}_2.$

4. $\mathscr{T}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) =$
$\mathscr{T}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \,\prime\, \mathscr{T}_{Æ^1(\zeta)}(\hat{\varsigma}_2) \,\prime\, \mathscr{T}_{Æ^2(\zeta)}(\hat{\varsigma}_1)$,
$\mathscr{I}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) =$
$\mathscr{I}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \,\prime\, \mathscr{I}_{Æ^1(\zeta)}(\hat{\varsigma}_2) \,\prime\, \mathscr{I}_{Æ^2(\zeta)}(\hat{\varsigma}_1)$,
$\mathscr{F}_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) =$
$\mathscr{F}_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \,\prime\, \mathscr{F}_{Æ^1(\zeta)}(\hat{\varsigma}_2) \,\prime\, \mathscr{F}_{Æ^2(\zeta)}(\hat{\varsigma}_1)$,
$\mu_{\Xi(\gamma,\zeta)}((\hat{\varrho}_1,\hat{\varsigma}_1),(\hat{\varrho}_2,\hat{\varsigma}_2)) =$
$\mu_{\Xi^1(\gamma)}(\hat{\varrho}_1,\hat{\varrho}_2) \,\prime\, \mu_{Æ^1(\zeta)}(\hat{\varsigma}_2) \,\prime\, \mu_{Æ^2(\zeta)}(\hat{\varsigma}_1)$, $(\hat{\varrho}_1,\hat{\varrho}_2) \in \mathscr{E}_1$ *and* $\hat{\varsigma}_1 \neq \hat{\varsigma}_2$.

Here $(\gamma,\zeta) \in \mathscr{Q}^1 \subseteq \mathscr{Q}^2$, $\mathbb{W}(\gamma,\zeta) = \mathbb{W}_1(\gamma)[\mathbb{W}_2(\zeta)]$ *are psvNHSGs of* $\mathscr{A}$.

Theorem 7. *The composition of two psvNHSGs is psvNHSG.*

Definition 18. *For two psvNHSGs* $\mathscr{A}^1 = (\mathscr{A}^1,\mathscr{Q}^1,Æ^1,\Xi^1)$ *and* $\mathscr{A}^2 = (\mathscr{A}^2,\mathscr{Q}^2,Æ^2,\Xi^2)$ *w.r.t.* $\mathscr{A}^1 = (\mathscr{V}_1,\mathscr{E}_2)$ *and* $\mathscr{A}^2 = (\mathscr{V}_2,\mathscr{E}_2)$. *Let* $\mathscr{A} = \mathscr{A}^1 \Re \mathscr{A}^2$ *be the union of* $\mathscr{A}^1$ *and* $\mathscr{A}^2$ *where* $\mathscr{A} = (Æ,\Xi,\mathscr{Q}^1 \Re \mathscr{Q}^2)$ *is pSVNHSS over* $\mathscr{V} = \mathscr{V}_1 \Re \mathscr{V}_2$, $\Xi = (\Xi^1 \Re \Xi^2, \mathscr{Q}^1 \Re \mathscr{Q}^2)$ *is pSVNHSS over* $\mathscr{E} = \mathscr{E}_1 \Re \mathscr{E}_2$ *where for* $\hat{\varrho},\hat{\varsigma} \in \mathscr{V}$, *PSVN-components are stated as*

1. $\mathscr{T}_{Æ_{\hat{\omega}}}(\hat{\varsigma}) =$
$$\begin{cases} \mathscr{T}_{Æ^1(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{T}_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{T}_{Æ^1(\hat{\omega})}(\hat{\varsigma}) \infty \mathscr{T}_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
$\mathscr{I}_{Æ_{\hat{\omega}}}(\hat{\varsigma}) =$
$$\begin{cases} \mathscr{I}_{Æ^1(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{I}_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{I}_{Æ^1(\hat{\omega})}(\hat{\varsigma}) \infty \mathscr{I}_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
$\mathscr{F}_{Æ_{\hat{\omega}}}(\hat{\varsigma}) =$
$$\begin{cases} \mathscr{F}_{Æ^1(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{F}_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{F}_{Æ^1(\hat{\omega})}(\hat{\varsigma}) \,\prime\, \mathscr{F}_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
$\mu_{Æ_{\hat{\omega}}}(\hat{\varsigma}) =$
$$\begin{cases} \mu_{Æ^1(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mu_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mu_{Æ^1(\hat{\omega})}(\hat{\varsigma}) \infty \mu_{Æ^2(\hat{\omega})}(\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
2. $\mathscr{T}_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) =$
$$\begin{cases} \mathscr{T}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{T}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{T}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \infty \mathscr{T}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
$\mathscr{I}_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) =$
$$\begin{cases} \mathscr{I}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{I}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{I}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \infty \mathscr{I}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
$\mathscr{F}_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) =$
$$\begin{cases} \mathscr{F}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{F}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{F}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \,\prime\, \mathscr{F}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$$
$\mu_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) =$
$$\begin{cases} \mu_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mu_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mu_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \infty \mu_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ;\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases}.$$

Definition 19. *For two psvNHSGs* $\mathscr{A}^1 = (\mathscr{A}^1,\mathscr{Q}^1,Æ^1,\Xi^1)$ *and* $\mathscr{A}^2 = (\mathscr{A}^2,\mathscr{Q}^2,Æ^2,\Xi^2)$ *w.r.t.* $\mathscr{A}^1 = (\mathscr{V}_1,\mathscr{E}_2)$ *and* $\mathscr{A}^2 = (\mathscr{V}_2,\mathscr{E}_2)$. *Let* $\mathscr{A} = \mathscr{A}^1 \Im \mathscr{A}^2$ *be the intersection of* $\mathscr{A}^1$ *and* $\mathscr{A}^2$ *where* $\mathscr{A} = (Æ,\Xi,\mathscr{Q}^1 \Re \mathscr{Q}^2)$ *is pSVNHSS over*

$\mathscr{V} = \mathscr{V}_1 \Im \mathscr{V}_2$, $\Xi = (\Xi^1 \Re \Xi^2, \mathscr{Q}^1 \Re \mathscr{Q}^2)$ is pSVNHSS over $\mathscr{E} = \mathscr{E}_1 \Im \mathscr{E}_2$ where for $\hat{\varrho}, \hat{\varsigma} \in \mathscr{V}$, PSVN-components can be given by

1. $\mathscr{T}_{\text{Æ}_{\hat{\omega}}}(\hat{\varsigma}) = \begin{cases} \mathscr{T}_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{T}_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{T}_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) \prime \mathscr{T}_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

   $\mathscr{I}_{\text{Æ}_{\hat{\omega}}}(\hat{\varsigma}) = \begin{cases} \mathscr{I}_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{I}_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{I}_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) \prime \mathscr{I}_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

   $\mathscr{F}_{\text{Æ}_{\hat{\omega}}}(\hat{\varsigma}) = \begin{cases} \mathscr{F}_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{F}_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{F}_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) \infty \mathscr{F}_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

   $\mu_{\text{Æ}_{\hat{\omega}}}(\hat{\varsigma}) = \begin{cases} \mu_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mu_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mu_{\text{Æ}^1(\hat{\omega})}(\hat{\varsigma}) \prime \mu_{\text{Æ}^2(\hat{\omega})}(\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

2. $\mathscr{T}_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) = \begin{cases} \mathscr{T}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{T}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{T}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \prime \mathscr{T}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

   $\mathscr{I}_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) = \begin{cases} \mathscr{I}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{I}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{I}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \prime \mathscr{I}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

   $\mathscr{F}_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) = \begin{cases} \mathscr{F}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mathscr{F}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mathscr{F}_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \infty \mathscr{F}_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases},$

   $\mu_{\Xi_{\hat{\omega}}}(\hat{\varrho}\hat{\varsigma}) = \begin{cases} \mu_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \circ \mathscr{Q}^2 \\ \mu_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^2 \circ \mathscr{Q}^1 \\ \mu_{\Xi^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) \prime \mu_{\Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) & ; \hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2 \end{cases}.$

Definition 20.*For two psvNHSGs $\mathscr{A}^1 = (\mathscr{A}^1, \mathscr{Q}^1, \text{Æ}^1, \Xi^1)$ and $\mathscr{A}^2 = (\mathscr{A}^2, \mathscr{Q}^2, \text{Æ}^2, \Xi^2)$ w.r.t. $\mathscr{A}^1 = (\mathscr{V}_1, \mathscr{E}_2)$ and $\mathscr{A}^2 = (\mathscr{V}_2, \mathscr{E}_2)$. Let $\mathscr{A} = \mathscr{A}^1 \Big/ \mathscr{A}^2$ be the join of $\mathscr{A}^1$ and $\mathscr{A}^2$ where $\mathscr{A} = (\text{Æ}^1 \Big/ \text{Æ}^2, \Xi^1 \Big/ \Xi^2, \mathscr{Q}^1 \Re \mathscr{Q}^2)$ is pSVNHSS over $\mathscr{V} = \mathscr{V}_1 \Re \mathscr{V}_2$, $\Xi = (\Xi^1 \Big/ \Xi^2, \mathscr{Q}^1 \Re \mathscr{Q}^2)$ is pSVNHSS over $\mathscr{E} = \mathscr{E}_1 \Re \mathscr{E}_2$ where*

1. $(\text{Æ}^1 \Big/ \text{Æ}^2, \mathscr{Q}^1 \Re \mathscr{Q}^2) = (\text{Æ}^1, \mathscr{Q}^1) \Re (\text{Æ}^2, \mathscr{Q}^2)$.
2. $(\Xi^1 \Big/ \Xi^2, \mathscr{Q}^1 \Re \mathscr{Q}^2) = (\Xi^1, \mathscr{Q}^1) \Re (\Xi^2, \mathscr{Q}^2), if\, \hat{\varrho}\hat{\varsigma} \in \mathscr{E}_1 \Re \mathscr{E}_2$.

when $\hat{\omega} \in \mathscr{Q}^1 \Im \mathscr{Q}^2$ and $\hat{\varrho}\hat{\varsigma} \in \mathscr{E}$ and uncertain parts are

$\mathscr{T}_{\Xi^1 \big/ \Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mathscr{T}_{\text{Æ}^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}), \mathscr{T}_{\text{Æ}^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma})\right\},$

$\mathscr{I}_{\Xi^1 \big/ \Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mathscr{I}_{\text{Æ}^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}), \mathscr{I}_{\text{Æ}^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma})\right\},$

$$\mathscr{F}_{\Xi^1 \big/ \Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mathscr{F}_{Æ^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}), \mathscr{F}_{Æ^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma})\right\},$$
$$\mu_{\Xi^1 \big/ \Xi^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mu_{Æ^1(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}), \mu_{Æ^2(\hat{\omega})}(\hat{\varrho}\hat{\varsigma})\right\}.$$

Definition 21. *The complement $\mathscr{A}^c = (\mathscr{A}^c, \mathscr{Q}^c, Æ^c, \Xi^c)$ of psvNHSG $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ is a psvNHSG for which $\hat{\varrho}, \hat{\varsigma} \in \mathscr{V}$ and $\hat{\omega} \in \mathscr{Q}$ and it satisfies the following conditions*

1. $\mathscr{Q}^c = \mathscr{Q}$.
2. $Æ^c(\hat{\omega}) = Æ(\hat{\omega})$.
3. $\mathscr{T}_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \mathscr{T}_{Æ(\hat{\omega})}(\hat{\varrho}) \prime \mathscr{T}_{Æ(\hat{\omega})}(\hat{\varsigma}) \circ \mathscr{T}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma})$.
4. $\mathscr{I}_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \mathscr{I}_{Æ(\hat{\omega})}(\hat{\varrho}) \prime \mathscr{I}_{Æ(\hat{\omega})}(\hat{\varsigma}) \circ \mathscr{I}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma})$.
5. $\mathscr{F}_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \mathscr{F}_{Æ(\hat{\omega})}(\hat{\varrho}) \prime \mathscr{F}_{Æ(\hat{\omega})}(\hat{\varsigma}) \circ \mathscr{F}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma})$.
6. $\mu_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \mu_{Æ(\hat{\omega})}(\hat{\varrho}) \prime \mu_{Æ(\hat{\omega})}(\hat{\varsigma}) \circ \mu_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma})$.

Definition 22. *If $\mathscr{A}^c = \mathscr{A}$ where $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ is a psvNHSG, then $\mathscr{A}$ is self complementary.*

Definition 23. *If $\Xi(\hat{\omega})$ is PSVNH-graph of $\mathscr{A}$, $\hat{\omega} \in \mathscr{Q}$, then it is complete with*
$$\mathscr{T}_{\Xi(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mathscr{T}_{Æ(\hat{\omega})}(\hat{\varrho}), \mathscr{T}_{Æ(\hat{\omega})}(\hat{\varsigma})\right\},$$
$$\mathscr{I}_{\Xi(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mathscr{I}_{Æ(\hat{\omega})}(\hat{\varrho}), \mathscr{I}_{Æ(\hat{\omega})}(\hat{\varsigma})\right\},$$
$$\mathscr{I}_{\Xi(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mathscr{I}_{Æ(\hat{\omega})}(\hat{\varrho}), \mathscr{I}_{Æ(\hat{\omega})}(\hat{\varsigma})\right\},$$
$$\mu_{\Xi(\hat{\omega})}(\hat{\varrho}\hat{\varsigma}) = \min\left\{\mu_{Æ(\hat{\omega})}(\hat{\varrho}), \mu_{Æ(\hat{\omega})}(\hat{\varsigma})\right\}.$$

Definition 24. *A psvNHSG $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ is strong psvNHSG if $\Xi(\hat{\omega})$ is SSVNH-graph of $\mathscr{A}$, $\hat{\omega} \in \mathscr{Q}$.*

Theorem 8. *For strong psvNHSGs $\mathscr{A}^1 = (\mathscr{A}^1, \mathscr{Q}^1, Æ^1, \Xi^1)$ and $\mathscr{A}^2 = (\mathscr{A}^2, \mathscr{Q}^2, Æ^2, \Xi^2)$ w.r.t. $\mathscr{A}^1 = (\mathscr{V}_1, \mathscr{E}_2)$ and $\mathscr{A}^2 = (\mathscr{V}_2, \mathscr{E}_2)$, then $\mathscr{A}^1[\mathscr{A}^2]$, their composition is strong psvNHSG.*

Theorem 9. *For strong psvNHSGs $\mathscr{A}^1 = (\mathscr{A}^1, \mathscr{Q}^1, Æ^1, \Xi^1)$ and $\mathscr{A}^2 = (\mathscr{A}^2, \mathscr{Q}^2, Æ^2, \Xi^2)$ w.r.t. $\mathscr{A}^1 = (\mathscr{V}_1, \mathscr{E}_2)$ and $\mathscr{A}^2 = (\mathscr{V}_2, \mathscr{E}_2)$, then $\mathscr{A}^1 \subseteq_{\mathbb{P}} \mathscr{A}^2$, their cartesian product is strong psvNHSG.*

Definition 25. *The complement $\mathscr{A}^c = (\mathscr{A}^c, \mathscr{Q}^c, Æ^c, \Xi^c)$ of strong psvNHSG $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ $\hat{\omega} \in Æ, \hat{\varrho}, \hat{\varsigma} \in \mathscr{V}$, is given by*

1. $\mathscr{Q}^c = \mathscr{Q}$.
2. $Æ^c(\hat{\omega})(\hat{\varrho}) = Æ(\hat{\omega})(\hat{\varrho})$.
3. $\mathscr{T}_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \begin{cases} \min\left\{\mathscr{T}_{Æ(\hat{\omega})}(\hat{\varrho}), \mathscr{T}_{Æ(\hat{\omega})}(\hat{\varsigma})\right\} & ; \mathscr{T}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = 0 \\ 0 & ; \mathscr{T}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) > 0 \end{cases}.$
4. $\mathscr{I}_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \begin{cases} \min\left\{\mathscr{I}_{Æ(\hat{\omega})}(\hat{\varrho}), \mathscr{I}_{Æ(\hat{\omega})}(\hat{\varsigma})\right\} & ; \mathscr{I}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = 0 \\ 0 & ; \mathscr{I}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) > 0 \end{cases}.$
5. $\mathscr{F}_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \begin{cases} \min\left\{\mathscr{F}_{Æ(\hat{\omega})}(\hat{\varrho}), \mathscr{F}_{Æ(\hat{\omega})}(\hat{\varsigma})\right\} & ; \mathscr{F}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = 0 \\ 0 & ; \mathscr{F}_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) > 0 \end{cases}.$
6. $\mu_{\Xi^c(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = \begin{cases} \min\left\{\mu_{Æ(\hat{\omega})}(\hat{\varrho}), \mu_{Æ(\hat{\omega})}(\hat{\varsigma})\right\} & ; \mu_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) = 0 \\ 0 & ; \mu_{\Xi(\hat{\omega})}(\hat{\varrho}, \hat{\varsigma}) > 0 \end{cases}.$

Theorem 10. *The complement $\mathscr{A}^c = (\mathscr{A}^c, \mathscr{Q}^c, Æ^c, \Xi^c)$ of strong psvNHSG $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ $\hat{\omega} \in Æ, \hat{\varrho}, \hat{\varsigma} \in \mathscr{V}$, is strong psvNHSG.*

Theorem 11. *If $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ and its complement $\mathscr{A}^c = (\mathscr{A}^c, \mathscr{Q}^c, Æ^c, \Xi^c)$ are strong psvNHSGs $\hat{\omega} \in Æ, \hat{\varrho}, \hat{\varsigma} \in \mathscr{V}$, then the union $\mathscr{A} \Re \mathscr{A}^c$ is itself complete psvNHSG.*

After establishing the foundational concepts and mathematical models for managing uncertainty and imprecise information through a theoretical framework, psvNHSG, we apply these theories to real-world decision-making problem to demonstrate their practicality and effectiveness. This application phase will help validate the theoretical results by showing how the proposed methods enhance accuracy and reliability in actual scenarios.

2.5 Application of psvNHSG in MADM-based Recruitment Process

The MADM can be used in the recruitment process to evaluate and compare job candidates based on multiple criteria, such as skills, experience, education, and cultural fit. Techniques like weighted scoring and decision matrices can help streamline and improve the selection process. However, it is crucial to ensure that the criteria are relevant and non-discriminatory, and that the decision-making process remains transparent and fair. To create a trustworthy recruitment system, an MADM-based algorithm is proposed and validated by applying it to a real-life recruitment process. The proposed algorithm and the application are the modified versions of algorithm and case study presented by Rahman et al. [25].

*Example 6.*Assume that a company is looking to hire someone to fill the assistant manager position that is currently unfilled. The recruitment committee has examined six candidates, $\mathscr{V} = \{\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \mathfrak{C}_4, \mathfrak{C}_5, \mathfrak{C}_6\}$. To choose one of these candidates, the committee needs to conduct additional analysis. Qualification (β_1), relevant experience in years (β_2), and computer skill (β_3) are the assessment indications. Their sub-parametric disjoint sets are: $\mathscr{Q}_1 = \{\beta_{11} = Graduate\}$, $\mathscr{Q}_2 = \{\beta_{21} = 5, \beta_{22} = 7, \beta_{23} = 10\}$ and $\mathscr{Q}_3 = \{\beta_{31} = MSoffice\}$ respectively such that $\mathscr{Q} = \mathscr{Q}_1 \subseteq \mathscr{Q}_2 \subseteq \mathscr{Q}_3 = \{\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3\}$ and $\mathscr{A} = \{(\mathbb{W}, \mathscr{Q})\} = \{(\mathbb{W}(\hat{\omega}_1)), (\mathbb{W}(\hat{\omega}_2)), (\mathbb{W}(\hat{\omega}_3))\}$ is psvNHSG. This selection is completed based on the algorithm (Figure 13 states its depiction) given below:

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Algorithm: An MADM-based recruitment process using aggregation operations of psvNHSG

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In this algorithm, the matrix operations of the pSVNHSS framework are employed to systematically construct the decision matrix, which represents the evaluations of alternatives with respect to multiple parameters. Through these matrix manipulations, the algorithm computes the scoring values by aggregating the neutrosophic information: truth, indeterminacy, and falsity degrees, associated with each alternative. These computed scores are then utilized to rank the alternatives in an objective and consistent manner, ensuring that the decision-making process effectively incorporates uncertainty and imprecision inherent in expert judgments.

1. Assume the set $\mathscr{V}$ as initial space consisting of candidates and the $\mathscr{Q}$ as a collection consisting of sub parametric valued tuples.
2. Consider two pSVNHSSs $(\text{Æ}, \mathscr{Q})$ and $(\Xi, \mathscr{Q})$.
3. On the basis of $(\text{Æ}, \mathscr{Q})$ and $(\Xi, \mathscr{Q})$, present psvNHSG $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, \text{Æ}, \Xi)$.
4. Present resultant psvNHSG $\mathbb{W}(\hat{\omega}) = \left[\mathbb{W}(\hat{\omega}_\kappa)\right._{\kappa}$ for $\hat{\omega} = \lfloor_{\kappa} \hat{\omega}_\kappa$ values of κ.
5. Determine I-Matrix on the basis of psvNHSG $\mathbb{W}(\hat{\omega})$.
6. After computing score values $\mathbb{S}_\kappa$ of $\mathfrak{C}_\kappa$ for all κ, compute the average score values by the utilizing $\mathbb{S}_\kappa = \frac{\mathscr{T}_\kappa + \mathscr{I}_\kappa \circ \mathscr{F}_\kappa + \mu_\kappa + 1}{4}$.
7. Recommend the candidate $\mathfrak{C}_\kappa$ such that $\mathfrak{C}_\kappa = \max_i \mathfrak{C}_i$.
8. In case of overlapping values of κ, select unique one $\mathfrak{C}_\kappa$.

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The step-wise depiction of the proposed algorithm is presented in Fig. 13. The psvNHSGs $\mathbb{W}(\hat{\omega}_1), \mathbb{W}(\hat{\omega}_2)$ and $\mathbb{W}(\hat{\omega}_3)$ w.r.t. sub-parametric values are given in Table 12 and stated in Fig. 14. The I-Matrices of psvNHSGs are: The psvNHSG thus constructed is represented in the form of following incidence matrix $\mathbb{W}(\hat{\omega})$ by considering $\hat{\omega} = \hat{\omega}_1 \,/\, \hat{\omega}_2 \,/\, \hat{\omega}_3$. The Table 13 presents the relevant score values along with their averages. It can easily be noticed from Table 13, the candidate $\mathfrak{C}_4$ has secured the greatest score thus it is recommended.

2.6 Comparison Analysis and Discussion

For human resource management, the issue of candidate selection is crucial for any firm. There are not many studies on this subject that take the possibility degree and graphical exploration into account in fuzzy and

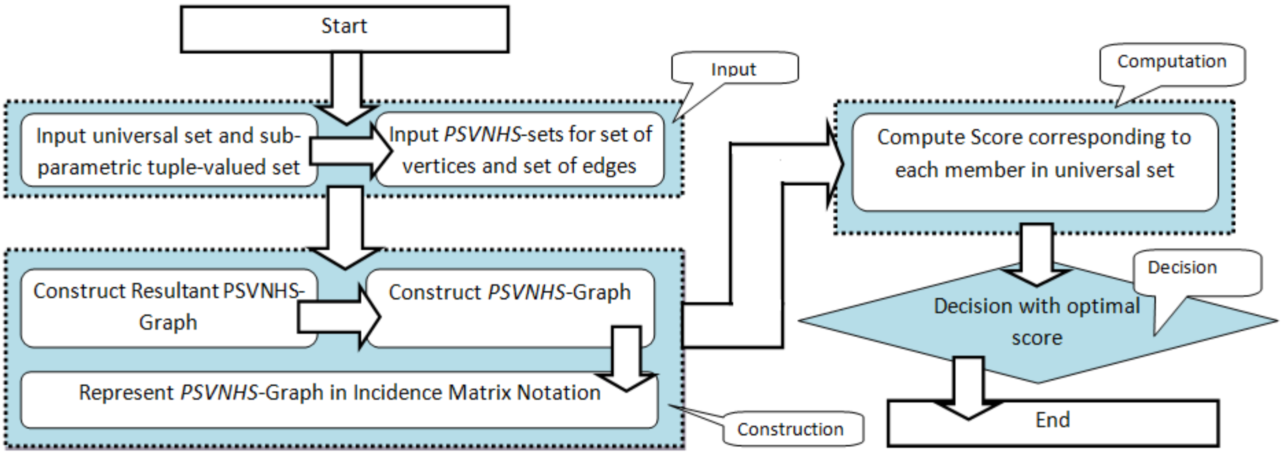

Fig. 13: Proposed Algorithm

Table 12: psvNHSG $\mathscr{A} = (\mathscr{A}, \mathscr{Q}, Æ, \Xi)$ demonstrated in Fig.14

Æ	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	
$\hat{\omega}_1$	(0.4,0.6,0.9,0.4)	(0.3,0.9,0.6,0.3)	(0.5,0.6,0.3,0.5)	(0.6,0.3,0.8,0.6)	(0.5,0.8,0.7,0.5)	(0.3,0.6,0.9,0.3)	
$\hat{\omega}_2$	(0.7,0.8,0.5,0.7)	(0.7,0.3,0.9,0.7)	(0.3,0.7,0.4,0.3)	(0.8,0.5,0.3,0.8)	(0,0,0,0)	(0.7,0.3,0.7,0.7)	
$\hat{\omega}_3$	(0.7,0.4,0.6,0.7)	(0.6,0.3,0.9,0.6)	(0.5,0.5,0.9,0.5)	(0.6,0.7,0.5,0.6)	(0.7,0.5,0.3,0.7)	(0.5,0.8,0.9,0.5)	
Ξ	$(\mathfrak{C}_1,\mathfrak{C}_2)$	$(\mathfrak{C}_1,\mathfrak{C}_3)$	$(\mathfrak{C}_1,\mathfrak{C}_4)$	$(\mathfrak{C}_1,\mathfrak{C}_5)$	$(\mathfrak{C}_2,\mathfrak{C}_3)$	$(\mathfrak{C}_2,\mathfrak{C}_4)$	$(\mathfrak{C}_2,\mathfrak{C}_5)$
$\hat{\omega}_1$	(0.2,0.4,0.7,0.2)	(0,0,1,0)	(0.3,0.2,0.5,0.3)	(0,0,1,0)	(0.3,0.5,0.4,0.3)	(0.2,0.2,0.7,0.2)	(0.3,0.3,0.5,0.3)
$\hat{\omega}_2$	(0.6,0.2,0.7,0.6)	(0.2,0.6,0.4,0.2)	(0.5,0.4,0.4,0.5)	(0,0,1,0)	(0,0,1,0)	(0.6,0.2,0.8,0.6)	(0,0,1,0)
$\hat{\omega}_3$	(0.5,0.2,0.8,0.5)	(0,0,1,0)	(0,0,1,0)	(0.5,0.3,0.4,0.5)	(0.4,0.2,0.7,0.4)	(0.4,0.2,0.6,0.4)	(0,0,1,0)
Ξ	$(\mathfrak{C}_2,\mathfrak{C}_6)$	$(\mathfrak{C}_3,\mathfrak{C}_4)$	$(\mathfrak{C}_3,\mathfrak{C}_5)$	$(\mathfrak{C}_3,\mathfrak{C}_6)$	$(\mathfrak{C}_4,\mathfrak{C}_5)$	$(\mathfrak{C}_5,\mathfrak{C}_6)$	
$\hat{\omega}_1$	(0,0,1,0)	(0,0,1,0)	(0.4,0.5,0.6,0.4)	(0.2,0.4,0.7,0.2)	(0.4,0.2,0.3,0.4)	(0.3,0.5,0.8,0.3)	
$\hat{\omega}_2$	(0.5,0.2,0.8,0.5)	(0.2,0.4,0.4,0.2)	(0,0,1,0)	(0.3,0.2,0.5,0.3)	(0,0,1,0)	(0,0,1,0)	
$\hat{\omega}_3$	(0,0,1,0)	(0,0,1,0)	(0.4,0.3,0.8,0.4)	(0.4,0.3,0.7,0.4)	(0.5,0.4,0.2,0.5)	(0.3,0.4,0.6,0.3)	

$$\mathbb{W}(\hat{\omega}_1) = \begin{pmatrix} (0,0,0,0) & (0.2,0.4,0.7,0.2) & (0,0,0,0) & (0.3,0.2,0.5,0.3) & (0,0,0,0) & (0,0,0,0) \\ (0.2,0.4,0.7,0.2) & (0,0,0,0) & (0.3,0.5,0.4,0.3) & (0.2,0.2,0.7,0.2) & (0.3,0.3,0.5,0.3) & (0,0,0,0) \\ (0,0,0,0) & (0.3,0.5,0.4,0.3) & (0,0,0,0) & (0,0,0,0) & (0.4,0.5,0.6,0.4) & (0.2,0.4,0.7,0.2) \\ (0.3,0.2,0.5,0.3) & (0.2,0.2,0.7,0.2) & (0,0,0,0) & (0,0,0,0) & (0.4,0.2,0.3,0.4) & (0,0,0,0) \\ (0,0,0,0) & (0.3,0.3,0.5,0.3) & (0.4,0.5,0.6,0.4) & (0.4,0.2,0.3,0.4) & (0,0,0,0) & (0.3,0.5,0.8,0.3) \\ (0,0,0,0) & (0,0,0,0) & (0.2,0.4,0.7,0.2) & (0,0,0,0) & (0.3,0.5,0.8,0.3) & (0,0,0,0) \end{pmatrix}$$

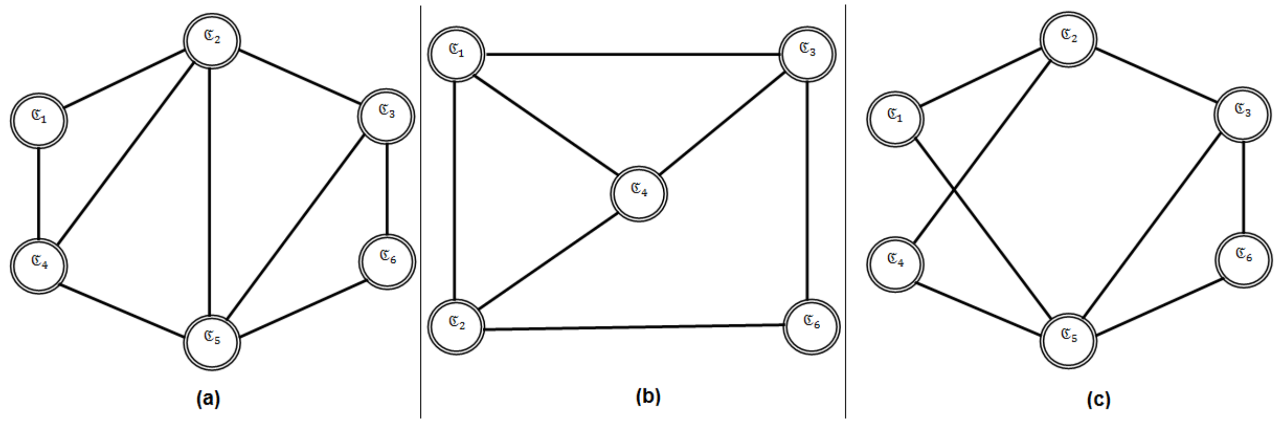

Fig. 14: Pictorial Depiction of Table 12 with (a):$\mathbb{W}(\hat{\omega}_1)$, (b): $\mathbb{W}(\hat{\omega}_2)$ and (c): $\mathbb{W}(\hat{\omega}_3)$.

$$\mathbb{W}(\hat{\omega}_2) = \begin{pmatrix} (0,0,0,0) & (0.6,0.2,0.7,0.6) & (0.2,0.6,0.4,0.2) & (0.5,0.4,0.4,0.5) & (0,0,0,0) & (0,0,0,0) \\ (0.6,0.2,0.7,0.6) & (0,0,0,0) & (0,0,0,0) & (0.6,0.2,0.8,0.6) & (0,0,0,0) & (0.5,0.2,0.8,0.5) \\ (0.2,0.6,0.4,0.2) & (0,0,0,0) & (0,0,0,0) & (0.2,0.4,0.4,0.2) & (0,0,0,0) & (0.3,0.2,0.5,0.3) \\ (0.5,0.4,0.4,0.5) & (0.6,0.2,0.8,0.6) & (0.2,0.4,0.4,0.2) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) \\ (0,0,0,0) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) \\ (0,0,0,0) & (0.5,0.2,0.8,0.5) & (0.3,0.2,0.5,0.3) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) \end{pmatrix}$$

$$\mathbb{W}(\hat{\omega}_3) = \begin{pmatrix} (0,0,0,0) & (0.5,0.2,0.8,0.5) & (0,0,0,0) & (0,0,0,0) & (0.5,0.3,0.4,0.5) & (0,0,0,0) \\ (0.5,0.2,0.8,0.5) & (0,0,0,0) & (0.4,0.2,0.7,0.4) & (0.4,0.2,0.6,0.4) & (0,0,0,0) & (0,0,0,0) \\ (0,0,0,0) & (0.4,0.2,0.7,0.4) & (0,0,0,0) & (0,0,0,0) & (0.4,0.3,0.8,0.4) & (0.4,0.3,0.7,0.4) \\ (0,0,0,0) & (0.4,0.2,0.6,0.4) & (0,0,0,0) & (0,0,0,0) & (0.5,0.4,0.2,0.5) & (0,0,0,0) \\ (0.5,0.3,0.4,0.5) & (0,0,0,0) & (0.4,0.3,0.8,0.4) & (0.5,0.4,0.2,0.5) & (0,0,0,0) & (0.3,0.4,0.6,0.3) \\ (0,0,0,0) & (0,0,0,0) & (0.4,0.3,0.7,0.4) & (0,0,0,0) & (0.3,0.4,0.6,0.3) & (0,0,0,0) \end{pmatrix}$$

$$\mathbb{W}(\hat{\omega}) = \begin{pmatrix} (0,0,0,0) & (0.2,0.2,0.8,0.2) & (0,0,0.4,0) & (0,0,0.5,0) & (0,0,0.4,0) & (0,0,0,0) \\ (0.2,0.2,0.8,0.2) & (0,0,0,0) & (0,0,0.7,0) & (0.2,0.2,0.8,0.2) & (0,0,0.5,0) & (0,0,0.8,0) \\ (0,0,0.4,0) & (0,0,0.7,0) & (0,0,0,0) & (0,0,0.4,0) & (0,0,0.8,0) & (0.2,0.2,0.7,0.2) \\ (0,0,0.5,0) & (0.2,0.2,0.8,0.2) & (0,0,0.4,0) & (0,0,0,0) & (0,0,0.3,0) & (0,0,0,0) \\ (0,0,0.4,0) & (0,0,0.5,0) & (0,0,0.8,0) & (0,0,0.3,0) & (0,0,0,0) & (0,0,0.8,0) \\ (0,0,0,0) & (0,0,0.8,0) & (0.2,0.2,0.7,0) & (0,0,0,0) & (0,0,0.8,0) & (0,0,0,0) \end{pmatrix}$$

Table 13: Presentation of scores along with choice values.

	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_\kappa$
$\mathfrak{C}_1$	0.250	0.200	0.150	0.125	0.150	0.250	1.125
$\mathfrak{C}_2$	0.200	0.250	0.075	0.200	0.125	0.050	0.900
$\mathfrak{C}_3$	0.200	0.075	0.250	0.200	0.050	0.225	1.000
$\mathfrak{C}_4$	0.125	0.200	0.150	0.250	0.175	0.250	1.150
$\mathfrak{C}_5$	0.150	0.125	0.050	0.175	0.250	0.050	0.800
$\mathfrak{C}_6$	0.250	0.050	0.175	0.250	0.050	0.250	1.025

Table 14: Analysis of preferential features of proposed study over predefined researches

Literature	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5	Measure 6	Measure 7
Broumi et al. [1]	⇈	⇈	⇈	⇊	⇊	⇊	⇊
Gani & Begum [2]	⇈	⇈	⇊	⇊	⇊	⇊	⇊
Thumbakara & George [8]	⇊	⇊	⇊	⇈	⇊	⇊	⇊
Shah et al. [9]	⇈	⇈	⇈	⇈	⇊	⇊	⇊
Alkhazaleh et al. [11]	⇈	⇊	⇊	⇈	⇊	⇈	⇊
Bashir et al. [12]	⇈	⇈	⇊	⇈	⇊	⇈	⇊
Karaaslan [13]	⇈	⇈	⇈	⇈	⇊	⇈	⇊
Rahman et al. [22]	⇈	⇈	⇊	⇈	⇈	⇈	⇊
Rahman et al. [24]	⇈	⇈	⇈	⇈	⇈	⇈	⇊
Saeed et al. [27]	⇈	⇈	⇈	⇈	⇈	⇊	⇊
psvNHSG	⇈	⇈	⇈	⇈	⇈	⇈	⇈

soft set-like settings. This study's preferred feature is that it can address the limitations of existing graphical structures regarding consideration of three-dimensional membership-graded settings, consideration of multi-argument domain settings, and consideration of possibility-graded settings. Since literature lacks any pertinent research regarding such application in the psvNHSG environment, the computation-based analysis is not practical, but the structural analysis is offered in Table 14 to highlight its admirable viewpoint and adaptability. In Table 14, the measures 1 to 7 are meant for "consideration of true membership grade", "consideration of false membership grade", "consideration of indeterminacy grade", "consideration of mapping with a single argument", "consideration of mapping with multi-argument", "consideration of entitlement of possibility degree", and "candidate selection ranking with graphical exploration", respectively. Additionally, the symbols ⇈ and ⇊ are meant for yes and no respectively.

3 Conclusions

When making decisions involving multiple attributes, it has been observed that experts sometimes present their advice as three-dimensional arguments (neutrosophic setting). There are also situations that emphasize the need to classify parameters into their respective disjoint sub-classes and to use the possibility degree to evaluate the acceptance level of expert judgments for potential solutions. This study characterizes the fundamental concepts, such as the properties, operations, products, and composition of psvNHSG, to expand the literature for the reflection of the possibility degree that resolves the hesitant nature of neutrosophic elements for each alternative under consideration. The existing literature on soft set-like models in graph theory is unable to address such issues. Essential properties, aggregation operations, and products are examined theoretically and with examples. Additionally, an approach is proposed that makes use of psvNHSG aggregates, and it is further clarified by talking about a real-world application for MADM model. While the proposed psvNHSG framework demonstrates strong potential for efficiently addressing complex decision-making problems characterized by uncertainty and vagueness, its current implementation has been confined to a single organizational case study, which limits the generalizability of the results. To enhance its practical relevance, future studies should investigate the framework's applicability across diverse fields such as healthcare, engineering design, supply chain optimization, and environmental management, where uncertainty and multi-criteria evaluation are prevalent. Moreover, the theoretical foundation of the framework can be further strengthened by extending it to dynamic, hierarchical, or weighted graph structures, allowing for more flexible and realistic modeling of relationships among parameters and alternatives. Despite its strengths, the framework faces certain limitations, particularly the computational complexity that increases with higher-order or large-scale datasets, and the subjective nature of assigning neutrosophic values, which may introduce inconsistencies or bias in evaluation. Addressing these challenges could lead to more efficient, scalable, and objective implementations in future research.

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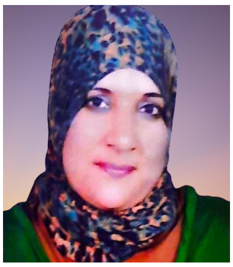

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