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Profile Monitoring for Com-Poisson Responses

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Abstract: In real case problems, the relationship between the response variable and one or more explanatory variables is called profile that should be monitored over time instead of the quality characteristic itself. Profile monitoring is used in such instances. Many researches have been done in profile monitoring but most of them assumed that the response variable follows normal distribution. In this paper, we will obtain the form of the Sum Com-Poisson likelihood equation and the parametric estimation. For a random sample $Y_1 \dots Y_N$, where $P(Yi = yi \mid \lambda, v, m)$, $i = 1, \dots, N$. where the Conway–Maxwell–Poisson (CMP or COM–Poisson).

Keywords: Profile monitoring, Statistical process control, Phase I, Com-Poisson responses

1 Introduction

In the area of statistical process control, sometimes the relationship between a response variable and predictor variables, which is called as profile, should be monitored over time. There are many classifications for profiles according to the type of the relationship which should be monitored. For example, simple linear profiles, non-linear profiles, polynomial profiles, etc. Most of the literature concerned with profile monitoring deals with Phase I and Phase II analysis of simple linear profiles. For example, see [1], [2], [3], [4], [5] and [6]. [7] Have proposed new linear profile schemes for shrinking variations in chemical processes, linear profile monitoring is an approach that describes the direct relationship between the process or product characteristics and further checks the stability of the relationship by monitoring relevant parameters. Research including [7] and [8] are dedicated to multiple linear profiles. References [8] and [9] have proposed methods for monitoring polynomial profiles in Phase I and Phase II, respectively. Monitoring of non-linear profiles is investigated by [10] and [11]. In all the aforementioned research it is assumed that the response variable follows normal distribution.

In the past decade, there has been an increasing research interest on profile monitoring. Most of the research assumes that a linear regression model can be used to properly describe the profile. Some of the research, such as [3], [4], focuses on Phase I methods. Other works, such as [1], [2], focus on Phase II methods. Detailed accounts of recent. Most recently, [7] proposed a self-starting starting Phase II control chart for monitoring linear profiles when the process parameters are not known or cannot be reasonably estimated due to lack of large Phase I samples.

Another recent paper on the Phase I method was by used linear mixed models to monitor the linear profiles in an attempt to account for any correlation structure within a profile, especially when the data are unbalanced or when there are missing data. All of the afore-mentioned research assumes that the response variable is continuous. However, in many industrial applications the main response variable of interest is binary, as in the case of whether a product can be classified as defective or non-defective. For instance, the compressive strength of an alloy fastener used in aircraft construction is an important predictor of the quality of the alloy fastener. Some of the research, focuses on Phase I methods. Other works, focus on Phase II methods. Detailed accounts of recent developments in linear profile monitoring can be found. Most recently,[12] proposed a self-starting Phase II control chart for monitoring linear profiles when the process parameters are not known or cannot be reasonably estimated due to lack of large Phase I samples. Another recent paper on the Phase I method used linear mixed models to monitor the linear profiles in an attempt to account for any correlation structure within a profile, especially when the data are unbalanced or when there are missing data. In another recent paper proposed Phase II methods for multiple linear regression profiles. As for other non-linear profiles, some recent works can be found. All of the afore-mentioned research assumes that the response variable is continuous.

However, the observed outcome of an alloy fastener being. Recently, [13] have proposed $5T^2$ based methods to monitor

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logistic profiles in which the response variable is binary. [14] evaluated two of the best methods of [15] in monitoring profiles with Poisson responses. In this paper, we will obtain the form of the Sum Com-Poisson profiles likelihood equation and parametric estimation. The rest of this paper is outlined as follows: Section 2 describes the procedure of parameter estimation when the response variable follows Com-Poisson distribution. Section 3. Conclusion and future research are given in the final section.

2 Parameter Estimation in Com-Poisson Profiles

Suppose that there are n independent experimental settings, and each setting contains two explanatory variables as follows: k_{ij} is the j^{th} observation of i^{th} setting level which is assumed to follow a Com-Poisson distribution with the mass probability function of k_{ij} is as follows:

$$\boldsymbol{P}\left(k_{ij},\lambda_i,\boldsymbol{\nu}_i\right)=\frac{\lambda_i^{k_{ij}}}{\left(k_{ij}!\right)^{\nu_i}Z\ \ (\lambda_i,\nu_i)},\ \ \boldsymbol{k_{ji}}=\mathbf{0},\mathbf{1},\mathbf{2},\ldots. \quad (1)$$

where $\boldsymbol{Z}(\boldsymbol{\lambda},\boldsymbol{\nu})=\sum_{j=0}^{\infty}\frac{\lambda^j}{(j!)^{\nu}};\quad \boldsymbol{\nu}\geq\mathbf{0}$

Hence, $\boldsymbol{E}\left(k_{ij}\right)=\lambda_i\frac{\partial\log Z(\lambda_i,\nu_i)}{\partial\lambda_i}\approx\lambda_i^{\frac{1}{\nu_i}}-\frac{\nu_i-1}{2\nu_i}$

$$\lambda_i=\left(\boldsymbol{E}\left(k_{ij}\right)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}$$

$$=\left(\boldsymbol{\mu}+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}$$

$$\boldsymbol{Var}\left(k_{ij}\right)=\frac{\partial\boldsymbol{E}(k_{ij})}{\partial\log\lambda_i}\approx\frac{\mathbf{1}}{\boldsymbol{\nu}_i}\lambda_i^{\mathbf{1}/\boldsymbol{\nu}_i}$$

In addition, let $\eta_{ij}=\beta_0+\beta_1x_{ij}$

By using Log link function (g) to make a relationship between λ_i , $\boldsymbol{\nu}_i$ and η_i

$$\text{Hence,}\eta_i=g\left(\lambda_i^{1/\nu_i}-\frac{\nu_i-1}{2\nu_i}\right)=\log\left(\boldsymbol{E}\left(k_{ij}\right)\right)=\beta_0+\beta_1x_i\,, \quad (2)$$

Consequently,

$$\left(\lambda_i^{\frac{1}{\nu_i}}-\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)=\exp\left(\eta_i\right),\qquad i=1,2,\ldots,n,$$

$$\lambda_i=\left(\exp(\eta_i)+\frac{\nu_i-1}{2\nu_i}\right)^{\nu_i} \quad (3)$$

Where β_0 and β_1 are the regression parameters.

Assume that there are m_i observations in i^{th} setting level. The definition of Y_i in Eq. (4) is as follows:

$$Y_i=\sum_{j=1}^{m_i}k_{ij} \quad (4)$$

Obviously, Y_i follows a Com-Poisson distribution with mean $m_i(\lambda_i^{\frac{1}{\nu_i}}-\frac{\nu_i-1}{2\nu_i})$ with the mass probability function of k_{ij} is as follows:

$$\boldsymbol{P}(Y_i,\lambda_i,\boldsymbol{\nu}_i)=\frac{\boldsymbol{\lambda_i}^{y_i}}{(y_i!)^{\boldsymbol{\nu}_i}\boldsymbol{Z}^{m_i}(\boldsymbol{\lambda_i},\boldsymbol{\nu}_i)}\sum_{\substack{k_1,\dots k_{m_i}=0\\ k_1,\dots k_{m_i}=y_i}}^{y_i}\binom{y_i}{k_1\dots k_{m_i}}^{\boldsymbol{\nu}_i},$$

$$\boldsymbol{y_i}=\mathbf{0},\mathbf{1},\mathbf{2},\dots.$$

Where $\binom{y_i}{k_1\dots k_{m_i}}=\frac{y_i!}{k_1!\dots k_{m_i}!}$

Since Y_i in each setting level is independent, the joint likelihood function of $Y_1, Y_2,\dots, Y_n$ is as follows:

$$L(\lambda_i,\boldsymbol{\nu}_i,y_i)=\prod_{i=1}^{n}\boldsymbol{P}(Y_i,\lambda_i,\boldsymbol{\nu}_i,m_i) \tag{5}$$

By taking the logarithm of both sides of Eq. (5) and replacing λ_i with $\left(\exp(\eta_i)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}$, the following equation is obtained:$\log[L(\lambda_i,\boldsymbol{\nu}_i,y_i)]=\sum_{i=1}^{n}Y_i\boldsymbol{\nu}_i\log\left(\exp(\eta_i)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)-\sum_{i=1}^{n}m_i\log\boldsymbol{Z}\left(\left(\exp(\eta_i)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i},\boldsymbol{\nu}_i\right)-\sum_{i=1}^{n}\boldsymbol{\nu}_i\log(Y_i!)+\sum_{i=1}^{n}\log\sum_{\substack{k_1,\dots k_{m_i}=0\\ k_1,\dots k_{m_i}=y_i}}^{y_i}\binom{y_i}{k_1\dots k_{m_i}}^{\boldsymbol{\nu}_i}$

where $\boldsymbol{Z}(\boldsymbol{\lambda},\boldsymbol{\nu})=\sum_{\boldsymbol{j=0}}^{\infty}\frac{\lambda^{\boldsymbol{j}}}{(j!)^{\boldsymbol{\nu}}};\quad \boldsymbol{\nu}\geq\mathbf{0}$

$$=\sum_{i=1}^{n}Y_i\boldsymbol{\nu}_i\log\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)-\sum_{i=1}^{n}m_i\log\boldsymbol{Z}\left(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i},\boldsymbol{\nu}_i\right)-\sum_{i=1}^{n}\boldsymbol{\nu}_i\log(Y_i!)+\sum_{i=1}^{n}\log\sum_{\substack{k_1,\dots k_{m_i}=0\\ k_1,\dots k_{m_i}=y_i}}^{y_i}\binom{y_i}{k_1\dots k_{m_i}}^{\boldsymbol{\nu}_i} \tag{6}$$

Taking the derivative of Eq. (6) with respect to β, $\boldsymbol{\nu}_i$ and obtaining the score function in Eq. (7).

$$\frac{\partial\log[L(\lambda_i,\boldsymbol{\nu}_i,y_i)]}{\partial\beta}=\sum_{i=1}^{n}\frac{y_i\boldsymbol{\nu}_i x_i\exp(x_i\,\beta)}{\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}}-\sum_{i=1}^{n}\frac{m_i\frac{\partial}{\partial\beta}Z\left(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i},\boldsymbol{\nu}_i\right)}{Z\left(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i},\boldsymbol{\nu}_i\right)}$$

$$\frac{\partial}{\partial\beta}Z\left(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i},\boldsymbol{\nu}_i\right)=\sum_{J=0}^{\infty}\frac{\partial}{j!\,\boldsymbol{\nu}_i\partial\beta}\left(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}\right)^{j}=\sum_{J=0}^{\infty}\frac{\partial}{j!\,\boldsymbol{\nu}_i\partial\beta}\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{j\nu}_i}$$

$$=\sum_{J=0}^{\infty}\frac{\boldsymbol{\nu}_i j}{j!\,\boldsymbol{\nu}_i}\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{j\nu}_i-1}x_i\exp(x_i\,\beta)$$

$$\frac{\partial\log[L(\lambda_i,\boldsymbol{\nu}_i,y_i)]}{\partial\beta}=\sum_{i=1}^{n}\frac{y_i\boldsymbol{\nu}_i x_i\exp(x_i\,\beta)}{\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}}-\sum_{i=1}^{n}\frac{m_i\boldsymbol{\nu}_i x_i\exp(x_i\,\beta)\sum_{J=0}^{\infty}\frac{j}{j!^{\boldsymbol{\nu}_i}}\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{j\nu}_i-1}}{Z\left(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{\nu}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i},\boldsymbol{\nu}_i\right)}$$

$$= \sum_{i=1}^{n} \frac{\boldsymbol{\nu}_i x_i \exp(x_i\,\beta)}{\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right) Z\left(\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i\right)}$$

$$\left[y_i Z\left(\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i\right) - \sum_{J=0}^{\infty} \frac{j m_i}{j!^{\boldsymbol{\nu}_i}} \left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{j\boldsymbol{\nu}_i} \right]$$

$$\frac{\partial \log\,[L(\lambda_i, \boldsymbol{\nu}_i, y_i)]}{\partial \beta} = \sum_{i=1}^{n} \frac{\boldsymbol{\nu}_i x_i \exp(x_i\,\beta) \sum_{J=0}^{\infty} \frac{(y_i - j m_i)}{j!^{\boldsymbol{\nu}_i}} \left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{j\boldsymbol{\nu}_i}}{\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right) Z\left(\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i\right)} \tag{7}$$

$$\frac{\partial \log\,[L(\lambda_i, \boldsymbol{\nu}_i, y_i)]}{\partial \boldsymbol{\nu}_i}$$

$$= \sum_{i=1}^{n} y_i \log\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)$$

$$+ \sum_{i=1}^{n} \frac{y_i \boldsymbol{\nu}_i \left(\frac{1}{2{\boldsymbol{\nu}_i}^2}\right)}{\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)} - \sum_{i=1}^{n} \log(y_i!) - \sum_{i=1}^{n} \frac{m_i \frac{\partial}{\partial \boldsymbol{\nu}_i} Z\left[\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i\right]}{Z(\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i)}$$

$$+ \sum_{i=1}^{n} \frac{\frac{\partial}{\partial \boldsymbol{\nu}_i} \sum_{\substack{k_1,\dots k_{m_i}=0 \\ k_1,\dots k_{m_i}=y_i}}^{y_i} \binom{y_i}{k_1 \dots k_{m_i}}^{\boldsymbol{\nu}_i}}{\sum_{\substack{k_1,\dots k_{m_i}=0 \\ k_1,\dots k_{m_i}=y_i}}^{y_i} \binom{y_i}{k_1 \dots k_{m_i}}^{\boldsymbol{\nu}_i}}$$

$$\frac{\partial \log\,[L(\lambda_i, \boldsymbol{\nu}_i, y_i)]}{\partial \boldsymbol{\nu}_i}$$

$$= \sum_{i=1}^{n} y_i \log\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)$$

$$+ \sum_{i=1}^{n} \frac{y_i}{\mathbf{2}\boldsymbol{\nu}_i \left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)} - \sum_{i=1}^{n} \log(y_i!) - \sum_{i=1}^{n} \frac{m_i \frac{\partial}{\partial \boldsymbol{\nu}_i} Z(\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i)}{Z(\left(\exp(x_i\,\beta) + \frac{\boldsymbol{\nu}_i - \mathbf{1}}{\mathbf{2}\boldsymbol{\nu}_i}\right)^{\boldsymbol{\nu}_i}, \boldsymbol{\nu}_i)}$$

$$+ \sum_{i=1}^{n} \frac{\sum_{\substack{k_1,\dots k_{m_i}=0 \\ k_1,\dots k_{m_i}=y_i}}^{y_i} \frac{\partial}{\partial \boldsymbol{\nu}_i} \binom{y_i}{k_1 \dots k_{m_i}}^{\boldsymbol{\nu}_i}}{\sum_{\substack{k_1,\dots k_{m_i}=0 \\ k_1,\dots k_{m_i}=y_i}}^{y_i} \binom{y_i}{k_1 \dots k_{m_i}}^{\boldsymbol{\nu}_i}}$$

$$\frac{\partial}{\partial \boldsymbol{\nu}_i} \binom{y_i}{k_1 \dots k_{m_i}}^{\boldsymbol{\nu}_i} = \binom{y_i}{k_1 \dots k_{m_i}}^{\boldsymbol{\nu}_i} ln \binom{y_i}{k_1 \dots k_{m_i}}$$

$$\frac{\partial}{\partial \boldsymbol{v}_i} Z\left[\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{\boldsymbol{v}_i},\boldsymbol{v}_i\right]$$

$$=\sum_{j=0}^{\infty}\frac{\partial}{\partial \boldsymbol{v}_i}\frac{\left[\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{\boldsymbol{v}_i}\right]^j}{j!^{\boldsymbol{v}_i}}$$

$$=\sum_{j=0}^{\infty}\frac{\partial}{\partial \boldsymbol{v}_i}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}=\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}\frac{\partial}{\partial \boldsymbol{v}_i}ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}$$

$$=\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}\frac{\partial}{\partial \boldsymbol{v}_i}\boldsymbol{v}_i ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]$$

$$=\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i} ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]$$

$$+\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}\boldsymbol{v}_i\frac{\partial}{\partial \boldsymbol{v}_i} ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]$$

$$=\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i} ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]$$

$$+\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i-1}\boldsymbol{v}_i\frac{\partial}{\partial \boldsymbol{v}_i}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]=$$

$$=\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i} ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]$$

$$+\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i-1}\frac{j\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j-1}}{j!\,2\boldsymbol{v}_i}$$

$$\frac{\partial}{\partial \boldsymbol{v}_i} Z\left[\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{\boldsymbol{v}_i},\boldsymbol{v}_i\right]$$

$$=\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j\boldsymbol{v}_i}}{j!^{\boldsymbol{v}_i}}\right] ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]+\sum_{j=0}^{\infty}\frac{j}{2\boldsymbol{v}_i}\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j\boldsymbol{v}_i-1}}{j!^{\boldsymbol{v}_i}}$$

$$= \sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}\left[ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]+\frac{j}{\mathbf{2}\boldsymbol{v}_i\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)}\right]$$

Then,

$$\frac{\partial \log\left[L\left(\lambda_i,\boldsymbol{v}_i,y_i\right)\right]}{\partial \boldsymbol{v}_i}$$

$$= \sum_{i=1}^{n} y_i \log\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)$$

$$+\sum_{i=1}^{n}\frac{y_i}{\mathbf{2}\boldsymbol{v}_i\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)}-\sum_{i=1}^{n}\log\left(y_i!\right)-\sum_{i=1}^{n}\frac{m_i}{Z(\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{\boldsymbol{v}_i},\boldsymbol{v}_i)}$$

$$\sum_{j=0}^{\infty}\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]^{\boldsymbol{v}_i}\left[ln\left[\frac{\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)^{j}}{j!}\right]+\frac{j}{\mathbf{2}\boldsymbol{v}_i\left(\exp(x_i\,\beta)+\frac{\boldsymbol{v}_i-\mathbf{1}}{\mathbf{2}\boldsymbol{v}_i}\right)}\right]$$

$$+\sum_{i=1}^{n}\frac{\sum_{\substack{k_1,\ldots k_{m_i}=0\\ k_1,\ldots k_{m_i}=y_i}}^{y_i}\binom{y_i}{k_1\ldots k_{m_i}}^{\boldsymbol{v}_i} ln\binom{y_i}{k_1\ldots k_{m_i}}}{\sum_{\substack{k_1,\ldots k_{m_i}=0\\ k_1,\ldots k_{m_i}=y_i}}^{y_i}\binom{y_i}{k_1\ldots k_{m_i}}^{\boldsymbol{v}_i}} \quad (8)$$

3 Conclusions and Future Research

In this paper, a new form of the Com-Poisson profiles equation and parametric estimation is obtained. For a random sample $Y_1 \ldots Y_N$, where $P(Yi = yi \mid \lambda, v, m)$, $i = 1, \ldots, N$. Given the complex nature fu-nction consequently, we couldn't make steps. Then, extending this work, the steps and applied for the new function proposed for Com-Poisson profile response can be considered as a new contribution.

Conflicts of Interest: The authors declare that there is no conflict of interest regarding the publication of this article.

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