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Inference of Progressively Censored Data from The Generalized Exponential Distribution

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Abstract: In this paper, we derive approximate moments of progressively type-II right censored order statistics from the generalized exponential distribution . Also, using these moments to derive the best linear unbiased estimates and maximum likelihood estimates of the location and scale parameters from the generalized exponential distribution. In addition, we use Monte-Carlo simulation method to obtain the mean square error of the best linear unbiased estimates and maximum likelihood estimates and make comparison between them. Finally, we will present numerical example to illustrate the inference procedures developed in this distribution.

Keywords: progressively type-II censored sample, generalized exponential distribution, approximate moments, best linear unbiased estimates, maximum likelihood estimates , Monte-Carlo Method

1 Introduction

The exponential distribution is one of the most significant and widely mathematics used distribution in statistical practice. It possesses several important statistical properties, and yet exhibits great mathematical tractability. Also, it has the ability to model failure rate which are quite common in reliability and biological studies.

In this paper, we derive approximate moments of progressively type-II right censored order statistics from the generalized exponential distribution (GED). Also, using these moments to derive the best linear unbiased estimates ($BLUEs$) and maximum likelihood estimates ($MLEs$) of the location and scale parameters from the generalized exponential distribution. Several interesting mathematical results for inference procedures have been developed by the authors, see for examples: [12]; [3]; [4]; [6];[5]; [11];[9]; [10]; [2]; [1]; [7]; [13]; [8]; [14].

In addition, we use Monte-Carlo simulation method to make comparison between the (MSE) of ($BLUEs$) and ($MLEs$). Finally, we will present numerical example to illustrate the inference procedures developed in this distribution. Let $X_{1:m:n}, X_{2:m:n}, \cdots, X_{m:m:n}$ be the progressively type-II right censored order statistics of size m from the sample of size n with censoring scheme $(R_1, R_2, \cdots, R_m)$ taken from (GED) whose probability function is given by:

$$f(x) = \alpha\lambda \left(1 - e^{-\lambda x}\right)^{\alpha-1} e^{-\lambda x}, x \geq 0, \alpha, \lambda > 0, \tag{1}$$

and distribution function is given by

$$F(x) = \left(1 - e^{-\lambda x}\right)^{\alpha}, x \geq 0, \alpha, \lambda > 0, \tag{2}$$

so, we can write the joint density function of $X_{1:m:n}, X_{2:m:n}, \cdots, X_{m:m:n}$ of a progressively type-II right censored sample from the generalized exponential distribution, with censoring scheme $(R_1, R_2, \cdots, R_m)$, in the form:

$$f_{X_{1:m:n}, X_{2:m:n}, \cdots X_{m:m:n}}(x_1, x_2, \cdots, x_m)$$
$$= c\prod_{i=1}^{m} \alpha\lambda \left(1 - e^{-\lambda x_i}\right)^{\alpha-1} e^{-\lambda x_i} \left[1 - \left(1 - e^{-\lambda x_i}\right)^{\alpha}\right]^{R_i}, \tag{3}$$

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$$0 < x_1 < x_2 < \cdots < x_m < \infty,$$

where

$$c = n(n - R_1 - 1)(n - R_1 - R_2 - 2) \cdots (n - R_1 - R_2 - \cdots R_{m-1} - m + 1).$$

by [3].

Since, the joint density function is more complicated to use it to find the moments, so we try to find relationship between the generalized exponential distribution and uniform distribution.

2 Progressively Type-II Right Censored Transformation

Let $U_{1:m:n}, U_{2:m:n}, \cdots, U_{m:m:n}$ be the progressively type-II right censored order statistics of size m from the sample of size n with censoring scheme $(R_1, R_2, \cdots, R_m)$ taken from the uniform $(0,1)$ distribution.

The exact moments of progressively type-II right censored order statistics from the uniform $(0,1)$ distribution can be written in the form:

$$\begin{aligned} E(U_{i:m:n}) &= 1 - \prod_{j=m-i+1}^{m} \alpha_j, i = 1, 2, \cdots, m, \\ Var(U_{i:m:n}) &= \left(\prod_{j=m-i+1}^{m} \alpha_j \right) \left(\prod_{j=m-i+1}^{m} \gamma_j - \prod_{j=m-i+1}^{m} \alpha_j \right), \\ Cov(U_{i:m:n}, U_{k:m:n}) &= \left(\prod_{j=m-i+1}^{m} \alpha_j \right) \left(\prod_{j=m-k+1}^{m} \gamma_j - \prod_{j=m-k+1}^{m} \alpha_j \right), k < i, \end{aligned} \tag{4}$$

where

$$\begin{aligned} a_i &= i + \sum_{j=m-i+1}^{m} R_j, i = 1, 2, \cdots, m, \\ \alpha_i &= \frac{a_i}{a_i + 1}, i = 1, 2, \cdots, m, \\ \beta_i &= \frac{1}{(a_i + 1)(a_i + 2)}, i = 1, 2, \cdots, m, \\ \gamma_i &= \alpha_i + \beta_i, i = 1, 2, \cdots, m. \end{aligned} \tag{5}$$

by[3].

These expressions enable us to derive the approximate means, variances, and covariances for the generalized exponential distribution using the the following theorems [12].

Theorem 1. *If X is a random variable with $E(X) = \mu$, $D^2(X) = \sigma^2$, and $y = \phi(x)$ then, for sufficiently small σ, and well-behaved ϕ*

$$E(Y) \simeq \phi(\mu) + \frac{1}{2}\sigma^2 \phi^{\cdot\cdot}(\mu), \tag{6}$$

and

$$D^2(Y) \simeq (\phi^{\cdot}(\mu))^2 \sigma^2. \tag{7}$$

Theorem 2. *If X and Y are random variables with $E(X) = \mu$, $E(Y) = \upsilon$, $D^2(X) = \sigma^2$, $D^2(Y) = \tau^2$ and $\rho(X,Y) = \rho$, and $Z = \phi(x,y)$ then, for sufficiently small σ and τ, and well behaved ϕ*

$$E(Z) \simeq \phi(\mu, \upsilon) + \frac{1}{2}\sigma^2 \frac{\partial^2 \phi}{\partial x^2} + \rho\sigma\tau \frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{2}\tau^2 \frac{\partial^2 \phi}{\partial y^2}, \tag{8}$$

and

$$D^2(Z) \simeq \sigma^2 \left(\frac{\partial \phi}{\partial x} \right)^2 + 2\rho\sigma\tau \left(\frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} \right) + \tau^2 \left(\frac{\partial \phi}{\partial y} \right)^2, \tag{9}$$

where all partial derivatives are evaluated at $x = \mu$ and $y = \upsilon$.

3 Deriving Moments Using Transformation

Since, the joint density function is more difficult to use it to find the moments, so we get relationship between the generalized exponential distribution and uniform distribution. Let

$$U \simeq \left(1-e^{-\lambda x}\right)^{\alpha}. \tag{10}$$

So by using Equations (6), (7) and (8), we derive the approximate moments in the form

$$E\left(X_{i:m:n}\right) \simeq \frac{-1}{\lambda}\ln(1-\mu^{\frac{1}{\alpha}})+\frac{1}{2}\sigma^{2}\left[\frac{\alpha\lambda\left(\frac{1}{\alpha}-1\right)\left(1-\mu^{\frac{1}{\alpha}}\right)\mu^{\frac{1}{\alpha}-2}+\lambda\mu^{2\left(\frac{1}{\alpha}-1\right)}}{\left(\alpha\lambda\left(1-\mu^{\frac{1}{\alpha}}\right)\right)^{2}}\right], \tag{11}$$

$$D^{2}\left(X_{i:m:n}\right) \simeq \frac{\sigma^{2}}{(\alpha\lambda)^{2}}\frac{\mu^{2\left(\frac{1}{\alpha}-1\right)}}{\left(1-\mu^{\frac{1}{\alpha}}\right)^{2}}, \tag{12}$$

$$Z=\frac{1}{\lambda^{2}}\ln\left(1-u_{i}^{\frac{1}{\alpha}}\right)\left(1-u_{j}^{\frac{1}{\alpha}}\right), \tag{13}$$

and

$$Cov(X_{i:m:n},X_{j:m:n}) \simeq E(Z)-E(X_{i:m:n})E(X_{j:m:n}). \tag{14}$$

We use these moments to derive the best linear unbiased estimators for the location (μ) and scale (σ) parameters of a (GED).

4 Estimation of Parameters

The best linear unbiased and maximum likelihood methods are used to obtain estimates of the location (μ) and scale (σ) parameters. Let the probability density function is given by :

$$f(x)=\frac{\alpha\lambda}{\sigma}\left(1-e^{-\lambda\left(\frac{x_i-\mu}{\sigma}\right)}\right)^{\alpha-1}e^{-\lambda\left(\frac{x_i-\mu}{\sigma}\right)}, x>\mu, \tag{15}$$

also the distribution function is given by:

$$F(x)=\left(1-e^{-\lambda\left(\frac{x_i-\mu}{\sigma}\right)}\right)^{\alpha}. \tag{16}$$

4.1 Best linear unbiased estimates$(BLUEs)$

Consider an arbitrary continues distribution $F(x)$. Suppose that the progressively censored order statistics can be represented by the linear transformation $\mathbf{Y}=\mu\mathbf{1}+\sigma\mathbf{X}$, where the vector $\mathbf{X}$ represent a vector of progressively type-II censored order statistics from the standard distribution $F(x)$, then the best linear unbiased estimators of μ and σ will be minimizing the generalized variance $Q(\theta)=(\mathbf{Y}-\mathbf{A}\theta)^{T}\sum^{-1}(\mathbf{Y}-\mathbf{A}\theta)$ with respect to θ where $\theta=(\mu,\sigma)^{T}$, $\mathbf{A}$ is the $p\times p$ matrix, $\mathbf{1}$ is $p\times 1$ vector with components all $1's$, μ is the mean vector of $\mathbf{X}$ and $\sum$ is the variance-covariance matrix of $\mathbf{X}$. The minimum occurs when

$$\mu^{*}=-\mu^{T}\Gamma Y=\sum_{i=1}^{m}A_{i}Y_{i:m:n}, \tag{17}$$

and

$$\sigma^{*}=\mathbf{1}^{T}\Gamma Y=\sum_{i=1}^{m}B_{i}Y_{i:m:n}, \tag{18}$$

where

$$\Gamma=\sum^{-1}\left(\mathbf{1}\mu^{T}-\mu\mathbf{1}^{T}\right)\sum^{-1}/\Delta, \tag{19}$$

and

$$\Delta = \left(\mathbf{1}^T \sum^{-1} \mathbf{1}\right)\left(\mu^T \sum^{-1} \mu\right) - \left(\mathbf{1}^T \sum^{-1} \mu\right)^2. \tag{20}$$

See[3].

From these results, we can get the variances and covariances of the estimators in the form:

$$Var(\mu^*) = \left(\sigma^2 \mu^T \sum^{-1} \mu\right)/\Delta, \tag{21}$$

$$Var(\sigma^*) = \left(\sigma^2 \mathbf{1}^T \sum^{-1} \mathbf{1}\right)/\Delta, \tag{22}$$

$$Cov(\mu^*, \sigma^*) = \left(-\sigma^2 \mu^T \sum^{-1} \mathbf{1}\right)/\Delta. \tag{23}$$

The coefficients A_i, B_i, $i = 1, 2, \cdots m$ which represented in Equations (17) and (18) satisfy the relations $\sum_{i=1}^{m} A_i = 1$, $\sum_{i=1}^{m} B_i = 0$. See [3].

4.2 Maximum likelihood estimates

Let $X_{1:m:n}, X_{2:m:n}, \cdots, X_{m:m:n}$ be the progressively type-II right censored order statistics of size m from the sample of size n with censoring scheme $(R_1, R_2, \cdots, R_m)$ taken from the generalized exponential distribution whose probability function is given by (15) and the communicative distribution function is given by (16), then the likelihood function can be written in the form:

$$L(\mu, \sigma) = C \prod_{i=1}^{m} f\left(X_{i:m:n}\right)\left[1 - F\left(X_{i:m:n}\right)\right]^{R_i}, \tag{24}$$

where C is normalizing constant, see[3].

The likelihood function to be maximized for estimators of μ and σ is given by:

$$L(\mu, \sigma) = (\text{constant})(\alpha\lambda)^m (\sigma)^{-m} \prod_{i=1}^{m} e^{-\frac{\lambda}{\sigma}(x_i - \mu)} \left[1 - e^{-\frac{\lambda}{\sigma}(x_i - \mu)}\right]^{\alpha - 1}$$
$$\left(1 - \left[1 - e^{-\frac{\lambda}{\sigma}(x_i - \mu)}\right]^{\alpha}\right)^{R_i}. \tag{25}$$

The log-likelihood function can be written in form:

$$\ln L(\mu, \sigma) = \ln \text{constant} + m \ln \alpha\lambda - m \ln \sigma - \sum_{i=1}^{m} \lambda \left[\frac{x_i - \mu}{\sigma}\right]$$
$$+ \sum_{i=1}^{m} (\alpha - 1) \ln \left[1 - e^{-\lambda\left(\frac{x_i - \mu}{\sigma}\right)}\right]$$
$$+ \sum_{i=1}^{m} R_i \ln \left(1 - \left[1 - e^{-\lambda\left(\frac{x_i - \mu}{\sigma}\right)}\right]^{\alpha}\right), \tag{26}$$

by differentiating the log-likelihood function given by (26) with respect to μ and σ. The resulting equations to be solved for maximum likelihood estimators μ and σ are given:

$$\frac{\lambda}{\widehat{\sigma}} - (\alpha - 1) \sum_{i=1}^{m} \frac{\lambda e^{-\lambda\left(\frac{x_i - \widehat{\mu}}{\widehat{\sigma}}\right)}}{\widehat{\sigma}\left[1 - e^{-\lambda\left(\frac{x_i - \widehat{\mu}}{\widehat{\sigma}}\right)}\right]}$$
$$+ \sum_{i=1}^{m} R_i \frac{\alpha\lambda e^{-\lambda\left(\frac{x_i - \widehat{\mu}}{\widehat{\sigma}}\right)} \left[1 - e^{-\lambda\left(\frac{x_i - \widehat{\mu}}{\widehat{\sigma}}\right)}\right]^{\alpha - 1}}{\widehat{\sigma}\left(1 - \left[1 - e^{-\lambda\left(\frac{x_i - \widehat{\mu}}{\widehat{\sigma}}\right)}\right]^{\alpha}\right)} = 0, \tag{27}$$

and

$$\frac{-m}{\widehat{\sigma}}+\sum_{i=1}^{m}\lambda\left[\frac{x_i-\widehat{\mu}}{\widehat{\sigma}^2}\right]-(\alpha-1)\sum_{i=1}^{m}\frac{\lambda\left(x_i-\widehat{\mu}\right)e^{-\lambda\left(\frac{x_i-\widehat{\mu}}{\widehat{\sigma}}\right)}}{\widehat{\sigma}^2\left[1-e^{-\lambda\left(\frac{x_i-\widehat{\mu}}{\widehat{\sigma}}\right)}\right]}$$
$$+\sum_{i=1}^{m}R_i\frac{\alpha\lambda e^{-\lambda\left(\frac{x_i-\widehat{\mu}}{\widehat{\sigma}}\right)}\left[1-e^{-\lambda\left(\frac{x_i-\widehat{\mu}}{\widehat{\sigma}}\right)}\right]^{\alpha-1}}{\widehat{\sigma}^2\left(1-\left[1-e^{-\lambda\left(\frac{x_i-\widehat{\mu}}{\widehat{\sigma}}\right)}\right]^{\alpha}\right)}=0. \quad (28)$$

Since Equations (27) and (28) cannot be solved analytically, so we can use MATLAB program to solve these equations.

5 Simulation Study

Let us consider the following table represented different schemes of progressively censored data:

Table 1: Sample sizes and censoring schemes from the generalized exponential distribution

m	n	$scheme$
5	15	$R_1=[2\ 0\ 4\ 0\ 4]$
6	20	$R_2=[4\ 0\ 4\ 0\ 2\ 4]$
7	25	$R_3=[4\ 0\ 4\ 4\ 2\ 2\ 2]$
8	30	$R_4=[4\ 0\ 8\ 0\ 8\ 0\ 0\ 2]$

So, by using Equations (17) and (18), the coefficients of the $BLUEs$ of μ and σ from the generalized exponential distribution using different schemes represented in table (1) about $\mu=0$ and $\sigma=1$ are obtained in the following tables $(2,3,4$ and $5)$:

Table 2: Coefficients of the Blues of μ and σ from the generalized exponential distribution using the first scheme with $\mu=0$ and $\sigma=1$

$sch1$		$\lambda=1,\alpha=0.5$		$\lambda=2,\alpha=0.5$		$\lambda=3,\alpha=0.5$	
m	n	A_i	B_i	A_i	B_i	A_i	B_i
5	15	0.3816	-0.3287	0.3964	-0.1394	0.2907	-0.0330
		0.3982	-0.3725	0.4550	-0.2061	0.4520	-0.1404
		0.4021	-0.4189	0.5374	-0.2929	0.6759	-0.2828
		0.4525	-0.5905	0.9263	-0.6511	1.7710	-0.9444
		-0.6345	1.7105	-1.3150	1.2895	-2.1896	1.4006
		$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$

Also, the variances and covariances of the estimators μ and σ can be represented in the following table 6:

MSE of μ and σ from the generalized exponential distribution with $\mu=0$ and $\sigma=1$ can be represented in the following table 7:

From the numerical results presented in tables $2,3,4,5$ and 7, we can conclude the following:

1-As a check of the entries of tables $2,3,4$ and 5, we see that $\sum_{i=1}^{n}A_i\simeq 1$, $\sum_{i=1}^{n}B_i\simeq 0$.

2-From Table (7), we see that as n increases, the mean square error $MSE(\mu^*)$ and $MSE(\sigma^*)$ decrease for all censoring schemes and all values of λ and α.

3-From Table (7), we see that as n increases, the mean square error$MSE(\widehat{\mu})$ and $MSE(\widehat{\sigma})$ decrease for all censoring schemes and all values of λ and α.

Table 3: Coefficients of the Blues of μ and σ from the generalized exponential distribution using the second scheme with $\mu = 0$ and $\sigma = 1$

*sch*2		$\lambda = 1, \alpha = 0.5$		$\lambda = 2, \alpha = 0.5$		$\lambda = 3, \alpha = 0.5$	
m	n	A_i	B_i	A_i	B_i	A_i	B_i
6	20	0.2772	−0.1975	0.2617	−0.0571	0.1106	0.0520
		0.3025	−0.2465	0.3189	−0.1183	0.2585	−0.0456
		0.3141	−0.2921	0.3811	−0.1837	0.4220	−0.1507
		0.3376	−0.3786	0.5216	−0.3199	0.7878	−0.3779
		0.4246	−0.6065	0.9307	−0.6936	1.9378	−1.0770
		−0.6560	1.7212	−1.4140	1.3725	−2.5167	1.5991
		$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$

Table 4: Coefficients of the Blues of μ and σ from the generalized exponential distribution using the third scheme with $\mu = 0$ and $\sigma = 1$

*sch*3		$\lambda = 1, \alpha = 0.5$		$\lambda = 2, \alpha = 0.5$		$\lambda = 3, \alpha = 0.5$	
m	n	A_i	B_i	A_i	B_i	A_i	B_i
7	25	0.1935	−0.0795	0.1739	−0.0053	0.0439	0.0752
		0.2226	−0.1213	0.2195	−0.0496	0.1482	0.0078
		0.2361	−0.1561	0.2576	−0.0892	0.2421	−0.0529
		0.2468	−0.1995	0.3087	−0.1422	0.3725	−0.1363
		0.2641	−0.2725	0.4064	−0.2397	0.6296	−0.2974
		0.3299	−0.4621	0.7075	−0.5197	1.4824	−0.8191
		−0.4931	1.2911	−1.0736	1.0457	−1.9187	1.2227
		$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$

Table 5: Coefficients of the Blues of μ and σ from the generalized exponential distribution using the fourth scheme with $\mu = 0$ and $\sigma = 1$

*sch*4		$\lambda = 1, \alpha = 0.5$		$\lambda = 2, \alpha = 0.5$		$\lambda = 3, \alpha = 0.5$	
m	n	A_i	B_i	A_i	B_i	A_i	B_i
8	30	0.1985	−0.1223	0.2207	−0.0620	0.1806	−0.0188
		0.2148	−0.1322	0.2308	−0.0711	0.2074	−0.0365
		0.2160	−0.1339	0.2341	−0.0771	0.2271	−0.0506
		0.2085	−0.1297	0.2362	−0.0844	0.2548	−0.0703
		0.1965	−0.1240	0.2372	−0.0918	0.2839	−0.0911
		0.1582	−0.0942	0.2430	−0.1107	0.3842	−0.1574
		0.1340	−0.0759	0.3172	−0.1826	0.7568	−0.3787
		−0.3265	0.8123	−0.7192	0.6797	−1.2948	0.8034
		$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$	$\simeq 1$	$\simeq 0$

6 Numerical Examples

A progressively type-II censored sample of size $m = 5$ from a sample of size $n = 15$ from the generalized exponential distribution with $\mu = 0$, $\sigma = 1$, $\lambda = 1$, $\alpha = 0.5$ with scheme $R_i = (2\ 0\ 4\ 0\ 4)$, was simulated using MATLAB program. The simulated progressively type-II right censored sample is given by : table 8

By making use of Equations (17) and (18), and using the coefficients A_i and B_i given in table (2) for $n = 15$ and $m = 5$, we get the $BLUEs$ of the μ and σ as follows:

$$\begin{aligned}\mu^* &= (0.3816 \times 0.0633) + (0.3982 \times 0.2537) + (0.4021 \times 0.6477) \\ &\quad + (0.4525 \times 0.8226) + (-0.6345 \times 1.1530) \\ &= 0.0263\end{aligned}$$

and

$$\begin{aligned}\sigma^* &= (-0.3287 \times 0.0633) + (-0.3725 \times 0.2537) + (-0.4189 \times 0.6477) \\ &\quad + (-0.5905 \times 0.8226) + (1.7105 \times 1.1530) \\ &= 1.0999\end{aligned}$$

Table 6: The variances and covariances of the estimators μ^* and σ^* from the generalized exponential distribution

λ	α	m	n	sch	$Var(\mu^*)$	$Var(\sigma^*)$	$Cov(\mu^*,\sigma^*)$
1	0.5	5	15	1	0.3232	0.3035	−0.2840
		6	20	2	0.3839	0.5020	−0.4104
		7	25	3	0.4042	0.7320	−0.4512
		8	30	4	0.4452	0.8475	−0.5146
2	0.5	5	15	1	2.3823	0.6583	0.2221
		6	20	2	3.0677	0.7188	−0.0767
		7	25	3	4.0511	0.7611	−0.3051
		8	30	4	6.0099	0.8842	0.3326
3	0.5	5	15	1	5.5772	0.7053	−0.7211
		6	20	2	7.1693	0.7298	−1.3334
		7	25	3	9.2102	0.7744	−1.1602
		8	30	4	9.7474	0.8912	−0.1788

Table 7: MSE of μ and σ from the generalized exponential distribution using different schemes with $\mu = 0$ and $\sigma = 1$

λ	α	m	n	sch	$MSE(\mu^*)$	$MSE(\sigma^*)$	$MSE(\widehat{\mu})$	$MSE(\widehat{\sigma})$
1	0.5	5	15	1	0.0455	0.3033	$2.1212e-004$	$2.2426e-003$
		6	20	2	0.0434	0.2902	$2.3355e-005$	$3.8585e-004$
		7	25	3	0.0211	0.2882	$1.0052e-005$	$2.8025e-004$
		8	30	4	0.0120	0.2712	$1.0056e-006$	$1.2514e-004$
2	0.5	5	15	1	0.0350	0.2919	$2.1525e-004$	$2.2446e-003$
		6	20	2	0.0335	0.2818	$2.3756e-005$	$4.2525e-004$
		7	25	3	0.0209	0.2512	$1.0080e-006$	$2.9090e-004$
		8	30	4	0.0019	0.2415	$2.2121e-006$	$2.2025e-004$
3	0.5	5	15	1	0.0330	0.2525	$1.999e-004$	$2.0202e-004$
		6	20	2	0.0229	0.2424	$2.2525e-005$	$2.3039e-005$
		7	25	3	0.0211	0.2025	$2.1212e-005$	$2.2626e-005$
		8	30	4	0.0015	0.1104	$1.2525e-006$	$1.3032e-006$

Table 8: Progressively type-II right censored sample generated from the generalized exponential distribution

$X_{i:5:15}$	0.0633	0.2537	0.6477	0.8226	1.1530
R_i	2	0	4	0	4

The standard error of the estimates $\mu^*$and σ^* are

$$SE(\mu^*) = \sigma^* (Var(\mu^*))^{\frac{1}{2}} = 1.0999 \times (0.3232)^{\frac{1}{2}} = 0.625301$$

$$SE(\sigma^*) = \sigma^* (Var(\sigma^*))^{\frac{1}{2}} = 1.0999 \times (0.3035)^{\frac{1}{2}} = 0.605944$$

Using the same data,we can get by simulation

$$\widehat{\mu} = 0.0000132$$

$$\widehat{\sigma} = 1.000035$$

Then, the best linear unbiased prediction for the failure following $Y^{(2\,0\,4\,0\,4)}_{5:5:15}$ may be determined simply by equating μ^* or σ^* based on the sample of size $m = 5$ to μ^* and σ^* based on the sample of size $n = 15$ with progressive censoring $(2\ 0\ 4\ 0\ 0\ 3)$ whose coefficients are given in the form:

Table 9: Coefficients of the Blues of μ and σ from the generalized exponential distribution using the fourth scheme with $\mu = 0$ and $\sigma = 1$

A_i	0.3313	0.3333	0.3177	0.2830	0.2524	−0.5176
B_i	−0.2396	−0.2393	−0.2293	−0.2009	−0.1790	1.0881

then

$$\mu^* = (0.3313 \times 0.0633) + (0.3333 \times 0.2537) + (0.3177 \times 0.6477) + (0.2830 \times 0.8226) + (0.2524 \times 1.1530) + (-0.5176 \times y^*_{6:6:15})$$

Upon equating this with $\mu^* = 0.0263$ and solving, we get the first -order approximation to the $BLUE$ of $y^*_{6:6:15}$as

$$y^*_{6:6:15} = \frac{0.83511679 - 0.0263}{0.5176} = 1.1562629038$$

*Remark.*We see the values of μ^*close to 0 and σ^*close to 1.

*Remark.*We will have the same prediction if we use the equality of σ^*instead of μ^* .We can repeat these steps to $y^*_{7:7:15}$,$y^*_{8:8:15}$,$\cdots$,$y^*_{15:15:15}$.

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