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A Quaternion-ESPRIT Source Localization Method with Uniform Loop and Dipole Pair Circular Array

Lanmei Wang[1,2,*], *Minggao Zou*[1,2], *Guibao Wang*[3] *and Zhihai Chen*[1,2]

[1] School of Physics and Optoelectronic Engineering, Xidian University, xian 710071, China
[2] Collaborative Innovation Center of Information Sensing and Understanding at Xidian University, China
[3] School of Physics and Telecommunication Engineering, Shaanxi University of Technology, hanzhong 723001, China

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Abstract: Owing to the advantages of quaternion in describing vector sensor array data, a quaternion model of uniform cocentered loop and dipole (CLD) pair circular array is deduced and constructed in this paper. Two sets of synchronous time data are used to construct correlation matrix. According to the relationships of the two sets data and that of the two steering vectors, the array steering vector and the frequency of signals are obtained. The direction of arrival (DOA) is derived by dot division operation and the least square method. The polarization parameters are obtained using the relationship between the dipole steering vector and magnetic loop vector. Without spectral peak searching and parameter matching, this method gives closed-form solution of frequency, DOA and polarization parameters. The proposed algorithm results in a reduction by half of memory requirements for representation of data covariance model and decreases the computational cost. Finally, simulation results verify that the performance of proposed method is superior to the long-vector approach because quaternion matrix operations can maintain the vectorial property of the vector sensor and provide a better subspace approximation than the long-vector approach.

Keywords: array signal processing; direction of arrival; ESPRIT; uniform circular array; polarization; quaternion

1 Introduction

Source localization using an electromagnetic vector sensor (EMVS) arrays are widely used in radar, sonar, navigation, geophysics, and acoustic tracking [1], since they have the capability of separating signals based on their polarization characteristics and spatial diversity. The problem of estimating signal polarizations along with arrival angles has been discussed previously in many literatures. The first direction-finding algorithms, explicitly exploiting all six electromagnetic components, have been developed by Nehorai and Paldi [2, 3] and Li [4], respectively. The cross-product-based DOA estimation algorithm was first adapted to ESPRIT by Wong and Zoltowski [5–8]. A uni-vector-sensor MUSIC algorithm was proposed in [9]. Many other algorithms have been devised to estimate the direction of arrival (DOA) and polarization parameters of multiple electromagnetic signals [10, 11], the maximum likelihood approach was presented in [10]; two distinct versions of estimate signal parameters via rotation invariance (ESPRIT) estimators were reported in [4] and [7] respectively; the multiple signal classification (MUSIC) technique was investigated in [8–11]. However, existing methods mostly combine the output of a vector sensor into a long-vector in series, which is called long-vector mode. This way of processing data originated from vector-sensors has the main advantage of allowing, together with a rather complicated parametrization of the data, the use of well-known matrix algebra techniques over the real or the complex field. However, the long-vector approaches have the drawback of destroying locally the vector-type of the signal and fail in high-accuracy estimation of DOA and polarization because of the reorganization of the data into a large vector. In recent years a few research have been made on estimating the DOA of EMVS within the algebraic system theory for quaternion and its extension [12–17]. Quaternion MUSIC (Q-MUSIC) technique was proposed based on the quaternion formalism of the two component vector sensor array in [12]. The three component vector sensor was expressed as a biquaternion number, then

* Corresponding author e-mail: lmwang@mail.xidian.edu.cn

biquaternion-based MUSIC (BQ-MUSIC) was proposed in [13]. The six component electromagnetic (EM) vector sensor array was represented by a quad-quaternion model in [14].

The advantages of using quaternion for EMVS is that the local vector nature of a EMVS array is preserved in multiple imaginary parts, and it could result in a more compact representation. The use of quaternion allows us to skip the parametrization step used in long-vector techniques as it intrinsically includes the vector dimension in the process. Quaternionic matrix operations can provide a better subspace approximation than the long-vector approach. Compared with long-vector methods, the quaternion and its extension method based estimators are shown to be more robust to model errors, while their computation efforts for estimating the data covariance matrices are lower [12–16].

Comparing with CLD pairs oriented along x axis, y axis and CDD (cocentered dipole and dipole) pairs, CLL (cocentered loop and loop) pairs oriented along x and y axis respectively, we can see that CLD pairs along the z axis is more easier to realize the decoupling of polarization and angle of arrival parameters because of its simple structure [18–24]. A frequency, DOA and polarization joint estimation method of uniform CLD pair circular array based on quaternion-ESPRIT is proposed in this paper. Two sets of synchronous time data are used to construct correlation matrix. According to the subspace theory, the array steering vector and the frequency of signals are obtained. The spatial steering vector composed of phase differences between adjacent array elements is estimated by dot division of the array steering vectors. The direction of arrival (DOA) is derived by the least square method. The key components of the proposed approach and results :1) can decouple frequency, DOA estimation from the polarization estimations, errors of frequency, DOA and polarization herein do not cumulate; 2) have lower computational efforts and higher accuracy estimations of DOA and polarization than that of long-vector methods; 3) have the advantage of parameter automatic matching and without spectral peak searching.

2 Quaternion and Array Models

2.1 Quaternion [17]

Quaternion is developed by William Hamilton in 1843. The quaternion has four components, i.e., one real part and three imaginary parts and can be represented in Cartesian form as:

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \quad a,b,c,d \in \mathrm{R} \tag{1}$$

where a, b, c and d are real numbers and i, j and k are complex operators which obey the following rules.

$$\begin{matrix} \mathrm{ij} = -\mathrm{ji} = \mathrm{k} & \mathrm{jk} = -\mathrm{kj} = \mathrm{i} \\ \mathrm{ki} = -\mathrm{ik} = \mathrm{j} & \mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = -1 \end{matrix} \tag{2}$$

From formulas (1) and (2), quaternion can also be expressed as the following form:

$$q = a + \mathrm{i}b + (c + \mathrm{i}d)\,\mathrm{j} = \alpha + \beta\mathrm{j} \tag{3}$$

The quaternion expressed in equation (3) is also known as the ”Cayley-Dickson representation”. The quaternion conjugate and the quaternion modulus are respectively given by

$$\begin{matrix} q^* = a - \mathrm{i}b - \mathrm{j}c - \mathrm{k}d \\ \|q\| = \sqrt{a^2 + b^2 + c^2 + d^2} \end{matrix} \tag{4}$$

The addition of two quaternions can be given by the addition of their corresponding real and imaginary parts, which satisfies the commutative property. However the multiplication is not commutative.

2.2 Signal and array models

K narrowband completely polarized electromagnetic plane wave source signals from far-field, impinge upon a uniform circular array (UCA), which is composed of $\mathrm{N}(\mathrm{N} > \mathrm{K})$ identical CLD pairs. The center of the UCA is the origin of the Cartesian coordinates. The radius R of the circle satisfies the Nyquist sampling criterion of $^{\mathrm{R}}/_{\lambda_{\min}} \le 1/\left(4\sin\left(\frac{2\pi}{\mathrm{N}}\right)\right)$, that is to say inter-element spacing less than or equal to half of minimal wavelength, where $\lambda_{\min}$ refers to the minimal signals wavelength of the incident signal and $\lambda_k = \frac{c}{f_k}$, with f_k be the frequency of signal. The zero CLD pair located on the cross point of the circle and the positive x-axis, along the anti-clockwise direction is respectively the zero, first ,second,... $(\mathrm{N}-1)$ th CLD pair. as shown in Figure 1. For the CLD pairs, the dipoles parallel to the z-axis is referred to as the z-axis dipoles and the loops parallel to the x-y plane as the x-y plane loops, respectively measuring the electric field components and the magnetic field components. The CLD pairs steering vector of the kth $(1 \le k \le \mathrm{K})!$ unit-power electromagnetic source signal is the following 2×1 vector [4, 23]:

$$\begin{bmatrix} h_{kz} \\ e_{kz} \end{bmatrix} = \begin{bmatrix} 0 & \sin\theta_k \\ -\sin\theta_k & 0 \end{bmatrix} \begin{bmatrix} \cos\gamma_k \\ \sin\gamma_k \mathrm{e}^{\mathrm{j}\eta_k} \end{bmatrix} \tag{5}$$

where $\theta_k \in [0, ^{\pi}/_2]$ is the signals elevation angle measured from the positive $z-$axis, $\gamma_k \in [0, ^{\pi}/_2]$ represents the auxiliary polarization angle, and $\eta_k \in [-\pi, \pi]$ symbolizes the polarization phase difference. The $z-$axis electric field e_{kz} and the $z-$axis magnetic field h_{kz} both involve the same factor $\sin\theta_k$, so polarization estimation based on CLD pairs is independent of the sources direction of arrival and it requires no prior information of azimuth and elevation angles.

The e_{kz} and h_{kz} can be expressed as follows with a quaternion c_k :

$$c_k = e_{kz} + \mathrm{i}h_{kz} = -\sin\theta_k \sin\gamma_k \mathrm{e}^{\mathrm{j}\eta_k} + \mathrm{i}\sin\theta_k \cos\gamma_k \tag{6}$$

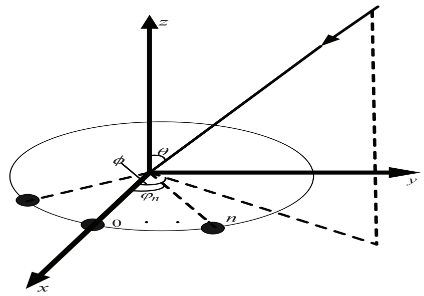

Fig. 1: Uniform circular array geometry.

The output of array response for the kth incident signal can be expressed as follows:

$$x_k(t) = \underbrace{c_k\mathbf{q}(\theta_k,\phi_k)}_{\mathbf{a}_1(\theta_k,\phi_k,\gamma_k,\eta_k)} s_k(t) \tag{7}$$

where $\phi_k (0 \le \phi_k \le 2\pi)$ is the azimuth of the kth incident signal, $s_k(t)$ is the kth incident signal, with $\mathbf{q}(\theta_k,\phi_k)$ is the spatial steering vector constituted by the phase differences between the array elements and the origin, i.e.,

$$\mathbf{q}\left(\theta_k,\phi_k\right) = \begin{bmatrix} e^{j2\pi R\sin\theta_k\cos(\phi_k-\varphi_0)/\lambda_k} \\ \vdots \\ e^{j2\pi R\sin\theta_k\cos(\phi_k-\varphi_{N-1})/\lambda_k} \end{bmatrix} \tag{8}$$

According to equation (8), the phase of $\mathbf{q}\left(\theta_k,\phi_k\right)$ can be expressed as:

$$\arg\left[\mathbf{q}(\theta_k,\phi_k)\right] = \frac{2\pi R}{\lambda_k}\begin{bmatrix} \beta_k \\ \alpha_k\sin\Delta+\beta_k\cos\Delta \\ \vdots \\ \alpha_k\sin\left[(N-1)\Delta\right]+\beta_k\cos\left[(N-1)\Delta\right] \end{bmatrix} \tag{9}$$

where $\Delta = {}^{2\pi}/_{N}$, $\alpha_k = \sin\theta_k\sin\phi_k$, $\beta_k = \sin\theta_k\cos\phi_k$. Now suppose the following matrix

$$\mathbf{W} = \frac{2\pi R}{\lambda_k}\begin{bmatrix} 0 & 1 \\ \sin\Delta & \cos\Delta \\ \vdots & \vdots \\ \sin\left[(N-1)\Delta\right] & \cos\left[(N-1)\Delta\right] \end{bmatrix} \tag{10}$$

Then formula (9) can be rewritten as:

$$\arg\left[\mathbf{q}(\theta_k,\phi_k)\right] = \mathbf{W}\cdot\begin{bmatrix}\alpha_k \\ \beta_k\end{bmatrix} \tag{11}$$

For the sake of simplicity, let $\mathbf{q}_k$ denotes $\mathbf{q}(\theta_k,\phi_k)$. Now let

$$\mathbf{q}_{1k} = \mathbf{q}_k(1,N-1), \mathbf{q}_{2k} = \mathbf{q}_k(2,N) \tag{12}$$

where $\mathbf{q}_{1k}$ and $\mathbf{q}_{2k}$ are the first and the last N-1 elements of $\mathbf{q}_k$, respectively.

Define

$$\Delta\mathbf{q}_k = \left[\mathbf{q}_{2k}./\mathbf{q}_{1k}\right] \tag{13}$$

where./ denotes dot division which is the division of the corresponding elements of two vectors. From equations (8) and (13), the phase of $\Delta\mathbf{q}_k$ can be expressed as:

$$\arg\left[\Delta\mathbf{q}_k\right] = \Delta\mathbf{W}\cdot\begin{bmatrix}\alpha_k \\ \beta_k\end{bmatrix} \tag{14}$$

where $\Delta\mathbf{W} = \mathbf{W}_2 - \mathbf{W}_1$,$\mathbf{W}_1 = \mathbf{W}(1:N-1,:)$,$\mathbf{W}_2 = \mathbf{W}(2:N,:)$, with $\mathbf{W}_1$ and $\mathbf{W}_2$ are the first and the last N-1 rows of $\mathbf{W}$, respectively.

3 Quaternion-ESPRIT Algorithm

The received data collected by the CLD UCA at time t can be represented as

$$\mathbf{X}_1(t) = \mathbf{A}_1\mathbf{S}(t) + \mathbf{N}_1(t) \tag{15}$$

The received data collected by the CLD UCA at time $t+\Delta T$ can be represented as

$$\mathbf{X}_2(t) = \mathbf{A}_2\mathbf{S}(t) + \mathbf{N}_2(t) = \mathbf{A}_1\boldsymbol{\Phi}\mathbf{S}(t) + \mathbf{N}_2(t) \tag{16}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} e^{j2\pi f_1\Delta T} & & \\ & \ddots & \\ & & e^{j2\pi f_K\Delta T} \end{bmatrix} \tag{17}$$

where$\mathbf{X}_i(t)$, $\mathbf{S}(t)$, $\mathbf{N}_i(t)$ and $\mathbf{A}_i$ is the received data, the incident signals, the zero-mean additive complex Gaussian noise and the steering vector matrix of incident signals, respectively, i.e.,

$$\mathbf{X}_i(t) = \begin{bmatrix} \mathbf{x}_{i,0}(t) \\ \mathbf{x}_{i,1}(t) \\ \vdots \\ \mathbf{x}_{i,M-1}(t) \end{bmatrix}, \mathbf{S}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_K(t) \end{bmatrix}$$

$$\mathbf{N}_i(t) = \begin{bmatrix} \mathbf{n}_{i,0}(t) \\ \mathbf{n}_{i,1}(t) \\ \vdots \\ \mathbf{n}_{i,M-1}(t) \end{bmatrix}, \mathbf{A}_i{}^{T} = \begin{bmatrix} \mathbf{a}_i^T(\theta_1,\phi_1,\gamma_1,\eta_1) \\ \mathbf{a}_i^T(\theta_2,\phi_2,\gamma_2,\eta_2) \\ \vdots \\ \mathbf{a}_i^T(\theta_K,\phi_K,\gamma_K,\eta_K) \end{bmatrix}$$

with ΔT is the constant time delay between the two sets of time samples.

The received data of overall array is

$$\mathbf{Z}(t) = \begin{bmatrix}\mathbf{A}_1 \\ \mathbf{A}_2\end{bmatrix}\mathbf{S}(t) + \mathbf{N}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) \tag{18}$$

where $\mathbf{A} = \left[\mathbf{a}(\theta_1,\phi_1,\gamma_1,\eta_1),\cdots,\mathbf{a}(\theta_K,\phi_K,\gamma_K,\eta_K)\right]$,with

$$\mathbf{a}(\theta_k,\phi_k,\gamma_k,\eta_k) = \left[\mathbf{a}_1^T(\theta_k,\phi_k,\gamma_k,\eta_k), \mathbf{a}_2^T(\theta_k,\phi_k,\gamma_k,\eta_k)\right]^T$$
$$\mathbf{a}_2(\theta_k,\phi_k,\gamma_k,\eta_k) = \mathbf{a}_1(\theta_k,\phi_k,\gamma_k,\eta_k)\, e^{j2\pi f_k\Delta T}$$

$k = 1,\cdots,K$

The correlation matrix of received data $\mathrm{Z}(t)$ is

$$\mathbf{R}_z = \mathrm{E}\left[\mathbf{Z}\mathbf{Z}^{\mathrm{H}}\right] = \mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}} + \sigma^2\mathbf{I} \tag{19}$$

with $E[\cdot]$ symbolizing the statistical mean, $(\cdot)^H$ denoting the complex conjugate transpose, σ^2 indicating the white noise power and $\mathbf{R}_s = \mathrm{E}\left[\mathbf{S}(t)\mathbf{S}^{\mathrm{H}}(t)\right]$ representing the source covariance matrix. Let $\mathbf{E}_s$ be the $\mathrm{N}\times\mathrm{K}$ matrix composed of the K eigenvectors corresponding to the K largest eigenvalues of $\mathbf{R}_z$ and let $\mathbf{E}_n$ denote the $\mathrm{N}\times(\mathrm{N}-\mathrm{K})$ matrix composed of the remaining $\mathrm{N}-\mathrm{K}$ eigenvectors of $\mathbf{R}_z$. According to the subspace theory, there exists $\mathbf{K}\times\mathbf{K}$ nonsingular matrix $\mathbf{T}$, and the signal subspace can be expressed explicitly as

$$\mathbf{E}_s = \mathbf{A}\mathbf{T} \tag{20}$$

According to the definition of signal subspace, the relationship between signal subspace and steering vector can be expressed explicitly as

$$\mathbf{E}_1 = \mathbf{A}_1\mathbf{T} \qquad \mathbf{E}_2 = \mathbf{A}_2\mathbf{T} = \mathbf{A}_1\boldsymbol{\Phi}\mathbf{T} \tag{21}$$

The following expression can be obtained by crunching matrix operation

$$\mathbf{E}_1^{\#}\mathbf{E}_2\mathbf{T}^{-1} = \mathbf{T}^{-1}\boldsymbol{\Phi} \tag{22}$$

where $\mathbf{E}_1^{\#} = \left(\mathbf{E}_1^{\mathrm{H}}\mathbf{E}_1\right)^{-1}\mathbf{E}_1^{\mathrm{H}}$

Let $\psi{=}\mathbf{E}_1^{\#}\mathbf{E}_2 = \left(\mathbf{E}_1^{\mathrm{H}}\mathbf{E}_1\right)^{-1}\mathbf{E}_1^{\mathrm{H}}\mathbf{E}_2$, then equation (22) can be rewritten as

$$\psi\mathbf{T}^{-1} = \mathbf{T}^{-1}\boldsymbol{\Phi} \tag{23}$$

Equation (23) implies that the estimation of Φ is a matrix whose diagonal elements are composed of the K eigenvalues of matrix ψ and the full-rank matrix is composed of the K eigenvectors of matrix ψ . The estimations of A_1, A_2 and A can be obtained:

$$\mathbf{A}_1{=}\mathbf{E}_1\mathbf{T}^{-1}, \quad \mathbf{A}_2 = \mathbf{E}_2\mathbf{T}^{-1}, \quad \mathbf{A}{=}\mathbf{E}_s\mathbf{T}^{-1} \tag{24}$$

From the formula (17), the estimation of frequency is given as:

$$f_k = \sin^{-1}\left[\frac{1}{2\pi\Delta\mathrm{T}}\arg\left(\hat{\Phi}_{kk}\right)\right] \tag{25}$$

3.1 The estimations of DOA

The estimations of DOA can be got from $\mathbf{A}_1$ as following procedure. Base on the subspace theory and formula (7), the following equation can be obtained:

$$\mathbf{A}_{1k}./\mathbf{q}_k = \delta \qquad \delta \text{ is constant.}$$

Let

$$\mathbf{Q}_{1k} = [\mathbf{A}_{1k}(2:\mathrm{N})], \mathbf{Q}_{2k} = [\mathbf{A}_{1k}(1:\mathrm{N}-1)], \mathbf{Q}_k = [\mathbf{Q}_{2k}./\mathbf{Q}_{1k}] \tag{26}$$

The estimation of spatial steering vector $\Delta\hat{\mathbf{q}}_k$ is obtained:

$$\Delta\hat{\mathbf{q}}_k = [\mathbf{Q}_{2k}./\mathbf{Q}_{1k}] \tag{27}$$

According to equation (13), $\Delta\hat{\mathbf{q}}_k$ in equation (27) can be expressed as follows:

$$\Omega = \arg[\Delta\hat{\mathbf{q}}_k] = \Delta\mathbf{W}\cdot\begin{bmatrix}\hat{\alpha}_k\\ \hat{\beta}_k\end{bmatrix} \tag{28}$$

The estimations ofα_k and β_k can be got from (28), i.e.,

$$\begin{bmatrix}\hat{\alpha}_k\\ \hat{\beta}_k\end{bmatrix} = [\Delta\mathbf{W}]^{\#}\Omega \tag{29}$$

where $[\Delta\mathbf{W}]^{\#} = \left[(\Delta\mathbf{W})^{\mathrm{H}}\Delta\mathbf{W}\right]^{-1}(\Delta\mathbf{W})^{\mathrm{H}}$

From formula (29) the estimation of signals are obtained:

$$\begin{cases}\hat{\theta}_{\mathrm{k}} = \arcsin\left(\sqrt{\hat{\alpha}_k^2+\hat{\beta}_k^2}\right)\\ \begin{cases}\hat{\phi}_{\mathrm{k}} = \arctan\left(\frac{\hat{\alpha}_k}{\hat{\beta}_k}\right), \hat{\beta}_k \geq 0\\ \hat{\phi}_{\mathrm{k}} = \pi + \arctan\left(\frac{\hat{\alpha}_k}{\hat{\beta}_k}\right), \hat{\beta}_k < 0\end{cases}\end{cases} \tag{30}$$

3.2 The estimations of DOA

From formulas (7) and (18), the matrix $\mathbf{A_1}$ can be expressed as another form:

$$\mathbf{A}_1{=}\mathbf{A}_e + \mathrm{i}\mathbf{A}_h \tag{31}$$

From the equation (6), (7) and (19), it can be seen that:

$$\mathbf{A}_e = \mathbf{A}_h\Omega \tag{32}$$

where

$$\Omega = \begin{bmatrix}-\tan\gamma_1 e^{j\eta_1} & & \\ & \ddots & \\ & & -\tan\gamma_K e^{j\eta_K}\end{bmatrix} \tag{33}$$

According to equation (33), the polarization parameters estimation are presented as

$$\gamma_k = \tan^{-1}(|\Omega_{kk}|)\ \eta_k = \arg(-\Omega_{kk}) \tag{34}$$

According to the quaternion structure characteristics of uniform CLD pair circular array steering vector, the dipole and magnetic loop steering vectors are obtained, then the estimation of polarization parameters are got by formulas (32) and (33).

4 Comparison of the computational costs

A full estimation of the computational complexity of the methods is difficult as it is dependent on hardware and software. Consequently, we only focus on one aspect of the algorithm: the estimation of the covariance matrix. This procedure, as it implies repetitive operations, best illustrates the complexity difference between the two algorithms. For a vector sensor array composed by N CLD pairs, M snapshot data is used to estimate covariance matrices:

$$\mathbf{R}_{\mathrm{Q}} = \frac{1}{\mathbf{M}} \sum_{i=1}^{\mathbf{M}} \mathbf{X}_{\mathrm{Q}i} \mathbf{X}_{\mathrm{Q}i}^{\mathrm{H}} = \frac{1}{\mathbf{M}} \sum_{i=1}^{\mathbf{M}} \mathbf{R}_{\mathrm{Q}i} \tag{35}$$

$$\mathbf{R}_{\mathrm{C}} = \frac{1}{\mathbf{M}} \sum_{i=1}^{\mathbf{M}} \mathbf{X}_{\mathrm{C}i} \mathbf{X}_{\mathrm{C}i}^{\mathrm{H}} = \frac{1}{\mathbf{M}} \sum_{i=1}^{\mathbf{M}} \mathbf{R}_{\mathrm{C}i} \tag{36}$$

where $\mathbf{R}_{\mathrm{Q}i} = \mathbf{X}_{\mathrm{Q}i} \mathbf{X}_{\mathrm{Q}i}^{\mathrm{H}}$ and $\mathbf{R}_{\mathrm{C}i} = \mathbf{X}_{\mathrm{C}i} \mathbf{X}_{\mathrm{C}i}^{\mathrm{H}}$.

For the quaternion representation $\mathbf{X}_{\mathrm{Q}i} \in \mathrm{H}^{\mathrm{N}}$, the matrix $\mathbf{R}_{Qi}$ has $\mathbf{N^2}$ quaternionic entries and can be represented at machine memory level on $\mathbf{4N^2}$ real fields. The multiplication of two quaternions implies 16 real multiplications and 12 real additions, that is a total of $16\mathbf{N^2}$ real multiplications and $\mathbf{N^2}$ real additions for a matrix $\mathbf{R}_{Qi}$. The estimate of $\mathbf{R}_{Q}$ needs $16\mathbf{N^2}M$ real multiplications,$12\mathrm{N^2M}+(\mathrm{M}-1)\,4\mathrm{N^2}= 16\mathrm{N^2M}-4\mathrm{N^2}$ real additions and $\mathbf{4N^2}$ real numbers divisions. For the long-vector representation $\mathbf{X}_{\mathrm{C}i} \in \mathrm{C}^{2\mathrm{N}}$, the matrix has $\mathbf{4N^2}$ complex entries and can be represented at machine memory level on $\mathbf{8N^2}$ real fields. The multiplication of two complexes implies 4 real multiplications and 2 real additions, that is a total of $\mathbf{16N^2}$ real multiplications and $\mathbf{8N^2}$ real additions for a matrix $\mathbf{R}_{Ci}$. The estimate of $\mathbf{R}_{C}$ needs $\mathbf{16N^2}M$ real multiplications,$8\mathrm{N^2M}+(\mathrm{M}-1)\,8\mathrm{N^2} = 16\mathrm{N^2M}-8\mathrm{N^2}$ real additions, $\mathbf{8N^2}$ real numbers divisions. According to the above analysis, the quaternion algorithm can reduce half of the memory requirements for data covariance model representation. The quaternionic approach demands $\mathbf{4N^2}$ real numbers additions more than and $\mathbf{4N^2}$ real numbers divisions less than the long-vector method. The computational complexity for division is several times more than for addition, implying higher computational cost for long-vector.

5 Simulation Results

In this section, some simulations are conducted to evaluate the performances on DOA and polarization estimation by the proposed method. Two uncorrelated equal-powered signals with parameters $(\theta_1, \phi_1, \gamma_1, \eta_1) = (72^\circ, 85^\circ, 30^\circ, 120^\circ)$ and $(\theta_2, \phi_2, \gamma_2, \eta_2) = (30^\circ, 43^\circ, 67^\circ, 80^\circ)$ impinging upon the UCA with N=14 CLD pair sensors. The radius of UCA is $0.5\lambda_{\min}$. The frequency ratio of two signals are $(f_1/f_s, f_2/f_s) = (0.2, 0.4)$. The signal-to-noise ratio (SNR) is from 0 to 45dB, 1024 snapshots are used in each of the 500 independent Monte Carlo simulation experiments. The results are shown in figures 2-11.

Experiment 1: the performance of RMSE.

The dotted line with star and solid line with circular data points in Figs. 2-5 respectively plot the root mean squared error (RMES) of azimuth, elevation, polarization phase difference (PPD) and auxiliary polarization angle (APA), respectively estimated by long-vector and the proposed quaternion method, at various signal-to-noise ratio (SNR) levels. The proposed quaternion procedure is better than long vector. The estimation precision at 0dB based on the quaternion model has improved larger than 0.14° for azimuth, 0.79° for elevation, 0.43° for PPD, 0.2° for APA, compared with that of the long-vector method. Moreover, the RMSE of azimuth, elevation, PPD and APA are reduced evidently as the SNR increases using the quaternion method. The enhanced performance is rooted in the special data model of quaternion. When under low SNR conditions, the proposed algorithm can has better performance than that of long-vector methods.

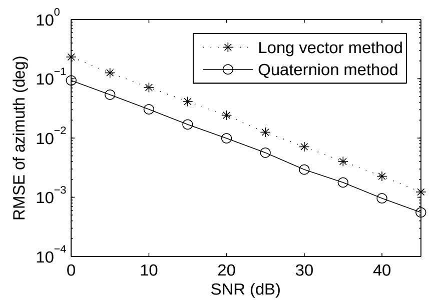

Fig. 2: RMSE of azimuth versus SNR.

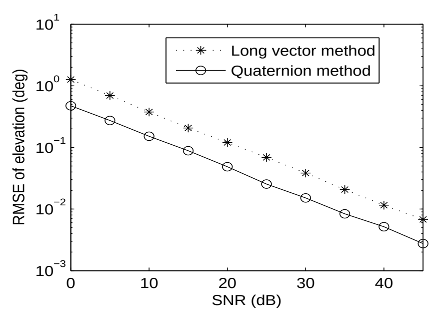

Fig. 3: RMSE of elevation versus SNR.

Experiment 2: the scatter diagrams plotted by the long vector and the proposed quaternion method.

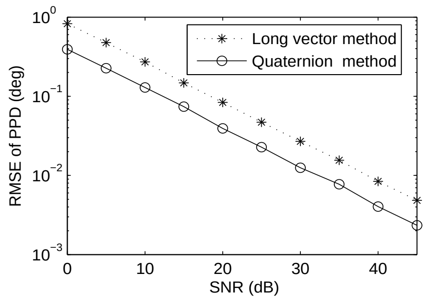

Fig. 4: RMSE of PPD versus SNR.

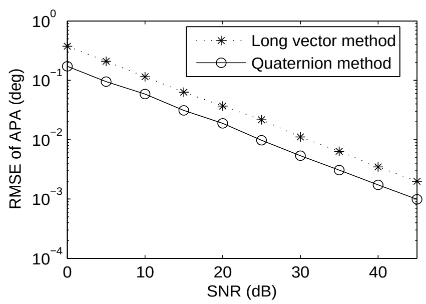

Fig. 5: RMSE of APA versus SNR.

In this experiment we show the scatter diagrams of DOA and polarization. Without loss of generality, the SNR is set at 15dB. The simulation results are shown in Figs. 6-9.

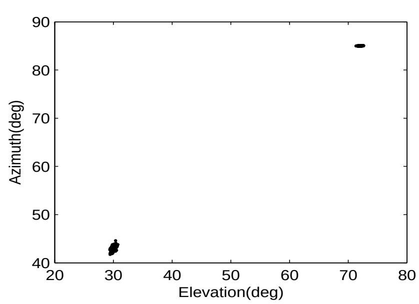

Fig. 6: Scatter diagram of DOA by long vector method.

Figs. 7 and 9 show that almost all estimated values are located in the vicinity of actual values under the application of proposed quaternion algorithm, the DOA and polarization estimated value errors are only 0.16° and 0.25° , respectively. On the contrary, in Fig. 6 and 8 the DOA and polarization estimated value errors with the long vector method are 0.63° and 0.7° , respectively. The

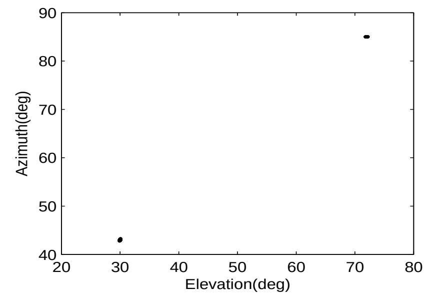

Fig. 7: Scatter diagram of DOA by quaternion method.

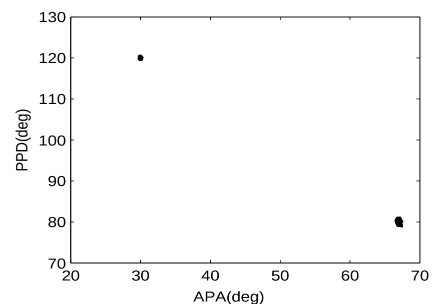

Fig. 8: Scatter diagram of polarization by long vector method.

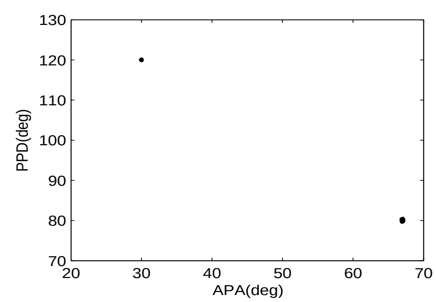

Fig. 9: Scatter diagram of polarization by quaternion method.

performance of quaternion algorithm is much better than that of long vector algorithm.

Experiment 3: the probability of success plotted by the long vector and the proposed quaternion method..

The probability of success of DOA and polarization estimations are given in this experiment. The result is shown in Figs 10-11.

The curves with star and circular data points in Figs. 10-11 respectively plot the probability of success of DOA and polarization, respectively estimated by the proposed long vector and quaternion method, at various signal-to-noise ratio (SNR) levels. The proposed quaternion procedure is better and more robust than long vector procedure.

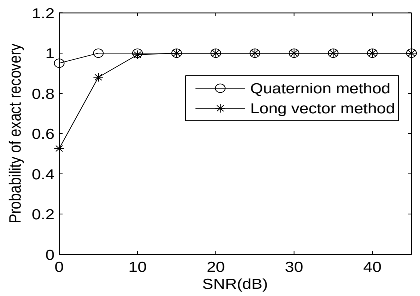

Fig. 10: Probability of exact recovery of DOA versus SNR.

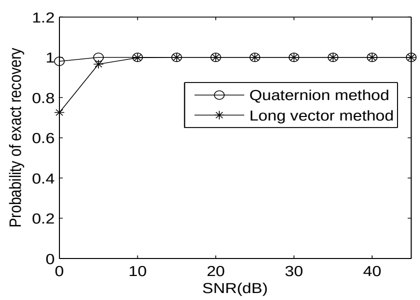

Fig. 11: Probability of exact recovery of polarization versus SNR.

6 Conclusions

The quaternion-ESPRIT algorithm for estimating frequency, DOA and polarization using UCA is proposed in this paper. The proposed algorithm can decouple frequency, DOA estimations from the polarization estimation, errors of frequency, DOA and polarization herein do not cumulate. The new method can obtain higher precision parameters than the traditional long vector data model method, because quaternion-ESPRIT algorithm retains the vector nature of vector sensor and provides a better subspace approximation. The computation efforts for estimating the data covariance matrices are lower than that of long-vector method.

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Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this article.

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Lanmei Wang was an associate professor at school of physics and optoelectronic engineering in xidian university, China. She received the Master of Science degree in radio physics in 2002 and the Ph.D.degree in information engineering in 2005, both from Xidian University, Xi'an Shaanxi, China. Her research interests includes in polarization sensitive array signal processing and the application of compressed sensing and frame theory in array signal processing

Minggao Zou was born in Jiangxi, China, in August of 1991. He is currently working toward a Masters degree in radio physics at Xidian University, Xian Shaanxi, China. His research interest lies in the parameter estimation of polarization sensitive array.

Guibao Wang was an associate professor at School of Physics and Telecommunication Engineering, Shaanxi University of Technology, China. He received the Master of Science degree in computational mathematics in 2003 and the Ph.D.degree in information engineering in 2014, both from Xidian University, Xian Shaanxi, China. His research interests focus on the polarization sensitive array signal processing.

Zhihai Chen was born in Shanxi, China, in July of 1989. He is currently working toward the Masters degree in radio physics at Xidian University, Xian Shaanxi, China. His research interest lies in the parameter estimation of electromagnetic vector sensor array processing based on the hypercomplex algebra and geometric algebra..