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The Design of Adaptive Control System with Reference Model for Population Dynamics

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Abstract: In the given work the adaptive control system is applied to a predator-prey system dynamics. It is assumed that considering system can adapt to changes of internal and external conditions. Researching control systems keep working capacity at unforeseen changes of properties of model of population, the D-factor (control laws) or environment by change of algorithm of the functioning, and the program of behavior or search of optimum conditions. By means of feedback mechanism model of the predator-prey system dynamics shown and rebuilt with conditions of the D - factor. Moreover, the problem for construction of an adaptive control system with reference model consists in a finding of such parameters with estimations of control at which the complex behavior can be reduced to the predicted. The method of locally-parametrical optimization (gradient method) of adaptive control with reference model is also discussed. This should develop the situation throughout the testing and improving new control methods and phases based on population dynamics model.

Keywords: Predator-prey system, adaptive control systems, locally-parametrical optimization, elimination process of outbreaks.

1 Introduction

The phytophagous insect that causes the greatest damage to trees is a growing problem in Siberian and Kazakhstan area forests. During outbreaks, the number of larvae of this species reach several thousand per tree, and they completely defoliate trees. The major objection is to seeking regulate ways, combining control theory and biosystems problem

The solution of Integrated Pest Management (IPM) tasks involves not only the construction of models that providing a satisfactory description of the population dynamics and/or the prediction of the number of pests. Nevertheless no less major task is the development of such methods of pest control, where ability of pest transfer to mass outbreak is excluded [1,2,3]. The first type of problem has been explained in following works [4,5,6,7] and the second case tasks have not been sufficiently researched [8,9,10].

The control of pest populations can be formulated and solved using specific various ways: by single impulse or periodic impulses without aftereffect (as it can be observed, for example, at use of pesticides (Isaev, Khlebopros, Nedorezov, etc.) [11,12], single pulsed or periodic impulse influences with aftereffects (for example, when artificial epizootics are organized within local populations, input of sterile males to the system, and other manipulations), and in addition with continuous control (with continuous input into the system of predators or parasites, with alter in the flow velocity in the flow system etc.).

In this paper we research a model of predator-prey system dynamics with effect of saturation, and the following results were obtained:

- for the original predator-prey model with saturation without control, the phase portraits of outbreaks were determined;

- it was focus on that permanent control of interacting populations can lead to realization of non-outbreak dynamic regimes, and stabilization of both population's sizes within fixed limits;

- the adaptation mechanism for the predator-prey model in which the populations adapt to the new conditions were studied.

This paper is organized as follows. The predator-prey model with predator saturation is reported in details in

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Section 1 as well as it is explain clearly the parameters and phase portraits of population dynamics. Moreover, adaptive role for the predator-prey model, its biological validity, and detailed methodology for solving adaptive control problems are given in this section. Second section examines following key areas: the adaptive control theory, stages and properties. Third section describes designing of a local parametric optimization method for research system and detailed explained adaptation mechanism. Simulation blocks and results of model behavior demonstrated in Section 4.

2 Description of the system

Models of predator-prey system dynamics of Volterra type have one common problem. The predator-prey model with saturation was taken as a base for studying the control mechanisms [13,14,15]:

$$\begin{aligned} \frac{dx}{dt} &= px(1-\beta x-\frac{z}{1+x^2}) \\ \frac{dz}{dt} &= cz(-\alpha-z+\frac{\gamma x^2}{1+x^2}) \end{aligned} \tag{1}$$

where $x(t), z(t)$ are the numbers of preys and predators in system respectively at time t, $\alpha, \beta, \gamma, p, c = const \geq 0$; px is growth rate of the prey population;

$p\beta x^2$ is death rate of the preys caused by the action of the intra-population self-regulatory mechanisms;

$\frac{pxz}{1+x^2}$ is rate of destruction of preys by predators;

αcz is mortality rate of predators;

cz^2 is mortality rate of predators caused by the action of intra-population competition;

$\frac{c\gamma zx^2}{1+x^2}$ is growth rate of the number of predators at the expense of eating prey.

Simulation results:

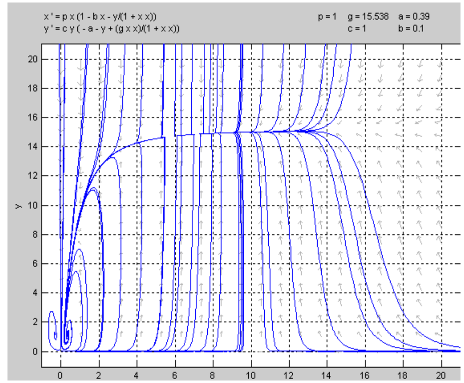
In Figure 1(a,b), the plot has several bends and kinks, indicating that the distribution of the data is probably anormal, and illustrated outbreak behaviour among population.

The main results can be summarized as follows outbreak regimes: fixed, permanent, reverse, and an outbreak proper. In fact, authors in [13] analysed the drawbacks of this tool as well. Figure 1. (a,b) shows the phase portraits of system (1).

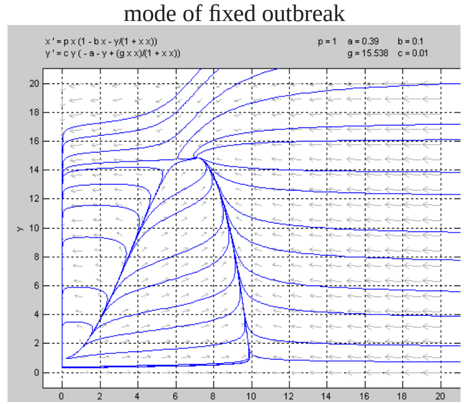
In the following case: $p = 1; \alpha = 0.39; \beta = 0.1; \gamma = 15.538; c = 1$ system (1) have next stationary points: for the stationary points:

$(x_1, z_1) = (0,0); (x_2, z_2) = \left(1/\beta, 0\right); (x_3, z_3) = \left(\sqrt{\frac{\alpha}{\gamma-\alpha}}, 0\right); (x_4, z_4) = (0,1); (x_6, z_6) = (5.92, 14.71)$– a saddle point; $(x_5, z_5) = (0.32, 1.06)$– it is stable focus, $(x_7, z_7) = (7.15, 14.85)$– it is stable node.

Obtained phase portraits observe a complex behavior of populations. An important feature of biological systems is their ability to switch from one operation mode to another, which corresponds to more stable steady state of the system.



(a) in case $p = 1,\ \alpha = 0.39,\ \beta = 0.1,\ \gamma = 15.538,\ c = 1$ the mode of fixed outbreak



b) in case $p = 1,\ \alpha = 0.39,\ \beta = 0.1,\ \gamma = 15.538,\ c = 0.01$ the mode of reverse outbreak

Fig. 1: Phase portraits of system (1)

On the phase plane, such a system has two (or more) stable stationary states. The regions of attraction of steady states are derived by separatrixes which come through an unstable stationary state of the saddle type. A system that has two or more stable steady states is called trigger mode [15,16,17,18].

It is important to note that trigger systems describe adequately one of the main features of biological systems - their ability to switch from one mode to another; for this reason, along with the oscillatory regimes trigger modes became widespread [19,20,21].

Additionally, we take a predator-prey model with one controlling parameter for two populations simultaneously:

$$\begin{aligned} \frac{dx}{dt} &= px(1-\beta x-\frac{z}{1+x^2}) - (x^3+k_1x) \\ \frac{dz}{dt} &= cz(-\alpha-z+\frac{\gamma x^2}{1+x^2}) - (z^3+k_2z) \end{aligned} \tag{2}$$

where(x^3+k_1x), (z^3+k_2z) – are the results of the regulatory impact of control factors, which correspond to the proportion of diseased or infected individuals in populations.

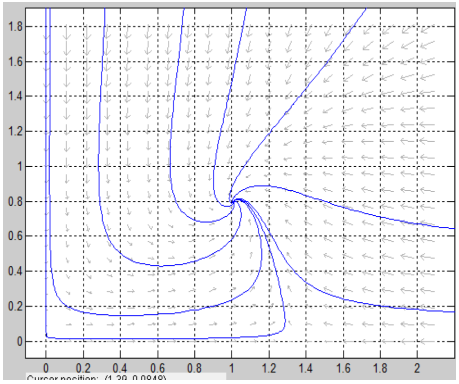


Fig. 2: Phase portraits of system (2)

As a controlling mechanism for phytophagous can be apply as insecticides (industrial or natural) harmless to mammals, and damaging only direct to phytophagous, and not to entomophagous.

For the following values of model (2) parameters $p = 1; \alpha = 0.39; \beta = 0.1; \gamma = 15.538; c = 1; k_1 = 0.1; k_2 = 2.5$, the next steady states were determined:

$(x_1, z_1) = (0, 0); (x_2, z_2) =$
$\left(\frac{1}{2}\left(\sqrt{p^2\beta^2 + 4p - 4k_1} - p\beta\right), 0\right); (x_4, z_4)$
$= \left(0, 1 - \frac{k_1}{p}\right);$

$(x_3, z_3) = \left(\frac{1}{\alpha - \gamma + k_2}\sqrt{(\alpha - \gamma + k_2)(\alpha + k_2)}, 0\right)$- is a saddle point,

$(x_5, z_5) = (0.382, 0.820)$- is stable focus.

Thus, a brief comparative analysis suggests that the control laws in the form of a one-parameter structurally stable mapping give the system a stable behavior. The structure of the phase trajectories of the system outlined a significant change. Phase trajectories show a stable node, and it means that the population system providing coexistence of species, that is plotted in figure 2.

Figure 2 depicted a phase portrait of the system (2), the reflective properties of the structural stability on a large scale for various initial values and the totally dynamics of the system is projected.

The nature of the phase portraits informs a stable node, i.e. stable coexistence of two populations not at zero. In addition the elimination process of the different outbreaks and finally this behavior is more predictable and controllable [22,23].

The application of the fold catastrophe (as a result of the epidemic process) can be explained as follows:

a) the phytophagous population significantly decreases;

b) the number of entomophages declined in the same period;

c) the disease gradually returns to the endemic level.

However, due to appropriate processing of chemical-biological or adverse conditions population phytophagous sharply reduced before the number of entomophages is significantly falling down. Consequently the path of mass reproduction passes through the fold, making a sharp transition to the endemic level. Furthermore the rate of transition depends on the extent of the fold.

3 Theory Basics in Adaptive control systems.

Adaptive control is a set of control theory methods that can generate control systems that have the ability to modify [24]:

a) Parameters of the controller;

b) Regulator of the structure depending on the change of the control object parameters;

c) External disturbances acting on the control object.

The purpose of control is to sustain the population level at the desired line or tendency and to provide the required dynamic characteristics.

The aim of adaptation is to ensure the stability of the system and the required dynamic characteristics in accordance with the control law, even if the system has non-stationary behavior (changing model parameters or some nonlinearities). The tasks of control mean to investigate the new adaptation mechanism.

Usually adaptive control systems considering capable of learning about the controlled process, i.e. development of a mathematical model of the process using on-line data and system identification. Adaptive control performs following parameters: control, state estimation, parameter estimation and include: model reference (MR) control, self-tuning (ST) control and gain scheduling. Additionally existing approaches: local parametric optimization; use of Lyapunov functions and hyperstability. The process of adaptive control can be proposed as a process of interaction of three subsystems: an object, a regulator and an adaptation unit. The last two blocks are combined into an adaptive controller. The method of solving the problems of adaptive control consists of the main 4 steps: choice of the control law, select the settings parameters; pick up the adaptation algorithm and research the adaptive system.

Firstly is the control law is achieving, which provides a principled opportunity to concrete goal. Secondly, the unknown parameters on which the ideal control law depends replacing on the parameters being tuned. At the end, a control algorithm is obtained, which does not include the uncertainty parameters, so it can be implemented by the regulator. When the defined parameters are selected, the goal of adaptation is set. In this paper we propose systems with a reference model. At the 3-rd stage of the uncertainty conditions, it is difficult to achieve the control goal at once, and then the adaptation algorithm sequentially changes the adjusted parameters, approaching the goal.

In this adaptive control system, results are obtained that draw the controllability property of the model. There are adaptive control systems with a reference model.

In such models, the main objective is to ensure that the tracking error converges to zero *e(t)=x(t)-*x_m*(t)*$\rightarrow$ 0 in t$\rightarrow\infty$, at, where *x(t)* - a configurable model (the initial model of a population with complex behavior), $x_m(t)$ - a reference model (the desired stable behavior of populations).

The task is to find such control parameters, in which complex behavior can be reduced to the predicted one.

In this paper, we research the method of adaptive control with a reference model: the method of local-parametric optimization (gradient method).

4 Designing of a Local Parametric Optimization method for research system

In this section, we first demonstrate that an adaptive control law using the basic model equation. This section offers the basic principles of constructing an adaptive control system for the predator-prey system. Adaptive is deal with a system in which populations can adapt to changes in internal and external conditions.

For the original predator-prey model (2), describing complex behavior, where the control law is represented in the form $u_1 = x^3 + k_1 x$.

Moreover, for determine the parameters k_1, k_2 , we construct the sensitivity function for the prey population. In the gradient method, the rate of change of the adjustable parameters is assigned proportional to the gradient to the adjustable parameters from the function.

In the beginning, the basic notation is given: $\nabla k_1 = -M_1 \frac{\partial J}{\partial k_1}$,

where k_1- is the vector of adjustable parameters;

M_1- some positive-definite matrix of gain factors;

J- function of optimization criterion;

∇k_1- changes on the regulator coefficient by k_1.

In the proposed equation, the term $\frac{\partial J}{\partial k_1}$ is the gradient of the optimization criterion function, which is used as a navigation tool in the adaptation process.

In the composition of the function J, a $k_1$vector was included as an argument to emphasize the dependence on this vector J. This function is defined in the following form $J(k) = 0,5\int_{t-\tau}^{t} e^2 dt$, where is the e error between the reference and adjustable model, $e = x_m - x$;.

The adaptation mechanism k_1 is computed by substituting the expression for the gradient value and the optimization criterion function:

$$\dot{\nabla k_1} = \frac{d}{dt}(\nabla k_1) = \frac{d}{dt}\left[-M_1 \frac{\partial}{\partial k_1}\left[0,5\int_{t-\tau}^{t} e^2 dt\right]\right] =$$
$$= -0,5M_1 \frac{\partial}{\partial k_1}\frac{d}{dt}\int_{t-\tau}^{t} e^2 dt = -0,5M_1 \frac{\partial}{\partial k_1}(e^2) = -M_1 e\frac{\partial e}{\partial k_1}.$$

The partial derivative of the error with respect to the k_1 parameter is equal: $\frac{\partial e}{\partial k_1} = \frac{\partial (x_m - x)}{\partial k_1} = -\frac{\partial x}{\partial k_1}$.

The transformation, the time derivative of the coefficient change k_1 became: $\dot{\nabla k_1} = M_1 e \frac{\partial x}{\partial k_1}$.

After determining the basic conditions, investigating process base on the gradient method, the optimization criterion function for the prey population.

The prey population equation is found as:

$$\frac{dx}{dt} = px(1 - \beta x - \frac{z}{1+x^2}) - (x^3 + k_1 x);$$

For the left and right parts, we calculate the partial derivative with respect to the parameter being tuned:

$$\frac{\partial}{\partial k_1}\left[\frac{dx}{dt}\right] = \frac{\partial}{\partial k_1}\left[px - p\beta x^2 - \frac{pxz}{1+x^2} - x^3 - k_1 x\right].$$

After these simplifications and differentiation of the equation the expression is reproduced below:

$$p\frac{\partial x}{\partial k_1} - 2p\beta x\frac{\partial x}{\partial k_1} - 3x^2\frac{\partial x}{\partial k_1} - k_1\frac{\partial x}{\partial k_1},$$

which can be transformed as follows $\frac{\partial}{\partial k_1}[\frac{dx}{dt}] = \frac{\partial x}{\partial k_1}(p - 2p\beta x - 3x^2 - k_1)$.

By solving for the partial derivative of the tunable parameter, we get the sensitivity function. In addition, substituting in the expressions for the adaptation mechanism is defined as:

$$\dot{\nabla k_1} = M_1 e \frac{x}{p - 2p\beta x - 3x^2 - k_1}.$$

Integrating both sides, the proposed adaptation mechanism is now equal:

$$\nabla k_1 = M_1 \int \frac{ex}{p - 2p\beta x - 3x^2 - k_1} dx + \nabla k_1(0) \qquad (3)$$

Similarly, calculate the sensitivity function for the predator population. A generalized scheme of an adaptive system with a reference model by the gradient method is given in Figure 3.

Block M.A - adaptation mechanism, for the adaptive control system for the model (2). Block A.S. - an adjustable system of the original predator-prey model. R.M. - reference model.

Adaptation significance for model

Adaptation of phytophagous is reviewed as follows. Any single adaptation of the phytophagous (in food, areal and biochemical reaction, climate temperature and conditions), is made to increase the control of phytophagous number. The advantage of phytophagous was reflected in the rate of progressive specialization; due to the more unilateral process is the adaptation than the faster it can be obtained. Influence of feedback affect make the predator-prey model is adapted, repurposed, retargeted to the new control law conditions.

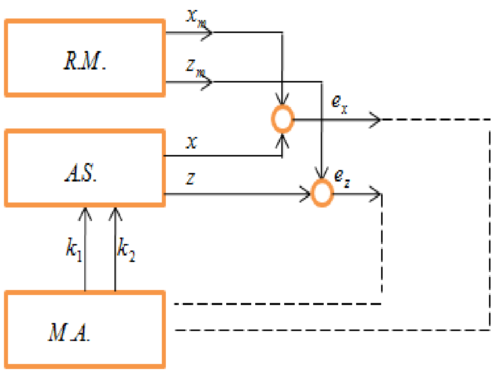


Fig. 3: General view of the adaptive system with the reference model

Ecosystems totally can be projected as complex adaptive systems, as “complex” properties mixed as multiple parts and connections, moreover an “adaptive” because their feedback subsystems return them the ability to flexible in a fluctuating environment.

5 Simulation case

As a simulation part of this paper, we illustrate the implementation of numerical methods through programming with the technical software package Matlab/Simulink. The flowchart below demonstrates the relationship between without and included control law on base model.

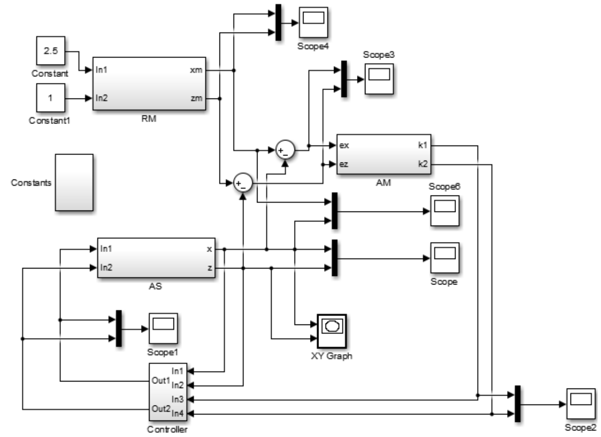
A flow chart of the adaptive control system with a reference model in the Matlab system is presented in Fig 4.

Keeping the constants the same and plotting the numerical solution for the predator and prey (figure 5), we observe that the solution is very well-behaved. These examples demonstrate that the stability characteristics of adaptation control are superior.

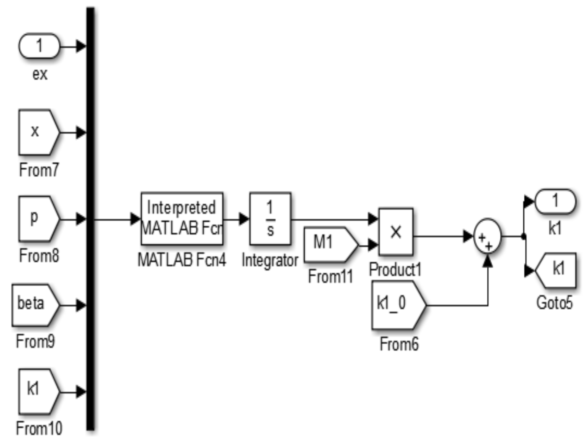
Figure 5 depicts three different scenarios of stabilization in a data set. In figure 5(a), data process within each of the two populations to explain the outbreak.

Results after influence control observed Stabilization of the population at a lower level shown in figure 5(b). This trend indicates, a gradual decrease of entomophagous followed (correspondingly) by phytophagy can be explained by a drop the food base for the entomophagy. For this case, we expect the group means to be significantly predictable.

The data in figure 5(c) demonstrates an adaptation process.



The general view of the adaptive control system



The detailed view of adaptation mechanism for model (3)

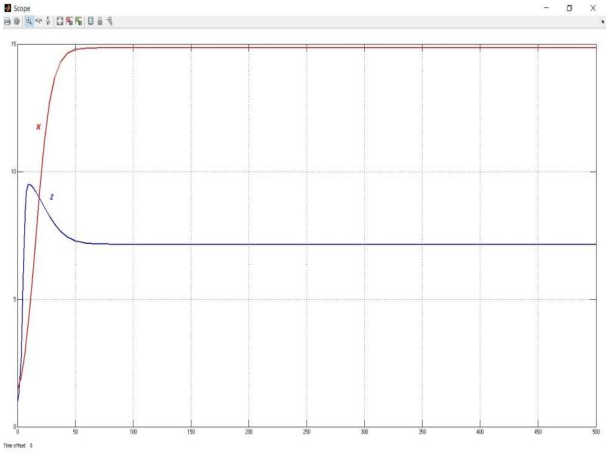
Fig. 4: Flow chart of an adaptive control system

6 Conclusion

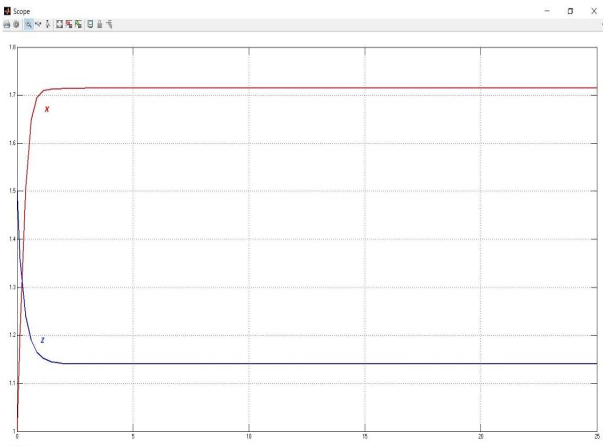
Attempts to pest control using chemicals highlighted the really and temporary affect, or do not bring any effect. Correspondingly, insects which the processing is carried out, rapidly adapt to them (for example, DDT).

Finally, using the experiments, an analysis of the main assessments of the adaptive control system with a reference model was made: population dynamics, the dynamics of the convergence error, the vector trajectory of the state and the dynamics of the control signals, the parameters of the adaptation mechanism. The properties of controllability and adaptability of the model highlighted. The common schema illustrates process of adaptive control system. This indicates the process of adaptation between the reference and adjustable model.

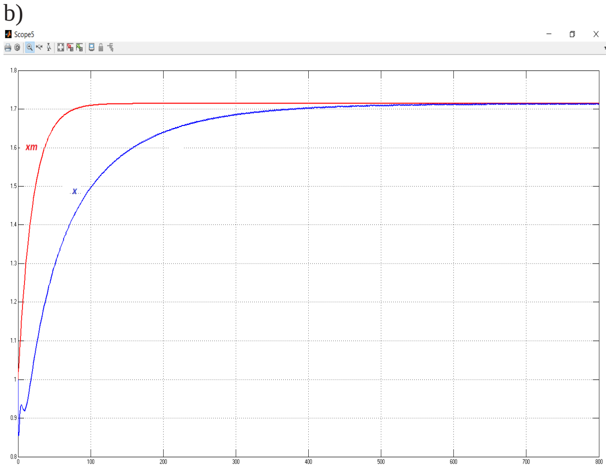
A further prominent feature of the data is that clearly signify the elimination process of outbreaks. Another important trend in the result is that modes of population level signify mean stabilization. Analytical and numerical results were performed and described. Efforts can be



a)



b)



c)

Fig. 5: Scenario of population with and without control law.

taken by both biologists and applied mathematics to deal with this situation. This is important because combining nonlinearities and ecological systems into the formulation and solution of modern control problems.

In conclusion, it is clear that more attention should be allocated to the researching and design of more control methods in this area.

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