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Supra Generalized Closed Soft Sets with Respect to an Soft Ideal in Supra Soft Topological Spaces

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Abstract: The concept of soft ideal was first introduced by Kandil et al. [13]. In 1999, Molodtsov [22] introduced the concept of soft sets as a general mathematical tool for dealing with uncertain objects. The concept of generalized closed soft sets in soft topological spaces was introduced by Kannan [15] in 2012. The notions of supra soft topological space were first introduced by Kandil et al. [14]. In this paper, we introduce the concept of supra generalized closed soft sets(supra g-closed soft for short) in a supra topological space (X,μ,E) and study their properties in detail. Also, we introduce the concept of supra generalized closed soft sets with respect to a soft ideal (supra-$\tilde{I}g$-closed soft for short) in a supra topological space (X,μ,E) and study their properties in detail, which is the extension of the concept of supra generalized closed soft sets.

Keywords: Soft sets, Soft topological space,Open soft, Closed soft, Supra soft topological space, Supra open soft, Supra closed soft, Supra g-closed soft, Supra g-open soft, Supra-$\tilde{I}g$-closed soft, Supra-$\tilde{I}g$-open soft sets, soft Ig-open sets and continuous soft mappings.

1 Introduction

The concept of soft sets was first introduced by Molodtsov [22] in 1999 as a general mathematical tool for dealing with uncertain objects. In [22,23], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.
After presentation of the operations of soft sets [20], the properties and applications of soft set theory have been studied increasingly [4,16,23,25]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1,3,6,18,19,20,21,23,24,32]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [7].
Recently, in 2011, Shabir and Naz [29] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [9] investigated the properties of open (closed) soft, soft nbd and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. Kandil et al. [13] introduced the notion of soft ideal in soft set theory. Also, they introduced the notion of soft local function. These concepts are discussed with a view to find new soft

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topologies from the original one. Kandil et al [14] introduced the notion of supra soft topological spaces, which is wider and more general than the class of soft topological spaces.
In 1970, Levine [18] introduced the notion of g-closed sets in topological spaces as a generalization of closed sets.
Indeed ideals are very important tools in general topology. It was the works of Newcomb [26], Rancin [27], Samuels [28] and Hamlet Jankovic [8, 10] which motivated the research in applying topological ideals to generalize the most basic properties in general Topology. S. Jafari and N. Rajesh introduced the concept of g-closed sets with respect to an ideal which is a extension of the concept of g-closed sets. Recently. K. Kannan [15] introduced the concept of g-closed soft sets in a soft topological spaces.
The main purpose of this paper is to introduce the notion of supra generalized closed soft sets(supra g-closed soft for short) in a supra topological space (X,μ,E) and study their properties in detail. We introduce the concept of supra generalized closed soft sets with respect to a soft ideal (supra-$\tilde{I}g$-closed soft for short) in a supra topological space (X,μ,E) and study their properties in detail, which is the extension of the concept of supra generalized closed soft sets. Also, we study the relationship between supra-$\tilde{I}g$-closed soft sets, supra-$\tilde{I}g$-open soft sets, supra g-closed soft sets and supra g-open soft sets. This paper, not only can form the theoretical basis for further applications of topology on soft set, but also lead to the development of information systems.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1.[22] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \phi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2.[20] Let $F_A,\ G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

(1)$A \subseteq B$, and
(2)$F(e) \subseteq G(e)$, $\forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3.[20] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4.[4] The complement of a soft set (F,A), denoted by $(F,A)'$, is defined by $(F,A)' = (F',A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e)$, $\forall e \in A$ and F' is called the soft complement function of F.
Clearly $(F')'$ is the same as F and $((F,A)')' = (F,A)$.

Definition 2.5.[29] The difference of two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E) - (G,E)$ is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6.[29] Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7.[20] A soft set (F,A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).

Definition 2.8.[20] A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

Definition 2.9.[20] The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e),\ e \in A - B, \\ G(e),\ e \in B - A, \\ F(e) \cup G(e),\ e \in A \cap B\ . \end{cases}$$

Definition 2.10.[20] The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.11.[33] Let I be an arbitrary indexed set and $L = \{(F_i,E), i \in I\}$ be a subfamily of $SS(X)_E$.

(1)The union of L is the soft set (H,E), where $H(e) = \bigcup_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i\in I}(F_i,E) = (H,E)$.
(2)The intersection of L is the soft set (M,E), where $M(e) = \bigcap_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i\in I}(F_i,E) = (M,E)$.

Definition 2.12.[29] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1)$\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,
(2)the union of any number of soft sets in τ belongs to τ,
(3)the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 2.13.[9] Let (X,τ,E) be a soft topological space. A soft set (F,A) over X is said to be closed soft set in X, if its relative complement $(F,A)'$ is an open soft set.

Definition 2.14.[9] Let(X,τ,E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X,\tau,E)$, or $OS(X)$ and the set of all closed soft sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition 2.15.[29] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E) i.e
$cl(F,E) = \tilde{\cap}\{(H,E) : (H,E)\ is\ closed\ soft\ set\ and\ (F,E)\tilde{\subseteq}(H,E)\}$.

Definition 2.16.[33] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) i.e
$int(G,E) = \tilde{\cup}\{(H,E) : (H,E)\ is\ an\ open\ soft\ set\ and\ (H,E)\tilde{\subseteq}(G,E)\}$.

Definition 2.17.[33] The soft set $(F,E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F,E) is denoted by x_e.

Definition 2.18.[33] The soft point x_e is said to be belonging to the soft set (G,A), denoted by $x_e \tilde{\in} (G,A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.19.[33] A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood (briefly: nbd) of the soft point $x_e \tilde{\in} X_E$ if there exists an open soft set (H,E) such that $x_e \tilde{\in} (H,E) \tilde{\subseteq} (G,E)$.
A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood of the soft (F,E) if there exists an open soft set (H,E) such that $(F,E)\tilde{\in}(H,E)\tilde{\subseteq}(G,E)$.
The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$, is the family of all its neighborhoods.

Definition 2.20.[29] Let (X,τ,E) be a soft topological space, $(F,E) \in SS(X)_E$ and Y be a non null subset of X. Then the sub soft set of (F,E) over Y denoted by (F_Y,E), is defined as follows:

$$F_Y(e) = Y \cap F(e)\ \forall e \in E.$$

In other words $(F_Y,E) = \tilde{Y} \tilde{\cap} (F,E)$.

Definition 2.21.[29] Let (X,τ,E) be a soft topological space and Y be a non null subset of X. Then

$$\tau_Y = \{(F_Y,E) : (F,E) \in \tau\}$$

is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Theorem 2.1.[29] Let (Y,τ_Y,E) be a soft subspace of a soft topological space (X,τ,E) and $(F,E) \in SS(X)_E$. Then

(1)If (F,E) is an open soft set in Y and $\tilde{Y} \in \tau$, then $(F,E) \in \tau$.
(2)(F,E) is an open soft set in Y if and only if $(F,E) = \tilde{Y} \tilde{\cap} (G,E)$ for some $(G,E) \in \tau$.
(3)(F,E) is a closed soft set in Y if and only if $(F,E) = \tilde{Y} \tilde{\cap} (H,E)$ for some (H,E) is τ-closed soft set.

Definition 2.22.[2] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets on X and Y respectively, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;

(1)If $(F,A) \in SS(X)_A$. Then the image of (F,A) under f_{pu}, written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that
$$f_{pu}(F)(b) = \begin{cases} \cup_{a \in p^{-1}(b) \cap A}\ u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$
for all $b \in B$.
(2)If $(G,B) \in SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that
$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$
for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition 2.23.[11,33] Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then

(1)The function f_{pu} is called continuous soft (cts-soft) if $f_{pu}^{-1}(G,B) \in \tau_1\ \forall\ (G,B) \in \tau_2$.
(2)The function f_{pu} is called open soft if $f_{pu}(G,A) \in \tau_2 \forall\ (G,A) \in \tau_1$.
(3)The function f_{pu} is called closed soft if $f_{pu}(G,A) \in \tau_2' \forall\ (G,A) \in \tau_1'$.
(4)The function f_{pu} is called semi open soft if $f_{pu}(G,A) \in SOS(Y) \forall\ (G,A) \in \tau_1$.

(5)The function f_{pu} is called semi continuous soft function (semi-cts soft) if $f_{pu}^{-1}(G,B) \in SOS(X) \forall\ (G,B) \in \tau_2$.
(6)The function f_{pu} is called irresolute soft if $f_{pu}^{-1}(G,B) \in SOS(X)[f_{pu}^{-1}(F,B) \in SCS(X)] \forall\ (G,B) \in SOS(Y)[(F,B) \in SCS(Y)]$.
(7)The function f_{pu} is called irresolute open (closed) soft if $f_{pu}(G,A) \in SOS(Y)[f_{pu}(F,A) \in SCS(Y)] \forall\ (G,A) \in SOS(X)[(F,A) \in SCS(Y)]$.

Theorem 2.2.[2] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$, the following statements hold,

(a) $f_{pu}^{-1}((G,B)') = (f_{pu}^{-1}(G,B))' \forall\ (G,B) \in SS(Y)_B$.
(b) $f_{pu}(f_{pu}^{-1}((G,B))) \tilde{\subseteq} (G,B) \forall\ (G,B) \in SS(Y)_B$. If f_{pu} is surjective, then the equality holds.
(c) $(F,A) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}((F,A))) \forall\ (F,A) \in SS(X)_A$. If f_{pu} is injective, then the equality holds.
(d) $f_{pu}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$. If f_{pu} is surjective, then the equality holds.
(e) $f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ and $f_{pu}(\tilde{\phi}_A) = \tilde{\phi}_B$.
(f)If $(F,A) \tilde{\subseteq} (G,A)$, then $f_{pu}(F,A) \tilde{\subseteq} f_{pu}(G,A)$.
(g)If $(F,B) \tilde{\subseteq} (G,B)$, then $f_{pu}^{-1}(F,B) \tilde{\subseteq} f_{pu}^{-1}(G,B)\ \forall\ (F,B),(G,B) \in SS(Y)_B$.
(h) $f_{pu}^{-1}[(F,B) \tilde{\cup} (G,B)] = f_{pu}^{-1}(F,B) \tilde{\cup} f_{pu}^{-1}(G,B)$ and $f_{pu}^{-1}[(F,B) \tilde{\cap} (G,B)] = f_{pu}^{-1}(F,B) \tilde{\cap} f_{pu}^{-1}(G,B)$ $\forall\ (F,B),(G,B) \in SS(Y)_B$.
(I) $f_{pu}[(F,A) \tilde{\cup} (G,A)] = f_{pu}(F,A) \tilde{\cup} f_{pu}(G,A)$ and $f_{pu}[(F,A) \tilde{\cap} (G,A)] \tilde{\subseteq} f_{pu}(F,A) \tilde{\cap} f_{pu}(G,A)$ $\forall\ (F,A),(G,A) \in SS(X)_A$. If f_{pu} is injective, then the equality holds.

Definition 2.24.[13] Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with the same set of parameters E. Then $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with the same set E if

(1) $(F,E) \in \tilde{I}$ and $(G,E) \in \tilde{I} \Rightarrow (F,E) \tilde{\cup} (G,E) \in \tilde{I}$,
(2) $(F,E) \in \tilde{I}$ and $(G,E) \tilde{\subseteq} (F,E) \Rightarrow (G,E) \in \tilde{I}$,
i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Theorem 2.3.[12] Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal, (X_2, τ_2, B) be a soft topological space and $f_{pu} : (X_1, \tau_1, A, \tilde{I}) \to (X_2, \tau_2, B)$ be a soft function. Then $f_{pu}(\tilde{I}) = \{f_{pu}((F,A)) : (F,A) \in \tilde{I}\}$ is a soft ideal on X_2.

Definition 2.25.[14] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq SS(X)_E$ is called a supra soft topology on X with a fixed set E if

(1) $\tilde{X}, \tilde{\phi} \in \mu$,
(2)the union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called a supra soft topological space (or supra soft spaces) over X.

Definition 2.26.[14] Let (X, μ, E) be a supra soft topological space over and $(F,E) \in SS(X)_E$. Then the supra soft interior of (G,E), denoted by $int^s(G,E)$ is the soft union of all supra open soft subsets of (G,E). Clearly $int^s(G,E)$ is the largest supra open soft set over X which contained in (G,E) i.e
$int^s(G,E) = \tilde{\cup}\{(H,E) : (H,E)\ is\ supra\ open\ soft\ set\ and\ (H,E) \tilde{\subseteq} (G,E)\}$.

Definition 2.27.[14] Let (X, μ, E) be a supra soft topological space over and $(F,E) \in SS(X)_E$. Then the supra soft closure of (F,E), denoted by $cl^s(F,E)$ is the soft intersection of all supra closed super soft sets of (F,E). Clearly $cl^s(F,E)$ is the smallest supra closed soft set over X which contains (F,E) i.e
$cl^s(F,E) = \tilde{\cap}\{(H,E) : (H,E)\ is\ supra\ closed\ soft\ set\ and\ (F,E) \tilde{\subseteq} (H,E)\}$.

3 Supra generalized closed soft sets

Kannan [15] introduced generalized closed soft sets in soft topological spaces. In this section we generalize the notions of generalized closed soft sets to supra soft topological spaces.

Definition 3.1. A soft set (F,E) is called a supra generalized closed soft set (supra g-closed soft) in a supra soft topological space (X, μ, E) if $cl^s(F,E) \tilde{\subseteq} (G,E)$ whenever $(F,E) \tilde{\subseteq} (G,E)$ and (G,E) is a supra open soft in X.

Example 3.1. Suppose that there are three cars in the universe X given by $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively.
Let $(F_1,E), (F_2,E)$ be two soft sets over the common universe X, which describe the composition of the cars, where
$F_1(e_1) = \{h_2, h_3\}, \quad F_1(e_2) = \{h_1, h_2\}$,
$F_2(e_1) = \{h_1, h_2\}, \quad F_2(e_2) = \{h_1, h_3\}$.
Then $\mu = \{\tilde{X}, \tilde{\phi}, (F_1,E), (F_2,E)\}$ is the supra soft topology over X. Hence the soft sets $(F_1,E), (F_2,E)$ are supra g-closed soft sets in (X, μ, E), but the set (G,E) where
$G(e_1) = \{h_2\}, \quad G(e_2) = \{h_1\}$ is not supra g-closed soft in (X, μ, E).

Remark 3.1. The soft intersection (resp. soft union) of any two supra g-closed soft sets is not supra g-closed soft in general as shown in the following examples.

Examples 3.1.

(1)In example 3.1, $(F_1,E),(F_2,E)$ are supra g-closed soft in (X,μ,E), but their soft intersection $(F_1,E)\tilde{\cap}(F_1,E)=(M,E)$ where $M(e_1)=\{h_2\}$, $M(e_2)=\{h_1\}$ is not supra g-closed soft.

(2)Suppose that there are four alternatives in the universe of houses $X=\{h_1,h_2,h_3,h_4\}$ and consider $E=\{e\}$ be the single parameter "quality of houses "to be the a linguistic variable. Let $(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E)$ be eight soft sets over the common universe X which describe the goodness of the houses, where $F_1(e)=\{h_1\}$, $F_2(e)=\{h_4\}$, $F_3(e)=\{h_1,h_4\}$, $F_4(e)=\{h_1,h_2\}$, $F_5(e)=\{h_2,h_4\}$, $F_6(e)=\{h_1,h_2,h_3\}$, $F_7(e)=\{h_2,h_3,h_4\}$, $F_8(e)=\{h_1,h_2,h_4\}$.
Then $\mu=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E)\}$ is the supra soft topology over X. Hence the sets $(F_1,E),(F_2,E)$ are supra g-closed soft sets in (X,μ,E), but their soft union $(F_1,E)\tilde{\cup}(F_1,E)=(H,E)$ where $H(e)=\{h_1,h_4\}$ is not supra g-closed soft.

Remark 3.2. Every supra closed soft set is supra g-closed soft. But the converse is not true in general as shown in the following example.

Example 3.2. In example 3.1, $(F_1,E),(F_2,E)$ are supra g-closed soft in (X,μ,E), but not supra closed soft over X.

Theorem 3.1. Let (X,μ,E) be a supra soft topological space and (F,E) be a supra g-closed soft in X. If $(F,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}cl^s(F,E)$, then (H,E) is a supra g-closed soft.

Proof. Let $(H,E)\tilde{\subseteq}(G,E)$ and $(G,E)\in\mu$. Since $(F,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}(G,E)$ and (F,E) is supra g-closed soft in X, then $cl^s(F,E)\tilde{\subseteq}(G,E)$. Hence $cl^s(H,E)\tilde{\subseteq}cl^s(F,E)\tilde{\subseteq}(G,E)$. Thus $cl^s(H,E)\tilde{\subseteq}(G,E)$. Therefore, (H,E) is a supra g-closed soft.

Theorem 3.2. Let (X,μ,E) be a supra soft topological space. Then (H,E) is supra g-closed soft in X if and only if $cl^s(H,E)\setminus(H,E)$ contains only null supra closed soft set.

Proof. Let (H,E) be a supra g-closed soft set, (F,E) be a non null supra closed soft set in X and $(F,E)\tilde{\subseteq}cl^s(H,E)\setminus(H,E)$. Then $(F,E)'$ is supra open soft, $(F,E)\tilde{\subseteq}cl^s(H,E)$ and $(F,E)\tilde{\subseteq}(H,E)'$. Hence $(H,E)\tilde{\subseteq}(F,E)'$. Since (H,E) is supra g-closed soft. Then $cl^s(H,E)\tilde{\subseteq}(F,E)'$. Hence $(F,E)\tilde{\subseteq}[cl^s(H,E)]'$. This means that $(F,E)\tilde{\subseteq}cl^s(H,E)\tilde{\cap}[cl^s(H,E)]'=\tilde{\phi}$. Thus $(F,E)=\tilde{\phi}$ which is a contradiction. Therefore, $cl^s(H,E)\setminus(H,E)$ contains only null supra closed soft set.
Conversely, assume that $cl^s(H,E)\setminus(H,E)$ contains only null supra closed soft set, $(H,E)\tilde{\subseteq}(G,E)$, (G,E) is supra open soft and suppose that $cl^s(H,E)\tilde{\nsubseteq}(G,E)$. Then $cl^s(H,E)\tilde{\cap}(G,E)'$ is a non null supra closed soft subset of $cl^s(H,E)\setminus(H,E)$ which is a contradiction. Thus (H,E) is supra g-closed soft in X. This completes the proof.

Corollary 3.1. Let (F,E) be supra g-closed soft set. Then (F,E) is supra closed soft if and only if $cl^s(F,E)\setminus(F,E)$ is supra closed soft.

Proof. If (F,E) is supra closed soft, then $cl^s(F,E)\setminus(F,E)=\tilde{\phi}$ is supra closed soft. Conversely, suppose that $cl^s(F,E)\setminus(F,E)$ is supra closed soft. Since (F,E) be supra g-closed soft set. Then $cl^s(F,E)\setminus(F,E)=\tilde{\phi}$ from Theorem 3.2. Hence $cl^s(F,E)=(F,E)$. Thus (F,E) is supra closed soft.

Definition 3.2. A soft set (F,E) is called a supra generalized open soft set (supra g-open soft) in a supra soft topological space (X,μ,E) if its relative complement $(F,E)'$ is supra g-closed soft in X .

Theorem 3.3. Let (X,μ,E) be supra soft topological space. Then the supra soft set (F,E) is supra g-open soft set if and only if $(F,E)\tilde{\subseteq}int^s(G,E)$ whenever $(F,E)\tilde{\subseteq}(G,E)$ and (F,E) is supra closed soft in X.

Proof. Let (F,E) be a supra g-open soft in X, $(F,E)\tilde{\subseteq}(G,E)$ and (F,E) is supra closed soft in X. Then $(F,E)'$ is supra g-closed soft from Definition 3.2 and $(G,E)'\tilde{\subseteq}(F,E)'$. Since (F,E) is supra g-open soft in X. Then $cl^s(G,E)'\tilde{\subseteq}(F,E)'$. Hence $(F,E)\tilde{\subseteq}[cl^s(G,E)']'=int^s(G,E)$. Conversely, let $(F,E)'\tilde{\subseteq}(H,E)$ and (H,E) is supra open soft in X. Then $(H,E)'\tilde{\subseteq}(F,E)$ and $(H,E)'$ is supra closed soft in X. Hence $(H,E)'\tilde{\subseteq}int^s(F,E)$ from the necessary condition. Thus $[int^s(F,E)]'=cl^s[(F,E)']\tilde{\subseteq}(H,E)$ and (H,E) is supra open soft in X. This means that $(F,E)'$ is supra g-closed soft in X. Therefore, (F,E) is supra g-open soft set from Definition 3.2. This completes the proof.

Example 3.3. In example 3.1, $(F_1,E)',(F_2,E)'$ are supra g-open soft in (X,μ,E).

Remark 3.3. Every supra open soft set is supra g-open soft. But the converse is not true in general as shown in the following example.

Example 3.4. In example 3.1, $(F_1,E)',(F_2,E)'$ are supra g-open soft in (X,μ,E), but not supra open soft over X.

Theorem 3.4. Let (X,μ,E) be a supra soft topological space and (F,E) be a supra g-open soft in X. If $int^s(F,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}(F,E)$, then (H,E) is a supra g-open soft.

Proof. Let $(G,E)\tilde{\subseteq}(H,E)$ and $(G,E)\in\mu$. Since $(G,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}(F,E)$ and (F,E) is supra g-open soft in

X, then $(G,E)\tilde{\subseteq}int^s(F,E)$. Hence $(G,E)\tilde{\subseteq}int^s(F,E)\tilde{\subseteq}int^s(H,E)$. Thus $(G,E)\tilde{\subseteq}int^s(H,E)$. Therefore, (H,E) is a supra g-open soft.

4 Supra generalized closed soft sets with respect to an soft ideal

Definition 4.1. A soft set $F_E \in SS(X,E)$ is called supra generalized closed soft with respect to a soft ideal $\tilde{I}$ (supra-$\tilde{I}g$-closed soft) in a supra soft topological space (X,μ,E) if $cl^sF_E \setminus G_E \in \tilde{I}$ whenever $F_E\tilde{\subseteq}G_B$ and $G_E \in \mu$.

Example 4.1. Let $X = \{h_1,h_2,h_3\}$ be the set of three houses under consideration and $E = \{e_1,e_2\}$ be the set of decision parameters which are stands for "wooden" and "green surroundings" respectively.
. Let $(F_1,E),(F_2,E)$ be two soft sets representing the attractiveness of the houses which Mr. A and Mr. B are going to buy, where
$F_1(e_1) = \{h_2,h_3\}, F_1(e_2) = \{h_1,h_2\}$,
$F_2(e_1) = \{h_1,h_2\}, F_2(e_2) = \{h_1,h_3\}$.
Then $\mu = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ is the supra soft topology over X. Let $\tilde{I} = \{\tilde{\phi},(I_1,E),(I_2,E),(I_3,E)\}$ be a soft ideal over X, where $(I_1,E),(I_2,E),(I_3,E)$ are soft sets over X defined by
$I_1(e_1) = \{h_1\}$, $F_1(e_2) = \phi$,
$I_2(e_1) = \phi$, $I_2(e_2) = \{h_3\}$, and
$I_3(e_1) = \{h_1\}$, $I_3(e_2) = \{h_3\}$.
So (F_1,E) is a supra-$\tilde{I}g$-closed soft.

Proposition 4.1. Every supra g-closed soft set is supra-$\tilde{I}g$-closed soft.

Proof. Let F_E be a supra g-closed soft set in a supra soft topological space (X,μ,E) and $F_E\tilde{\subseteq}G_E$ such that $G_E \in \mu$. Since F_E is supra g-closed soft, then $cl^s(F,E)\tilde{\subseteq}G_E$ and hence $cl^s(F,E) \setminus G_E = \phi \in \tilde{I}$. Consequently F_E is a supra-$\tilde{I}g$-closed soft set.

The following example shows that the converse of the above proposition is not true in general.

Example 4.2. Suppose that there are three cars in the universe X given by $X = \{h_1,h_2,h_3\}$. Let $E = \{e_1,e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively.
Let Let $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$ be four soft sets over the common universe X, which describe the composition of the cars, where
$F_1(e_1) = \{h_1\}$, $F_1(e_2) = X$,
$F_2(e_1) = \{h_1,h_2\}$, $F_2(e_2) = X$,
$F_3(e_1) = \{h_3\}$, $F_3(e_2) = X$,and
$F_4(e_1) = \{h_1,h_3\}$, $F_4(e_2) = X$.
Then $\mu = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$ is the supra soft topology over X. Let $\tilde{I} = \{\tilde{\phi},(I_1,E),(I_2,E),(I_3,E)\}$ be a soft ideal over X, where $(I_1,E),(I_2,E),(I_3,E)$ are soft sets over X defined by
$I_1(e_1) = \{h_2\}$, $F_1(e_2) = \phi$,
$I_2(e_1) = \{h_3\}$, $I_2(e_2) = \phi$, and
$I_3(e_1) = \{h_2,h_3\}$, $I_3(e_2) = \phi$.
So (F_1,E) is a supra-$\tilde{I}g$-closed soft but it is not supra g-closed soft.

Theorem 4.1. A supra soft set (G,E) is a supra-$\tilde{I}g$-closed soft set in a supra soft topological space (X,μ,E) if and only if there exist a supra closed soft set (F,E) such that $(F,E)\tilde{\subseteq}cl^s(G,E) \setminus (G,E)$ implies $(F,E) \in \tilde{I}$.

Proof. $(\Rightarrow)$ Suppose that (G,E) be a supra-$\tilde{I}g$-closed soft set and (F,E) be a supra closed soft set such that $(F,E)\tilde{\subseteq}cl^s(G,E) \setminus (G,E)$. Then $(G,E)\tilde{\subseteq}(F,E)'$. By our assumption, $cl^s(G,E) \setminus (F,E)' \in \tilde{I}$. But $(F,E)\tilde{\subseteq}cl^s(G,E)\tilde{\cap}(F,E) = cl^s(G,E) \setminus (F,E)'$. Thus $(F,E) \in \tilde{I}$ from Definition 2.24.
$(\Leftarrow)$ Conversely, assume that $(G,E)\tilde{\subseteq}(H,E)$ and $(H,E) \in \mu$. Then $cl^s(G,E) \setminus (H,E) = cl^s(G,E)\tilde{\cap}(H,E)' = cl^s(G,E)\tilde{\cap}cl^s((H,E)')$ is a supra closed soft set in (X,μ,E) and $cl^s(G,E) \setminus (H,E)\tilde{\subseteq}cl^s(G,E) \setminus (H,E)$. By assumption $cl^s(G,E) \setminus (H,E) \in \tilde{I}$. This implies that (G,E) is a supra-$\tilde{I}g$-closed soft.

Theorem 4.2. If (F,E) and (G,E) are supra-$\tilde{I}g$-closed soft sets in a supra soft topological space (X,μ,E), then $(F,E)\tilde{\cup}(G,E)$ is also supra-$\tilde{I}g$-closed soft in (X,μ,E).

Proof. Suppose that F_E and G_E are supra-$\tilde{I}g$-closed soft in (X,μ,E). Let $F_E\tilde{\cup}G_E\tilde{\subseteq}H_E$ and $H_E \in \tau$, then $F_E\tilde{\subseteq}H_E$ and $G_E\tilde{\subseteq}H_E$. By assumption $cl^sF_E \setminus H_E \in \tilde{I}$ and $cl^sG_E \setminus H_E \in \tilde{I}$. It follows that $[cl^sF_E \setminus H_E]\tilde{\cup}[cl^sG_E \setminus H_E] = cl^s[(F_E\tilde{\cup}G_E] \setminus H_E \in \tilde{I}$. Thus $F_E\tilde{\cup}G_E$ is a supra-$\tilde{I}g$-closed soft.

Theorem 4.3. If F_E is supra-$\tilde{I}g$-closed soft in a supra soft topological space (X,μ,E) and $F_E\tilde{\subseteq}G_E\tilde{\subseteq}cl^sF_E$, then G_B is supra-$\tilde{I}g$-closed soft in (X,μ,E).

Proof. Let F_E be a supra-$\tilde{I}g$-closed soft, $F_E\tilde{\subseteq}G_E\tilde{\subseteq}cl^sF_E$ in (X,μ,E) and $G_E\tilde{\subseteq}H_E$ such that $H_E \in \mu$. Then $F_E\tilde{\subseteq}H_E$. Since F_E is a supra-$\tilde{I}g$-closed soft, then $cl^sF_E \setminus H_E \in \tilde{I}$. Now, $G_E\tilde{\subseteq}cl^sF_E$ implies that $cl^sG_E\tilde{\subseteq}cl^sF_E$. So $cl^sG_E \setminus H_E\tilde{\subseteq}cl^sF_E \setminus H_E$. Thus $cl^sG_E \setminus H_E \in \tilde{I}$. Consequently, G_E is a supra-$\tilde{I}g$-closed soft in (X,μ,E). This completes the proof.

Remark 4.1. The soft intersection of two supra-$\tilde{I}g$-closed soft sets need not be a supra-$\tilde{I}g$-closed soft set as shown in the following example.

Example 4.3. Suppose that there are three dresses in the universe X given by $X = \{h_1,h_2,h_3\}$. Let $E = \{e_1(cotton),e_2(woollen)\}$ be the set of parameters showing the material of the dresses.

Let $(F_1,E),(F_2,E),(F_3,E)$ be three soft sets over the common universe X, which describe the composition of the dresses, where

$F_1(e_1) = \{h_2\}, F_1(e_2) = \{h_1\}$,
$F_2(e_1) = \{h_2\}, F_2(e_2) = \{h_2\}$, and
$F_3(e_1) = \{h_2\}, F_3(e_2) = \{h_1,h_2\}$.

Then $\mu = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ is the supra soft topology over X. Let $\tilde{I} = \{\tilde{\phi},(I_1,E),(I_2,E),(I_3,E)\}$ be a soft ideal over X, where $(I_1,E),(I_2,E),(I_3,E)$ are soft sets over X defined by

$I_1(e_1) = \{h_1\}$, $F_1(e_2) = \phi$,
$I_2(e_1) = \phi$, $I_2(e_2) = \{h_1\}$, and
$I_3(e_1) = \{h_1\}$, $I_3(e_2) = \{h_1\}$.

So the soft sets $(G,E),(H,E)$ which defined by $G(e_1) = \{h_1,h_2\}$, $G(e_2) = \phi$ and $H(e_1) = \{h_2,h_3\}$, $H(e_2) = \phi$ are supra-$\tilde{I}g$-closed soft sets. But their soft intersection $(G,E)\tilde{\cap}(H,E) = (K,E)$, where $K(e_1) = \{h_2\}$, $K(e_2) = \phi$ is not supra-$\tilde{I}g$-closed soft.

Theorem 4.4. If H_E is supra-$\tilde{I}g$-closed soft set and F_E is a supra closed soft in a supra soft topological space (X,μ,E). Then $H_E\tilde{\cap}F_E$ is a supra-$\tilde{I}g$-closed soft in (X,μ,E).

Proof. Assume that $H_E\tilde{\cap}F_E\tilde{\subseteq}G_E$ and $G_E \in \mu$. Then $H_E\tilde{\subseteq}G_E\tilde{\cup}F_E^{'}$. Since H_E is a supra-$\tilde{I}g$-closed soft set. It follows that $cl^sH_E \setminus [G_E\tilde{\cup}F_E^{'}] \in \tilde{I}$. Now, $cl^s[H_E\tilde{\cap}F_E]\tilde{\subseteq}cl^sH_E\tilde{\cap}cl^sF_E = cl^sH_E\tilde{\cap}F_E = [cl^sH_E\tilde{\cap}F_E] \setminus F_E^{'}$. Thus $cl^s[H_E\tilde{\cap}F_E] \setminus G_E\tilde{\subseteq}[cl^sH_E\tilde{\cap}F_E] \setminus [G_E\tilde{\cup}F_E^{'}]\tilde{\subseteq}cl^sH_E \setminus [G_E\tilde{\cup}F_E^{'}] \in \tilde{I}$. Hence $H_E\tilde{\cap}F_E$ is a supra-$\tilde{I}g$-closed soft set.

5 Soft generalized open sets with respect to soft ideal

Definition 5.1. A soft set $F_E \in SS(X,E)$ is called supra generalized open soft set with respect to a soft ideal $\tilde{I}$ (supra-$\tilde{I}g$-open soft) in a supra soft topological space (X,μ,E) if and only if its relative complement $F_E^{'}$ is a supra-$\tilde{I}g$-closed soft in (X,μ,E).

Theorem 5.1. A supra soft set (F,E) is a supra $\tilde{I}g$-open soft set in a supra soft topological space (X,μ,E) if and only if $G_E \setminus I_E\tilde{\subseteq}int^sF_E$ for some $I_E \in \tilde{I}$, whenever $G_E\tilde{\subseteq}F_E$ and G_E is supra closed soft in (X,μ,E).

Proof. $(\Rightarrow)$ Suppose that F_E is a supra-$\tilde{I}g$-open soft set. Let $G_E\tilde{\subseteq}F_E$ such that G_E is a supra closed soft. We have $F_E^{'}\tilde{\subseteq}G_E^{'}$, $F_E^{'}$ is a supra-$\tilde{I}g$-closed soft and $G_E^{'} \in \mu$. It follows that $cl^sF_E^{'} \setminus G_E^{'} \in \tilde{I}$ from Definition 3.1. This implies that $cl^sF_E^{'} \setminus G_E^{'} = I_E \in \tilde{I}$ for some $I_E \in \tilde{I}$, and then $cl^sF_E^{'} \setminus G_E^{'} = cl^sF_E^{'}\tilde{\cap}G_E = I_E \in \tilde{I}$, so $[cl^sF_E^{'}\tilde{\cap}G_E]\tilde{\cup}G_E^{'} = I_E\tilde{\cup}G_E^{'}$. This implies that $cl^sF_E^{'}\tilde{\subseteq}cl^sF_E^{'}\tilde{\cup}G_E^{'} = I_E\tilde{\cup}G_E^{'}$. Hence $cl^sF_E^{'}\tilde{\subseteq}G_E^{'}\tilde{\cup}I_E$ for some $I_E \in \tilde{I}$. So $(G_E^{'}\tilde{\cup}I_E)^{'}\tilde{\subseteq}[cl^sF_E^{'}]^{'} = int^sF_E$. Therefore, $G_E \setminus I_E = G_E\tilde{\cap}I_E^{'}\tilde{\subseteq}int^sF_E$.

$(\Leftarrow)$ Conversely, assume that F_E be a supra soft set. We want to prove that F_E is a supra-$\tilde{I}g$-open soft set. It is sufficient to prove that $F_E^{'}$ is a supra-$\tilde{I}g$-closed soft set. So, let $F_E^{'}\tilde{\subseteq}G_E$ such that $G_E \in \mu$. Hence $G_E^{'}\tilde{\subseteq}F_E$. By assumption, $G_E^{'} \setminus I_E\tilde{\subseteq}int^sF_E = [cl^sF_E^{'}]^{'}$ for some $I_E \in \tilde{I}$. Hence $cl^sF_E^{'}\tilde{\subseteq}[G_E^{'} \setminus I_E]^{'} = G_E\tilde{\cup}I_E$. Thus $cl^sF_E^{'} \setminus G_E\tilde{\subseteq}[G_E^{'} \setminus I_E]^{'} = [G_E\tilde{\cup}I_E] \setminus G_E = [G_E\tilde{\cup}I_E]\tilde{\cap}G_E^{'} = I_E\tilde{\cap}G_E^{'}\tilde{\subseteq}I_E \in \tilde{I}$. This shows that $cl^sF_E^{'} \setminus G_E \in \tilde{I}$. Therefore, $F_E^{'}$ is a supra-$\tilde{I}g$-closed soft set and hence F_E is a supra-$\tilde{I}g$-open soft set. This completes the proof.

Definition 5.2. Two soft sets F_E and G_E are said to be supra soft separated sets in a supra soft topological space (X,μ,E) if $cl^sF_E\tilde{\cap}G_E = \tilde{\phi}$ and $F_E\tilde{\cap}cl^sG_E = \tilde{\phi}$.

Theorem 5.2. If A_E and B_E are supra soft separated and supra-$\tilde{I}g$-open soft sets in a supra soft topological space (X,τ,E), then $A_E\tilde{\cup}B_E$ is a supra-$\tilde{I}g$-open soft in (X,μ,E).

Proof. Suppose that A_E and B_E are supra soft separated and supra-$\tilde{I}g$-open soft sets in a supra soft topological space (X,τ,E) and F_E be a supra closed soft set such that $F_E\tilde{\subseteq}A_E\tilde{\cup}B_E$. Then $F_E\tilde{\cap}cl^sA_E\tilde{\subseteq}[A_E\tilde{\cup}B_E]\tilde{\cap}cl^sA_E = A_E$ and $F_E\tilde{\cap}cl^sB_E\tilde{\subseteq}B_E$ from Definition 5.2. $[(F_E\tilde{\cap}cl^sA_E] \setminus D_E\tilde{\subseteq}int^sA_E$ and $[F_E\tilde{\cap}cl^sB_E] \setminus C_E\tilde{\subseteq}int^sB_E$ for some $D_E,C_E \in \tilde{I}$ from Theorem 5.1. This means that $[F_E\tilde{\cap}cl^sA_E] \setminus int^sA_E \in I$ and $(F_E\tilde{\cap}cl^sB_E) \setminus int^sA_E \in I$. Then $[(F_E\tilde{\cap}cl^sA_E) \setminus int^sA_E]\tilde{\cup}[(F_E\tilde{\cap}cl^sB_E) \setminus int^sB_E] \in \tilde{I}$. Hence $[F_E\tilde{\cap}(cl^sA_E\tilde{\cup}cl^sB_E)] \setminus [int^sA_E\tilde{\cup}int^sB_E] \in \tilde{I}$. But $F_E = F_E\tilde{\cap}(A_E\tilde{\cup}B_E)\tilde{\subseteq}F_E\tilde{\cap}[cl^s(A_E\tilde{\cup}B_E)]$, and we have $F_E \setminus int^s(A_E\tilde{\cup}B_E)\tilde{\subseteq}(F_E\tilde{\cap}(cl^s[A_E\tilde{\cup}B_E])) \setminus int^s(A_E\tilde{\cup}B_E)\tilde{\subseteq}(F_E\tilde{\cap}[cl^s(A_E\tilde{\cup}B_E)] \setminus int^sA_E\tilde{\cup}int^sB_E) \in \tilde{I}$. Now, take $G_E = F_E \setminus int^s(A_E\tilde{\cup}B_E) \in \tilde{I}$. Then $F_E \setminus G_E = F_E \setminus [F_E \setminus int^s(A_E\tilde{\cup}B_E)]\tilde{\subseteq}int^s(A_E\tilde{\cup}B_E)$. Hence $F_E \setminus G_E\tilde{\subseteq}int^s(A_E\tilde{\cup}B_E)$ for some $G_E \in \tilde{I}$. Therefore, $A_E\tilde{\cup}B_E$ is a supra-$\tilde{I}g$-open soft in (X,μ,E) from Theorem 5.1.

Corollary 5.1. If A_E and B_E are supra-$\tilde{I}g$-closed soft sets in a supra soft topological space (X,μ,E) such that $A_E^{'}$ and $B_E^{'}$ are supra soft separated sets, then $A_E\tilde{\cap}B_E$ is a supra-$\tilde{I}g$-closed soft in (X,μ,E).

Proof. Obvious from Theorem 5.2.

Theorem 5.3. If A_E and B_E are supra-$\tilde{I}g$-open soft sets in a supra soft topological space (X,μ,E), then $A_E\tilde{\sqcap}B_E$ is a supra-$\tilde{I}g$-open soft in (X,μ,E).

Proof. Let A_E and B_E are supra-$\tilde{I}g$-open soft sets in a supra soft topological space (X,μ,E), then $A_E^{'}$ and $B_E^{'}$ are supra-$\tilde{I}g$-closed soft sets. Hence $(A_E\tilde{\sqcap}B_E)^{'}=A_E^{'}\tilde{\sqcup}B_E^{'}$ is a supra-$\tilde{I}g$-closed soft from Theorem 4.2. Therefore, $A_E\tilde{\sqcup}B_E$ is a supra-$\tilde{I}g$-closed soft.

Theorem 5.4. Let A_E be a supra-$\tilde{I}g$-open soft in a supra soft topological space (X,μ,E) such that $int^sA_E\tilde{\sqsubseteq}B_E\tilde{\sqsubseteq}$ for some $B_E\in SS(X)_E$. Then B_E is a supra-$\tilde{I}g$-open soft in (X,μ,E).

Proof. Let A_E be a supra-$\tilde{I}g$-open soft in a supra soft topological space (X,μ,E) such that $int^sA_E\tilde{\sqsubseteq}B_E\tilde{\sqsubseteq}$ for some $B_E\in SS(X)_E$. Then $A_E^{'}\tilde{\sqsubseteq}B_E^{'}\tilde{\sqsubseteq}(int^sA_E)^{'}=cl^s(A_E^{'})$ and $A_E^{'}$ is a supra-$\tilde{I}g$-closed soft. Hence $B_E^{'}$ is a supra-$\tilde{I}g$-closed soft from Theorem 4.3. Therefore, B_E is a supra-$\tilde{I}g$-open soft in (X,μ,E).

Theorem 5.5. A soft set A_E is a supra-$\tilde{I}g$-closed soft in a supra soft topological space (X,μ,E) if and only if $cl^sA_E\setminus A_E$ is a supra-$\tilde{I}g$-open soft.

Proof. $(\Rightarrow)$ Let $F_E\tilde{\sqsubseteq}cl^sA_E\setminus A_E$ and F_E is a supra closed soft set. Then $F_E\in\tilde{I}$ from Theorem 4.1. Hence, there exists $I_E\in\tilde{I}$ such that $F_E\setminus I_E=\tilde{\phi}$. Thus $F_E\setminus I_E=\tilde{\phi}\tilde{\sqsubseteq}int^s[cl^sA_E\setminus A_E]$. Therefore, $cl^sA_E\setminus A_E$ is a supra-$\tilde{I}g$-open soft from Theorem 5.1.

$(\Leftarrow)$ Let $A_E\tilde{\sqsubseteq}G_E$ such that $G_E\in\mu$. Then $cl^sA_E\tilde{\sqcap}G_E^{'}\tilde{\sqsubseteq}cl^sA_E\tilde{\sqcap}A_E^{'}=cl^sA_E\setminus A_E$. By hypothesis, $[cl^sA_E\tilde{\sqcap}G_E^{'}]\setminus I_E\tilde{\sqsubseteq}int^s[cl^sA_E\setminus A_E]=\tilde{\phi}$, for some $I_E\in\tilde{I}$ from Theorem 5.1. This implies that $cl^sA_E\tilde{\sqcap}G_E^{'}\tilde{\sqsubseteq}I_E\in\tilde{I}$. Therefore, $cl^sA_E\setminus G_E\in\tilde{I}$. Thus A_E is a supra-$\tilde{I}g$-closed soft.

Theorem 5.6. Let (X_1,μ_1,A), (X_2,μ_2,B) be supra soft topological spaces. Let $f_{pu}:SS(X_1)_A\to SS(X_2)_B$ be closed and continuous soft function. If $A_E\in SS(X,E)$ is a supra-$\tilde{I}g$-closed soft in (X,μ_1,A), then $f_{pu}(A_E)$ is a supra-$f_{pu}(\tilde{I})g$-closed soft in (X_2,μ_2,B), where $f_{pu}(\tilde{I})=\{f_{pu}(I_E):I_E\in\tilde{I}\}$.

Proof. Let $A_E\in SS(X)_A$ be a supra-$\tilde{I}g$-closed soft in (X,μ_1,A) and $f_{pu}(A_E)\tilde{\sqsubseteq}G_E$ for some $G_E\in\mu_2$. Then $A_E\tilde{\sqsubseteq}f_{pu}^{-1}(G_E)$. It follow that $cl^sA_E\setminus f_{pu}^{-1}(G_E)\in\tilde{I}$ from Definition 3.1. Hence $f_{pu}(cl^sA_E)\setminus G_E\in f_{pu}(\tilde{I})$ from Theorem 2.2. Since f_{pu} is a closed soft function, then $f_{pu}cl^s(A_E)$ is a supra closed soft in μ_2 from Definition 2.23. Thus $cl^s(f_{pu}(A_E))\tilde{\sqsubseteq}cl^s[f_{pu}cl^s(A_E)]=f_{pu}cl^s(A_E)$. This implies that $cl^s(f_{pu}(A_E))\setminus G_E\tilde{\sqsubseteq}f_{pu}(cl^sA_E)\setminus G_E\in f_{pu}(\tilde{I})$. Therefore, $f_{pu}(A_E)$ is a supra-$f_{pu}(I)g$-closed soft in (X_2,μ_2,B). This completes the proof.

6 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of soft topological spaces. We introduce the notions of supra g-closed soft, supra g-open sets, supra-$\tilde{I}g$-closed soft sets, supra-$\tilde{I}g$-open soft sets and have established several interesting properties. Because there exists compact connections between soft sets and information systems [25,30], we can use the results deducted from the studies on soft topological space to improve these kinds of connections. We see that this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.
Granular computing is a recent approach in the field of computer science that uses topological structure as granulation models. The suggested approach for supra-$\tilde{I}g$-closed soft sets give new methods for generating the classes of subsets whose lower and upper approximations are contained in elementary sets which in turn help in the process of decision making under both quantities and qualitative information.

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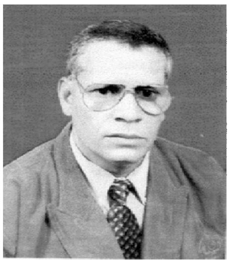

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