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Discrete Burr Type XII Beta Distribution: Properties and Applications

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Abstract: In this article, we attempt to introduce a new count data model which is obtained by compounding discrete Burr type XII distribution with Beta distribution of first kind. The proposed model has several properties such as it can be nested to different compound distributions on specific parameter settings. We shall first study some basic distributional and moment properties of the new distribution. Then, certain structural properties of the distribution such as its unimodality and hazard rate behavior are discussed. Finally, real data set is analyzed to investigate the suitability of the proposed distribution in modeling count.

Keywords: Discrete Burr type XII Distribution, Beta distribution, compound distribution, count data, reliability.

1 Introduction

Compound distribution arises when all or some parameters of a distribution known as parent distribution vary according to some probability distribution called the compounding distribution, for instance negative binomial distribution can be obtained from Poisson distribution when its parameter λ follows gamma distribution. If the parent distribution is discrete then resultant compound distribution will also be discrete and if the parent distribution is continuous then resultant compound distribution will also be continuous i,e. the support of the original (parent) distribution determines the support of compound distributions.

In several research papers it has been found that compound distributions are very flexible and can be used efficiently to model different types of data sets. With this in mind many compound probability distributions have been constructed. In the early 1970s, Dubey [10] derived a compound gamma, beta and F distribution by compounding a gamma distribution with another gamma distribution and reduced it to the beta Ist and 2nd kind and to the F distribution by suitable transformations. Sankaran [1] introduced a compound of Poisson distribution with that of Lindley distribution for modeling count data. Gerstenkorn [11,12] proposed several compound distributions, he obtained compound of gamma distribution with exponential distribution by treating the parameter of gamma distribution as an exponential variate and also obtained compound of polya with beta distribution. Ghitany, Al-Mutairi and Nadarajah [2,3] introduced zero-truncated Poisson-Lindley distribution, who used the distribution for modeling count data in the case where the distribution has to be adjusted for the count of missing zeros. Zamani and Ismail [4] constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. Recently, Ahmad, Rashid and Jan [17] introduced a new class of generalized complementary compound lifetime distributions which is obtained by compounding generalized Lindley distribution with power series distribution.

In this paper we propose a new count data model by compounding two parameter discrete Burr type XII distributions with Beta distribution of first kind, as there is a need to find more plausible discrete probability models or survival models in medical science and other fields, to fit to various discrete data sets. It is well known in general that a compound model is more flexible than the ordinary model and it is preferred by many data analysts in analyzing statistical data. Moreover, it presents beautiful mathematical exercises and broadened the scope of the concerned model being compounded.

2 Material and Methods

A discrete analogue of the continuous Burr type XII distribution was introduced by Krishna and Punder [6], and is defined

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by the probability mass function (pmf):

$$f_1(x;q,\gamma)=q^{\log(1+x^{\gamma})}-q^{\log(1+(x+1)^{\gamma})}, \quad x=0,1,2,..., \tag{1}$$

Where $\gamma>0$ and $0<q<1$ is its parameter. The first and the second moments of the discrete Burr type XII random variable X are given by

$$E(X)=\sum_{x=1}^{\infty} q^{\log(1+x^{\gamma})}$$

$$E(X^2)=2\sum_{x=1}^{\infty} xq^{\log(1+x^{\gamma})}+E(X)$$

There are various types of life time models such as exponential, Pareto and Gamma that are used in reliability and life testing. Although many alternatives and generalizations, it is fair to say that the Beta distribution of first kind provides the premier family of continuous distributions on bounded support. The probability density function of generalized beta distribution is given by

$$f(x;\delta,\alpha,\beta)=\frac{\delta x^{\delta\alpha-1}(1-x^{\delta})^{\beta-1}}{B(\alpha,\beta)}; \qquad 0<x<1 \tag{2}$$

If we put $\delta=1$, the equation (2) reduces to beta distribution of first kind with probability density function as:

$$f(x;\alpha,\beta)=\frac{1}{B(\alpha,\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}; \qquad 0<x<1, \tag{3}$$

where α and β are positive real quantities and the variable X satisfies $0\le x\le 1$. The quantity $B(\alpha,\beta)$ is the beta function. Equation (3) is also known as the standard beta or classical beta distribution.
where $\alpha,\beta>0$ are shape parameters. The raw moments of Beta distribution of first kind (BD(I)) are given by

$$E(X^r)=\int_0^1 x^r f_2(X;\alpha,\beta)dx$$

$$E(X^r)=\frac{\Gamma(\alpha+r)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+r)\Gamma(\alpha)} \tag{4}$$

Beta distribution is not very popular among statisticians because researchers have not analyzed and investigated it systematically in much detail. Beta distribution is similar to the Minimax [15,16] distribution but unlike Minimax distribution it has not closed form of cumulative distribution function.

Usually the parameter q in DBXIID is fixed constant but here we have considered a problem in which the probability parameter q is itself a random variable following BD with pdf (3).

3 Proposed Model

If $X|q\sim$ DBXII(q,γ) where q is itself a random variable following Beta distribution BD(α,β), then determining the distribution that results from marginalizing over q will be known as a compound of discrete Burr type XII distribution with that of Beta distribution, which is denoted by DBXIIBD$(X;\alpha,\beta,\gamma)$. It may be noted that proposed model will be a discrete since the parent distribution DBXIID is discrete.

Theorem 3.1: The probability mass function of a compound of DBXII(q,γ) with BD(α,β) is given by

$$f_{DBXIIBD}(X;\alpha,\beta,\gamma)=\frac{1}{B(\alpha,\beta)}[B(\beta,\log(1+x^{\gamma})+\alpha)-B(\beta,\log(1+(x+1)^{\gamma})+\alpha)]$$

Where $x=0,1,2,......$ and $\alpha,\beta,\gamma>0$

Proof: Using the definition (3), the pmf of a compound of DBXII(q,γ) with BD(α,β) can be obtained as

$$f_{DBXIIBD}(X;\alpha,\beta,\gamma)=\int_0^1 f_1(x|q)\,f_2(q)\,dq$$

$$f_{DBXIIBD}(X\,;\,\alpha,\beta,\gamma)=\frac{1}{B(\alpha,\beta)}\int_0^1 (q^{\log(1+x^{\gamma})}-q^{\log(1+(x+1)^{\gamma})})\,q^{\alpha-1}(1-q)^{\beta-1}\,dq$$

$$f_{DBXIIBD}(X\,;\,\alpha,\beta,\gamma)=\frac{1}{B(\alpha,\beta)}\left[B(\beta,\log(1+x^{\gamma})+\alpha)-B(\beta,\log(1+(x+1)^{\gamma})+\alpha)\right]$$

$$f_{DBXIIBD}(X\,;\alpha,\beta,\gamma)=\frac{1}{B(\alpha,\beta)}\left[\frac{\Gamma(\beta)\,\Gamma\left(\log(1+x^{\gamma})+\alpha\right)}{\Gamma\left(\beta+\log(1+x^{\gamma})+\alpha\right)}-\frac{\Gamma(\beta)\,\Gamma\left(\log(1+(x+1)^{\gamma})+\alpha\right)}{\Gamma\left(\beta+\log(1+(x+1)^{\gamma})+\alpha\right)}\right] \tag{5}$$

Where $x=0,1,2,...$ and $\alpha,\beta,\gamma>0$. From here a random variable X following a compound of DBXIID with BD will be symbolized by $\text{DBXIIBD}(X\,;\alpha,\beta,\gamma)$.

Fig.1(a) to fig.1(i) provides a pmf plot of the proposed model $\text{DBXIIBD}(X\,;\alpha,\beta,\gamma)$ for different values of parameters. It is evident that the proposed model is right skewed with unimodel behavior.

The Cumulative distribution function of the $\text{DBXIIBD}(X\,;\alpha,\beta,\gamma)$ is given by

$$F(x)=1-\frac{1}{B(\alpha,\beta)}B(\beta,\log(1+x^{\gamma})+\alpha),\qquad x=0,1,2,...\text{ and }(\alpha>0,\beta>0,\gamma>0)\ ,$$

where $B(\beta,\log(1+x^{\gamma})+\alpha)=\dfrac{\Gamma(\beta)\,\Gamma\left(\log(1+x^{\gamma})+\alpha\right)}{\Gamma\left(\beta+\log(1+x^{\gamma})+\alpha\right)}$

Fig.2 (a) to fig.2(i) provides a CDF plot of the proposed model $\text{DBXIIBD}(X\,;\alpha,\beta,\gamma)$ for different values of parameters. The initial rise of the CDF plot decreases as α increases but as β increases, initial rise of the CDF plot also increases.

3.1 Simulation

In order to generate a sequence of random numbers $x_1,x_2,....,x_n$ of the $\text{DBXIIBD}(X\,;\alpha,\beta,\gamma)$ random variable X with pmf $p(X=x_i)=p_i,\sum_{i=0}^{k}p_i=1$ and a cdf F(x), where k may be finite or infinite can be described as

Step1: Generate a random number u from uniform distribution U(0,1).

Step2: Generate random number x_i based on

$if\ u\le p_0=F(r_0)\ then\ X=x_0$

$if\ p_0<u\le p_0+p_1=F(r_1)\ then\ X=x_1$

.

.

$.if\ \sum_{j=0}^{k-1}p_j<u\le\sum_{j=0}^{k}p_j=F(r_k)\ then\ X=x_k$

In order to generate n random numbers from discrete Burr type XII Beta model, $x_1,x_2,....,x_n$, repeat step 1 to step 2 n times.

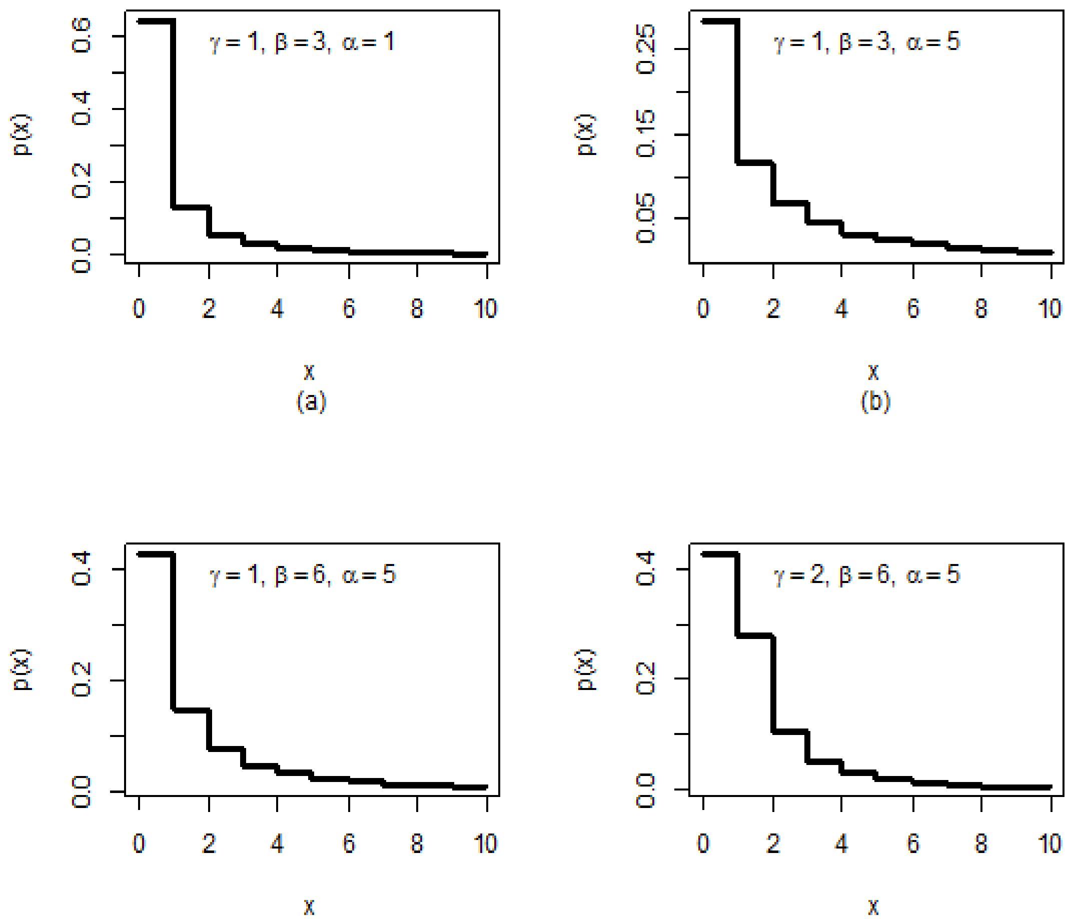

Fig. 1: pmf plot of Discrete Burr type XII Beta distribution

4 Nested Distributions

In this particular section we show that the proposed model can be nested to different models under specific parameter setting.

Proposition 4.1: If $X \sim DBXIIBD\left(X;\alpha,\beta,\gamma\right)$ then by setting $\alpha=\beta=1$ we obtain a compound of DBXII distribution with uniform distribution.

Proof: For $\alpha=\beta=1$ in BD (I) reduces to Uniform (0,1) distribution, therefore a compound DPD with uniform distribution is followed from (5) by simply putting $\alpha=\beta=1$ in it.

$$f_{DPUD}\left(X;\gamma\right)=B(1,\log(1+x^{\gamma})+1)-B(1,\log(1+(x+1)^{\gamma})+1) \quad \text{for } x=0,1,2,...,\gamma>0$$

which is probability mass function of a compound of DPD with uniform distribution.

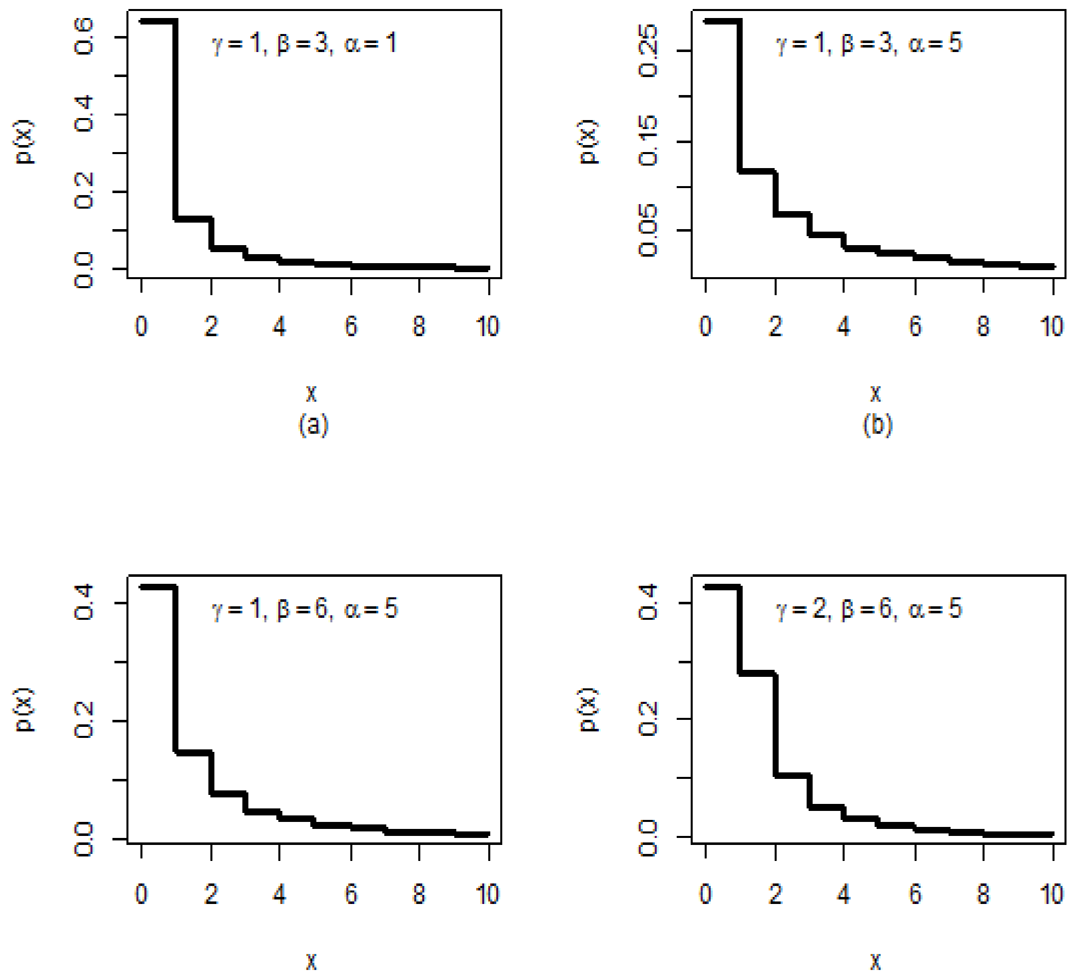

Fig. 2: CDF plot of Discrete Burr type XII Beta distribution

5 Reliability Measures of Compound Discrete Burr type XII Beta Distribution

If $X \sim DBXIIBD\left(X;\alpha,\beta,\gamma\right)$, then the various reliability measures of a random variable X are given by

(a) **Survival Function.**

$$s(x)=\frac{1}{B(\alpha,\beta)}B(\beta,\log(1+x^{\gamma})+\alpha),\ \ x=0,1,2...\ \text{ and }\ \alpha>0,\beta>0\,,$$

where $B(\beta,\log(1+x^{\gamma})+\alpha)=\dfrac{\Gamma(\beta)\,\Gamma\left(\log(1+x^{\gamma})+\alpha\right)}{\Gamma\left(\beta+\log(1+x^{\gamma})+\alpha\right)}$

(b) Rate of Failure Function.

$$r(x)=\frac{p(x)}{s(x)}=\frac{[B(\beta,\log(1+x^{\gamma})+\alpha)-B(\beta,\log(1+(x+1)^{\gamma})+\alpha)]}{B(\beta,\log(1+x^{\gamma})+\alpha)}, \quad x=0,1,2\ldots \text{ and } \alpha>0,\beta>0,\gamma>0,$$

Where $B(\beta,\log(1+x^{\gamma})+\alpha)=\dfrac{\Gamma(\beta)\,\Gamma\left(\log(1+x^{\gamma})+\alpha\right)}{\Gamma\left(\beta+\log(1+x^{\gamma})+\alpha\right)}$

(c) Second Rate of Failure Function.

$$h(x)=\log\left(\frac{s(x)}{s(x+1)}\right)=\log\left(\frac{B(\beta,\log(1+x^{\gamma})+\alpha)}{B(\beta,\log(1+(x+1)^{\gamma})+\alpha)}\right), \quad x=0,1,2\ldots \text{ and } \alpha>0,\beta>0,\gamma>0,$$

Where, B (.) refers to the beta function defined by $B(a,b)=\dfrac{\Gamma a\,\Gamma b}{\Gamma(a+b)}$

Fig. 3 provides the first rate of failure plot for DBXIIB distribution for different values of parameters.

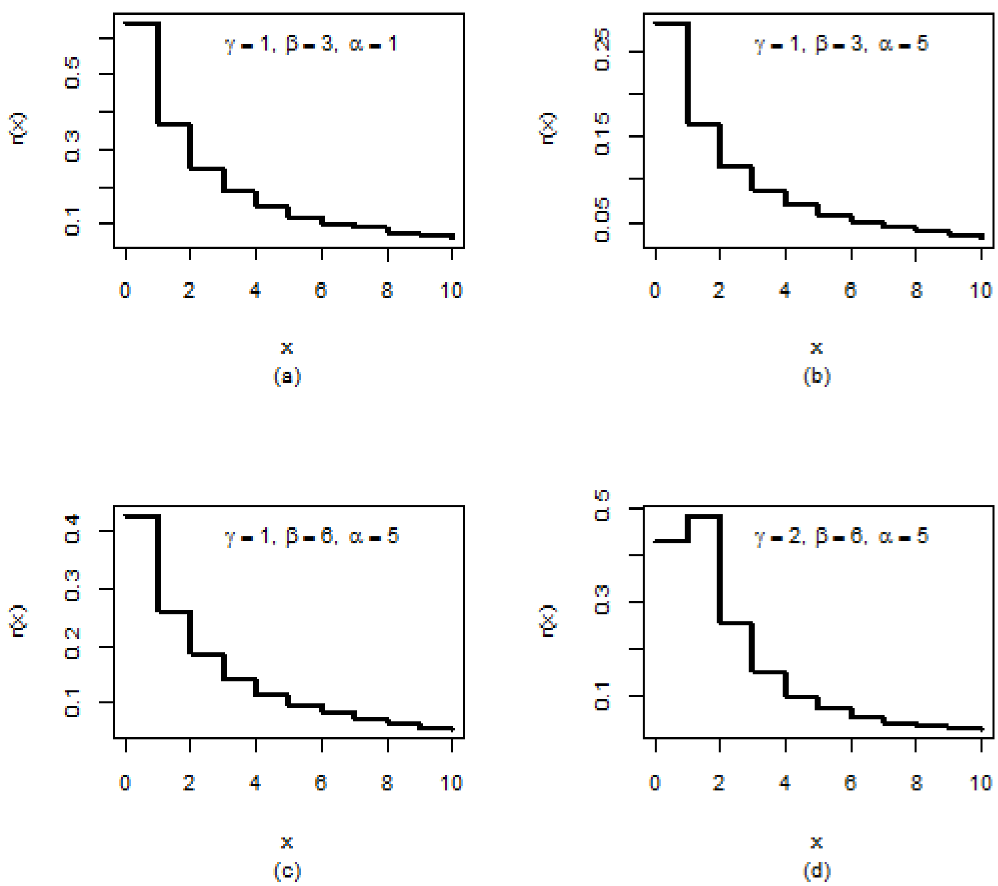

Fig. 3: r(x) plot of Discrete Burr type XII Beta distribution

6 Moment Generating and Probability Generating Functions of $DBXIIBD(\alpha,\beta,\gamma)$

(a) The moment generating function of the Compound discrete Burr type XII Beta distribution is

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}p(x)$$

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}\left[\frac{1}{B(\alpha,\beta)}[B(\beta,\log(1+x^{\gamma})+\alpha)-B(\beta,\log(1+(x+1)^{\gamma})+\alpha)]\right]$$

$$M_x(t)=\sum_{x=0}^{\infty}e^{tx}[\psi(x;\beta,\alpha,\gamma)-\psi(x+1;\beta,\alpha,\gamma)],$$

Where $\psi(x;\beta,\alpha,\gamma)=\frac{1}{B(\alpha,\beta)}B(\beta,\log(1+x^{\gamma})+\alpha)$

$$M_x(t)=\begin{pmatrix}\psi(0;\beta,\alpha,\gamma)+e^{t}\psi(1;\beta,\alpha,\gamma)+e^{2t}\psi(2;\beta,\alpha,\gamma)+e^{3t}\psi(3;\beta,\alpha,\gamma)+....-\{\psi(1;\beta,\alpha,\gamma)\\+e^{t}\psi(2;\beta,\alpha,\gamma)+e^{2t}\psi(3;\beta,\alpha,\gamma)+e^{3t}\psi(4;\beta,\alpha,\gamma)+......\}\end{pmatrix}$$

$$M_x(t)=\psi(0;\gamma,\beta,\alpha)+(e^{t}-1)\psi(1;\gamma,\beta,\alpha)+(e^{2t}-e^{t})\psi(2;\gamma,\beta,\alpha)+(e^{3t}-e^{2t})\psi(3;\gamma,\beta,\alpha)+....$$

$$M_x(t)=1+\sum_{x=1}^{\infty}(e^{xt}-e^{(x-1)t})\psi(x;\beta,\alpha,\gamma)$$

Differentiating $M_x(t)$ r times with respect to t

$$M_x^{(r)}(t)=\sum_{x=1}^{\infty}(x^{r}e^{xt}-(x-1)^{r}e^{(x-1)t})\psi(x;\beta,\alpha,\gamma)$$

First four moments of the proposed model are given by

$$\mu_1^{'}=\sum_{x=1}^{\infty}\psi(x;\beta,\alpha,\gamma)$$

$$\mu_2^{'}=\sum_{x=1}^{\infty}(2x-1)\psi(x;\beta,\alpha,\gamma)$$

$$\mu_3^{'}=\sum_{x=1}^{\infty}(3x^2-3x+1)\psi(x;\beta,\alpha,\gamma)$$

$$\mu_4^{'}=\sum_{x=1}^{\infty}(4x^3-6x^2+4x-1)\psi(x;\beta,\alpha,\gamma)$$

(b) Probability generating function of the Compound discrete Burr type XII Beta distribution is

$$G_{[x]}(t)=\sum_{x=0}^{\infty}t^{x}p(x)$$

$$G_{[x]}(t)=\sum_{x=0}^{\infty}t^{x}\left[\frac{1}{B(\alpha,\beta)}[B(\beta,\log(1+x^{\gamma})+\alpha)-B(\beta,\log(1+(x+1)^{\gamma})+\alpha)]\right]$$

$$G_{[x]}(t)=\sum_{x=0}^{\infty}t^{x}[\psi(x;\beta,\alpha,\gamma)-\psi(x+1;\beta,\alpha,\gamma)]$$

Where $\psi(x;\beta,\alpha)=\frac{1}{B(\alpha,\beta)}\left\{B(\beta,\log(1+x^{\gamma})+\alpha)\right\}$

$$G_{[x]}(t)=\psi(0;\gamma,\beta,\alpha)+(t-1)\psi(1;\gamma,\beta,\alpha)+t(t-1)\psi(2;\gamma,\beta,\alpha)+t^{2}(t-1)\psi(3;\gamma,\beta,\alpha)+....$$

$$G_{[x]}(t)=1+(t-1)\sum_{x=1}^{\infty}t^{x-1}\psi(x;\beta,\alpha,\gamma)$$

Differentiating $G_{[x]}(t)$ with respect to t

$$G^{'}_{[x]}(t)=\sum_{x=1}^{\infty}((t-1)(x-1)t^{x-2}+t^{x-1})\psi(x;\beta,\alpha,\gamma)$$

$$G^{'}_{[x]}(t)=\sum_{x=1}^{\infty}(t^{x-2}(xt-x+1))\psi(x;\beta,\alpha,\gamma)$$

$$G^{''}_{[x]}(t)=\sum_{x=1}^{\infty}(x-1)t^{x-3}\left\{(t-1)(x-2)+2t\right\}\psi(x;\beta,\alpha,\gamma)$$

At t=1, $G^{'}_{[x]}(t)$, $G^{''}_{[x]}(t)$ gives first and second factorial moments

$$E(x)=G^{'}_{[x]}(1)=\sum_{x=1}^{\infty}\psi(x;\beta,\alpha,\gamma)$$

$$E(x^{2})=G^{'}_{[x]}(1)+G^{''}_{[x]}(1)=\beta\sum_{x=1}^{\infty}(2x-1)\psi(x;\beta,\alpha,\gamma)$$

7 Parameter Estimation of DBXIIBD

In this section the estimation of parameters of DBXIIBD$(X;\alpha,\beta,\gamma)$ model will be discussed through method of moments and maximum likelihood estimation.

7.1 Moments Method of Estimation

In order estimate three unknown parameters of DBXIIBD$(X;\alpha,\beta,\gamma)$ model by the method of moments, we need to equate first three sample moments with their corresponding population moments.

$m_1 = \gamma_1$; $m_2 = \gamma_2$; $m_3 = \gamma_3$

Where $\gamma_i = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the i[th] sample moment and m_i is the i[th] corresponding population moment and the solution for $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ may be obtained by solving above equations simultaneously through numerical methods.

7.2 Maximum Likelihood Method of Estimation

The estimation of parameters of DBXIIBD$(X;\alpha,\beta,\gamma)$ model via maximum likelihood estimation method requires the log likelihood function of DBXIIBD$(X;\alpha,\beta,\gamma)$

$$£(X;\alpha,\beta,\gamma) = \log L(X;\alpha,\beta,\gamma) = \sum_{i=1}^{n} \log(B(\beta, \log(1+x^{\gamma})+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)) - n\log(B(\beta,\alpha)) \quad (9)$$

The maximum likelihood estimate of $\Theta = \left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right)^T$ can be obtained by differentiating (9) with respect unknown parameters α, β and γ respectively and then equating them to zero.

$$\frac{\partial}{\partial\beta} £(X;\alpha,\beta,\gamma) = \sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\beta}\left(B(\beta, \log(1+x^{\gamma})+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)\right)}{\left(B(\beta, \log(1+x^{\gamma})+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)\right)}\right) - n\frac{\frac{\partial}{\partial\beta}(B(\beta,\alpha))}{B(\beta,\alpha)} \quad (10)$$

$$\frac{\partial}{\partial\alpha} £(X;\gamma,\alpha,\beta) = \sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\alpha}\left(B(\beta, \log(1+x^{\gamma})+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)\right)}{\left(B(\beta, \log(1+x^{\gamma})+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)\right)}\right) - n\frac{\frac{\partial}{\partial\alpha}(B(\beta,\alpha))}{B(\beta,\alpha)} \quad (11)$$

$$\frac{\partial}{\partial\gamma} £(X;\gamma,\alpha,\beta) = \sum_{i=1}^{n}\left(\frac{\frac{\partial}{\partial\gamma}\left(B(\beta, \log(1+x^{\gamma})+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)\right)}{\left(B(\beta, \log(1+x)+\alpha) - B(\beta, \log(1+(x+1)^{\gamma})+\alpha)\right)}\right) \quad (12)$$

These two derivative equations cannot be solved analytically, therefore $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ will be obtained by maximizing the log likelihood function numerically using Newton-Raphson method which is a powerful technique for solving equations iteratively and numerically.

8 Application of Discrete Burr type XII Beta distribution

In this section, we present a real data set to examine the fit of the proposed model. MLE based on the likelihood Eqs. (10) , (11) and (12) was used to obtain the parameter estimates of the proposed distribution, although it is also possible to perform a direct search of the maximum likelihood function to obtain the maximum likelihood estimators. This can be done using appropriate software such as R Studio statistical software. In this section an attempt has been made to fit to data relating to automobile claims as given in table 1 (Automobile claims frequencies data in Willmot[7]), using discrete Burr type XII Beta distribution (DBXIIBD) in comparison with some compound discrete models like, Poisson Akasha distribution (PAD) [9], Poisson Lindley distribution (PLD) [8] , Poisson Sujatha distribution (PSD) [5] and other classical discrete models.

Table 1: Automobile claim data studied by Willmot [7]

Count	0	1	2	3	4	5
Observed	3719	212	38	7	3	1

The ML estimates provided by the fitdistr procedure in R studio are given in the table 2.

Table 2: Estimated parameters by ML method for fitted distributions for Counts of Automobile claim data studied by Willmot [7]

Distribution	Parameter Estimates	Model function
Discrete Burr type XII Beta	$\beta = 399.9$ $\alpha = 7.86$ $\gamma = 1.0$	$p(x) = \frac{1}{B(\alpha,\beta)}[B(\beta,\log(1+x^{\gamma})+\alpha) - B(\beta,\log(1+(x+1)^{\gamma})+\alpha)],$ $x = 0,1,2,...$ for $\beta > 0, \alpha > 0, \gamma > 0$
Poisson	$\lambda = 0.08$	$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \lambda > 0; \quad x = 0,1,2,...$
Poisson Akasha	$\theta = 12.48$	$p(x) = \frac{\theta^{3}(x^{2}+3x+(\theta^{2}+2\theta+2))}{(\theta^{2}+2)(\theta+1)^{x+3}} \quad x = 0,1,2,... \quad \theta > 0$
Poisson Lindley	$\theta = 13.08$	$p(x) = \frac{\theta^{2}(x+\theta+2)}{(\theta+1)^{x+3}} \; x = 0,1,2,...., \; \theta > 0$
Poisson Sujatha	$\theta = 13.30$	$p(x) = \frac{\theta^{3}}{\theta^{2}+\theta+2}\frac{x^{2}+(\theta+4)x+(\theta^{2}+3\theta+4)}{(\theta+1)^{x+3}}, \; x = 0,1,2,... \; \theta > 0$
Discrete Rayleigh	$q = 0.12$	$p(x) = q^{x^{2}} - q^{(x+1)^{2}}, \; 0 < q < 1; x = 0,1,2,...$

Table 3: Table for goodness of fit for Counts of Automobile claim data (Willmot [7])

X	Observed	DBXIIBD	Poisson	DRayleigh	PLD	PAD	PSD
0	3719	3725.62	3667.00	3509.20	3678.71	3678.13	3678.71
1	212	202.06	300.36	470.03	278.44	278.75	278.44
2	38	35.09	12.30	0.78	21.11	21.34	21.11
3	7	9.91	0.34	0.00	1.60	1.65	1.60
4	3	3.67	0.01	0.00	0.12	0.13	0.12
5	1	3.66	0.00	0.00	0.01	0.01	0.01
P-values	-	0.201	<0.01	<0.01	0.0003	0.000001	0.000071

The p-value of Pearson's Chi-square statistic is 0.21 for discrete Burr type XII Beta distribution and <0.01for Poisson, discrete Rayleigh, Poisson Lindley, Poisson Akasha, and Poisson Sujatha distributions, respectively (see Table 3). This reveals that Poisson, discrete Rayleigh, PLD, PAD and PSD distributions are not good fit at all, whereas discrete Burr type XII Beta model being the significant model for automobile claim data. The null hypothesis that data come from discrete inverse Burr type XII Beta distribution is accepted.

Table 4: AIC, BIC and Negative loglikelihood (-logL) values for fitted distributions

Criterion	DBXIIBD	Poisson	DRayleigh	PLD	PAD	PSD
-logl	1128.03	1194.9	1535.85	1154.92	1154.13	1154.61
AIC	2260.05	2391.802	3073.705	2311.842	2310.258	2311.217
BIC	2272.63	2398.091	3079.994	2318.131	2316.547	2317.506

We have compared discrete Burr type XII Beta distribution with Poisson, discrete Rayleigh, Poisson Lindley, Poisson Akasha, and Poisson Sujatha distributions using the Akaike information criterion (AIC), given by Akaike [13] and the Bayesian information criterion (BIC), given by Schwarz [14] . Generic function calculating Akaike's 'An Information Criterion' for one or several fitted model objects for which a log-likelihood value can be obtained, according to the formula -2*log-likelihood + k*npar, where npar represents the number of parameters in the fitted model, and k = 2 for the usual AIC, or k = log(n) (n being the number of observations) for the so called BIC or SBC (Schwarz's Bayesian criterion). From table 4, Comparing the fits using AIC and BIC criterion, it is obvious that AIC and BIC criterion favors discrete Burr type XII Beta distribution in comparison with the Poisson, discrete Rayleigh, Poisson Lindley, Poisson Akasha, and Poisson Sujatha distributions, in the case of Counts of Automobile claim data.

9 Conclusion

In this paper, a new model is proposed by compounding discrete Burr type XII distribution (DBXII) with Beta distribution (BD) and it has been shown that proposed model can be nested to different compound distributions. Some important probabilistic properties and the problem of estimation of its parameters are studied. The proposed model is well competitive of some well known compound and classical discrete distributions.

References

[1] Sankaran, M. The Discrete Poisson-Lindley Distribution. *Biometrics.*, **26**, 145-149(1970).

[2] Ghitany, M.E., D.K. Al-Mutairi and S. Nadarajah, 2008a. Zero-truncate Poisson-Lindley distribution and its application. Math. Comput. Simulat., **79**, 279-287(2008a). http://portal.acm.org/citation.cfm?id=1461225.

[3] Ghitany, M.E., B. Atieh and S. Nadarajah, 2008b. Lindley distribution and its application. Math. Comput. Simulat., **78**, 493-506(2008b). http://portal.acm.org/citation.cfm?id=1377430.

[4] Zamani, H. and Ismail, N. (2010). Negative Binomial–Lindley Distribution And Its Application. *J. Mathematics And Statistics*., **1**, 4–9(2010).

[5] Shanker R, Fesshaye H (2016) On Poisson-Sujatha Distribution and its Applications to Model Count Data from Biological Sciences. *Biom Biostat Int J* **3(4)**: 00069(2016). DOI: 10.15406/bbij.2016.03.00069

[6] T. Krishna, H., & Singh Pundir, P. (2009). Discrete Burr and discrete Pareto distributions. Statistical Methodology, 6(2), 177-188(2009). doi:10.1016/j.stamet.2008.07.001

[7] Willmot, G.E.. The Poisson-inverse Gaussian distribution as an alternative to the negative binomial. Scand. Actuar. J., 113–127(1987).

[8] Shanker, R., Hagos, F. (2015). On Poisson-Lindley distribution and Its applications to Biological Sciences. Biometrics and Biostatistics International Journal., **2(4)**, 1-5(2015).

[9] Shanker, R., Hagos, F., Tesfazghi, T. (2016). On Poisson-Akash Distribution and its Applications. Biometrics and Biostatistics International Journal , **3(5)**, 1-5(2016).

[10] Dubey, D.S.. Compound gamma, beta and F distributions, *Metrika.*, **16(1)**, 27-31(1970).

[11] Gerstenkorn, T. A compound of the Polya distribution with the beta one, *Random Oper. and Stoch. Equ.*, **4(2)**, 103-110(1996).

[12] Gerstenkorn, T. (1993). A compound of the generalized gamma distribution with the exponential one, *Recherches surles deformations.*, **16(1)**, 5-10.

[13] Akaike, H. A new look at the statistical model identification. IEEE Trans. Autom.Control **19**, 716–723(1974).

[14] Schwarz, G. Estimating the dimension of a model. Ann. Stat.,**5**, 461–464(1987).

[15] Jones, M.c.(2009). Minimax's distribution: A beta-type distribution with some tractability advantages. *Statistical methodology,**6**(1), 70-81doi.org/10.1016/j.stamet.2008.04.001*

[16] Kumaraswamy, p. (1980). A generalized probability density functions for double-bounded random processes. *Journal of hydrology, **46**(1), 79-88.http//dx.doi.org/10.1016/0022-1694 (80)90036-0*

[17] *Ahmad*, Z., Rashid, A., & Jan, T. R. Generalized Version of Complementary Lindley Power Series Distribution. Pakistan Journal of Statistics and Operation Research, **14(1)**, 139-155(2018).

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