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Variation of Distribution of Concentration of Implanted Impurity with into Account Mismatch-Induced Stresses under Influence of Variation of Substrate Temperature

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Abstract: In this paper, an analysis is made of the mutual effect of radiation damage to the heterostructure during ion doping, substrate temperature during doping, and mismatch-induced stresses.

Keywords: Heterostructure; Ion doping; Mismatch-induced stresses; Influence of heating.

1 Introduction

Wide use of heterostructures for the manufacturing of devices of solid-state elec-tronics leads to the need to improve their properties. The presence of mechanical stresses in heterostructures can lead to the formation of misfit dislocations. To de-crease the magnitude of mechanical stresses, the selection of heterostructure materials is carried out with as little mismatch of the lattice constants [1-3]. Another way to reduce the magnitude of mismatch-induced stresses is the use of buffer layers between the main layers of the heterostructure with intermediate lattice constant values [4,5].

In the framework of this paper, a two-layer heterostructure consisting of a sub-strate and an epitaxial layer is considered. The impurity is implanted into this hetero-structure as shown in Fig. 1. The purpose of this paper is to analyze the mutual ef-fect of radiation damage to a heterostructure during ion doping, substrate temperature during doping and mismatch-induced stresses.

2 Method of Solution

To solve our aim, we determine the spatio-temporal distributions of radiation defects in the heterostructure Under consideration and analyze them. The spatio-temporal distribution of the concentration of radiation defects is determined by solving the following system of equations [6-10]

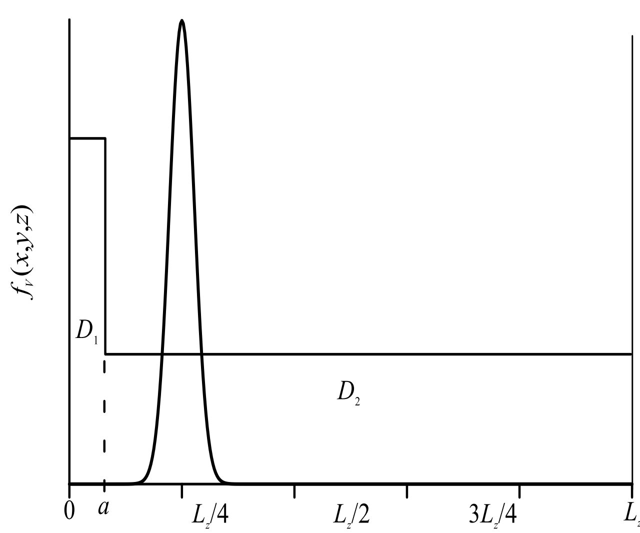

Fig. 1: The two-layer structure under consideration.

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$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right] \qquad (1)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)\,V^2(x,y,z,t)-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]$$

With boundary and initial conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0, \qquad (2)$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ I(x,y,z,0)=$$

$$=f_I(x,y,z),\ V(x,y,z,0)=f_V(x,y,z).$$

Here the following notations are introduced: Ω is the atomic weight of the impurity; the symbol denotes a surface gradient; $V(x,y,z,t)$ is the space-time distribution of the concentration of radiation vacancies; V^* is the equilibrium distribution of vacancies; $I(x,y,z,t)$ is the space-time distribution of the concentration of interstitial atoms; I^* is the equilibrium distribution of interstitial atoms; $\int\limits_0^{L_z} I(x,y,z,t)\,dz$ and $\int\limits_0^{L_z} V(x,y,z,t)\,dz$ are the surface concentration of points at the interface between the layers (in this case it was assumed that the axis Oz is perpendicular to the interface between the layers of the heterostructure); $\mu(x,y,z,t)$ is the chemical potential due to the presence of mechanical stresses; $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, $D_{VS}(x,y,z,T)$ are the coefficients of the volume and surface diffusion of interstitial atoms and vacancies; the terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to the formation of divacancies and analogous complexes of interstitial atoms (see, for example, [10] and the corresponding references in this paper); $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of recombination of point defects and generation of their simplest complexes, respectively, k is the Boltzmann constant; $T(x,y,z,t)$ is the temperature.

Spatio-temporal distributions of the concentrations of divacancies $\Phi_V(x,y,z,t)$ and analogous complexes of interstitial atoms $\Phi_I(x,y,z,t)$ are determined using the following system of equations [9, 10]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$
$$+k_I(x,y,z,T)I(x,y,z,t) \tag{3}$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$
$$+k_V(x,y,z,T)V(x,y,z,t)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial\,\Phi_V\,(x,y,z,t)}{\partial\,x}\right|_{x=0}=0,\ \left.\frac{\partial\,\Phi_V\,(x,y,z,t)}{\partial\,x}\right|_{x=L_x}=0,\ \left.\frac{\partial\,\Phi_V\,(x,y,z,t)}{\partial\,y}\right|_{y=0}=0,\tag{4}$$

$$\left.\frac{\partial\,\Phi_V\,(x,y,z,t)}{\partial\,y}\right|_{y=L_y}=0,\ \left.\frac{\partial\,\Phi_V\,(x,y,z,t)}{\partial\,z}\right|_{z=0}=0,\ \left.\frac{\partial\,\Phi_V\,(x,y,z,t)}{\partial\,z}\right|_{z=L_z}=0,$$

Here $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

Chemical potential μ_1 in Eq.(1) could be determine by the following relation [7]

$$\mu=E(z)\Omega\sigma_{ij}\,[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2,\tag{5}$$

where $E(z)$ is the Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial\,u_i}{\partial\,x_j}+\frac{\partial\,u_j}{\partial\,x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The Eq. (3) could be transform to the following form

$$\mu\,(x,y,z,t)=\left[\frac{\partial\,u_i(x,y,z,t)}{\partial\,x_j}+\frac{\partial\,u_j(x,y,z,t)}{\partial\,x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial\,u_i(x,y,z,t)}{\partial\,x_j}+\frac{\partial\,u_j(x,y,z,t)}{\partial\,x_i}\right]-\right.$$

$$\left.-\varepsilon_0\delta_{ij}+\frac{\sigma\,(z)\delta_{ij}}{1-2\sigma\,(z)}\left[\frac{\partial\,u_k(x,y,z,t)}{\partial\,x_k}-3\varepsilon_0\right]-K\,(z)\beta\,(z)[T\,(x,y,z,t)-T_0]\,\delta_{ij}\right\}\frac{\Omega}{2}E\,(z),$$

where σ is Poisson coefficient; $\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincide (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations [6]

$$\rho\,(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{xx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{xy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{xz}(x,y,z,t)}{\partial z}$$

$$\rho\,(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{yx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{yy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{yz}(x,y,z,t)}{\partial z}$$

$$\rho\,(z)\,\partial^2 u_z(x,y,z,t)\quad\partial\sigma_{zx}(x,y,z,t)\quad\partial\sigma_{zy}(x,y,z,t)\quad\partial\sigma_{zz}(x,y,z,t)$$

where $\sigma_{ij}=\frac{E\,(z)}{2[1+\sigma\,(z)]}\left[\frac{\partial\,u_i(x,y,z,t)}{\partial\,x_j}+\frac{\partial\,u_j(x,y,z,t)}{\partial\,x_i}-\frac{\delta_{ij}}{3}\frac{\partial\,u_k(x,y,z,t)}{\partial\,x_k}\right]+K\,(z)\delta_{ij}\times$ $\times\frac{\partial\,u_k(x,y,z,t)}{\partial\,x_k}-\beta(z)K\,(z)[T(x,y,z,t)-T_r]$, $\rho\,(z)$ is the density of materials of heterostructure, δ_{ij} Is the Kronecker symbol. With account the relation for σ_{ij} last system of equation could be written as

$$\rho(z)\frac{\partial^2\,u_x(x,y,z,t)}{\partial\,t^2}=\left\{K\,(z)+\frac{5E\,(z)}{6[1+\sigma\,(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial\,x^2}+\left\{K\,(z)-\frac{E\,(z)}{3[1+\sigma\,(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right]\times$$
$$\times\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$
$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$
$$\times K(z)\beta(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\times \quad (6)$$
$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}$$
$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\right.$$
$$\left.+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}+$$
$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}-$$
$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Conditions for the system of Eq. (6) could be written in the form

$$\frac{\partial\vec{u}(0,y,z,t)}{\partial x}=0;\ \frac{\partial\vec{u}(L_x,y,z,t)}{\partial x}=0;\ \frac{\partial\vec{u}(x,0,z,t)}{\partial y}=0;\ \frac{\partial\vec{u}(x,L_y,z,t)}{\partial y}=0;$$
$$\frac{\partial\vec{u}(x,y,0,t)}{\partial z}=0;\ \frac{\partial\vec{u}(x,y,L_z,t)}{\partial z}=0;\ \vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0.$$

We determine spatio-temporal distributions of concentrations of radiation defects by solving the Eqs.(1) and (3) framework standard method of averaging of function corrections [11]. Previously we transform the Eqs.(1) and (3) to the following form with account initial distributions of the considered concentrations

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)-$$

$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_I(x,y,z)\delta(t) \qquad (1a)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)-$$

$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_V(x,y,z)\delta(t)$$

$$\frac{\partial\Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t) \qquad (3a)$$

$$\frac{\partial\Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t).$$

Farther we replace concentrations of radiation defects in right sides of Eqs. (1*a*) and (3*a*) on their not yet known average values $\alpha_{1\rho}$. In this situation we obtain equations for the first-order approximations of the required concentrations in the following form

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\right]+$$
$$+f_I\,(x,y,z)\delta(t)-\alpha_{1I}^2k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T) \tag{1b}$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\right]+$$
$$+f_V\,(x,y,z)\delta(t)-\alpha_{1V}^2k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+$$
$$+f_{\Phi_I}\,(x,y,z)\delta(t)+k_I(x,y,z,T)I\,(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \tag{3b}$$

$$\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_VS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_VS}}{kT}\nabla_S\mu(x,y,z,t)\right]+$$
$$+f_{\Phi_V}\,(x,y,z)\delta(t)+k_V(x,y,z,T)V\,(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t).$$

Integration of the left and right sides of the Eqs. (1*b*) and (3*b*) on time gives us possibility to obtain relations for above approximation in the final form

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,\tau)d\tau+\alpha_{1I}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,\tau)\,d\tau+$$
$$+f_I\,(x,y,z)-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau \tag{1c}$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,\tau)d\tau+\alpha_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,\tau)\,d\tau+$$
$$+f_V\,(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_IS}}{kT}\nabla_S\mu(x,y,z,\tau)\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_IS}}{kT}\nabla_S\mu(x,y,z,\tau)\,d\tau\times$$
$$\times\alpha_{1\Phi_I}z+f_{\Phi_I}\,(x,y,z)+\int_0^t k_I(x,y,z,T)I\,(x,y,z,\tau)\,d\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)d\tau \tag{5c}$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_VS}}{kT}\nabla_S\mu(x,y,z,\tau)\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_VS}}{kT}\nabla_S\mu(x,y,z,\tau)\,d\tau\times$$

$$\times\alpha_{1\Phi_V} z + f_{\Phi_V}(x,y,z) + \int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau + \int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\tau\,.$$

We determine average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation [11]

$$\alpha_{1\rho} = \frac{1}{\Theta L_x L_y L_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t\,. \tag{7}$$

Substitution of the relations (1*c*) and (3*c*) into relation (7) gives us possibility to obtain required average values in the following form

$$\alpha_{1I} = \sqrt{\frac{(a_3+A)^2}{4a_4^2} - 4\left(B + \frac{\Theta a_3 B + \Theta^2 L_x L_y L_z a_1}{a_4}\right)} - \frac{a_3+A}{4a_4},$$

$$\alpha_{1V} = \frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x - \alpha_{1I}S_{II00} - \Theta L_x L_y L_z\right],$$

Where $S_{\rho\rho ij} = \int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{\rho,\rho}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$, $a_4 = S_{II00}\times$ $\times\left(S_{IV00}^2 - S_{II00}S_{VV00}\right)$, $a_3 = S_{IV00}S_{II00} + S_{IV00}^2 - S_{II00}S_{VV00}$, $a_2 = \int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_V(x,y,z)\,d\,z\,d\,y\,d\,x\,\times$ $\times S_{IV00}S_{IV00}^2 + S_{IV00}\Theta L_x^2 L_y^2 L_z^2 + 2S_{VV00}S_{II00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x - \Theta L_x^2 L_y^2 L_z^2 S_{VV00} -$ $- S_{IV00}^2\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x$, $a_1 = S_{IV00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x$, $a_0 = S_{VV00}\times$ $\times\left[\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]^2$, $A = \sqrt{8y + \Theta^2\frac{a_3^2}{a_4^2} - 4\Theta\frac{a_2}{a_4}}$, $B = \frac{\Theta a_2}{6a_4} + \sqrt[3]{\sqrt{q^2+p^3} - q} -$ $-\sqrt[3]{\sqrt{q^2+p^3} + q}$, $q = \frac{\Theta^3 a_2}{24a_4^2}\left(4a_0 - \Theta L_x L_y L_z\frac{a_1 a_3}{a_4}\right) - \Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2 - \Theta^2\frac{a_3^2}{a_4}\right) -$ $-\frac{\Theta^3 a_2^3}{54a_4^3} - L_x^2 L_y^2 L_z^2\frac{\Theta^4 a_1^2}{8a_4^2}$, $p = \Theta^2\frac{4a_0 a_4 - \Theta L_x L_y L_z a_1 a_3}{12a_4^2} - \frac{\Theta a_2}{18a_4}$,

$$\alpha_{1\Phi_I} = \frac{R_{I1}}{\Theta L_x L_y L_z} + \frac{S_{II20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi_I}(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$\alpha_{1\Phi_V} = \frac{R_{V1}}{\Theta L_x L_y L_z} + \frac{S_{VV20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi_V}(x,y,z)\,d\,z\,d\,y\,d\,x$$

Where $R_{\rho i} = \int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_I(x,y,z,T)I_1^i(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t\,.$

We determine approximations of the second and higher orders of concentrations of radiation defects framework standard iterative procedure of method of averaging of function corrections [11]. Framework this procedure to determine approximations of the *n*-th order of Concentrations of radiation defects we replace the required concentrations in the Eqs. (1*c*) and (3*c*) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to the following transformation of the appropriate equations.

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)]^2-k_{I,V}(x,y,z,T)\times$$

$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\times\right.$$

$$\left.\times\frac{D_{IS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\right\} \qquad (3d)$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)[\alpha_{1V}+V_1(x,y,z,t)]^2-k_{I,V}(x,y,z,T)\times$$

$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\times\right.$$

$$\left.\times\frac{D_{VS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\right\}$$

$$\frac{\partial \Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial y}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)]\,dW\right\}+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)]\,dW\right\}+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial z}\right]+f_{\Phi_I}(x,y,z)\delta(t) \qquad (3d)$$

$$\frac{\partial \Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial y}\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$
$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_V(x,y,z,T)V(x,y,z,t)+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial z}\right]+f_{\Phi_V}(x,y,z)\delta(t).$$

Integration of the left and the right sides of Eqs. (1*d*) and (3*d*) gives us possibility to obtain relations for the required concentrations in the final form.

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+$$
$$+\frac{\partial}{\partial z}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]^2d\tau-$$
$$-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu\,(x,y,z,\tau)\times$$
$$\times\Omega\frac{D_{IS}}{kT}\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]\times$$
$$\times\Omega\frac{D_{IS}}{kT}d\,W\,d\tau \qquad (1e)$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+$$
$$+\frac{\partial}{\partial z}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{V,V}(x,y,z,T)\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]^2d\tau-$$
$$-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu\,(x,y,z,\tau)\times$$
$$\times\Omega\frac{D_{VS}}{kT}\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]\times$$
$$\times\Omega\frac{D_{VS}}{kT}d\,W\,d\tau$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial \Phi_{1I}(x,y,z,\tau)}{\partial y}\times$$

$$\times D_{\Phi_I}(x,y,z,T)\,d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,\tau)}{\partial z}\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t \nabla_S\mu(x,y,z,\tau)\times$$

$$\times\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\times$$

$$\times\nabla_S\mu(x,y,z,\tau)\,d\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)\,d\tau+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau \quad (5e)$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,\tau)}{\partial x}\,d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial \Phi_{1V}(x,y,z,\tau)}{\partial y}\times$$

$$\times D_{\Phi_V}(x,y,z,T)\,d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,\tau)}{\partial z}\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t \nabla_S\mu(x,y,z,\tau)\times$$

$$\times\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\times$$

$$\times\nabla_S\mu(x,y,z,\tau)\,d\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\tau+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau+$$

$$+f_{\Phi_V}(x,y,z).$$

Average values of the second-order approximations of required approximations by using the following standard relation [1]

$$\alpha_{2\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]dz\,dy\,dx\,dt\cdot \quad (8)$$

Substitution of the relations (1e) and (3e) into relation (8) gives us possibility to obtain relations for required average values $\alpha_{2\rho}$

$$\alpha_{2\Phi I}=0,\ \alpha_{2\Phi V}=0,\ \alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}},$$

where $b_4=\frac{1}{\Theta L_xL_yL_z}S_{IV00}^2S_{VV00}-\frac{1}{\Theta L_xL_yL_z}S_{VV00}^2S_{II00}$, $b_3=-\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}(2S_{VV01}+S_{IV10}+$

$$+\Theta L_xL_yL_z)+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+S_{IV01}+\Theta L_xL_yL_z\right)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}(2S_{VV01}+S_{IV10}+$$

$$+\Theta L_xL_yL_z)-\frac{S_{IV00}^2S_{IV10}}{\Theta^3L_x^3L_y^3L_z^3},\ b_2=\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}(S_{VV02}+S_{IV11}+C_V)-(S_{IV10}-2S_{VV01}+\Theta L_xL_y\times$$

bove

$$\times L_z)^2+\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)+\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+2S_{IV01}+\Theta L_xL_y\times\right.$$

$$\left.\times L_z\right)\left(2S_{VV01}+\Theta L_xL_yL_z+S_{IV10}\right)-\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(C_V-S_{VV02}-S_{IV11}\right)+\frac{C_IS_{IV00}^2}{\Theta^2L_x^2L_y^2L_z^2}-\frac{2S_{IV10}}{\Theta L_xL_yL_z}\times$$

$$\times S_{IV00}S_{IV01},\ b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)+\frac{S_{IV01}}{\Theta L_xL_yL_z}\left(\Theta L_xL_y\times\right.$$

$$\left.\times L_z+2S_{II10}+S_{IV01}\right)\left(2S_{VV01}+S_{IV10}+\Theta LL_yL_z\right)-\frac{S_{IV10}S_{IV01}^2}{\Theta L_xL_yL_z}-\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(3S_{IV01}+2S_{II10}+\right.$$

$$\left.+\Theta L_xL_yL_z\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_IS_{IV00}S_{IV01},\ b_0=\frac{S_{II00}}{\Theta L_xL_yL_z}\left(S_{IV00}+S_{VV02}\right)^2-\frac{S_{IV01}}{L_xL_yL_z}\times$$

$$\times\frac{1}{\Theta}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_IS_{IV01}^2-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\times$$

$$\times\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right),\ C_I=\frac{\alpha_{1I}\alpha_{1V}}{\Theta L_xL_yL_z}S_{IV00}+\frac{\alpha_{1I}^2S_{II00}}{\Theta L_xL_yL_z}-\frac{S_{II20}S_{II20}}{\Theta L_xL_yL_z}-\frac{S_{IV11}}{\Theta L_xL_yL_z}$$

$$,C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2S_{VV00}-S_{VV02}-S_{IV11},\ E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\ F=\frac{\Theta a_2}{6a_4}+$$

$$+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r},\ r=\frac{\Theta^3b_2}{24b_4^2}\left(4b_0-\Theta L_xL_yL_z\frac{b_1b_3}{b_4}\right)-\frac{\Theta^3b_2^3}{54b_4^3}-b_0\frac{\Theta^2}{8b_4^2}\times$$

$$\times\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-L_x^2L_y^2L_z^2\frac{\Theta^4b_1^2}{8b_4^2},\ s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}.$$

Farther we determine solutions of Eqs.(6), i.e. components of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections we replace the required functions in the right sides of the equations by their not yet known average values α_i. The substitution leads to the following result

$$\rho(z)\frac{\partial^2u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\rho(z)\frac{\partial^2u_{1y}(x,y,z,t)}{\partial t^2}=$$

$$=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}$$

$$\rho(z)\frac{\partial^2u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta$$

Approximations of the second and higher orders of components of displacement vector could be determined by using standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$ [11]. The replacement leads to the following result:

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$
$$\times K(z)\beta(z)+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$
$$\times K(z)\beta(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\times$$
$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2\,[1+\sigma(z)]}\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}+\right.$$
$$\left.+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}+$$
$$+\frac{E(z)}{6[1+\sigma(z)]}\frac{\partial}{\partial z}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]-$$
$$-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\left.\right]\left.\right\}\frac{E(z)}{1+\sigma(z)}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

Integration of the left and right sides of the above relations on time t leads to the following result.

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{K(z)-\right.$$
$$\left.-\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{1+\sigma(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\,\left\{K(z)+\right.$$
$$\left.+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\times$$
$$\times\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\tau\,d\vartheta\times$$
$$\times\frac{1}{\rho(z)}-\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\right]-$$
$$-\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\tau\,d\vartheta+u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial x}\int\limits_0^\infty\int\limits_0^\vartheta T\left(x,y,z,\tau\right)d\,\tau\,d\,\vartheta$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^t\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial x\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\times$$
$$\times\frac{1}{1+\sigma(z)}+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{1}{\rho(z)}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\times$$
$$\times\frac{\partial^2}{\partial y^2}\int\limits_0^t\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\int\limits_0^t\int\limits_0^\vartheta T\,(x,y,z,\tau)d\,\tau\,d\,\vartheta-\left\{\frac{E(z)}{6[1+\sigma(z)]}-\right.$$
$$\left.-K(z)\right\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta-\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\frac{1}{1+\sigma(z)}-K(z)\frac{\beta(z)}{\rho(z)}\int\limits_0^\infty\int\limits_0^\vartheta T\,(x,y,z,\tau)\,d\,\tau\,d\,\vartheta-\frac{K(z)}{\rho(z)}\times$$
$$\times\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\left\{\frac{5E(z)}{12[1+\sigma(z)]}+\right.$$
$$\left.+K(z)\right\}-\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial}{\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\right]\right\}\times$$
$$\times\frac{1}{2\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial y\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial y^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial x\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial y\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\right]\frac{1}{\rho(z)}+\frac{1}{\rho(z)}\times$$
$$\times\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial}{\partial x}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial}{\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.\right.$$

$$+\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\}+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{1+\sigma(z)}\Bigg[6\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\}-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0z}.$$

We describe the temperature field by solution of the second Fourier law [12]

$$c(T)\frac{\partial T(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[\lambda(x,y,z,T)\frac{\partial T(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(x,y,z,T)\frac{\partial T(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\lambda(x,y,z,T)\frac{\partial T(x,y,z,t)}{\partial z}\right]+p(x,y,z,t), \tag{9}$$

with boundary and initial conditions

$$\left.\frac{\partial T(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial T(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial T(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial T(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ T(x,y,z,0)=f_T(x,y,z). \tag{10}$$

Here $T(x,y,z,t)$ is the space-time temperature distribution; $c(T)=c_{ass}[1-\eta\ exp(-T(x,y,z,t)/T_d)]$ (in the case of greatest interest, when the temperature is comparable in magnitude to or greater than the Debye temperature, we can assume that $c(T)\approx c_{ass}$) [12]; λ is the coefficient of thermal conductivity, the value of which is determined by the properties of the heterostructure materials and temperature; the temperature dependence of the thermal conductivity in the temperature region of greatest interest can be approximated as follows: $\lambda(x,y,z,T)=\lambda_{ass}(x,y,z)\ [1+\mu\ (T_d/T(x,y,z,t))^{\varphi}]$ (see, for example, [12]); $p(x,y,z,t)$ is the volumetric power density in the heterostructure; $\alpha(x,y,z,T)=\lambda(x,y,z,T)/c(T)$ is the coefficient of thermal diffusivity.

At the first stage, we determine the space-time distribution of the temperature field. For this, following [13-15], we represent the temperature-independent factor of the thermal diffusivity as a sum of constant and variable components: $\alpha_{ass}(x,y,z)=\lambda_{ass}(x,y,z)/c_{ass}=\alpha_{0ass}[1+\varepsilon_T g_T(x,y,z)]$.

Next, we seek the solution of (9) in the form of the following power series [13-15]

$$T(x,y,z,t)=\sum_{i=0}^{\infty}\varepsilon_T^i\sum_{j=0}^{\infty}\mu^j T_{ij}(x,y,z,t). \tag{10}$$

Substitution of this series into equation (9) allows us to obtain a system of equations for the initial approximation of the temperature $T_{00}(x,y,z,t)$, and also the correction functions $T_{ij}(x,y,z,t)$ ($i\geq1$, $j\geq1$). These equations are given in the Appendix. The substitution of the series (10) into the boundary and initial conditions for the space-time temperature distribution makes it possible to obtain boundary and initial conditions for the functions $T_{ij}(x,y,z,t)$ ($i\geq1$, $j\geq1$). These conditions are given in the Appendix. The solutions of the equations for the functions $T_{ij}(x,y,z,t)$ ($i\geq0$, $j\geq0$) were obtained by standard methods [16,17] in the second approximation with respect to the parameters ε and μ and are given in the Appendix.

Within the framework of this work, the required concentrations of radiation defects, as well as the components of the displacement vector and the temperature field, are determined in second approximations within the framework of the solution methods used. The approximation data are usually sufficient for obtaining qualitative conclusions and obtaining some quantitative results. The results of the analytical calculations were verified by comparing them with the results of numerical simulation.

3 Discussion

In this section, we will analyze the redistribution of radiation defects, taking into account the relaxation of mismatch-induced stresses and different values of substrate temperature.

As a result of the analysis of the relaxation of mismatch-induced stresses and the spatio-temporal distributions of radiation defects, it was found that the energy of the implanted ions should be chosen in such a way that the maximum of the distribution of interstitial atoms (see Fig. 1) would be inside the substrate. Then near the interface between layers of heterostructure, the number of radiation vacancies will exceed the number of interstitial atoms. In this case, value of the mechanical stresses decreases with the simultaneous decrease in the number of radiation vacancies. Dependences of the displacement vector and the concentration of vacancies on the coordinate perpendicular to the interface between heterostructure layers are shown in Fig. 2 and 3. Apparently, the decrease in the number of vacancies is a consequence of the condensation of the epitaxial layer under the influence of mismatch-induced stresses. Simultaneously with the compaction of the epitaxial layer, the value of the mechanical stresses decreases due to, apparently, an increase in the density of the atoms of the epitaxial layer.

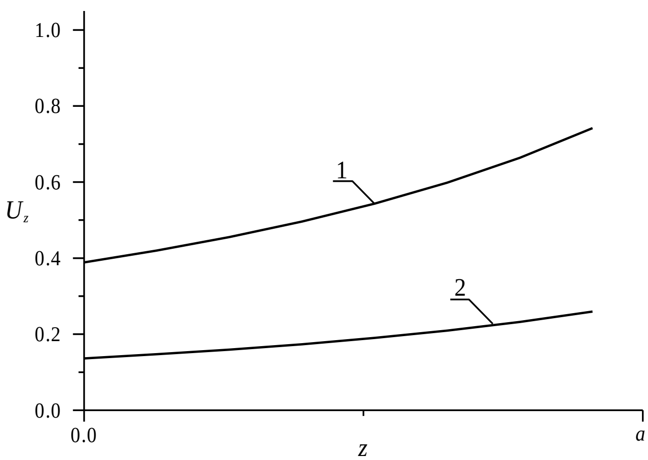

Fig.2. the normalized dependences of the displacement vector component uz on the z coordinate for the epitaxial layers before the radiation treatment (curve 1) and after the radiation treatment (curve 2).

Increasing of the temperature of the substrate stimulates the acceleration of the diffusion of the material of the epitaxial layer deep into the substrate. The diffusion leads to a smearing of the boundary between the layers of the

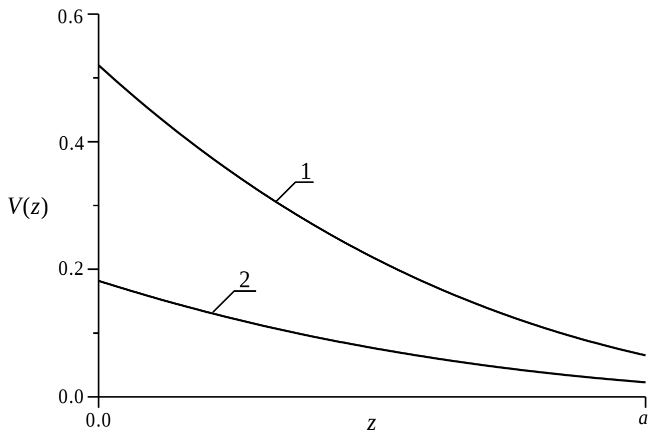

Fig.3. Normalized dependences of vacancy concentrations on the z coordinate in the unstressed (curve 1) and stressed (curve 2) defective materials.

heterostructure. Simultaneously, the presence of a smoother boundary makes it possible to reduce the magnitude of the mechanical stresses in the vicinity of the interface between the layers of the heterostructure due to a smoother change in its properties in a direction perpendicular to the interface of the heterostructure under consideration. In Fig. 4 shows the dependence of the value of the displacement vector component on the coordinate deep into the heterostructure for a sharp and smooth interface. The resulting decrease in the value of the mismatch-induced stresses is, probably, a consequence of the fact that, due to the intermixing of the layers, the heterostructure properties in the neighborhood of the interface change more smoothly. An increase in the temperature of the substrate also leads to spreading of distributions of the concentrations of radiation defects (see Fig.5).

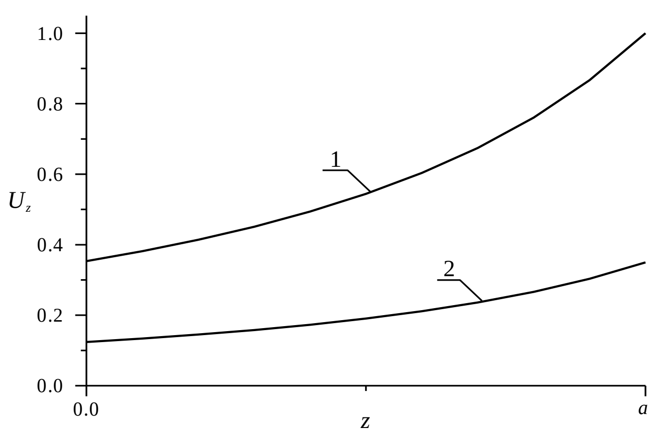

Fig. 4. The normalized dependences of the component of the displacement vector uz on the coordinate z for the sharp (curve 1) and smooth (curve 2) interface between layers of the heterostructure.

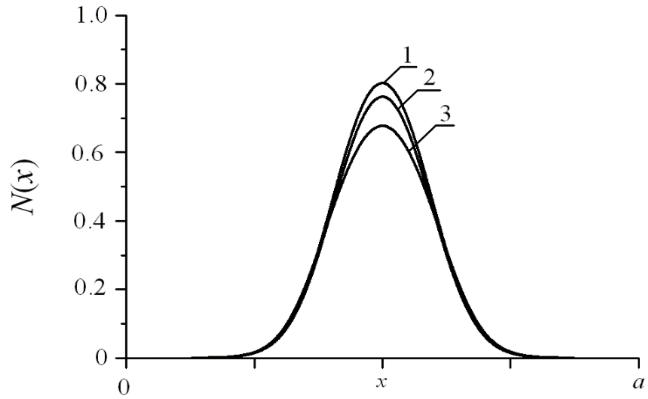

Fig. 5: Distributions of radiation defects at different values of temperature of substrate. Increasing of number of thecurve corresponds to increasing of temperature of substrate.

4 Conclusions

In this paper, an analysis is made of the mutual effect of radiation damage to the heterostructure during ion doping, substrate temperature during doping, and mechanical stresses. Analytical methods for analyzing the processes of mass and heat transfer are proposed.

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Appendix

Equations for functions $T_{ij}(x,y,z,t)$ ($i\geq 0$, $j\geq 0$)

$$\frac{\partial T_{00}(x,y,z,t)}{\partial t}=\alpha_{0ass}\left[\frac{\partial^2 T_{00}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{00}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{00}(x,y,z,t)}{\partial z^2}\right]+\frac{p(x,y,z,t)}{v_{ass}}$$

$$\frac{\partial T_{i0}(x,y,z,t)}{\partial t}=\alpha_{0ass}\left[\frac{\partial^2 T_{i0}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{i0}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{i0}(x,y,z,t)}{\partial z^2}\right]+\alpha_{0ass}\times$$

$$\times\left\{\frac{\partial}{\partial z}\left[g_T(x,y,z,T)\frac{\partial T_{i-10}(x,y,z,t)}{\partial z}\right]+g_T(x,y,z,T)\frac{\partial^2 T_{i-10}(x,y,z,t)}{\partial y^2}+\right.$$

$$\left.+g_T(x,y,z,T)\frac{\partial^2 T_{i-10}(x,y,z,t)}{\partial x^2}\right\},\ i\geq 1$$

$$\frac{\partial T_{01}(x,y,z,t)}{\partial t}=\alpha_{0ass}\left[\frac{\partial^2 T_{01}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{01}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{01}(x,y,z,t)}{\partial z^2}\right]+\left[\frac{\partial^2 T_{00}(x,y,z,t)}{\partial x^2}+\right.$$
$$+\frac{\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi}(x,y,z,t)}\left[\frac{\partial^2 T_{00}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{00}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{00}(x,y,z,t)}{\partial z^2}\right]-\left\{\left[\frac{\partial T_{00}(x,y,z,t)}{\partial x}\right]^2+\right.$$
$$+\left[\frac{\partial T_{00}(x,y,z,t)}{\partial y}\right]^2+\frac{\partial^2 T_{00}(x,y,z,t)}{\partial y^2}+\left.\frac{\partial^2 T_{00}(x,y,z,t)}{\partial z^2}\right]\frac{\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi}(x,y,z,t)}-\left\{\left[\frac{\partial T_{00}(x,y,z,t)}{\partial x}\right]^2+\right.$$
$$\left.+\left[\frac{\partial T_{00}(x,y,z,t)}{\partial y}\right]^2+\left[\frac{\partial T_{00}(x,y,z,t)}{\partial z}\right]^2\right\}\frac{\varphi\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi+1}(x,y,z,t)}$$

$$\frac{\partial T_{02}(x,y,z,t)}{\partial t}=\alpha_{0ass}\left[\frac{\partial^2 T_{02}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{02}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{02}(x,y,z,t)}{\partial z^2}\right]+\frac{\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi}(x,y,z,t)}\times$$
$$\times\left[\frac{\partial^2 T_{01}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{01}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{01}(x,y,z,t)}{\partial z^2}\right]-\frac{\varphi\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi+1}(x,y,z,t)}\left[\frac{\partial T_{00}(x,y,z,t)}{\partial x}\times\right.$$
$$\left.\times\frac{\partial T_{01}(x,y,z,t)}{\partial x}+\frac{\partial T_{00}(x,y,z,t)}{\partial y}\frac{\partial T_{01}(x,y,z,t)}{\partial y}+\frac{\partial T_{00}(x,y,z,t)}{\partial z}\frac{\partial T_{01}(x,y,z,t)}{\partial z}\right]$$

$$\frac{\partial T_{11}(x,y,z,t)}{\partial t}=\alpha_{0ass}\frac{\partial^2 T_{11}(x,y,z,t)}{\partial x^2}+\alpha_{0ass}\frac{T_{01}(x,y,z,t)}{T_{00}(x,y,z,t)}\left\{g_T(x,y,z,T)\frac{\partial^2 T_{00}(x,y,z,t)}{\partial x^2}+\right.$$
$$\left.+g_T(x,y,z,T)\frac{\partial^2 T_{00}(x,y,z,t)}{\partial y^2}+\frac{\partial}{\partial z}\left[g_T(x,y,z,T)\frac{\partial T_{00}(x,y,z,t)}{\partial z}\right]\right\}+\left\{\frac{\partial^2 T_{01}(x,y,z,t)}{\partial x^2}+\right.$$
$$+\frac{\partial}{\partial x}\left[g_T(x,y,z,T)\frac{\partial T_{01}(x,y,z,t)}{\partial x}\right]+[1+g_T(x,y,z,T)]\frac{\partial^2 T_{01}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{01}(x,y,z,t)}{\partial z^2}\times$$
$$\times[1+g_T(x,y,z,T)]\Big\}\alpha_{0ass}+\alpha_{0ass}T_d^{\varphi}\left[\frac{T_{10}(x,y,z,t)}{T_{00}^{\varphi+1}(x,y,z,t)}\frac{\partial^2 T_{00}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T_{00}(x,y,z,t)}{\partial y^2}\times\right.$$
$$\left.\times\frac{T_{10}(x,y,z,t)}{T_{00}^{\varphi+1}(x,y,z,t)}+\frac{T_{10}(x,y,z,t)}{T_{00}^{\varphi+1}(x,y,z,t)}\frac{\partial^2 T_{00}(x,y,z,t)}{\partial z^2}\right]+\frac{\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi}(x,y,z,t)}\left[\frac{\partial^2 T_{10}(x,y,z,t)}{\partial x^2}+\right.$$
$$\left.+\frac{\partial^2 T_{10}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T_{10}(x,y,z,t)}{\partial z^2}\right]+\frac{\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi}(x,y,z,t)}\left\{g_T(x,y,z,T)\frac{\partial^2 T_{10}(x,y,z,t)}{\partial x^2}+\right.$$
$$\left.+g_T(x,y,z,T)\frac{\partial^2 T_{10}(x,y,z,t)}{\partial y^2}+\frac{\partial}{\partial z}\left[g_T(x,y,z,T)\frac{\partial T_{00}(x,y,z,t)}{\partial z}\right]\right\}-\left[\frac{\partial T_{10}(x,y,z,t)}{\partial x}\times\right.$$
$$\left.\times\frac{\partial T_{00}(x,y,z,t)}{\partial x}+\frac{\partial T_{10}(x,y,z,t)}{\partial y}\frac{\partial T_{00}(x,y,z,t)}{\partial y}+\frac{\partial T_{10}(x,y,z,t)}{\partial z}\frac{\partial T_{00}(x,y,z,t)}{\partial z}\right]\frac{\varphi\alpha_{0ass}T_d^{\varphi}}{T_{00}^{\varphi+1}(x,y,z,t)}-$$
$$-\alpha_{0ass}T_d^{\varphi}\varphi\frac{g_T(x,y,z,T)}{T_{00}^{\varphi+1}(x,y,z,t)}\left\{\left[\frac{\partial T_{00}(x,y,z,t)}{\partial x}\right]^2+\left[\frac{\partial T_{00}(x,y,z,t)}{\partial y}\right]^2+\left[\frac{\partial T_{00}(x,y,z,t)}{\partial z}\right]^2\right\}$$

Conditions for functions $T_{ij}(x,y,z,t)$ ($i\geq0$, $j\geq0$)

$$\left.\frac{\partial T_{ij}(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial T_{ij}(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial T_{ij}(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial T_{ij}(x,y,z,t)}{\partial y}\right|_{x=L_y}=0$$

,

$$\left.\frac{\partial T_{ij}(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial T_{ij}(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,$$ $T_{00}(x,y,z,0)=f_T(x,y,z)$, $T_{ij}(x,y,z,0)=0$, $i\geq1$, $j\geq1$.

Solutions of equations for functions $T_{ij}(x,y,z,t)$ ($i\geq0$, $j\geq0$) with account appropriate boundary and initial conditions could be written as

$$T_{00}(x,y,z,t)=\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_T(u,v,w)\,d\,w\,d\,v\,d\,u+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\times$$

$$\times\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}c_n(w)f_T(u,v,w)\,d\,w\,d\,v\,d\,u+\frac{1}{L_xL_yL_z}\int\limits_0^{t}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}\frac{p(u,v,w,\tau)}{v_{ass}}\,d\,w\,d\,v\,d\,u\,d\,\tau+$$

where $c_n(\chi)=cos\,(\pi n\chi/L)$, $$e_{nT}(t)=\exp\left[-\pi^2n^2\alpha_{0ass}t\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}+\frac{1}{L_z^2}\right)\right];$$

$$T_{i0}(x,y,z,t)=2\frac{\pi\alpha_{0ass}}{L_x^2L_yL_z}\sum_{n=1}^{\infty}nc_n(x)c_n(y)c_n(z)e_{nT}(t)\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}s_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}g_T(u,v,w,T)\times$$

$$\times c_n(w)\frac{\partial T_{i-10}(u,v,w,\tau)}{\partial u}\,d\,w\,d\,v\,d\,u\,d\,\tau+2\frac{\pi\alpha_{0ass}}{L_xL_y^2L_z}\sum_{n=1}^{\infty}nc_n(x)c_n(y)c_n(z)e_{nT}(t)\int\limits_0^{t}e_{nT}(-\tau)\times$$

$$\times\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}s_n(v)\int\limits_0^{L_z}g_T(u,v,w,T)c_n(w)\frac{\partial T_{i-10}(u,v,w,\tau)}{\partial v}\,d\,w\,d\,v\,d\,u\,d\,\tau+2\frac{\pi\alpha_{0ass}}{L_xL_yL_z^2}\sum_{n=1}^{\infty}nc_n(x)\times$$

$$\times c_n(y)c_n(z)\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}s_n(w)g_T(u,v,w,T)\frac{\partial T_{i-10}(u,v,w,\tau)}{\partial w}\,d\,w\,d\,v\,d\,u\,d\,\tau\times$$

$$\times e_{nT}(t),$$ $i\geq1$,

Where $s_n(\chi)=sin\,(\pi\ n\ \chi/L)$;

$$T_{01}(x,y,z,t)=\alpha_{0ass}\frac{2\pi T_d^{\varphi}}{L_x^2L_yL_z}\sum_{n=1}^{\infty}nc_n(x)c_n(y)c_n(z)e_{nT}(t)\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}s_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}c_n(w)\times$$

$$\times\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial u^2}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}+\alpha_{0ass}\frac{2\pi T_d^{\varphi}}{L_xL_y^2L_z}\sum_{n=1}^{\infty}nc_n(x)c_n(y)c_n(z)e_{nT}(t)\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}c_n(u)\times$$

$$\times\int\limits_0^{L_y}s_n(v)\int\limits_0^{L_z}c_n(w)\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial v^2}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}+\alpha_{0ass}\frac{2\pi T_d^{\varphi}}{L_xL_yL_z^2}\sum_{n=1}^{\infty}nc_n(x)c_n(y)c_n(z)e_{nT}(t)\times$$

$$\times\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}s_n(w)\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial u^2}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}-2\frac{\varphi\alpha_{0ass}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)\times$$

$$\times T_d^{\varphi}c_n(z)e_{nT}(t)\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}c_n(w)\left[\frac{\partial T_{00}(u,v,w,\tau)}{\partial u}\right]^2\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}-$$

$$-2T_d^{\varphi}\frac{\varphi\alpha_{0ass}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int\limits_0^{t}e_{nT}(-\tau)\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}c_n(w)\left[\frac{\partial T_{00}(u,v,w,\tau)}{\partial v}\right]^2\times$$

$$\times\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}-2T_d^{\varphi}\frac{\varphi\alpha_{0ass}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\times$$

$$\times\left[\frac{\partial T_{00}(u,v,w,\tau)}{\partial w}\right]^2\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)};$$

$$T_{02}(x,y,z,t)=\alpha_{0ass}\frac{2\pi T_d^{\varphi}}{L_x^2L_yL_z}\sum_{n=1}^{\infty}n\,c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}s_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\times$$

$$\times\frac{\partial^2T_{01}(u,v,w,\tau)}{\partial u^2}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}+\frac{\alpha_{0ass}T_d^{\varphi}}{L_xL_y^2L_z}\sum_{n=1}^{\infty}n\,c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\times$$

$$\times 2\pi\int_0^{L_y}s_n(v)\int_0^{L_z}c_n(w)\frac{\partial^2T_{01}(u,v,w,\tau)}{\partial v^2}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}+\alpha_{0ass}\frac{2\pi T_d^{\varphi}}{L_xL_yL_z^2}\sum_{n=1}^{\infty}n\,c_n(x)c_n(y)e_{nT}(t)\times$$

$$\times c_n(z)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}s_n(w)\frac{\partial^2T_{01}(u,v,w,\tau)}{\partial w^2}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}-2T_d^{\varphi}\frac{\pi\alpha_{0ass}}{L_x^2L_yL_z}\times$$

$$\times\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\frac{\partial T_{00}(u,v,w,\tau)}{\partial u}\frac{\partial T_{01}(u,v,w,\tau)}{\partial u}\times$$

$$\times\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}-2T_d^{\varphi}\frac{\pi\alpha_{0ass}}{L_xL_y^2L_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\times$$

$$\times\frac{\partial T_{00}(u,v,w,\tau)}{\partial v}\frac{\partial T_{01}(u,v,w,\tau)}{\partial v}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}-2\frac{\pi\alpha_{0ass}}{L_xL_yL_z^2}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\times$$

$$\times T_d^{\varphi}\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\frac{\partial T_{00}(u,v,w,\tau)}{\partial w}\frac{\partial T_{01}(u,v,w,\tau)}{\partial w}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}$$

$$T_{11}(x,y,z,t)=\frac{2\alpha_{0ass}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}\{g_T(u,v,w,T)\times$$

$$\times\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial w^2}+g_T(u,v,w,T)\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial v^2}+\frac{\partial}{\partial w}\left[g_T(w)\frac{\partial T_{00}(u,v,w,\tau)}{\partial w}\right]\Bigg\}c_n(w)\times$$

$$\times\frac{T_{01}(u,v,w,\tau)}{T_{00}(u,v,w,\tau)}d\,w\,d\,v\,d\,u\,d\,\tau+\frac{2\alpha_{0ass}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\times$$

$$\times\int_0^{L_z}c_n(w)\left\{\frac{\partial^2T_{01}(u,v,w,\tau)}{\partial u^2}+[1+g_T(u,v,w,T)]\frac{\partial^2T_{01}(u,v,w,\tau)}{\partial u^2}+[1+g_T(u,v,w,T)]\times\right.$$

$$\left.\frac{\partial^2T_{01}(u,v,w,\tau)}{\partial v^2}+\frac{\partial}{\partial w}\left[g_T(u,v,w,T)\frac{\partial T_{01}(u,v,w,\tau)}{\partial w}\right]\right\}d\,w\,d\,v\,d\,u\,d\,\tau+2\frac{\alpha_{0ass}T_d^{\varphi}}{L_xL_yL_z}\times$$

$$\times\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}\left[\frac{T_{10}(u,v,w,\tau)}{T_{00}^{\varphi+1}(u,v,w,\tau)}\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial u^2}+\right.$$

$$\left.+\frac{T_{10}(u,v,w,\tau)}{T_{00}^{\varphi+1}(u,v,w,\tau)}\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial v^2}+\frac{T_{10}(u,v,w,\tau)}{T_{00}^{\varphi+1}(u,v,w,\tau)}\frac{\partial^2T_{00}(u,v,w,\tau)}{\partial w^2}\right]c_n(w)d\,w\,d\,v\,d\,u\,d\,\tau+$$

$$+2\frac{\alpha_{0ass}T_d^{\varphi}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\left[\frac{\partial^2T_{10}(u,v,w,\tau)}{\partial u^2}+\right.$$

$$\left.+\frac{\partial^2T_{10}(u,v,w,\tau)}{\partial v^2}+\frac{\partial^2T_{10}(u,v,w,\tau)}{\partial w^2}\right]\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}+2\frac{\alpha_{0ass}T_d^{\varphi}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\times$$

$$\times \int_0^t e_{nT}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w)\left\{\frac{\partial}{\partial w}\left[g_T(u,v,w,T)\frac{\partial T_{00}(u,v,w,\tau)}{\partial w}\right]+\frac{\partial^2 T_{00}(u,v,w,\tau)}{\partial u^2}\times\right.$$

$$\left.\times\, g_T(u,v,w,T)+g_T(u,v,w,T)\frac{\partial^2 T_{00}(u,v,w,\tau)}{\partial v^2}\right\}\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi}(u,v,w,\tau)}-2\varphi\frac{\alpha_{0ass}T_d^{\varphi}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)\times$$

$$\times\, c_n(y)c_n(z)e_{nT}(t)\int_0^t\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z}\left[\frac{\partial T_{10}(u,v,w,\tau)}{\partial u}\frac{\partial T_{00}(u,v,w,\tau)}{\partial u}+\frac{\partial T_{10}(u,v,w,\tau)}{\partial v}\times\right.$$

$$\left.\times\frac{\partial T_{00}(u,v,w,\tau)}{\partial v}+\frac{\partial T_{10}(u,v,w,\tau)}{\partial w}\frac{\partial T_{00}(u,v,w,\tau)}{\partial w}\right]c_n(w)\,e_{nT}(-\tau)\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}-$$

$$-2\varphi\frac{\alpha_{0ass}T_d^{\varphi}}{L_xL_yL_z}\sum_{n=1}^{\infty}c_n(x)c_n(y)c_n(z)e_{nT}(t)\int_0^t e_{nT}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z}\left\{\left[\frac{\partial T_{00}(u,v,w,\tau)}{\partial u}\right]^2+\right.$$

$$\left.+\left[\frac{\partial T_{00}(u,v,w,\tau)}{\partial v}\right]^2+\left[\frac{\partial T_{00}(u,v,w,\tau)}{\partial w}\right]^2\right\}c_n(w)\frac{d\,w\,d\,v\,d\,u\,d\,\tau}{T_{00}^{\varphi+1}(u,v,w,\tau)}$$