Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100219

On the Rotation and Axial Magnetic Field Effects of a Non-Homogeneous Composite Infinite Cylinder of Orthotropic Material

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Received: 1 Aug. 2014, Revised: 30 Jan. 2015, Accepted: 15 Feb. 2015
Published online: 1 Mar. 2016

Abstract: In this paper, the rotation and magnetic field effects problem on an orthotropic cylinder containing: (i) an isotropic core and (ii) a rigid core; is considered. The elastic constants and density are taken as a power function of the radial coordinate. Analytical expressions for components of the displacement, radial, hoop and axial stresses in different cases are obtained. The results obtained are illustrated numerically. A comparison between both cases are clarified numerically and then presented graphically.

Keywords: Composite, Rotation, Axial magnetic field, Orthotropic, Rigid core, Isotropic.

1 Introduction

An interaction between the elastic properties, magnetic field, and orthotropy has more attentions because of its utilitarian aspects in diverse field, especially, Geophysics, Geology, Biology, Engineering, Acoustics,... *etc*. Some problems of a homogenous nonisotropic cylinder were investigated in the plane strain by Chakravorty [1]. Chatterjee [2] solved some problems of plane strain in a non-homogeneous isotropic cylinder. Mukhopadhyay [3] investigated the effect of non-homogeneity on the stresses in a rotation of a nonhomogenous aeolotropic cylindrical shell. On the other hand, Mukhopadhyay [4] pointed out the present problem in the case of isotropic cylinder of homogenous density. In [5] Rudincki and Reoloffs, derived the stress and pore pressure induced by a plane strain shear. El-Naggar *et al.* [6] illustrated the rotation problem of an orthotropic cylinder containing (i) an isotropic core and (ii) a rigid core is considered. Mukhopadhyay [7] studied the effects of thermal relaxations and viscosity on an unbounded body with a spherical cavity subjected to a periodic loading on the boundary. Abd-Alla *et al.* [8] investigated the propagation of Rayleigh waves in magneto-thermoelastic half space of a homogeneous orthotropic material under the effects of the rotation, initial stress, and gravity field. Abd-Alla *et al.* [9] pointed out Love waves propagation in a non-homogeneous orthotropic magnetoelastic layer under initial stress overlying a semi-infinite medium. Abd-Alla *et al.* [10] discussed the rotation and gravity field effects on Stoneley waves in a non-homogeneous orthotropic elastic medium. Abd-Alla *et al*. [11] explained thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder. Abd-Alla *et al.* [12] investigated the propagation of S-wave phenomena in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field. Recently, Abo-Dahab *et al.* [13] discussed propagation of S-waves in a non-homogeneous anisotropic incompressible medium with the influences of gravity field, initial stress, magnetic field, and rotation. This paper deals the problem of plane-strain in a non-homogeneous orthotropic infinite circular cylinder viz. (i) rotation and magnetic field about its axis of a non-homogeneous infinite circular cylinder containing an isotropic core and (ii) rotation and magnetic field about its axis of a non-homogeneous infinite cylinder containing a

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rigid core. In both problems we assumed that the elastic constants and density are power functions of the radial coordinate. In both cases, the stresses have been calculated. Finally, a comparison between both cases is made to clarify the new parameters effects on the phenomena. The results obtained have been calculated numerically and presented by figures to explaine the physical meaning.

2 Formulation of the problem

Let us consider the cylindrical coordinates (r, θ, z) with the z-axis coinciding with the axis of the cylinder and let us consider the elastic medium is rotating uniformly with angular velocity $\underline{\Omega} = \Omega\, \underline{n}$, where, $\underline{n}$ representing the direction of the axis of rotation $\underline{\Omega} = (0, 0, \Omega)$. Both media are under the primary magnetic field $\overrightarrow{H_0}$ acting on z-axis, $\overrightarrow{H_0} = (0, 0, H_0)$. To consider the strains symmetrical about the z-axis, we have only the radial displacement $u_r = u$ and these are independent on θ. The plane strain perpendicular to the z-axis, u is a function of r only. The stresses components are given by

$$\begin{aligned} \tau_{rr} &= c_{11}\frac{du}{dr} + c_{12}\frac{u}{r}, \\ \tau_{\theta\theta} &= c_{12}\frac{du}{dr} + c_{22}\frac{u}{r}, \\ \tau_{zz} &= c_{13}\frac{du}{dr} + c_{23}\frac{u}{r}, \\ \tau_{r\theta} &= \tau_{rz} = \tau_{\theta z} = 0 \end{aligned} \tag{1}$$

where, c_{ij} are the elastic constants.

The equation of motion in the rotating frame has two additional terms (i) $\underline{\Omega} \wedge (\underline{\Omega} \wedge \underline{u})$ is the centripetal acceleration due to time varying motion only and (ii) $2\underline{\Omega} \wedge \underline{\dot{u}}$ is the Coriolis acceleration is neglected. And it has another term F_r when we add magnetic field. Then the equation becomes.

$$\frac{d\tau_{rr}}{dr} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) + \rho\,\Omega^2 r + F_r = 0 \tag{2}$$

where, Ω is the uniform angular velocity, ρ is the density of the cylinder material and F_r is radial component of Lorentz's force. The electromagnetic field is governed by Maxwell's equations under consideration that the medium is a perfect electric conductor and absence of the displacement current (SI) [8]:

$$\begin{aligned} \mathbf{j} = curl\mathbf{h} \quad &, \quad curl\mathbf{E} = -\mu_e\frac{\partial \mathbf{h}}{\partial t} \\ div\mathbf{h} = 0 \quad &, \quad div\mathbf{E} = 0 \\ \mathbf{E} &= -\mu_e(\frac{\partial \mathbf{u}}{\partial t} \wedge \mathbf{H}) \end{aligned} \tag{3}$$

Taking $\mathbf{h} = curl\,(\mathbf{u} \wedge \mathbf{H}_0)$, $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$, $\mathbf{F} = \mu_e(\mathbf{j} \wedge \mathbf{H})$

where, $\mathbf{h}$ is the perturbed magnetic field over the primary magnetic field vector. we characterize the elastic constants c_{ij} and the density ρ and μ_e is the magnetic permeability of non-homogeneous material by

$$c_{ij} = \alpha_{ij} r^{2m};\ \rho = \rho_0 r^{2m};\ \mu_e = \mu_{e_0} r^{2m} \tag{4}$$

where α_{ij}, ρ_0, *and* μ_{e_0} are constants and m is a rational number.

Substituting Eqs. (1) and (4) in Eq. (2), we obtain

$$r^2\frac{d^2u}{dr^2} + (2M+1)\,r\frac{du}{dr} + \frac{1}{\alpha_{11}}(2M\alpha_{12} - \frac{\alpha_{22} + \mu_{e_0}H_0^2}{1 + \frac{\mu_{e_0}H_0^2}{\alpha_{11}}})\,u = -\frac{1}{\alpha_{11} + \mu_{e_0}H_0^2}\rho_0\,\Omega^2\,r^3 \tag{5}$$

The complete solution of (5) is

$$u = Ar^{n-M} + Br^{-(n+M)} - pr^3 \tag{6}$$

where, A and B are constants, $n^2 = M^2 - \frac{1}{\alpha_{11}}\left(2M\alpha_{12} - \frac{\alpha_{22}+\mu_{e_0}H_0^2}{1+\frac{\mu_{e_0}H_0^2}{\alpha_{11}}}\right)$ and $p = -\frac{\rho_0\,\Omega^2}{(\alpha_{11}+\mu_{e_0}H_0^2)[(M+3)^2-n^2]}$, $M = \frac{m}{1+\frac{\mu_{e_0}H_0^2}{\alpha_{11}}}$

From Eqs. (1) and (6) we get

$$\begin{aligned} \tau_{rr} = A&[(n-M)\alpha_{11} + \alpha_{12}]\,r^{M+n-1} \\ &+ B[-(n+M)\alpha_{11} + \alpha_{12}]\,r^{M-n-1} \\ &- (3\alpha_{11} + \alpha_{12})\,p\,r^{2(M+1)} \end{aligned} \tag{7}$$

$$\begin{aligned} \tau_{\theta\theta} = A&[(n-M)\alpha_{12} + \alpha_{22}]\,r^{M+n-1} \\ &+ B[-(n+M)\alpha_{12} + \alpha_{22}]\,r^{M-n-1} \\ &- (3\alpha_{12} + \alpha_{22})\,p\,r^{2(M+1)} \end{aligned} \tag{8}$$

$$\begin{aligned} \tau_{zz} = A&[(n-M)\alpha_{13} + \alpha_{23}]\,r^{M+n-1} \\ &+ B[-(n+M)\alpha_{13} + \alpha_{23}]\,r^{M-n-1} \\ &- (3\alpha_{13} + \alpha_{23})\,p\,r^{2(M+1)} \end{aligned} \tag{9}$$

The corresponding equation (2) in a non-homogeneous isotropic case: i.e., $\lambda = \lambda_0 r^{2m}$ *and* $\mu = \mu_0 r^{2m}$ are given by:

$$\begin{aligned} r^2\frac{d^2u}{dr^2} &+ (2M_1+1)\,r\frac{du}{dr} \\ &+ \frac{1}{\lambda_0 + 2\mu_0}(2M_1\lambda_0 - (\lambda_0 + 2\mu_0))\,u = \\ &- \frac{1}{\lambda_0 + 2\mu_0 + \mu_{e_0}H_0^2}\rho_0\,\Omega^2\,r^3 \end{aligned} \tag{10}$$

where, $M_1 = \frac{m}{1+\frac{\mu_{e_0} H_0^2}{\lambda_0 + 2\mu_0}}$

The corresponding displacements and stress in a non-homogeneous isotropic case as the following form:

$$u = Cr^{L-M_1} + Dr^{-(L+M_1)} - qr^3 \tag{11}$$

$$\tau_{rr} = \frac{2\mu_0}{1-2\nu}[C[(1-\nu)(L-M_1)+\nu]\, r^{M_1+L-1} - D[(1-\nu)(L+M_1)-\nu]\, r^{M_1-L-1} - (3-2\nu)q\, r^{2(M_1+1)}] \tag{12}$$

$$\tau_{\theta\theta} = \frac{2\mu_0}{1-2\nu}[C[(1-\nu)+\nu(L-M_1)]\, r^{M_1+L-1} + D[(1-\nu)-\nu(L+M_1)]\, r^{M_1-L-1} - (2\nu+1)q\, r^{2(M_1+1)}] \tag{13}$$

$$\tau_{zz} = \frac{2\mu_0 \nu}{1-2\nu}[C\,(L-M_1+1)\, r^{M_1+L-1} + D\,(1-L-M_1)\, r^{M_1-L-1} - 4q\, r^{2(M_1+1)}] \tag{14}$$

where, C, D, λ_0 *and* μ_0 are constants,

$$L^2 = M_1^2 + \frac{1}{1-\nu}[1-(2M_1+1)\,\nu],$$

$$q = -\frac{\rho_0\, \Omega^2}{(\lambda_0 + 2\mu_0 + \mu_{e_0} H_0^2)[(M_1+3)^2 - L^2]}$$

where, μ_0 is the rigidity modulus, and ν is the Poisson's ratio of the material. For a complete cylinder $D = 0$.

3 Rotation of a circular cylinder containing a non-homogeneous isotropic core

In this section, we consider the rotation and magnetic field of a circular composite cylinder which is composed of a non-homogeneous isotropic material up to radius, the region bounded by the radii $r = a$ and $r = b(b > a)$ being composed of non-homogeneous orthotropic material. Let us denote the region $0 \leqslant r \leqslant a$ by region I and the region $a \leqslant r \leqslant b$ by region II. The boundary conditions are:

$$(\tau_{rr})_{II} = 0 \quad on \quad r = b$$

$$[(\tau_{rr})_I]_{r=a} = [(\tau_{rr})_{II}]_{r=a}$$

$$[(u)_I]_{r=a} = [(u)_{II}]_{r=a} \tag{15}$$

From Eqs. (6), (7), (11), (12), and (15), we get

$$A = \frac{h_1(b)[k_1k_3 - k_4 f_2(a)] + f_2(b)[(h_1(a) - h_2)k_4 - k_1h_3]}{f_1(b)[k_1k_3 - k_4 f_2(a)] + f_2(b)[k_4 f_1(a) - k_1k_2]}$$

$$B = \frac{f_1(b)[k_1h_3 - (h_1(a) - h_2)k_4] + h_1(b)[k_4 f_1(a) - k_1k_2]}{f_1(b)[k_1k_3 - k_4 f_2(a)] + f_2(b)[k_4 f_1(a) - k_1k_2]}$$

$$C = \frac{f_1(b)[h_3 f_2(a) - (h_1(a) - h_2)k_3]}{f_1(b)[k_1k_3 - k_4 f_2(a)] + f_2(b)[k_4 f_1(a) - k_1k_2]} + \frac{f_2(b)[(h_1(a)-h_2)k_2 - h_3 f_1(a)] + h_1(b)[k_3 f_1(a) - k_2 f_2(a)]}{f_1(b)[k_1k_3 - k_4 f_2(a)] + f_2(b)[k_4 f_1(a) - k_1k_2]} \tag{16}$$

where,

$$\begin{aligned}
f_1(r) &= [(n-M)\alpha_{11} + \alpha_{12}]\, r^{M+n-1} \\
f_2(r) &= [-(n+M)\alpha_{11} + \alpha_{12}]\, r^{M-n-1} \\
h_1(r) &= (3\alpha_{11} + \alpha_{12})p\, r^{2(M+1)} \\
h_2 &= \frac{2\mu_0(3-2\nu)}{1-2\nu}\, q\, a^{2(M_1+1)} \\
h_3 &= (p-q)\, a^3 \\
k_1 &= \frac{2\mu_0}{1-2\nu}[(1-\nu)(L-M_1)+\nu]\, a^{M_1+L-1} \\
k_2 &= a^{n-M}, \quad k_3 = a^{-(n+M)}, \quad k_4 = a^{L-M_1}
\end{aligned}$$

Having obtained the constants from (16), the stresses in region II $(a \leqslant r \leqslant b)$ can be calculated from (7)-(9), and the stresses in region I $(0 \leqslant r \leqslant a)$ can also be calculated from from (12)-(14).

In the associated non-homogeneous isotropic problem, that is, when the whole cylinder is composed of non-homogeneous isotropic material, the boundary conditions are given by :

$$\tau_{rr} = 0 \quad on \quad r = b \tag{17}$$

From Eqs. (12) and (17) the constant C is given by

$$C = \frac{(3-2\nu)q}{[(1-\nu)(L-M_1)+\nu]}\, b^{M_1-L+3} \tag{18}$$

where, $D = 0$ (since the cylinder is complete). The stresses in the non-homogeneous isotropic case are obtained from Eqs. (12)-(14) and (18).

4 Rotation of a circular cylinder containing a rigid core

Here we consider the rotation and magnetic field of a circular cylinder of radius $r = b$ containing a rigid core of radius $r = a$ having the same axis of the cylinder.

In the case, the boundary conditions are

$$\begin{aligned} u &= 0 \quad on \quad r = a, \\ \tau_{rr} &= 0 \quad on \quad r = b. \end{aligned} \tag{19}$$

From Eqs. (6), (7), and (19), the constants are given as:

$$\begin{aligned} A &= \frac{a^3 p\, f_2(b) - h_1(b)\, a^{-(n+M)}}{a^{n-M} f_2(b) - a^{-(n+M)} f_1(b)} \\ B &= \frac{h_1(b)\, a^{n-M} - a^3 p\, f_1(b)}{a^{n-M} f_2(b) - a^{-(n+M)} f_1(b)} \end{aligned} \tag{20}$$

The stresses can be calculated from Eqs. (7)-(9) and (20). In the corresponding non-homogeneous isotropic case, from Eqs. (11), (12), and (19), the constants are given by

$$\begin{aligned} C &= \frac{[(1-\nu)(L+M_1)-\nu]\, q a^3 b^{M_1-L} + (3-2\nu)\, q a^{-(L+M_1)} b^{2M_1+3}}{[(1-\nu)(L+M_1)-\nu]\,(b/a)^{M_1-L} + [(1-\nu)(L-M_1)+\nu](b/a)^{M_1+L}} \\ D &= \frac{[(1-\nu)(L-M_1)+\nu]\, q a^3 b^{M_1+L} - (3-2\nu)\, q a^{L-M_1} b^{2M_1+3}}{[(1-\nu)(L+M_1)-\nu]\,(b/a)^{M_1-L} + [(1-\nu)(L-M_1)+\nu](b/a)^{M_1+L}} \end{aligned} \tag{21}$$

From (12)-(14) and (21) the corresponding stresses can be calculated.

5 Numerical results and discussions

For the numerical calculation in different cases, we use the data of [5] for the orthotropic case:

$$\alpha_{11} = 7.288.\alpha_{13}, \quad \alpha_{22} = 1.948.\alpha_{13}, \quad \alpha_{12} = 0.638.\alpha_{13}$$
$$\alpha_{23} = 0.655.\alpha_{13} \quad \alpha_{13} = 4.308.10^9\, N/m^2$$

And we take $\nu = 0.25$ consequently, $\mu_0 = 10.465567$, for the isotropic case. The values of radial displacement u, and stresses τ_{rr}, $\tau_{\theta\theta}$ and τ_{zz} for elastic medium are studied for the effects of the rotation and magnetic field. The output is plotted in Figs. 1-18.

Fig. (1) shows the variations of radial displacement u , radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of nonhomogeneity m. In both figures, it clears that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the displacement increases with the increasing of nonhomogeneity while it decreases with an increasing of radius r, as well it is found that radial stress increases with increasing of nonhomogeneity and radius, hoop stress decreases with increasing of nonhomogeneity and radius, while axial stress decreases with increasing of nonhomogeneity and it increases with increasing of r.

Fig. (2) shows the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of magnetic field H. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the displacement decreases with the increasing of magnetic field while it decreases with an increasing of radius r, as well it is found that radial stress increases with increasing of magnetic field and radius, hoop stress decreases with increasing of magnetic field and radius, while axial stress increases with increasing of magnetic field and it increases with increasing of r.

Fig. (3) displays the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of rotation Ω. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the displacement increases with the increasing of rotation while it decreases with an increasing of radius r, as well it is found that radial stress decreases with increasing of rotation and radius, hoop stress increases with increasing of rotation, but it decreases with increasing of radius, while axial stress decreases with increasing of rotation and it increases with increasing of r.

Fig. (4) plots the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r which it has oscillatory behavior in the whole range of the $r-axis$ for different values of nonhomogeneity m. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement decreases with the increasing of nonhomogeneity and radius r, as well it finds that radial stress, hoop stress and axial stress decreases increase with the increasing of nonhomogeneity, while the components of stress intersected at $r = 0.75$.

Fig. (5) clears the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of magnetic field H. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement, radial stress, hoop stress and axial stress decreases increase with increasing of magnetic field, while the radial displacement decreases with increasing of radius, as well the components of stresses has oscillatory behavior in the whole range of the $r-axis$, which it decreases and increases gradually.

Fig. (6) shows that the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of rotation Ω. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement, radial stress, hoop stress and axial stress decreases with increasing of

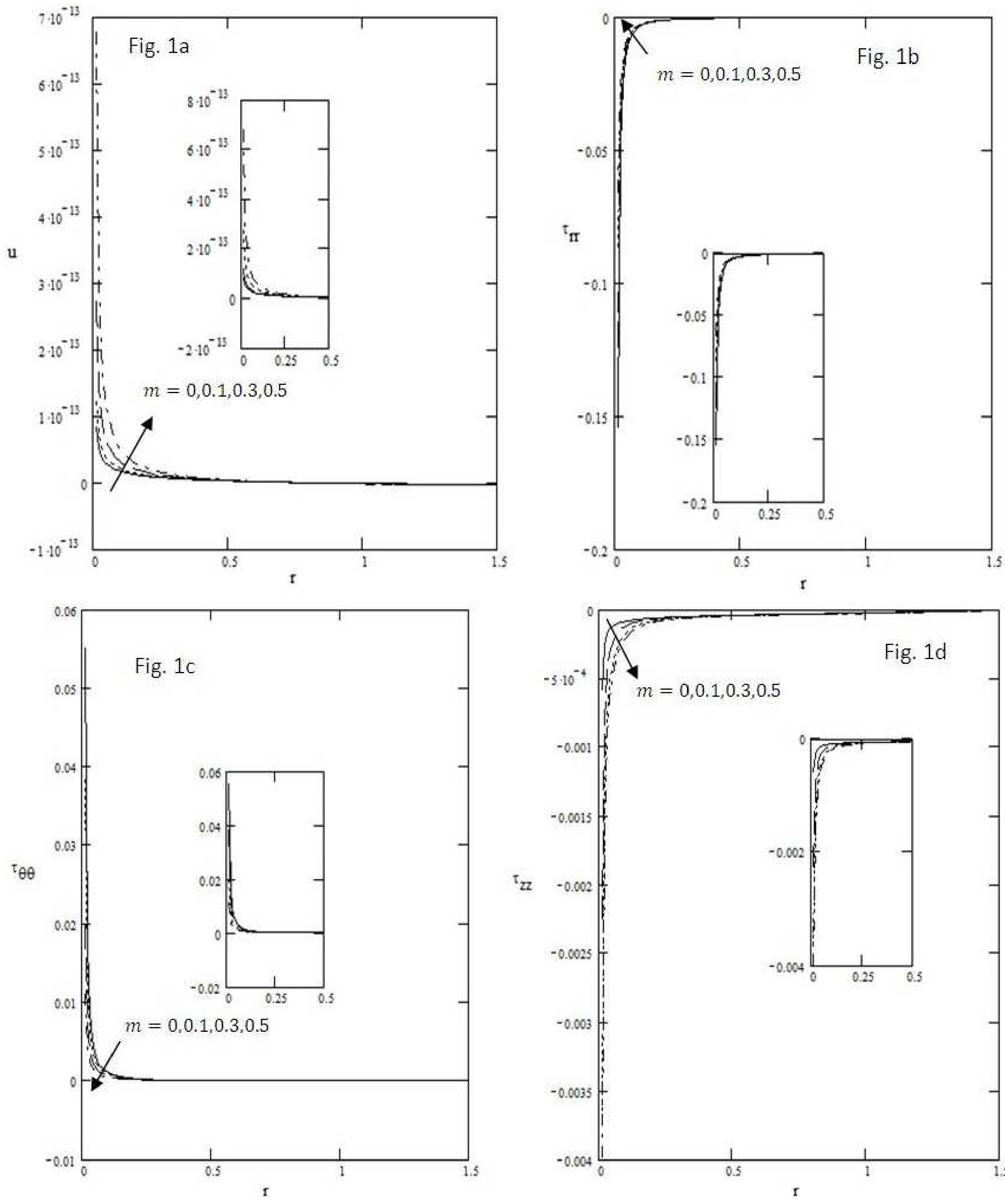

Fig. 1: Variations of u , τ_{rr} , $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of m.

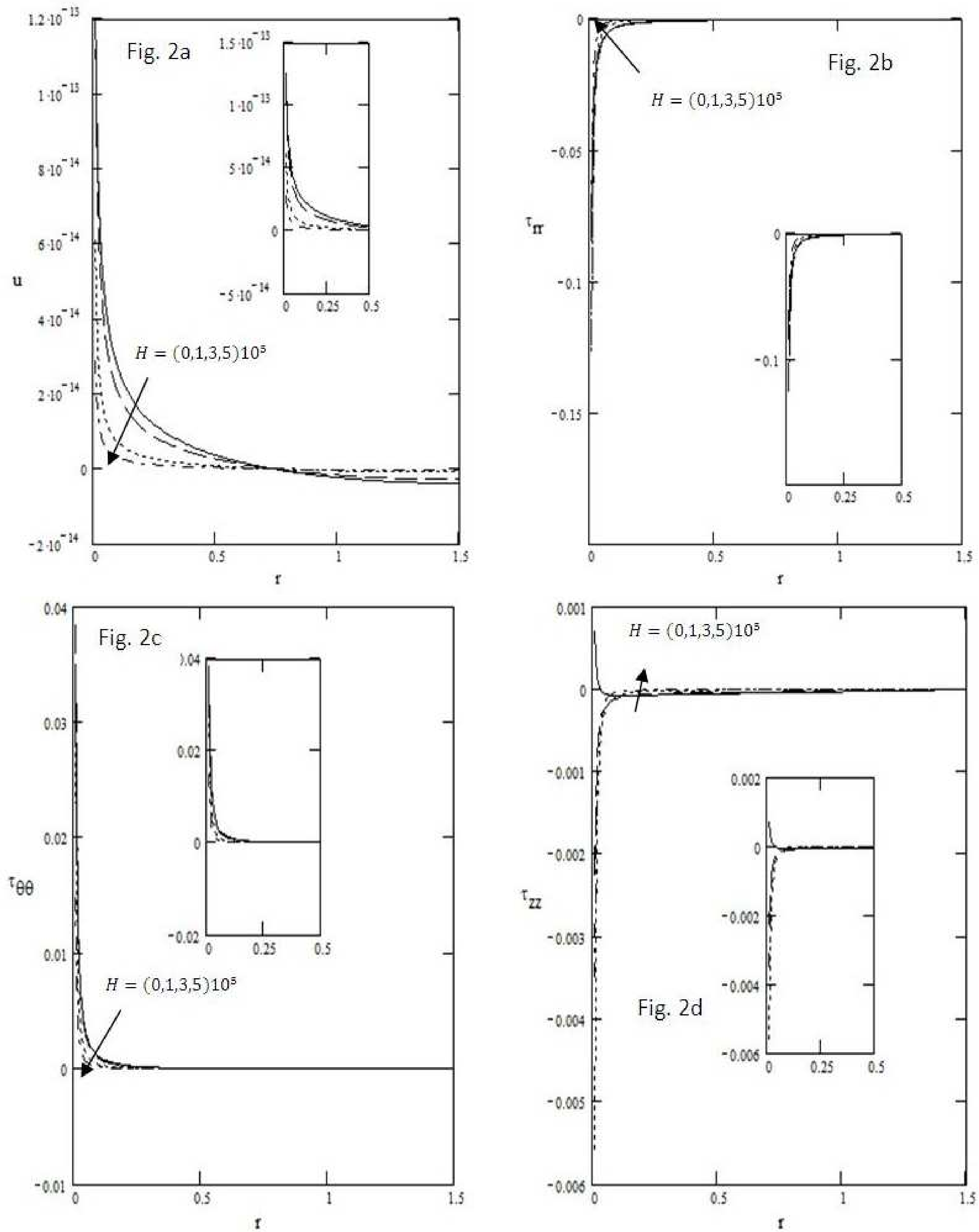

Fig. 2: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of H.

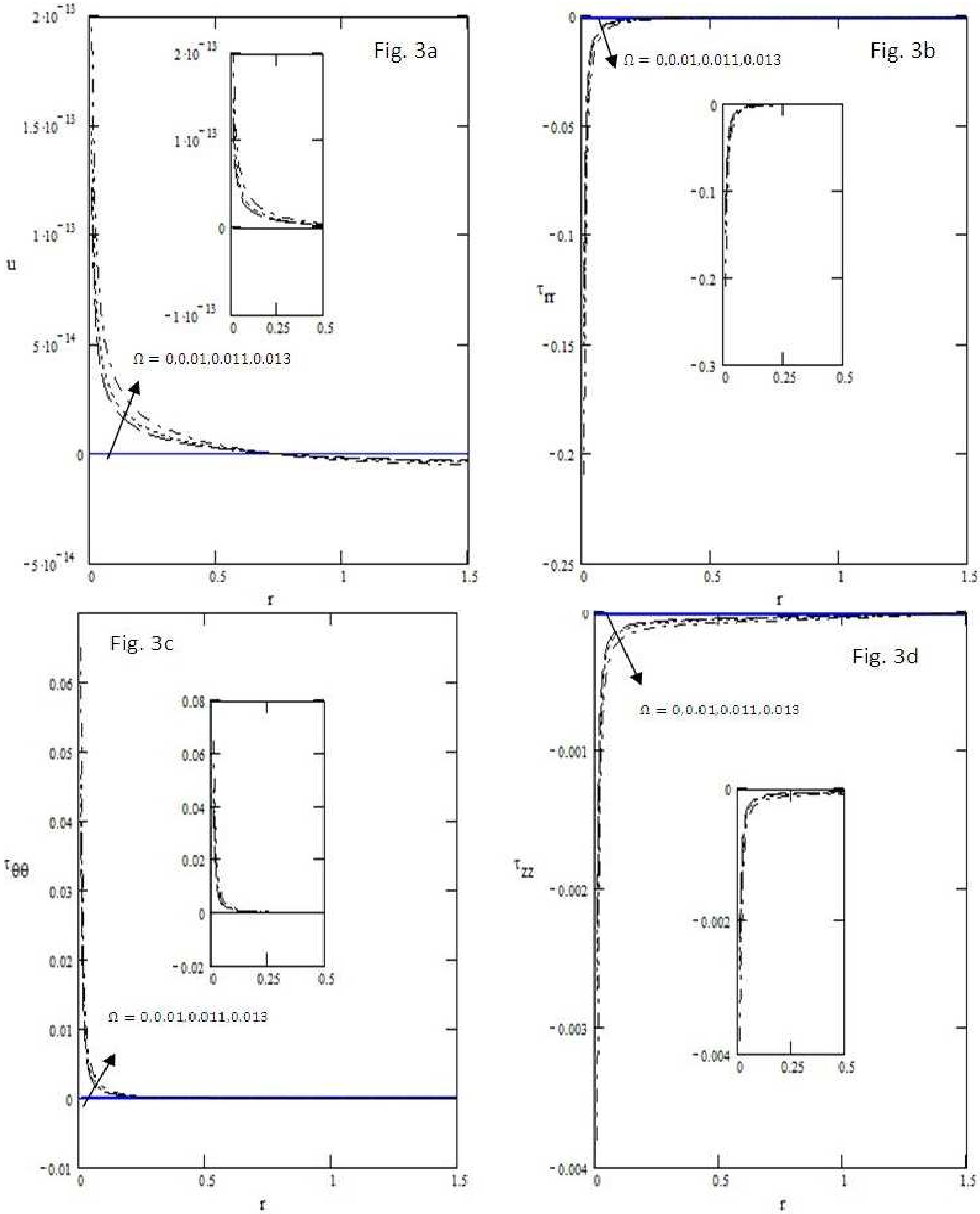

Fig. 3: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of Ω.

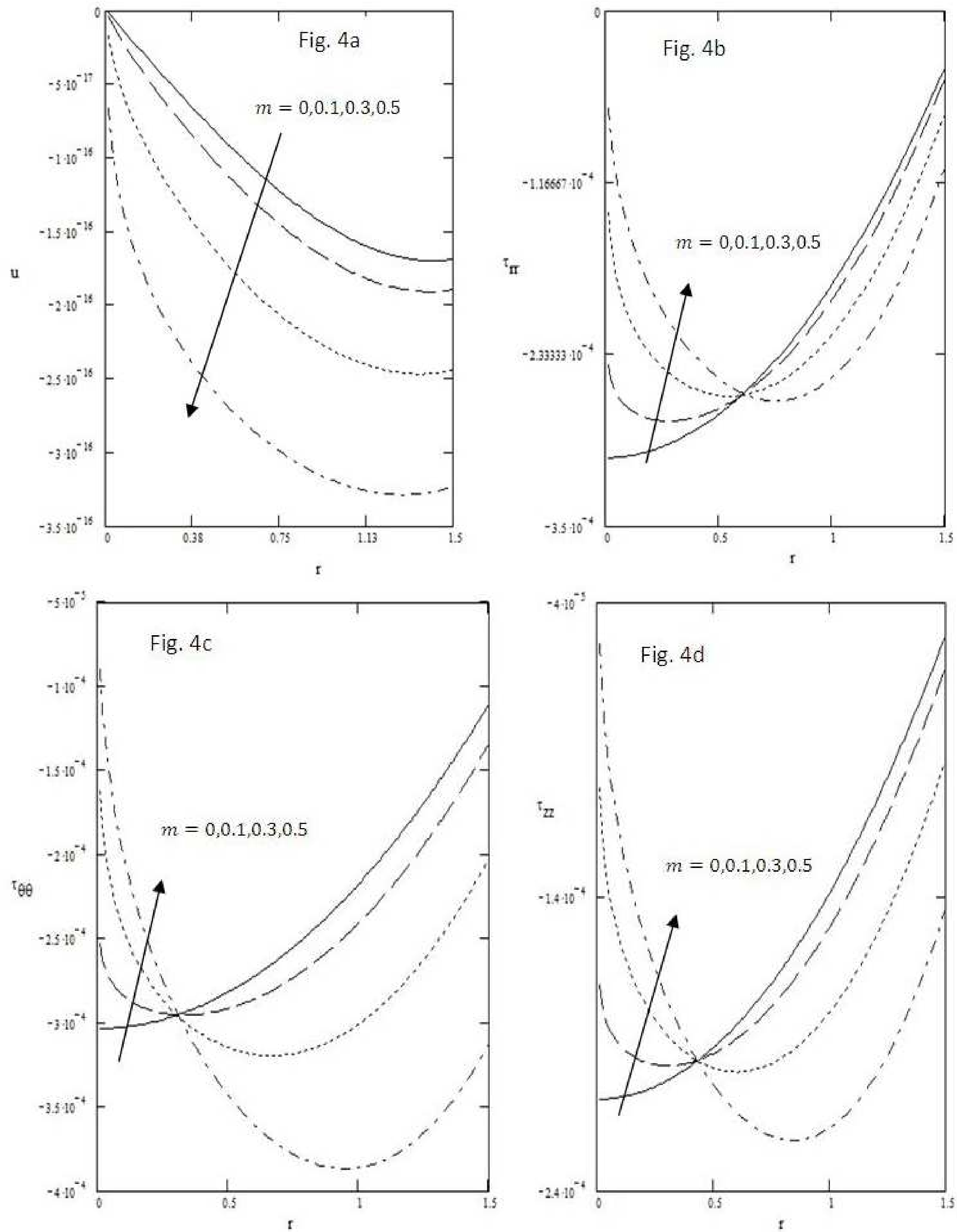

Fig. 4: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of m.

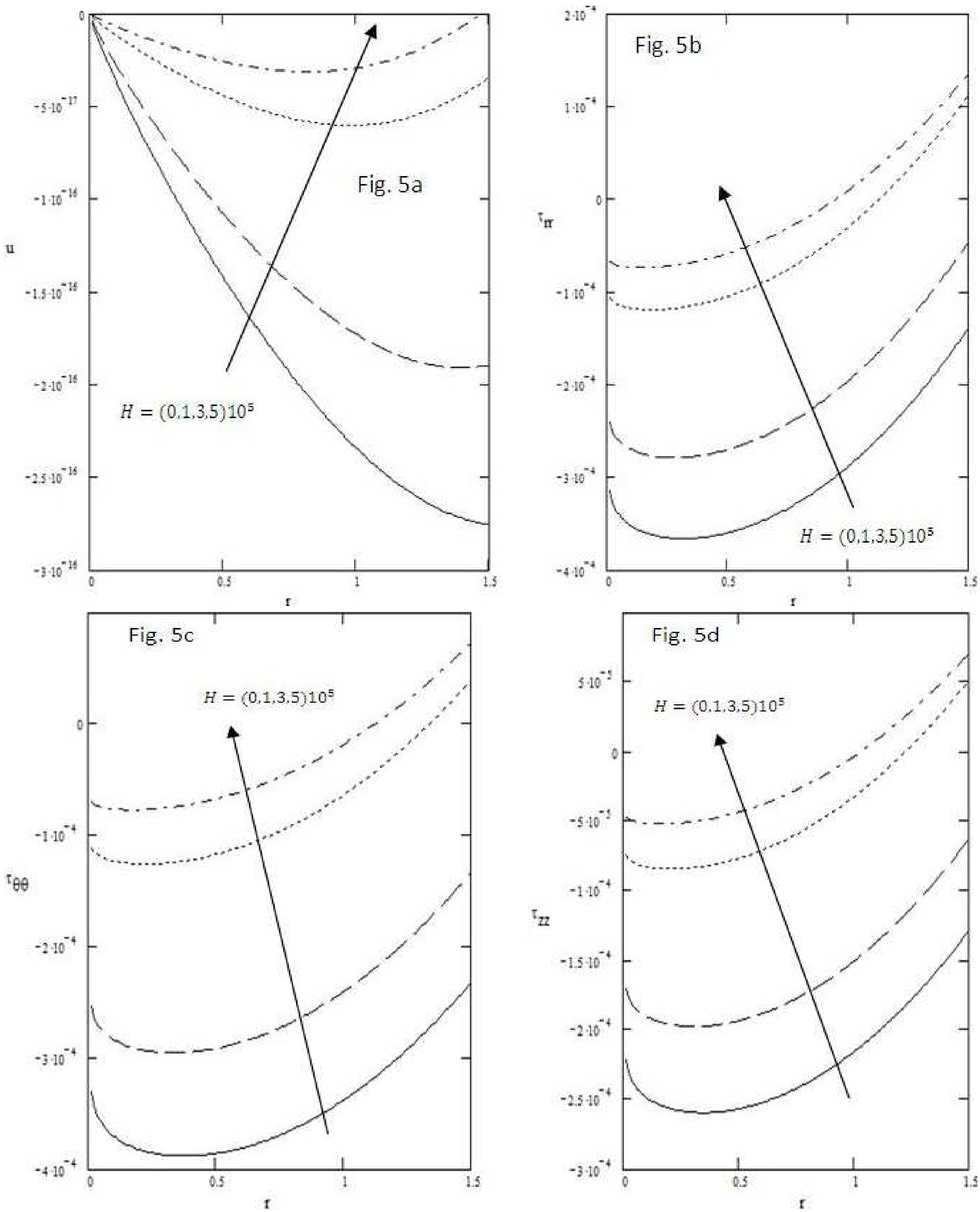

Fig. 5: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of H.

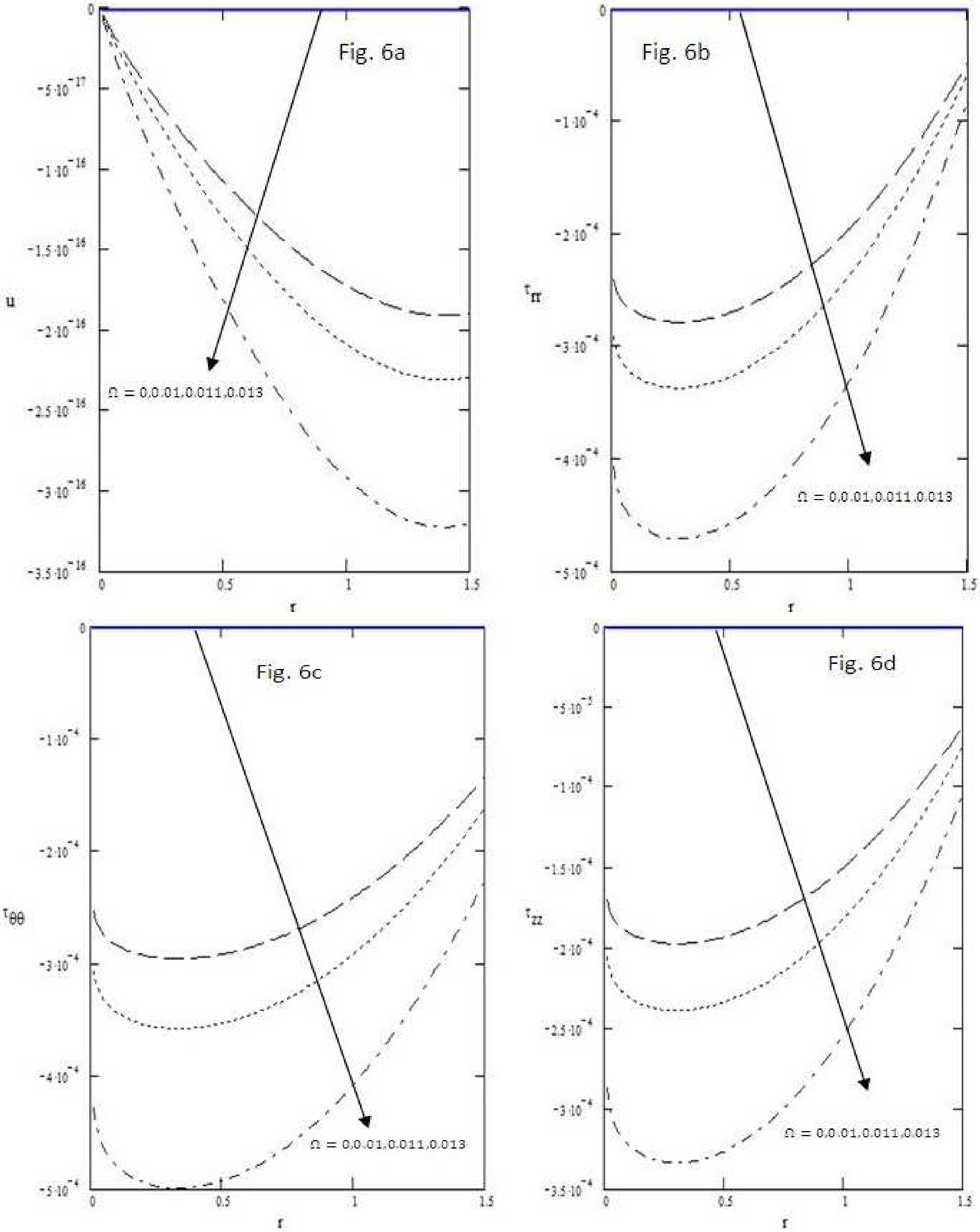

Fig. 6: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of Ω.

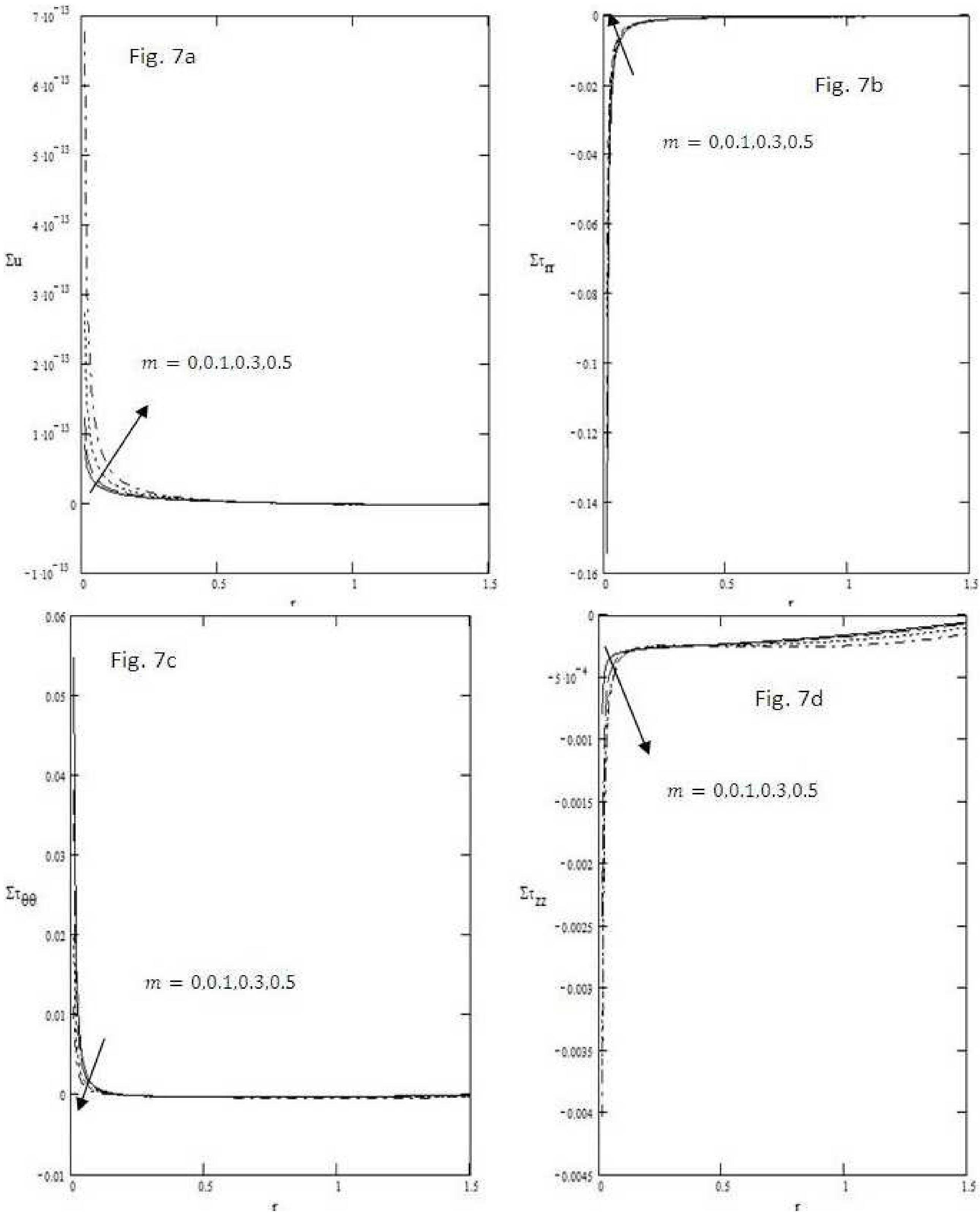

Fig. 7: Variations of $\sum u$,$\sum \tau_{rr}$, $\sum \tau_{\theta\theta}$,$\sum \tau_{zz}$ with respect to r with variation of m.

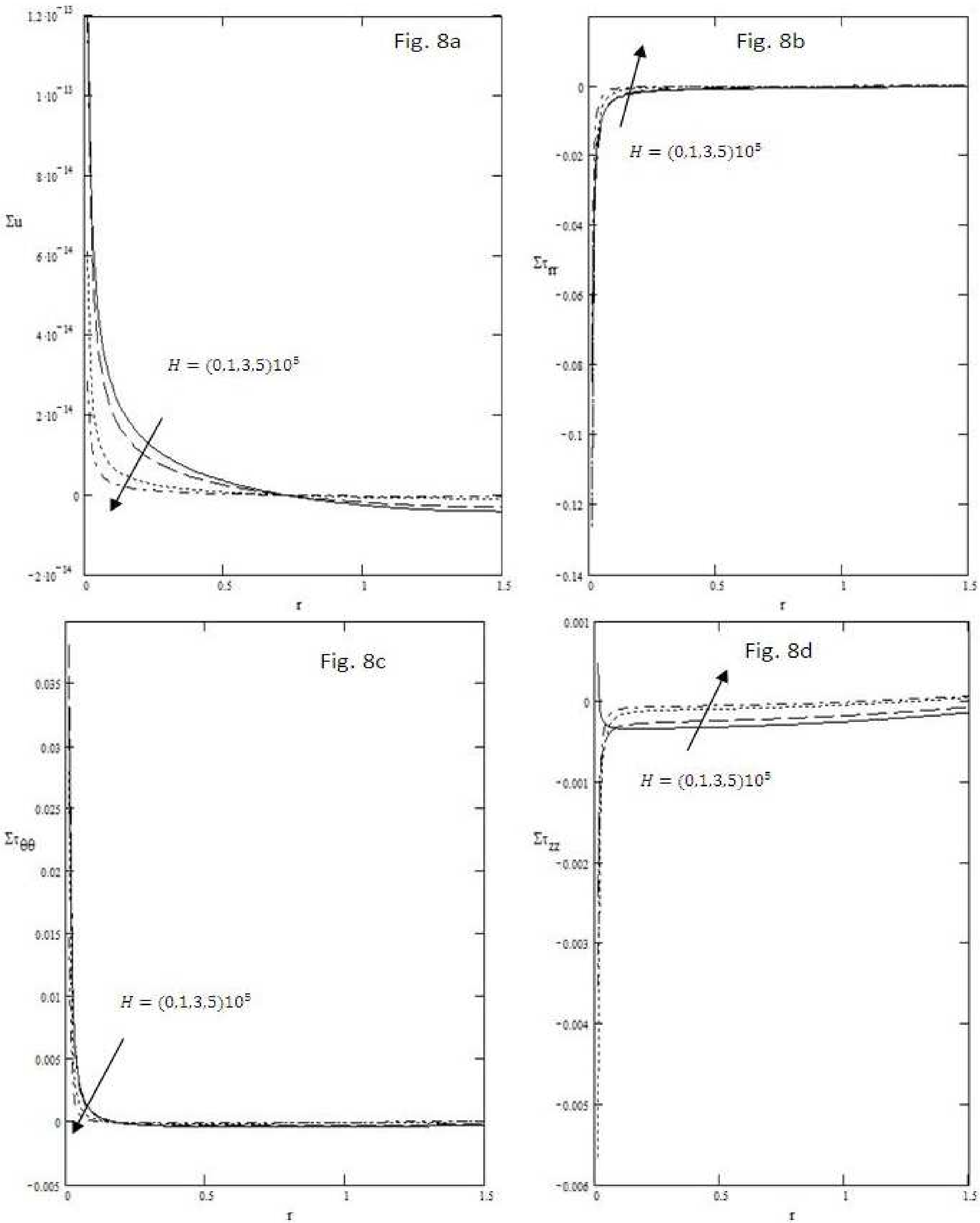

Fig. 8: Variations of $\sum u$,$\sum \tau_{rr}$, $\sum \tau_{\theta\theta}$,$\sum \tau_{zz}$ with respect to r with variation of H.

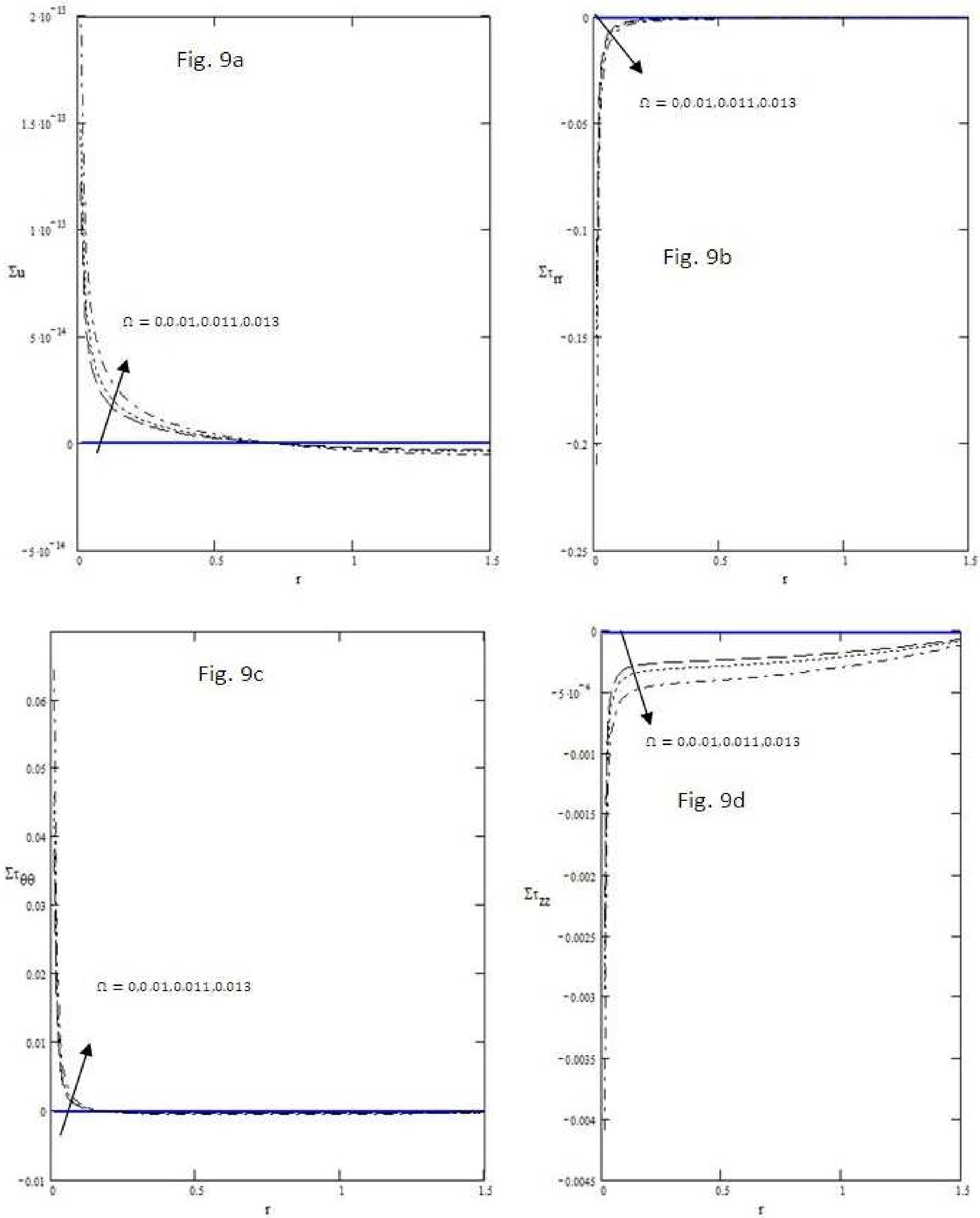

Fig. 9: Variations of $\sum u$,$\sum \tau_{rr}$, $\sum \tau_{\theta\theta}$,$\sum \tau_{zz}$ with respect to r with variation of Ω.

Non-homogeneous Isotropic Case

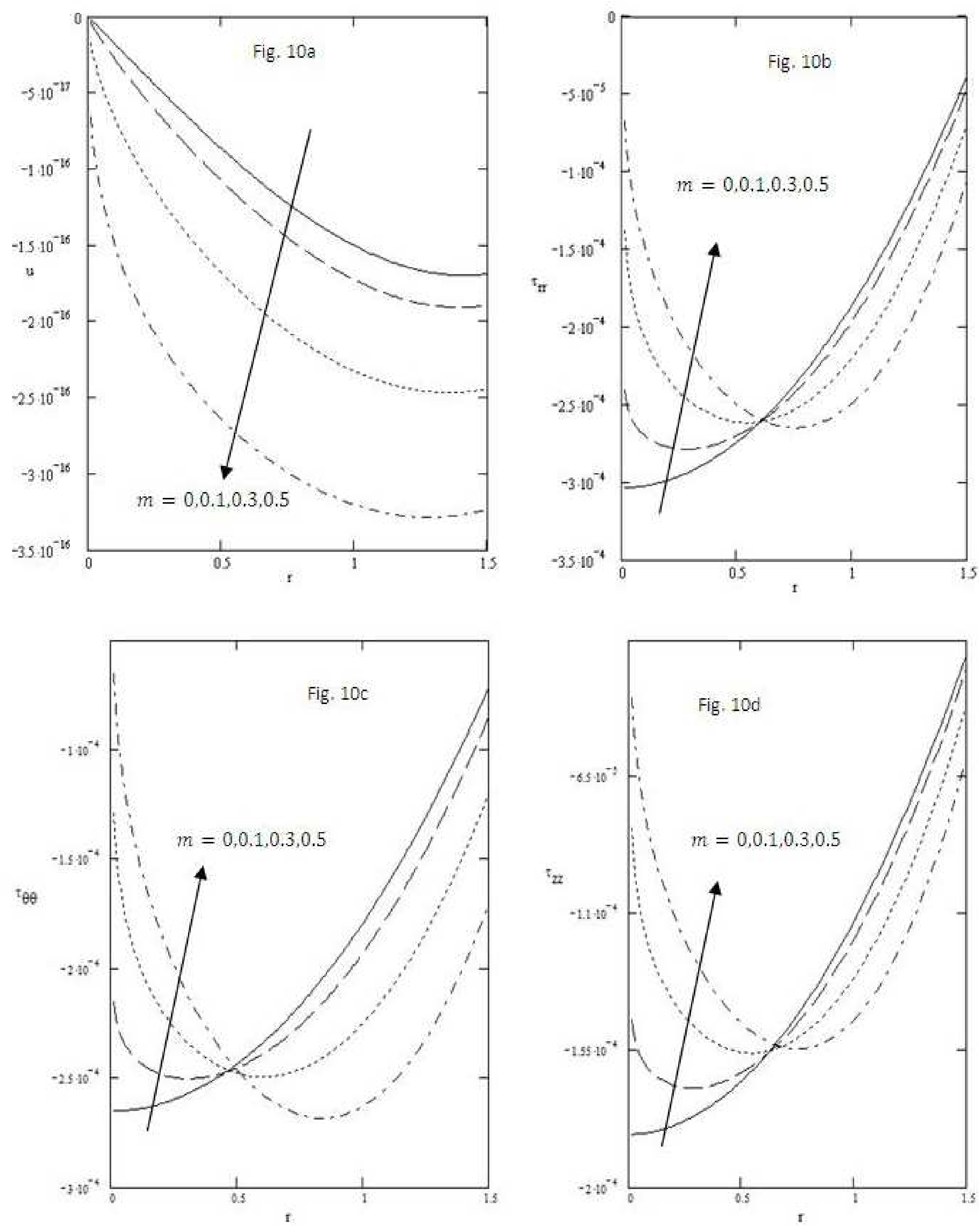

Fig. 10: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of m.

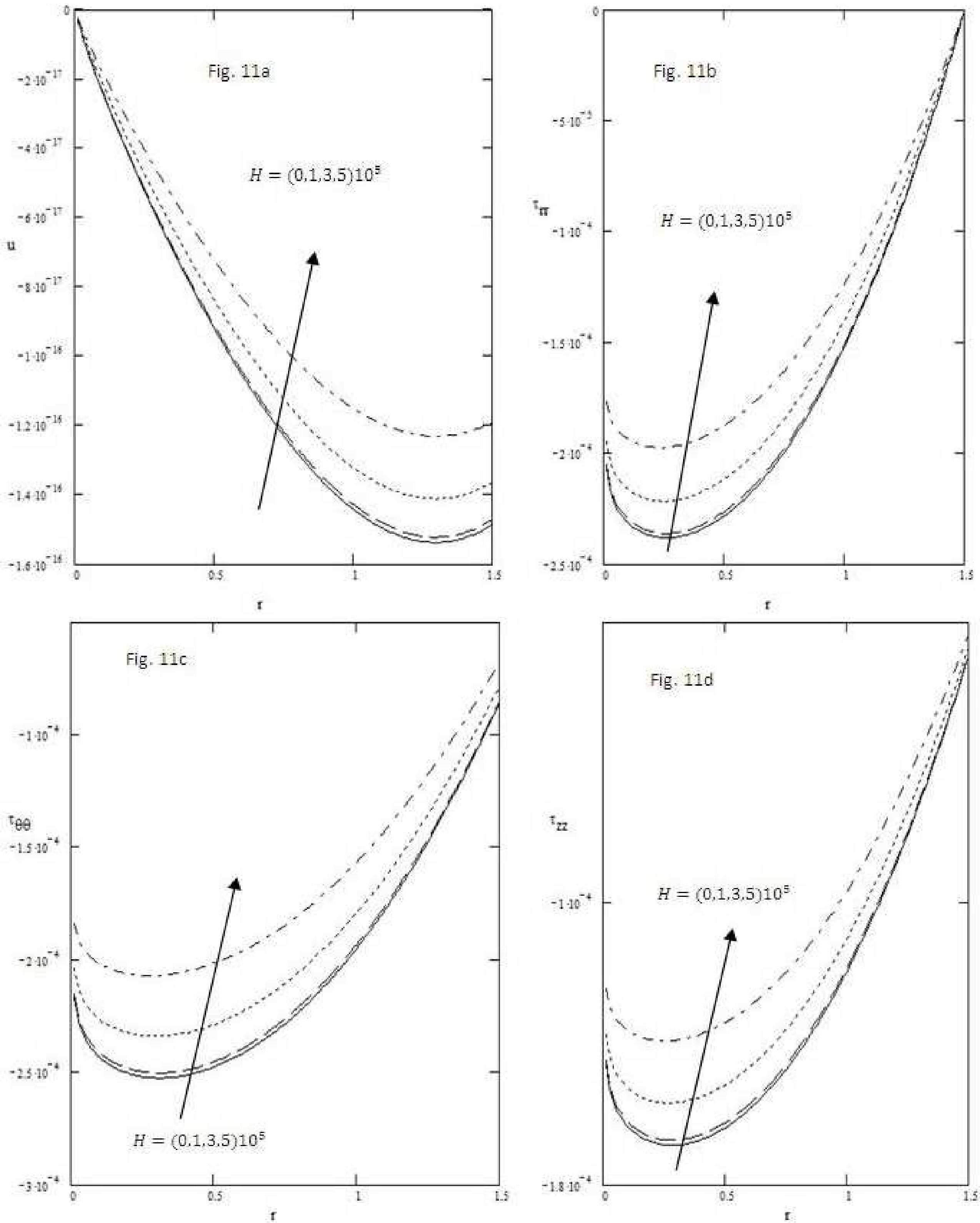

Fig. 11: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of H.

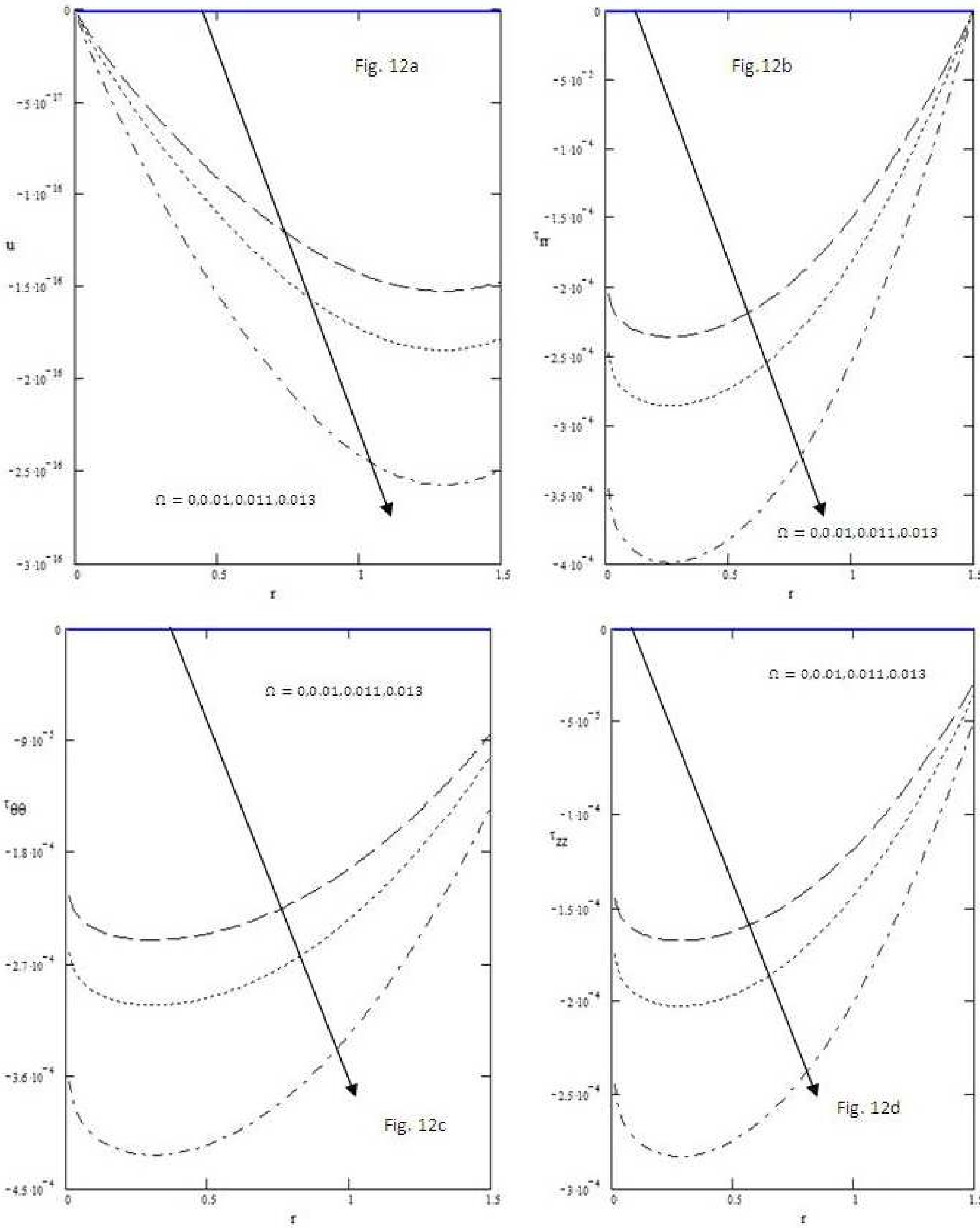

Fig. 12: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of Ω.

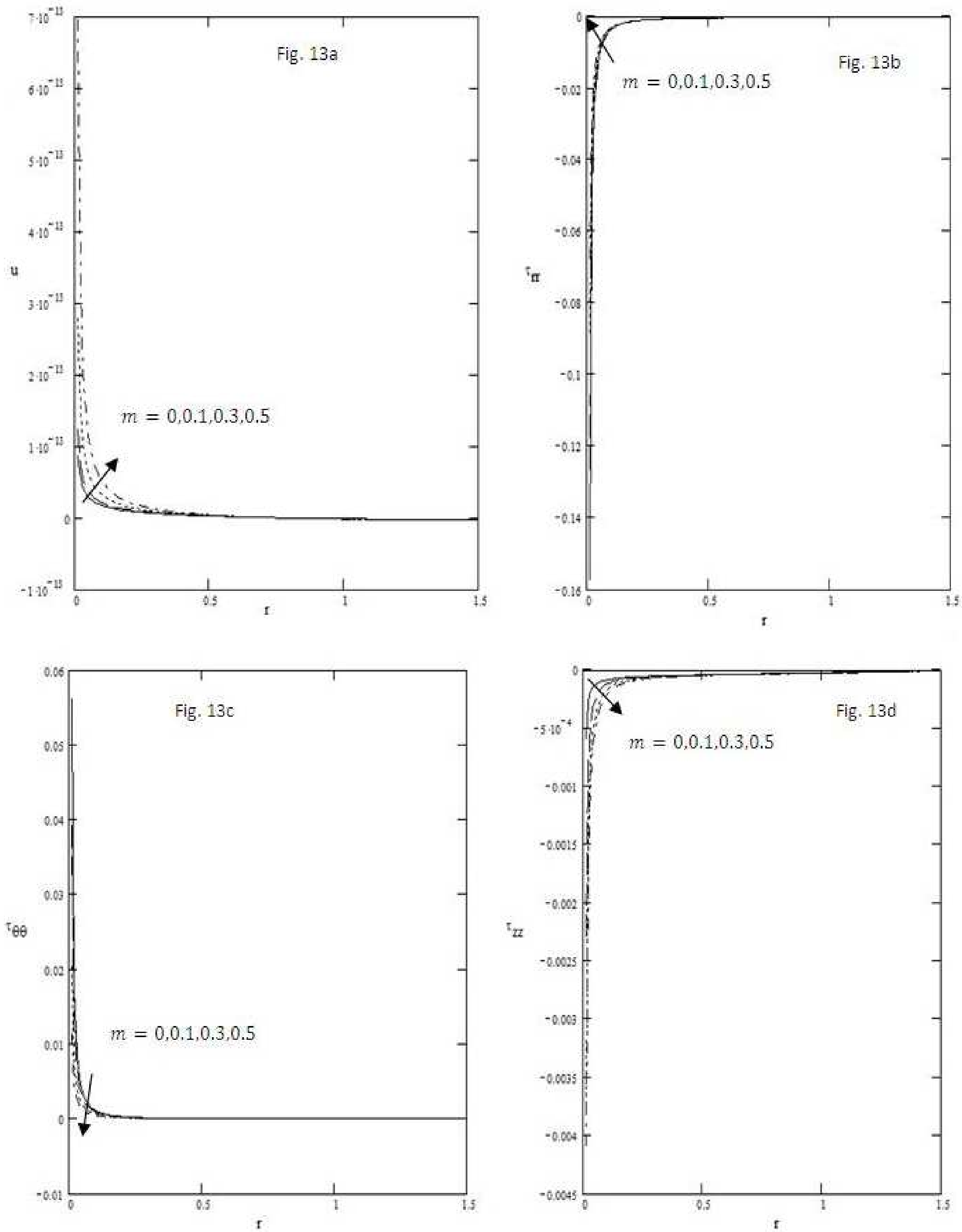

Fig. 13: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of m.

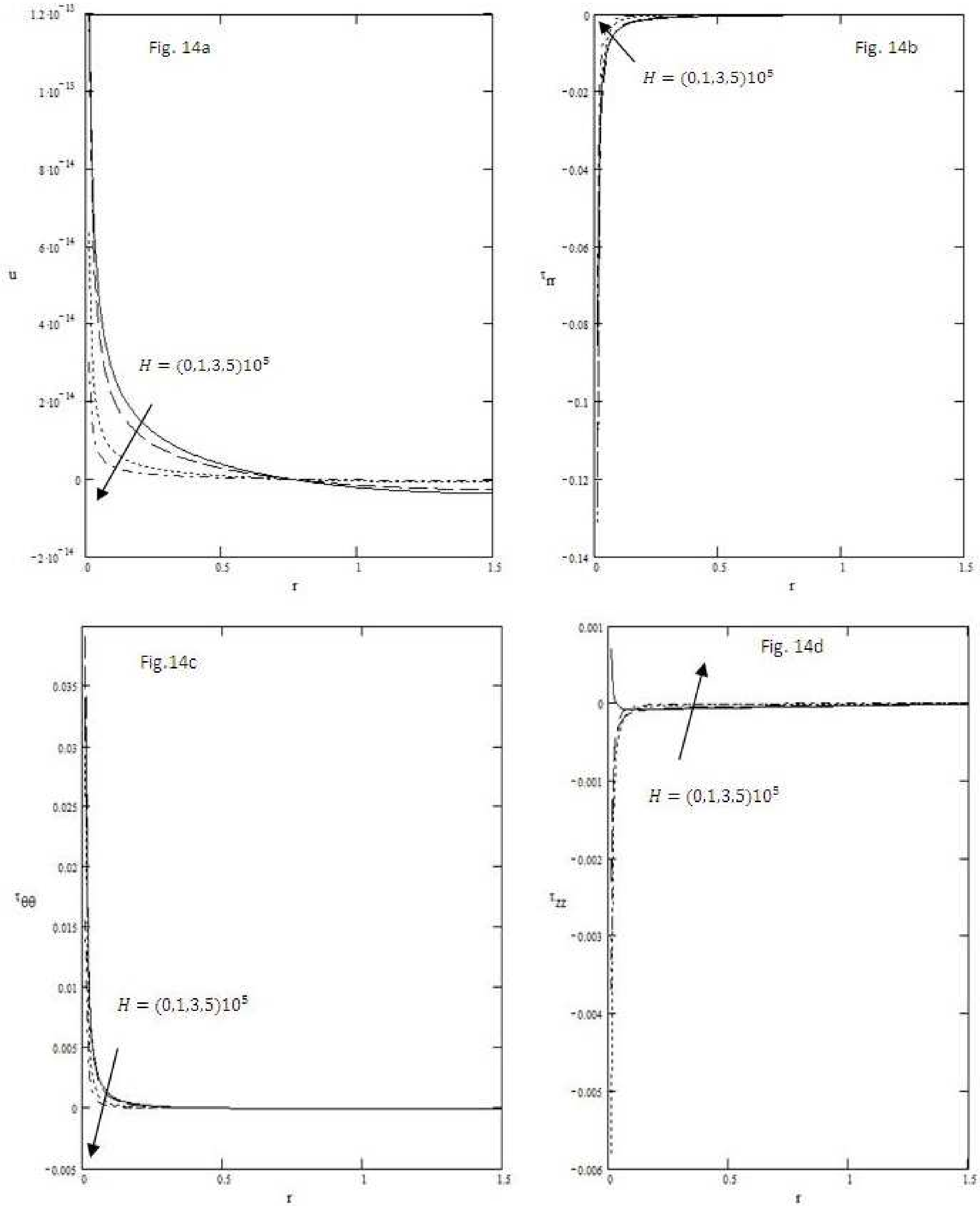

Fig. 14: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of H.

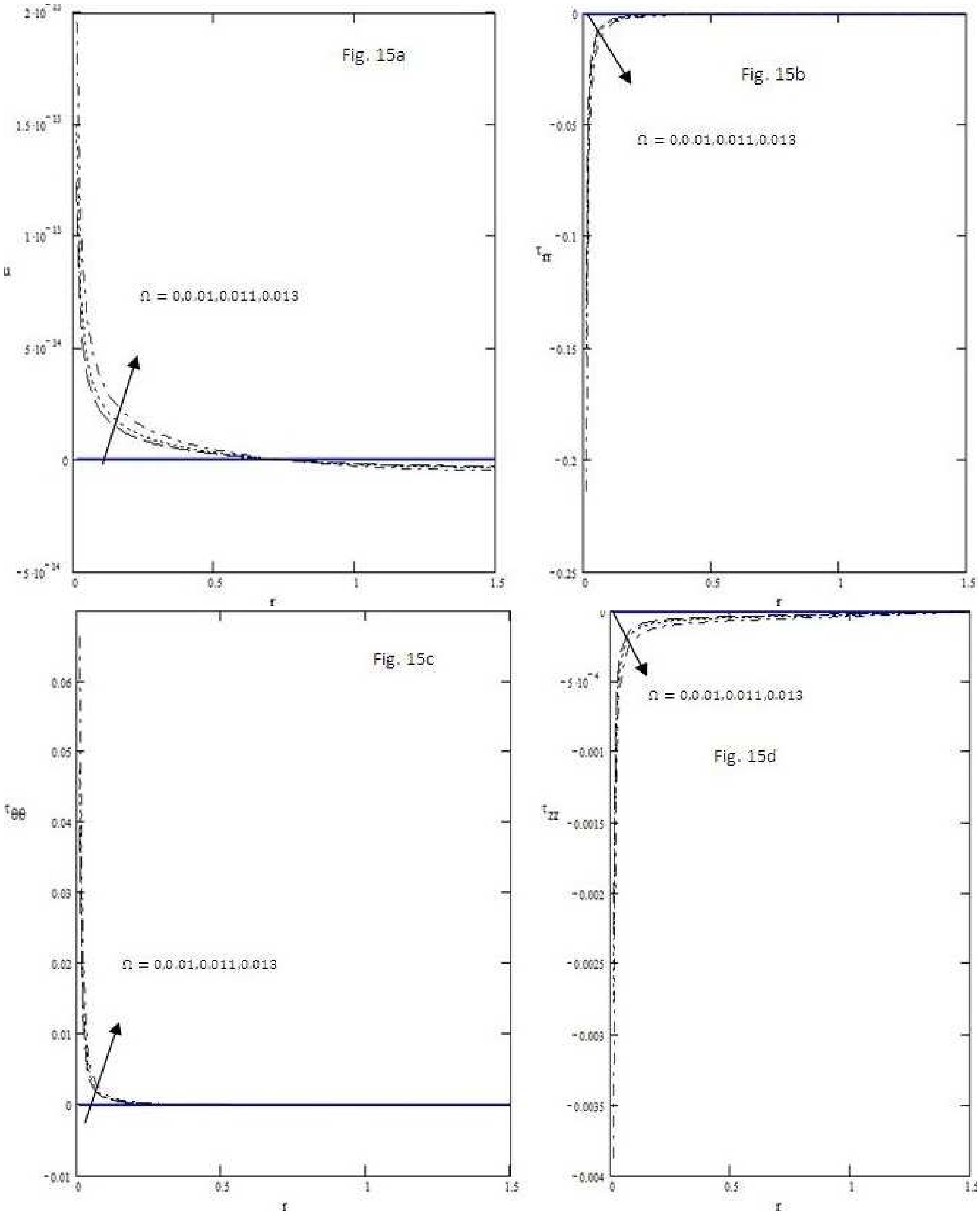

Fig. 15: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of Ω.

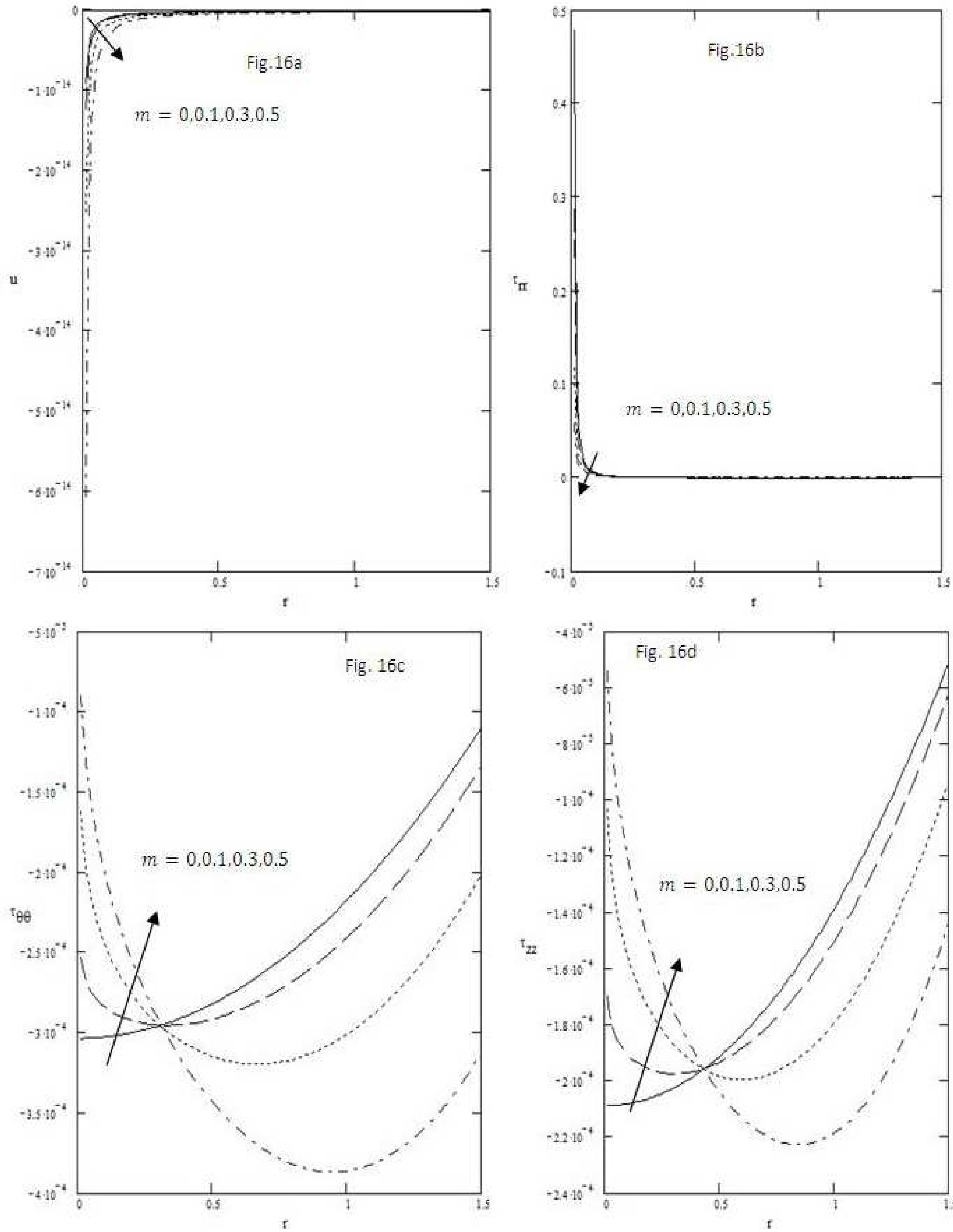

Fig. 16: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of m.

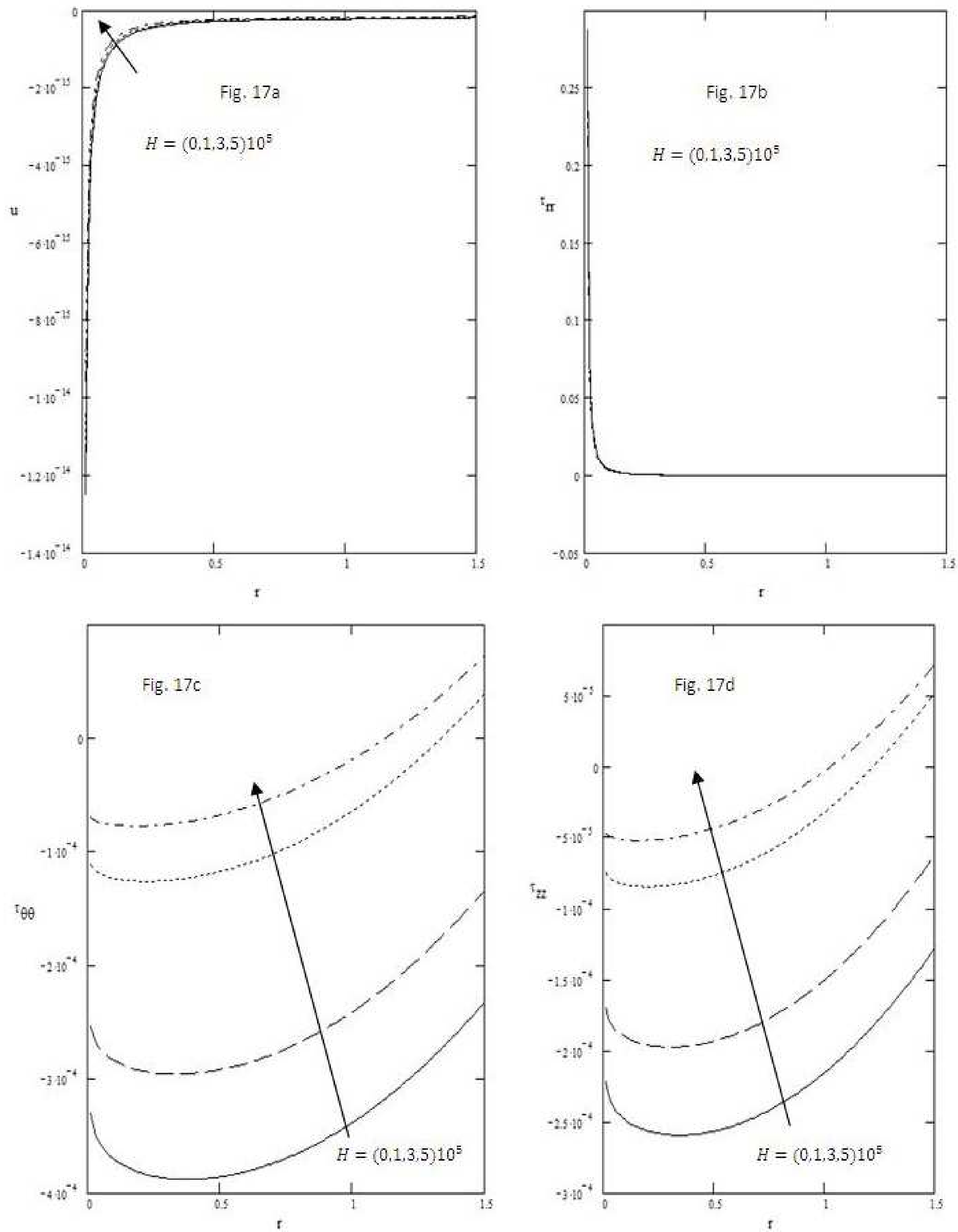

Fig. 17: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of H.

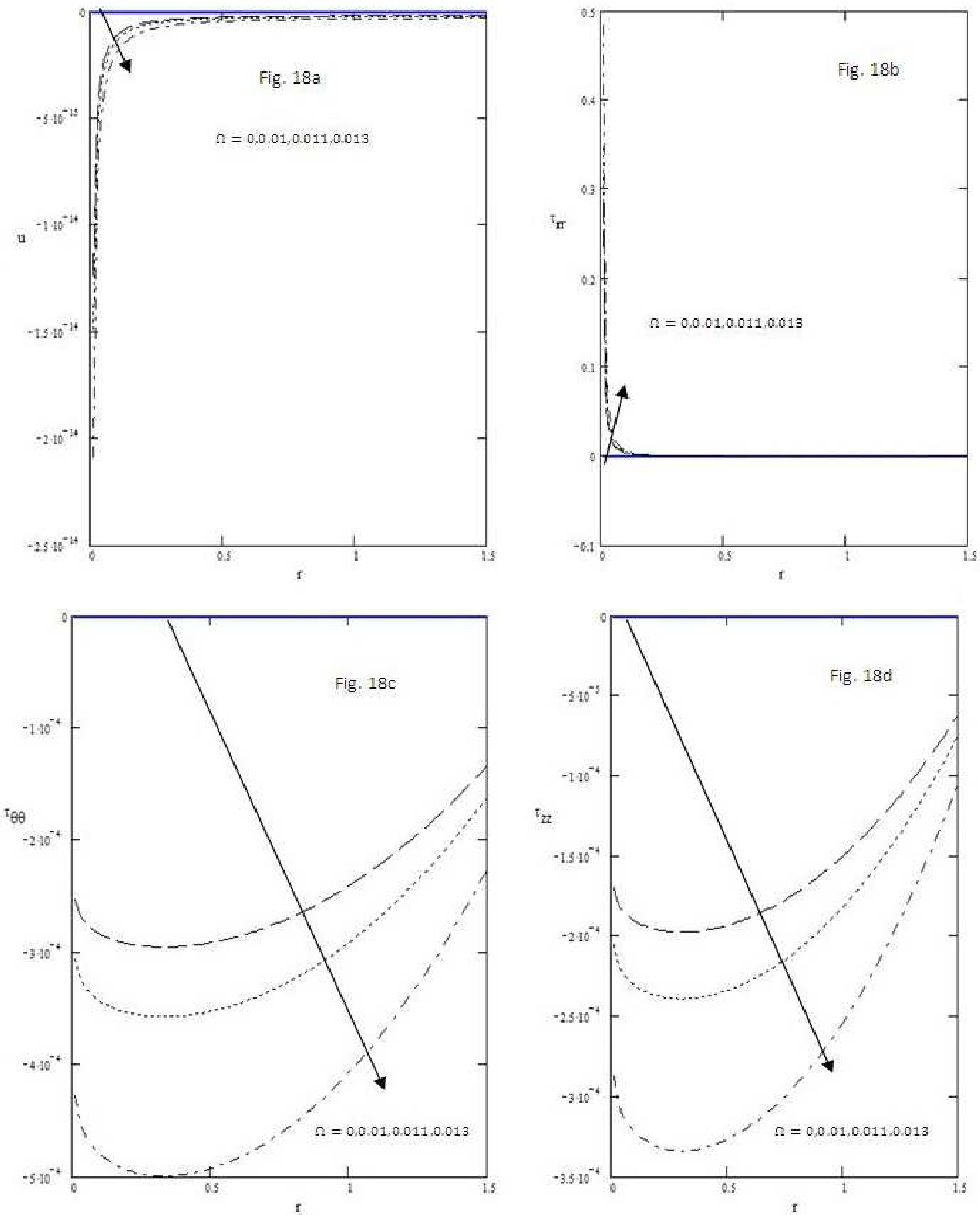

Fig. 18: Variations of u, τ_{rr}, $\tau_{\theta\theta}$, τ_{zz} with respect to r with variation of Ω.

rotation, while the radial displacement decreases with increasing of radius, as well the components of stresses has oscillatory behavior in the whole range of the r-axis, which it decreases and increases gradually.

Fig. (7) displays the variations of radial displacement $\sum u$, radial stress $\sum \tau_{rr}$ hoop stress $\sum \tau_{\theta\theta}$ and axial stress $\sum \tau_{zz}$ with respect to the radial r for different values of nonhomogeneity m. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the displacement increases with the increasing of nonhomogeneity while it decreases with an increasing of radius r, as well it is found that radial stress increases with increasing of nonhomogeneity and radius, hoop stress decreases with increasing of nonhomogeneity and radius, while axial stress decreases with increasing of nonhomogeneity and it increases with increasing of r.

Fig. (8) shows the variations of radial displacement $\sum u$, radial stress $\sum \tau_{rr}$, hoop stress $\sum \tau_{\theta\theta}$ and axial stress $\sum \tau_{zz}$ with respect to the radial r for different values of magnetic field H. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement decreases with the increasing of magnetic field while it decreases with an increasing of radius r , as well it is found that radial stress increases with increasing of magnetic field and radius, hoop stress decreases with increasing of magnetic field and radius, while axial stress increases with increasing of magnetic field and it increases with increasing of r.

Fig. (9) clears the variations of radial displacement $\sum u$, radial stress $\sum \tau_{rr}$, hoop stress $\sum \tau_{\theta\theta}$ and axial stress $\sum \tau_{zz}$ with respect to the radius r for different values of rotation Ω. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the displacement increases with an increasing of rotation while it decreases with an increasing of radius r, as well as it is found that radial stress decreases with increasing of rotation, while it increases with increasing of radius. Also, it is seen that hoop stress increases with increasing of rotation, but it decreases with increasing of radius tends to zero, while axial stress decreases with an increasing of rotation and it increases with increasing of radius r.

Non-homogeneous isotropic.

Fig. (10) shows the variations of radial displacement u , radial stress τ_{rr} , hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r which it has oscillatory behavior in the whole range of the $r-axis$ for different values of nonhomogeneity m. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded regio,n of space. It is observed that the radial displacement decreases with the increasing of nonhomogeneity, while it increases and decreases with increasing of radius r, as well it is found that radial stress, hoop stress and axial stress decreases increase with the increasing of nonhomogeneity, while the components of stress intersected at $r = 1.0, 0.5, 0.75$, respectively.

Fig. (11) displays that the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of magnetic field H . In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement, radial stress, hoop stress and axial stress decreases increase with increasing of magnetic field, while the radial displacement decreases with increasing of radius, as well the components of stresses decrease and increases gradually.

Fig. (12) shows the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of rotation Ω. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement, radial stress, hoop stress and axial stress decreases with increasing of rotation, while the radial displacement decreases with increasing of radius, as well the components of stresses has oscillatory behavior in the whole range of the $r-axis$, which it decreases and increases gradually.

Fig. (13) show the variations of radial displacement u, radial stress τ_{rr} , hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of nonhomogeneity m. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement increases with the increasing of nonhomogeneity while it decreases with an increasing of radius r, as well it is found that radial stress increases with increasing of nonhomogeneity and radius, hoop stress decreases with increasing of nonhomogeneity and radius, while the axial stress decreases with increasing of nonhomogeneity and it increases with increasing of r.

Fig. (14) shows the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of magnetic field H . In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space. It is observed that the radial displacement decreases with the increasing of magnetic field while it decreases with an increasing of radius r , as well it is found that radial stress increases with increasing of magnetic field and radius, hoop stress decreases with increasing of magnetic field and radius, while axial stress increases with increasing of magnetic field and it increases with increasing of r.

Fig. (15) displays the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of rotation Ω . It is observed that the displacement increases with the increasing of rotation while it decreases with an increasing of radius r, as well it is found that radial stress decreases with increasing of rotation, while it increases with increasing of radius, hoop stress increases with increasing of rotation, but it decreases with increasing of radius, while axial stress decreases with increasing of rotation and it increases with increasing of r.

Fig. (16) displays the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of nonhomogeneity m. In both figures, it is clear that the radial displacement and radial stress have a non zero value only in a bounded region of space, while the hoop stress and axial stress have an oscillatory behavior in the whole range of the $r-axis$. It is observed that the radial displacement decreases with the increasing of nonhomogeneity, while it increases with increasing of radius r, as well it is found that radial stress decreases with increasing of nonhomogeneity and radius, while the components of hoop stress and axial stress increase with the increasing of nonhomogeneity and it intersected at $r=0.4$.

Fig. (17) displays the variations of radial displacement u, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of magnetic field H. In both figures, it is clear that the radial displacement, radial stress, hoop stress and axial stress have a non zero value only in a bounded region of space, while the hoop stress and axial stress have an oscillatory behavior in the whole range of the $r-axis$. It is observed that the radial displacement increases with the increasing of magnetic field., while it increases with increasing of radius r , as well it is found that there is no effect of magnetic field and radius on the radial stress, while the components of hoop stress and axial stress increase with increasing of magnetic field.

Fig. (18) shows the variations of radial displacement$u$, radial stress τ_{rr}, hoop stress $\tau_{\theta\theta}$ and axial stress τ_{zz} with respect to the radial r for different values of rotation Ω. In both figures, it is clear that the radial displacement, hoop stress and axial stress have a non zero value only in a bounded region of space, while the hoop stress and axial stress have an oscillatory behavior in the whole range of the $r-axis$. It is observed that the radial displacement decreases with the increasing of rotation Ω, while the radial stress increases with increasing of rotation and it decreases with increasing of radius r, as well it is found that while the components of hoop stress and axial stress decrease with increasing of rotation Ω.

Finally, If the magnetic field neglected, the relevant results obtained tend to Abd-Alla *et al.* [11].

6 Conclusion

Due to the complicated nature of the governing equations of the magnetoelastic theory, the done works in this field are unfortunately limited. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the elastic medium without any restrictions on the actual physical quantities that appear in the governing equations of the considered problem. Important phenomena are observed in these computations.

– It was found that for large values of the magnetic field, rotation and nonhomogeneity give close results. The case is quite different when we consider small value of the rotation. The solutions obtained in the context of elasticity theory, however, exhibit the behavior of speeds of wave propagation.
– Comparing Figs. (1)-(18) for elastic medium for homogeneous, non-homogeneous, isotropic and orthotropic medium, it was found that the radial displacement and the components of stresses have the same behavior nearly in both media.
– The results presented in this paper will be very helpful for researchers concerning with material science, designers of new materials, as well as for those working on the development of a theory of hyperbolic propagation of elasticity. Study of the phenomenon of rotation, magnetic field and nonhomogeneity is also used to improve the conditions of oil extractions.

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