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# Effects of viscosity and stiffness on amplitude ratios in microstretch viscoelastic media

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The present investigation is concerned with the reflection and transmission of plane waves at an imperfect interface between two microstretch visoelastic half-spaces of different properties. It is shown that there exist four waves which comprises of two sets of coupled waves. The expressions for the reflection and transmission coefficients which are the ratios of the amplitude of reflected and transmitted waves to the angle of incident wave are obtained and deduced for normal force stiffness,transverse force stiffness, transverse couple stiffness, microstress stiffness and perfect bonding. The numerical results obtained have been illustrated graphically to understand the behavior of amplitude ratios versus angle of incidence of longitudinal displacement wave(LD-wave), longitudinal microstretch wave(LMS-wave)and coupled transverse displacement and microrotational wave(CD I-wave). It is found that the amplitude ratios of various reflected and transmitted waves are affected by the stiffness and viscosity of the media. Some special cases of interest have been deduced from the present investigation.
**Key Words** : Microstretch viscoelastic Solid, Normal and Transverse Force Stiffness, Transverse couple stiffness,Microstress stiffness, Perfect bonding, Amplitude Ratios.
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## 1 Introduction

Studies of propagation of elastic waves at an interface have long been of interest to researchers in the fields of geophysics, acoustics and nondestructive evaluation. Common to all these studies is the investigation of the degrees of interaction among the interface that manifest themselves in the forms of reflection and transmission agents and give rise to geometric dispersion. These interactions depend among other factors, upon the mechanical properties,geometric arrangements,number and nature of the interfacial conditions and on the loading conditions.

The theory of microstretch elastic solids has been introduced by Eringen(1971, 1991, 1999). This theory is a special case of the micromorphic theory. In the framework of micromorphic theory a material point is endowed with three deformable directors. When the directors are constrained to have only breathing-type microdeformations, then the body is a microstretch continuum (1999). The material points of this continua can stretch and contract independently of their translations and rotations. The theory is expected to find applications in the treatment of the mechanics of composite materials reinforced with chopped fibers and various porous materials. The theory of microstrech continua is a generalization of the theory of micropolar continua. The problem of micropolar viscoelastic waves has been discussed by McCarthy and Eringen(1969). Manole(1988,1992) and Gale(2000) presented some theorems on viscoelastic medium. Source problems on micropolar viscoelasticity is discussed by Kumar and Choudhary(2001,2005). Kumar(2000) investigated wave propagation in micropolar viscoelastic generalized thermoelastic solid. Recently, Singh and Kumar (2007) investigated reflection of wave at viscoelastic-micropolar elastic interface.

An actual interface between two elastic solids is much more complicated and has physical properties different from those of the substrates. There are two classical elastic boundary conditions for solid/solid interface. One boundary condition for welded interface and other is slip boundary condition. A generalization of this concept is that of an imperfectly bonded interface for which displacement across a surface need not be continuous.

Imperfect bonding considered in the present investigation is to mean that the stress components are continuous and small displacement field is not. The small vector difference in the displacement is assumed to depend linearly on the traction vector. Significant work has been done to describe the physical conditions on the interface by different mechanical boundary conditions by different investigators. Notable among them are Jones and Whitter(1967), Murty(1975), Nayfeh and Nassar(1978), Rokhlin *et.al.*(1980), Rokhlin(1984), Baik and Thomson(1984), Achenbach et.al.(1985), Lavrentyev and Rokhlin(1998). Recently various authors have used the imperfect conditions at an interface to study various types of problems(2001,2006(a),2006(b)).

In the present investigation, we studied the reflection and transmission of microstretch viscoelastic plane waves between two microstretch viscoelastic half-spaces and deduced the different cases.

## 2 Basic equations

Following Eringen(1999), the constitutive relations and field equations in microstretch solid in absence of body forces and body couples are given by

$$(\lambda_I+2\mu_I+K_I)\nabla(\nabla\cdot\overrightarrow{u})-(\mu_I+K_I)\nabla\times\nabla\times\overrightarrow{u}+K_I\nabla\times\overrightarrow{\phi}+\lambda_{0I}\nabla\phi^* = \rho\frac{\partial^2\overrightarrow{u}}{\partial t^2}, \quad (2.1)$$

$$(\alpha_I+\beta_I+\gamma_I)\nabla(\nabla\cdot\overrightarrow{\phi})-\gamma_I\nabla\times(\nabla\times\overrightarrow{\phi})+K_I\nabla\times\overrightarrow{u}-2K_I\overrightarrow{\phi}=\rho j\frac{\partial^2\overrightarrow{\phi}}{\partial t^2}, \quad (2.2)$$

$$\alpha_{0I}\nabla^2\phi^*-\lambda_{1I}\phi^*-\lambda_{0I}\nabla\cdot\overrightarrow{u}=\frac{\rho j_0}{2}\frac{\partial^2\phi^*}{\partial t^2}, \quad (2.3)$$

$$t_{ij}=\lambda_I\, u_{r,r}\delta_{ij}+\mu_I(u_{i,j}+u_{j,i})+K_I(u_{j,i}-\epsilon_{ijr}\phi_r)+\lambda_{0I}\delta_{ij}\phi^*, \quad (2.4)$$

$$m_{ij}=\alpha_I\phi_{r,r}\delta_{ij}+\beta_I\phi_{i,j}+\gamma_I\phi_{j,i}+b_{0I}\epsilon_{mji}\phi^*_{,m} \qquad (i,j,k,l=1,2,3), \quad (2.5)$$

$$\lambda_k=\alpha_{0I}\phi^*_{,k}+b_{0I}\epsilon_{klm}\phi_{l,m} \quad (2.6)$$

Assuming the viscoelastic nature of the material, described by Voigt(1987) model of linear viscoelasticity, by replacing the microstretch elastic constants, $\lambda,\mu,K,\alpha,\beta,\gamma,\alpha_0,\lambda_0,\lambda_1,b_0$ with $\lambda_I,\mu_I,K_I,\alpha_I,\beta_I,\gamma_I,\alpha_{0I},\lambda_{0I},\lambda_{1I},b_{0I}$

$$\lambda_I=\lambda+\frac{\partial}{\partial t}\lambda_v,\quad \mu_I=\mu+\frac{\partial}{\partial t}\mu_v,\quad K_I=K+\frac{\partial}{\partial t}K_v,\quad \alpha_I=\alpha+\frac{\partial}{\partial t}\alpha_v,$$

$$\beta_I=\beta+\frac{\partial}{\partial t}\beta_v,\quad \gamma_I=\gamma+\frac{\partial}{\partial t}\gamma_v,\quad \lambda_{0I}=\lambda_0+\frac{\partial}{\partial t}\lambda_{0v},\qquad \mu_I=\mu+\frac{\partial}{\partial t}\mu_v,$$

$$\lambda_{1I}=\lambda_1+\frac{\partial}{\partial t}\lambda_{1v},\quad b_{0I}=b_0+\frac{\partial}{\partial t}b_{0v}, \quad (2.7)$$

where $\lambda,\mu,K,\alpha,\beta,\gamma,\alpha_0,b_0,\lambda_1,\lambda_0,\lambda_v,\mu_v,K_v,\alpha_v,\beta_v,\gamma_v,\alpha_{0v},b_{0v},\lambda_{1v},\lambda_{0v}$- material constants, $\rho$-density, $\overrightarrow{u}$- displacement vector, $\overrightarrow{\phi}$-microrotation vector, $\lambda_k$-microsretch tensor, $\phi^*$-scalar point microstretch function, $j$-microinteria, $j_0$-microinteria of microcomponents, $\epsilon_{klr}$- alternate tensor, $t_{ij}$-components of force stress tensor, $m_{ij}$-components of couple stress tensor, $\delta_{ij}$- Kronecker delta.

## 3 Formulation and Solution of the Problem

We consider two homogeneous, isotropic microstretch viscoelastic half-spaces being in contact with each other at the plane surface which we designate as the plane z=0 of rectangular cartesian co-ordinate system OXYZ.We consider microstretch viscoelastic plane waves in xz-plane with wave front parallel to y-axis and all the field variables depend only on x,z,t.

For the two dimensional problem,the components of displacement and microrotation are given by

$$\overrightarrow{u} = (u_1, 0, u_3), \quad \overrightarrow{\phi} = (0, \phi_2, 0), \tag{3.1}$$

The components of displacement $u_1$, $u_3$ are related by the potential functions $q(x, z, t)$ and $\psi(x, z, t)$ as

$$u_1 = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{3.2}$$

Making use of equations (3.1)-(3.2)in equations (2.1)-(2.3), assuming the time harmonic behavior as $exp(i\omega t)$ and eliminating $\phi^*$ and $\phi_2$ from the resulting equations,we obtain

$$(\nabla^4 + A\omega^2\nabla^2 + B\omega^4)q = 0 \tag{3.3}$$

$$(\nabla^4 + C\omega^2\nabla^2 + D\omega^4)\psi = 0 \tag{3.4}$$

where

$$A = \frac{-\lambda_I}{\alpha_{0I}\omega^2} + \frac{\rho j_0}{2\alpha_{0I}} + \frac{\rho\alpha_{0I}\omega^2 + \lambda_{0I}^2}{(\lambda_I + K_I)\alpha_{0I}\omega^2}, \quad B = \frac{-\rho\lambda_{1I}}{(\lambda_I + K_I)\alpha_{0I}} + \frac{\rho^2 j_0}{2(\lambda_I + K_I)\alpha_{0I}},$$

$$C = \frac{\rho}{\mu_I + K_I} + \frac{\rho j}{\gamma_I} + \frac{(p_I - 2)q_I}{\omega^2}, \quad D = \frac{\rho}{\mu_I + K_I}[\frac{\rho j}{\gamma_I} - \frac{2q_I}{\omega^2}],$$

$$p_I = \frac{K_I}{\mu_I + K_I}, \quad q_I = \frac{K_I}{\gamma_I}.$$

The general solution of equation (3.3) and (3.4) can be written as

$$\overline{q} = \overline{q}_1 + \overline{q}_2, \quad \overline{\psi} = \overline{\psi}_1 + \overline{\psi}_2 \tag{3.5}$$

where the potentials $\overline{q}_1$, $\overline{q}_2$,$\overline{\psi}_1$, $\overline{\psi}_2$ are solutions of wave equations:

$$[\nabla^2 + \frac{\omega^2}{V_j^2}]\overline{q}_j = 0, \quad j = 1, 2, \quad [\nabla^2 + \frac{\omega^2}{V_j^2}]\overline{\psi}_j = 0, \quad j = 3, 4 \tag{3.6}$$

$$V_j^{-2} = \frac{[A + (-1)^j(A^2 - 4B)^{\frac{1}{2}}]}{2}, \quad j = 1, 2 \tag{3.7}$$

$$V_j^{-2} = \frac{[C + (-1)^j(C^2 - 4D)^{\frac{1}{2}}]}{2}, \quad j = 3, 4 \tag{3.8}$$

The roots of equations (3.7) correspond to longitudinal displacement wave(LD)and longitudinal microstretch wave(LMS) whereas roots of equations (3.8) correspond to transverse shear wave and transverse microrotaional wave(CD-I and CD-II).

## 4 Reflection and Transmission

We consider microstretch viscoelastic wave(LD-or LMS-or-CD I-or CD II-) propagating through the medium $M$, which we designate as the region$z > 0$ and incident at the plane $z = 0$ with its direction of propagating with angle $\theta_0$ normal to the surface. Corresponding to each incident wave, we get waves in the medium M as reflected waves and transmitted in medium $M'$. We write all the variables without a prime in the region $z > 0$ (medium $M$) and attach a prime to denote the variables in the region $z < 0$ (medium $M'$)as shown in Fig.(a)(geometry of the problem) is given in Appendix I.

## 5 Boundary Conditions

We consider two bonded microstretch viscoelastic half-spaces as shown in Fig(a). (Appendix I). If the bonding is imperfect and the size and spacing between the imperfections is much smaller than the wave-length then at the interface, these can be described by using boundary conditions at $z = 0$ (Lavrentyev and Rokhlin(1998)) as

$$(i)(t_{33})_{\acute{M}} = K_n[(u_3)_M - (u_3)_{\acute{M}}], \tag{5.1}$$

$$(ii)(t_{31})_{\acute{M}} = K_t[(u_1)_M - (u_1)_{\acute{M}}], \tag{5.2}$$

$$(iii)(m_{32})_{\acute{M}} = K_c[(\phi_2)_M - (\phi_2)_{\acute{M}}], \tag{5.3}$$

$$(iv)\lambda_3 = K_\lambda[(\phi^*)_M - (\phi^*)_{\acute{M}}] \tag{5.4}$$

$$(v)(t_{33})_M = (t_{33})_{\acute{M}}, \tag{5.5}$$

$$(v)(t_{31})_M = (t_{31})_{\acute{M}}, \tag{5.6}$$

$$(vi)(m_{32})_M = (m_{32})_{\acute{M}}, \tag{5.7}$$

$$(vi)(\lambda_3)_M = (\lambda_3)_{\acute{M}}, \tag{5.8}$$

where $K_n$, $K_t$, $K_c$ and $K_\lambda$ are the normal force stiffness,transverse force stiffness, transverse couple stiffness and microstress stiffness and coefficients of a unit layer thickness and having dimension $\frac{N}{m^3}$(normal force stiffness, transverse force stiffness) and $\frac{N}{m}$ (transverse couple stiffness, microstress stiffness).

Appropriate potentials satisfying the boundary conditions (5.1) - (5.8)in medium $M$ and $M'$ can be written as

**Medium** $M$:

$$q = B_0 \exp(-\overrightarrow{A_0} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_0} \cdot \overrightarrow{r})] + B_1 \exp(-\overrightarrow{A_1} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_1} \cdot \overrightarrow{r})]$$
$$+ B_2 \exp(-\overrightarrow{A_2} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_2} \cdot \overrightarrow{r})] \tag{5.9}$$

$$\phi^* = a_1 B_0 \exp(-\overrightarrow{A_0} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_0} \cdot \overrightarrow{r})] + a_1 B_1 \exp(-\overrightarrow{A_1} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_1} \cdot \overrightarrow{r})]$$
$$+ a_2 B_2 \exp(-\overrightarrow{A_2} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_2} \cdot \overrightarrow{r})] \qquad (5.10)$$

$$\psi = B_0 \exp(-\overrightarrow{A_0} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_0} \cdot \overrightarrow{r})] + B_3 \exp(-\overrightarrow{A_3} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_3} \cdot \overrightarrow{r})]$$
$$+ B_4 \exp(-\overrightarrow{A_4} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_4} \cdot \overrightarrow{r})] \qquad (5.11)$$

$$\phi_2 = EB_0 \exp(-\overrightarrow{A_0} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_0} \cdot \overrightarrow{r})] + EB_3 \exp(-\overrightarrow{A_3} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_3} \cdot \overrightarrow{r})]$$
$$+ EB_4 \exp(-\overrightarrow{A_4} \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_4} \cdot \overrightarrow{r})] \qquad (5.12)$$

**Medium** $\acute{M}$:

$$q' = B_1' \exp(-\overrightarrow{A_1}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_1}' \cdot \overrightarrow{r})] + B_2' \exp(-\overrightarrow{A_2}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_2}' \cdot \overrightarrow{r})] \qquad (5.13)$$

$$\phi^{*'} = a_1' B_1' \exp(-\overrightarrow{A_1}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_1}' \cdot \overrightarrow{r})] + a_2' B_2' \exp(-\overrightarrow{A_2}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_2}' \cdot \overrightarrow{r})] \qquad (5.14)$$

$$\psi' = B_3' \exp(-\overrightarrow{A_3}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_3}' \cdot \overrightarrow{r})] + B_4' \exp(-\overrightarrow{A_4}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_4}' \cdot \overrightarrow{r})] \qquad (5.15)$$

$$\phi_2' = E' B_3' \exp(-\overrightarrow{A_3}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_3}' \cdot \overrightarrow{r})] + F' B_4' \exp(-\overrightarrow{A_4}' \cdot \overrightarrow{r}) \exp[i(\omega t - \overrightarrow{P_4}' \cdot \overrightarrow{r})] \qquad (5.16)$$

The propagation vector $\overrightarrow{P_j}$, $\overrightarrow{P_j}'$ and attenuation vector $\overrightarrow{A_j}$ ,$\overrightarrow{A_j}'$ are given by

$$\overrightarrow{A_j} = -K_I \hat{x} - dV_{jI} \hat{z}, \quad \overrightarrow{P_j} = K_R \hat{x} + dV_{jR} \hat{z}, \quad j = 1, 2, 3, 4 \qquad (5.17)$$

$$\overrightarrow{A_j}' = -K_I \hat{x} + dV_{jI}' \hat{z}, \quad \overrightarrow{P_j}' = K_R \hat{x} - dV_{jR}' \hat{z}, \quad j = 1, 2, 3, 4 \qquad (5.18)$$

where

$$dV_j = dV_{jR} + idV_{jI} = p.v.(\frac{\omega^2}{V_j^2} - k^2)^{\frac{1}{2}} \quad j = 1, 2, 3, 4 \qquad (5.19)$$

$$dV_j' = dV_{jR}' + idV_{jI}' = p.v.(\frac{\omega^2}{\acute{V}_j^2} - k^2)^{\frac{1}{2}} \quad j = 1, 2, 3, 4 \qquad (5.20)$$

and $K = K_R + iK_I$ is the complex wave number.
The subscript R and I denote the real and imaginary parts of the corresponding complex number and $p.v.$ stands for the principal value of the complex number.

(i)For incident LD-wave and LMS-wave:

$$\overrightarrow{A_0} = -K_I \hat{x} + dV_{1I} \hat{z}, \quad \overrightarrow{P_0} = K_R \hat{x} - dV_{1R} \hat{z}, ]$$

and $B_0 = 0$ in equation (5.11)and (5.12).

(ii)For incident CD I-wave and CD II-wave:

$$\overrightarrow{A_0} = -K_I\hat{x} + dV_{2I}\hat{z}, \quad \overrightarrow{P_0} = K_R\hat{x} - dV_{2R}\hat{z},$$

and $B_0 = 0$ in equation (5.9) and (5.10).

The phase velocities of coupled longitudinal displacement and longitudinal microstretch wave and coupled transverse displacement and microrotational wave can be written as

$$\overrightarrow{c_j} = \frac{\overrightarrow{P_j}}{|\overrightarrow{P_j}|^2}, \quad j = 1, 2, 3, 4$$

where

$$|\overrightarrow{P_j}| = [\frac{1}{2}\{\ Re(K_{pj}^2) + \{(Re(K_{pj}^2))^2 + \frac{(Im(K_{pj}^2))^2}{\cos^2\gamma_j^\star}\}\}],$$

where

$$K_{pj}^2 = \frac{\omega^2}{V_j^2}.$$

The complex wave number $k$ in microstretch viscoelastic medium $(M)$ is given by

$$k = |\overrightarrow{P_j}|\sin\theta_j - i|\overrightarrow{A_j}|\sin(\theta_j - \gamma_j^\star), \quad j = 0, 1, 2, 3, 4$$

where

$$|\overrightarrow{A_j}| = [\frac{1}{2}\{\ -Re(K_{pj}^2) + \{(Re(K_{pj}^2))^2 + \frac{(Im(K_{pj}^2))^2}{\cos^2\gamma_j^\star}\}\}]^{\frac{1}{2}}.$$

where $\gamma_j^\star$ is the angle between propagation and attenuation vector. Similar results hold for microstretch viscoelastic medium $\acute{M}$.
Coupling constants are given by

$$a_1 = -\frac{\rho}{\lambda_{0I}}[(\frac{\lambda_I + K_I}{\rho})(1 + dV_1^2) + c^2], \quad a_2 = -\frac{\rho}{\lambda_{0I}}[(\frac{\lambda_I + K_I}{\rho})(1 + dV_2^2) + c^2],$$

$$E = \frac{k^2}{c_3^2}[b^2(1 + dV_3^2) - c^2], \quad F = \frac{k^2}{c_3^2}[b^2(1 + dV_4^2) - c^2],$$

Similarly,coupling constants for microstretch viscoelastic medium $M'$ are obtained.
Making use of potentials (5.9)-(5.16) in boundary conditions (5.1)-(5.8) and with the help of eqs. (2.4)-(2.6) and (3.1)-(3.2), we get a system of eight non-homogeneous equations, which can be written as

$$\sum_{m=1}^{8} a_{mn}Z_n = Y_m(n = 1, 2...8), \tag{5.21}$$

where

$$a_{1i} = iK_n dV_i (i = 1,2), \quad a_{1i} = iK_n k(i = 3,4),$$

$$a_{15} = -\lambda_I'(k^2 + d\acute{V}_1^2) - (2\acute{\mu}_I + \acute{K}_I)d\acute{V}_1^2 + iK_n d\acute{V}_1 + \lambda_{0I}' a_1'$$

$$a_{16} = -\lambda_I'(k^2 + d\acute{V}_2^2) - (2\acute{\mu}_I + \acute{K}_I)d\acute{V}_2^2 + iK_n d\acute{V}_2 + \lambda_{0I}' a_2'$$

$$a_{1i} = (2\acute{\mu}_I + \acute{K}_I)kd\acute{V}_i - ikK_n(i = 7,8), \quad a_{2i} = iK_t k(i = 1,2),$$

$$a_{2i} = -iK_t dV_i(i = 3,4), \quad a_{2i} = (2\acute{\mu}_I + \acute{K}_I)kd\acute{V}_i - ikK_t(i = 5,6),$$

$$a_{27} = (\acute{\mu}_I + \acute{K}_I)d\acute{V}_3{}^2 - \acute{\mu}_I k^2 - \acute{K}_I \acute{E} - iK_t d\acute{V}_3$$

$$a_{28} = (\acute{\mu}_I + \acute{K}_I)d\acute{V}_4{}^2 - \acute{\mu}_I k^2 - \acute{K}_I \acute{F} - iK_t d\acute{V}_4$$

$$a_{31} = 0, \quad a_{32} = -K_c E, \quad a_{33} = -K_c F, \quad a_{34} = 0 \quad a_{35} = -ib_{0I}' k a_1',$$

$$a_{36} = -ib_{0I}' k a_2', \quad a_{37} = (i\acute{\gamma}_I d\acute{V}_3 + K_c)\acute{E}, a_{38} = (i\acute{\gamma}_I d\acute{V}_4 + K_c)\acute{F},$$

$$a_{41} = -K_\lambda a_1, \quad a_{42} = -K_\lambda a_2, \quad a_{43} = 0 \quad a_{44} = 0, \quad a_{45} = (i\alpha_{0I}' dV_1' + K_\lambda)a_1',$$

$$a_{46} = (i\alpha_{0I}' dV_1' + K_\lambda)a_2', \quad a_{47} = 0, \qquad a_{48} = 0,$$

$$a_{51} = -\lambda_I k^2 - (\lambda_I + 2\mu_I + K_I)dV_1^2 + \lambda_{0I} a_1,$$

$$a_{52} = -\lambda_I k^2 - (\lambda_I + 2\mu_I + K_I)dV_2^2 + \lambda_{0I} a_2,$$

$$a_{5i} = -(2\mu_I + K_I)kdV_i(i = 3,4),$$

$$a_{55} = [-\acute{\lambda}_I(k^2 + d\acute{V}_1^2) + (2\acute{\mu}_1 + \acute{K}_1)d\acute{V}_1^2 - \acute{\lambda}_{0I}\acute{a}_1]$$

$$a_{56} = [-\acute{\lambda}_I(k^2 + d\acute{V}_2^2) + (2\acute{\mu}_1 + \acute{K}_1)d\acute{V}_2^2 - \acute{\lambda}_{0I}\acute{a}_2]$$

$$a_{5i} = -(2\acute{\mu}_I + \acute{K}_I)kd\acute{V}_i(i = 7,8), \quad a_{6i} = -(2\mu_I + K_I)kdV_i(i = 1,2)$$

$$a_{63} = (\mu_I + K_I)dV_3^2 - \mu_I k^2 - K_I E, \quad a_{64} = (\mu_I + K_I)dV_4^2 - \mu_I k^2 - K_I F,$$

$$a_{6i} = -(2\acute{\mu}_I + \acute{K}_I)kd\acute{V}_i(i = 5, 6) \quad a_{67} = [-(\acute{\mu}_I + \acute{K}_I)d\acute{V}_3^2 + \acute{\mu}_I k^2 + \acute{K}_I \acute{E}]$$

$$a_{68} = [-(\acute{\mu}_I + \acute{K}_I)d\acute{V}_4^2 + \acute{\mu}_I k^2 + \acute{K}_I \acute{F}], \quad a_{71} = -ib_{0I}ka_1, \quad a_{72} = -ib_{0I}ka_2,$$

$$a_{73} = -i\gamma_I dV_3 E, \quad a_{74} = -i\gamma_I dV_4 F, \quad a_{75} = \acute{b}_{0I} ik\acute{a}_1, \quad a_{76} = \acute{b}_{0I} ik\acute{a}_2,$$

$$a_{77} = -i\acute{\gamma}_I d\acute{V}_3 \acute{E}, \quad a_{78} = -i\acute{\gamma}_I d\acute{V}_4 \acute{F}, \quad a_{81} = -i\alpha_{0I} dV_1 a_1, \quad a_{82} = -i\alpha_{0I} dV_2 a_2,$$

$$a_{83} = 0, \quad a_{84} = 0, \quad a_{85} = -i\acute{\alpha}_{0I} d\acute{V}_1 \acute{a}_1, \quad a_{86} = -i\acute{\alpha}_{0I} d\acute{V}_2 \acute{a}_2, \quad a_{87} = 0, \quad a_{88} = 0.$$

For incident longitudinal displacement wave (LD-wave):

$$Y_1 = a_{11}, Y_2 = -a_{21}, Y_3 = 0, Y_4 = 0, Y_5 = -a_{51}, Y_6 = a_{61}, Y_7 = -a_{71}, Y_8 = 0, \tag{5.22}$$

For incident longitudinal microstretch wave (LMS-wave):

$$Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = 0, Y_4 = 0, Y_5 = -a_{52}, Y_6 = a_{62}, Y_7 = -a_{72}, Y_8 = 0, \tag{5.23}$$

For incident coupled transverse displacement and microrotaional wave(CD I-wave):

$$Y_1 = -a_{13}, Y_2 = a_{23}, Y_3 = 0, Y_4 = 0, Y_5 = a_{53}, Y_6 = -a_{63}, Y_7 = a_{73}, Y_8 = 0, \tag{5.24}$$

For incident coupled transverse displacement and microrotaional wave(CD II-wave):

$$Y_1 = -a_{14}, Y_2 = a_{24}, Y_3 = 0, Y_4 = 0, Y_5 = a_{54}, Y_6 = -a_{64}, Y_7 = a_{74}, Y_8 = 0, \tag{5.25}$$

$$Z_1 = \frac{B_1}{B_0}, Z_2 = \frac{B_2}{B_0}, Z_3 = \frac{B_3}{B_0}, Z_4 = \frac{B_4}{B_0}, Z_5 = \frac{\acute{B}_1}{B_0}, Z_6 = \frac{\acute{B}_2}{B_0}, Z_7 = \frac{\acute{B}_3}{B_0}, Z_8 = \frac{\acute{B}_4}{B_0} \tag{5.26}$$

where $Z_1, Z_2, Z_3, Z_4$, are amplitudes ratios's of reflected longitudinal displacement and longitudinal microstretch wave(LD-wave and LMS-wave) making an angle $\theta_1$, $\theta_2$, a set of coupled transverse displacement and transverse microrotational waves (CDI-wave and CD-II)making an angle $\theta_3, \theta_4$ and $Z_5, Z_6, Z_7, Z_8$ are amplitudes ratios's of transmitted longitudinal displacement wave and longitudinal microstretch wave(LD-wave and LMS-wave) making an angle $\acute{\theta}_1, \acute{\theta}_2$, a set of coupled transverse displacement and transverse microrotational waves (CD I-wave and CD II-wave) $\acute{\theta}_3, \acute{\theta}_4$.

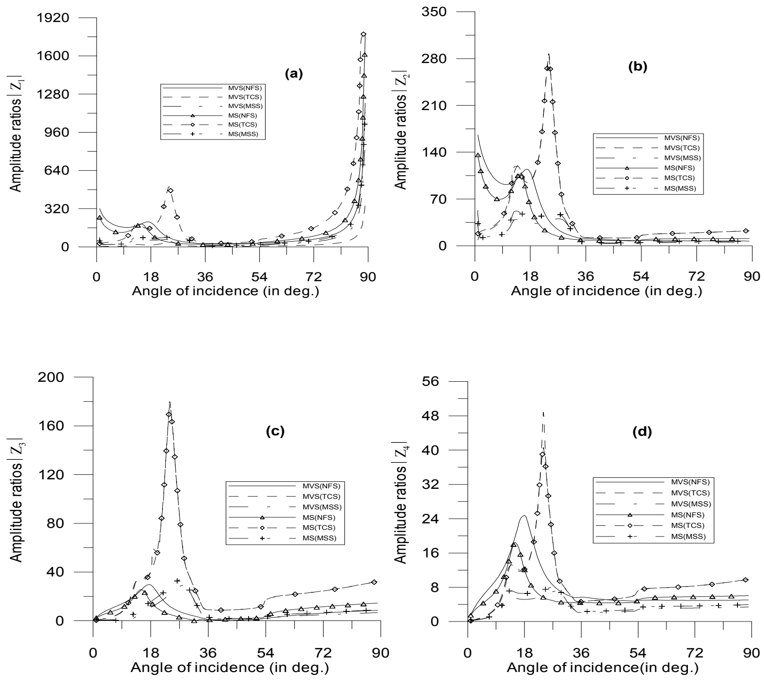



Figure (1) shows the angle of incidence with (a) Amplitude Ratio $|Z_1|$, (b) Amplitude Ratio $|Z_2|$, (c) Amplitude Ratio $|Z_3|$, (d) Amplitude Ratio $|Z_4|$ for LD-Wave

## 6 Particular Cases

CASE I: Normal Force Stiffness $K_n \neq 0, K_t \longrightarrow \infty, K_c \longrightarrow \infty, K_\lambda \longrightarrow \infty$ correspond to the case of normal force stiffness and we obtain a system of eight non-homogeneous equations as given by (5.21) with the changed values of $a_{mn}$ as

$$a_{21} = k, \quad a_{22} = k, \quad a_{23} = -dV_3, \quad a_{24} = -dV_4, \quad a_{25} = -k, \quad a_{26} = -k,$$

$$a_{27} = -d\acute{V}_3, \quad a_{28} = -d\acute{V}_4, \quad a_{31} = 0, \quad a_{32} = 0, \quad a_{33} = -E, \quad a_{34} = -F,$$

$$a_{35} = 0, \quad a_{35} = 0, \quad a_{37} = \acute{E}, \quad a_{38} = \acute{F}, \quad a_{41} = -a_1, \quad a_{42} = -a_2,$$

$$a_{43} = 0, \quad a_{44} = 0, \quad a_{45} = \acute{a_1}, \quad a_{46} = \acute{a_2}, \quad a_{47} = 0, \quad a_{48} = 0.$$

**CASE II: Transverse Force Stiffness**

$K_n \longrightarrow \infty, K_t \neq 0, K_c \longrightarrow \infty, K_\lambda \longrightarrow \infty,$

boundary conditions reduces to the transverse force stiffness, obtaining a system of eight non-homogeneous equations as given by equation (5.21) with modified values of

$a_{mn}$ as

$$a_{11} = dV_1, \quad a_{12} = dV_2, \quad a_{13} = k, \quad a_{14} = k, \quad a_{15} = d\acute{V}_1, \quad a_{16} = d\acute{V}_2,$$

$$a_{17} = -k, \quad a_{18} = -k, \quad a_{31} = 0, \quad a_{32} = 0, \quad a_{33} = -E, \quad a_{34} = -F,$$

$$a_{35} = 0, \quad a_{35} = 0, \quad a_{37} = \acute{E}, \quad a_{38} = \acute{F}, \quad a_{41} = -a_1, \quad a_{42} = -a_2,$$

$$a_{43} = 0, \quad a_{44} = 0, \quad a_{45} = \acute{a_1}, \quad a_{46} = \acute{a_2}, \quad a_{47} = 0, \quad a_{48} = 0.$$

**CASE III: Transverse couple Stiffness**

$K_n \longrightarrow \infty, K_t \longrightarrow \infty, K_c \neq 0, K_\lambda \longrightarrow \infty$, boundary conditions reduces to the transverse couple stiffness, obtaining a system of eight non-homogeneous equations as given by equation (5.21) with modified values of $a_{mn}$ as

$$a_{11} = dV_1, \quad a_{12} = dV_2, \quad a_{13} = k, \quad a_{14} = k, \quad a_{15} = d\acute{V}_1, \quad a_{16} = d\acute{V}_2,$$

$$a_{17} = -k, \quad a_{18} = -k, \quad a_{21} = k, \quad a_{22} = k, \quad a_{23} = -dV_3, \quad a_{24} = -dV_4,$$

$$a_{25} = -k, \quad a_{26} = -k, \quad a_{27} = -d\acute{V}_3, \quad a_{28} = -d\acute{V}_4, \quad a_{41} = -a_1,$$

$$a_{42} = -a_2, \quad a_{43} = 0, \quad a_{44} = 0, \quad a_{45} = \acute{a_1}, \quad a_{46} = \acute{a_2}, \quad a_{47} = 0, \quad a_{48} = 0.$$

**CASE IV: Microstress Stiffness**

$K_n \longrightarrow \infty, K_t \longrightarrow \infty, K_c \longrightarrow \infty, K_\lambda \neq 0$ boundary conditions reduces to the microstress stiffness, obtaining a system of eight non-homogeneous equations as given by equation (5.21)with modified values of $a_{mn}$ as

$$a_{11} = dV_1, \quad a_{12} = dV_2, \quad a_{13} = k, \quad a_{14} = k, \quad a_{15} = d\acute{V}_1, \quad a_{16} = d\acute{V}_2,$$

$$a_{17} = -k, \quad a_{18} = -k, \quad a_{21} = k, \quad a_{22} = k, \quad a_{23} = -dV_3, \quad a_{24} = -dV_4,$$

$$a_{25} = -k, \quad a_{26} = -k, \quad a_{27} = -d\acute{V}_3, \quad a_{28} = -d\acute{V}_4, \quad a_{31} = 0, \quad a_{32} = 0,$$

$$a_{33} = -E, \quad a_{34} = -F, \quad a_{35} = 0, \quad a_{36} = 0, \quad a_{37} = \acute{E}, \quad a_{38} = \acute{F}.$$

**CASE V:Perfect Bonding**

$K_n \longrightarrow \infty, K_t \longrightarrow \infty, K_c \longrightarrow \infty, K_\lambda \longrightarrow \infty$ correspond to the case of perfect bonding and we obtain a system of eight non-homogeneous equations as given by equation (5.21) with the changed values of $a_{mn}$ as

$$a_{14} = k, \quad a_{15} = d\acute{V}_1, \quad a_{16} = d\acute{V}_2, \quad a_{17} = -k, \quad a_{18} = -k, \quad a_{21} = k, \quad a_{22} = k,$$

$$a_{23} = -dV_3, \quad a_{24} = -dV_4, \quad a_{25} = -k, \quad a_{26} = -k, \quad a_{27} = -d\acute{V}_3, \quad a_{28} = -d\acute{V}_4,$$

$$a_{31} = 0, \quad a_{32} = 0, \quad a_{33} = -E, \quad a_{34} = -F, \quad a_{35} = 0, \quad a_{36} = 0, \quad a_{37} = \acute{E},$$

$$a_{38} = \acute{F}, \quad a_{41} = -a_1, \quad a_{42} = -a_2, \quad a_{43} = 0, \quad a_{44} = 0, \quad a_{45} = \acute{a}_1,$$

$$a_{46} = \acute{a}_2, \quad a_{47} = 0, \quad a_{48} = 0.$$

## 7 Special Cases

(i)If we neglect the effect of viscosity,that is ,when $\chi_1 = \chi$,where $\chi = \lambda, \mu, K, \alpha, \beta, \gamma$, $\alpha_0, \lambda_0, \lambda_1, b_0$, we obtain the expressions for reflection coefficients $|Z_i|, i = 1, 2...8$ in microstretch elastic medium for (a )normal force stiffness (b) transverse force stiffness (c) transverse couple stiffness (d) microstress stiffness (e) perfect bonding.

(ii)If we neglect stretch and micropolarity effect in medium $M$, $\acute{M}$ i.e. $\lambda_0, \acute{\lambda_0}, \alpha_0, \acute{\alpha_0}, \lambda_1$,
$\acute{\lambda_1} \to 0$ and $K$,$\acute{K} \to 0$ ,we obtain the expressions of reflection and transmission coefficients at viscoelastic /viscoelastic media at imperfect interface.

## 8 Numerical results and discussion

In order to illustrate theoretical results obtained in the proceeding sections, we now present some numerical results. Materials chosen for this purpose are Magnesium crystal(microstretch solid)(medium $M$)and Aluminum-epoxy composite(microstretch solid)(medium $M^{'}$),the physical data for which are given as

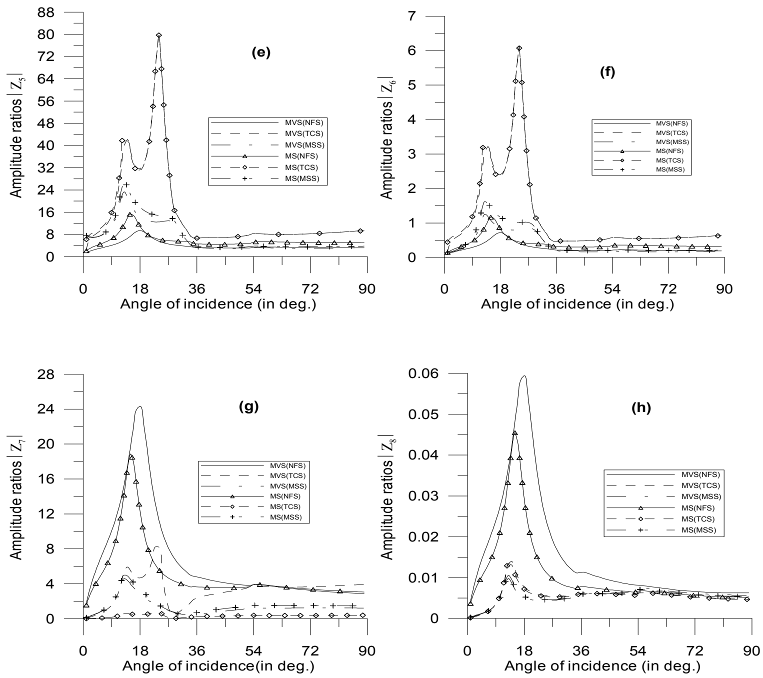



Figure (2) shows the angle of incidence with (e) Amplitude Ratio $|Z_5|$, (f) Amplitude Ratio $|Z_6|$, (g) Amplitude Ratio $|Z_7|$, (h) Amplitude Ratio $|Z_8|$ for LD-Wave

MAGNESIUM

$$\lambda = 9.4 \times 10^{10} Nm^{-2}, \mu = 4.0 \times 10^{10} Nm^{-2}, K = 1.0 \times 10^{10} Nm^{-2}, \rho = 1.74 \times 10^{3} Kgm^{-3},$$

$$\alpha = 0.89 \times 10^{-9} N, \beta = 0.7 \times 10^{-9} N, \gamma = 0.779 \times 10^{-9} N, j = 0.2 \times 10^{-19} m^2,$$

$$j_0 = 0.185 \times 10^{-19} m^2, b_0 = 0.6 \times 10^{5} N, \lambda_1 = 0.5 \times 10^{10} Nm^{-2},$$

$$\lambda_0 = 0.5 \times 10^{10} Nm^{-2}, \alpha_0 = 0.72 \times 10^{-9} N$$

ALUMINUM

$$\acute{\lambda} = 7.59 \times 10^{9} Nm^{-2}, \acute{\mu} = 1.89 \times 10^{9} Nm^{-2}, \acute{K} = 0.0149 \times 10^{9} Nm^{-2},$$

$$\acute{\rho} = 2.19 \times 10^{3} Kgm^{-3}, \acute{\alpha} = 0.03 \times 10^{5} N, \acute{\beta} = 0.026 \times 10^{5} N, \acute{\gamma} = .0268 \times 10^{5} N,$$

$$\acute{j_0} = .00189 \times 10^{-4} m^2, b_0 = 0.5 \times 10^{5} N, \lambda_1 = 0.037 \times 10^{9} Nm^{-2},$$

$$\lambda_0 = 0.037 \times 10^9 Nm^{-2}, \alpha_0 = 0.61 \times 10^5 N, \acute{j} = 0.00196 \times 10^{-4} m^2, .$$

with non-dimensional interface parameters as

$$\frac{K_n}{k\lambda} = 5, \quad \frac{K_t}{k\lambda} = 10, \quad \frac{K_c}{k\gamma} = 15, \quad \frac{K_\lambda}{k\gamma} = 12, \quad \frac{\omega}{\acute{\omega_0}} = 10, \quad \text{and} \quad \acute{\omega_0} = \sqrt{\frac{\acute{K}}{\acute{\rho}\acute{j}}}.$$

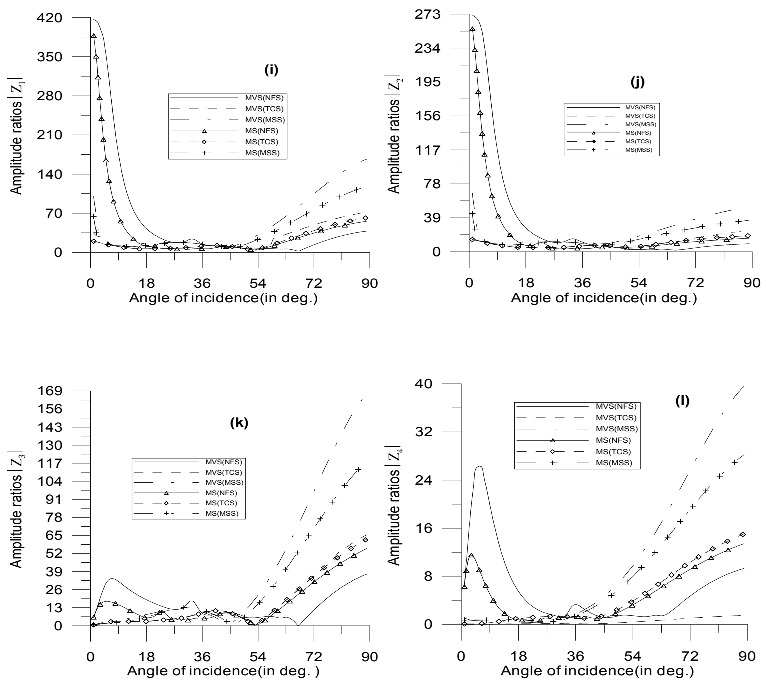



Figure (3) shows the angle of incidence with (i) Amplitude Ratio $|Z_1|$, (j) Amplitude Ratio $|Z_2|$, (k) Amplitude Ratio $|Z_3|$, (l) Amplitude Ratio $|Z_4|$ for LMS-Wave

For a particular model of microstretch viscoelastic solid the relevant parameters are expressed as

$$\lambda_I = \lambda(1 + iQ_1^{-1}), \quad \mu_I = \mu(1 + iQ_2^{-1}), \quad K_I = K(1 + iQ_3^{-1}), \quad \gamma_I = \gamma(1 + iQ_4^{-1}),$$

$$\alpha_I = \alpha(1 + iQ_5^{-1}), \quad \beta_I = \beta(1 + iQ_6^{-1}), \quad b_{0I} = b_0(1 + iQ_7^{-1}), \quad \lambda_{0I} = \lambda_0(1 + iQ_8^{-1}),$$

$$\alpha_{0I} = \alpha_0(1 + iQ_9^{-1}), \quad \lambda_{1I} = \lambda_1(1 + iQ_{10}^{-1}), .$$

where $Q_i$ and $\acute{Q}_i$ (1,...,10) are chosen as

$$Q_1 = 5, \quad Q_2 = 10, \quad Q_3 = 12, \quad Q_4 = 15, \quad Q_5 = 20, \quad Q_6 = 14, \quad Q_7 = 16,$$

$$Q_8 = 20, \quad Q_9 = 14, \quad Q_{10} = 16.$$

same are chosen for $\acute{Q}_i$.

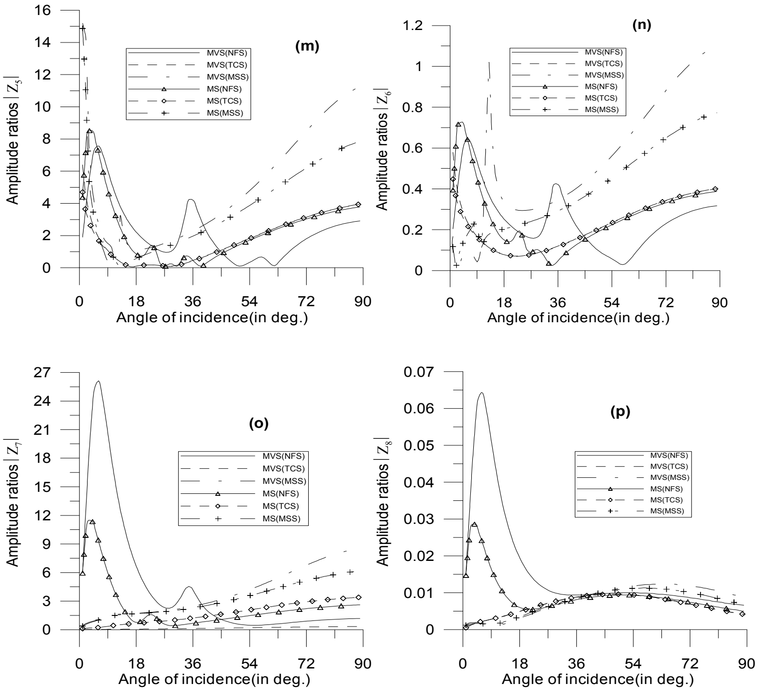



Figure (4) shows the angle of incidence with (m) Amplitude Ratio$|Z_5|$, (n) Amplitude Ratio$|Z_6|$, (o) Amplitude Ratio$|Z_7|$, (p) Amplitude Ratio $|Z_8|$ for LMS-Wave

A computer programme has been developed and amplitude ratios of various reflected and transmitted waves has been computed. The variations of amplitude ratios for normal force stiffness (NFS),transverse couple stiffness(TCS) and microstretch stiffness (MSS) for microstretch viscoelasticity (MSV) and microstretch elasticity (MS)have been shown .The solid line ,small dashed line,dash dot dash line is for MSV and solid line with center symbol 'triangle', small dashed line with center symbol 'diamond',dash dot dash line with center symbol 'plus'for MS respectively. The variations of the amplitude ratios for MSV(NFS),MSV(TCS),MSV(MSS), MS(NFS),MS(TCS)and MS(MSS) with angle of incidence of the incident LD-wave, LMS-wave and CD I-wave are shown graphically in figures 1-6.These variations are shown from normal incidence to grazing incidence i.e. $\theta_0 = 0^0$ to $\theta_0 = 90^0$.

## 8.1 Incident LD-wave

Fig.1(a) and 1(b) show the variations of amplitude ratios $|Z_i|$ (i=1,2) with the angle of incidence. In the initial range,the variations of amplitude ratios $|Z_i|$ (i=1,2) for both MVS and MS in case of all boundary stiffnesses look similar with difference in their magnitude. On reaching the grazing incidence,$|Z_1|$ attain maximum value whereas $|Z_2|$ attain minimum value and almost stable in the intermediate.

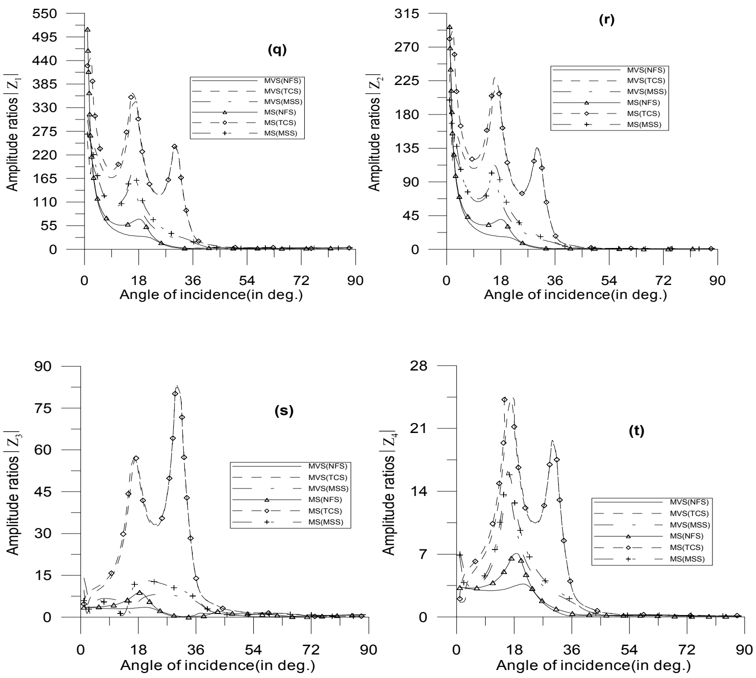



Figure (5) shows the angle of incidence with (q) Amplitude Ratio$|Z_1|$, (r) Amplitude Ratio$|Z_2|$, (s) Amplitude Ratio$|Z_3|$, (t) Amplitude Ratio $|Z_4|$ for CD-I Wave

The trend of variations of $|Z_i|$ (i=3,...,8)for both microstrecth viscoelastic medium and microstrecth medium is almost same i.e the distribution of curves for both media in case of NFS,TCS, MSS moving with hand-to-hand and ups and downs.It is evident from figures 1(c,d) and 2(e,f,g,h) that in range $45^0 \leq \theta_0 \leq 90^0$ $|Z_i|$ (i=3,...,8) follow the stable which shows that with the increase in the angle of incidence the amplitude ratios $|Z_i|$ (i=3,...,8) is stable or stationary irrespective of the properties of media.

### 8.2 Incident LMS-wave

The variations of amplitude ratios $|Z_1|$ and $|Z_2|$ from the normal incidence i.e.$\theta_0 = 0^0$ start with sharp decrease in case of both NFS and TCS whereas for MSS it start with small decrease for both media. With further increase in angle of incidence,all curves show small variations upto $\theta_0 = 54^0$ and increase in the remaining.(fig. 3(i,j)).

It is depicted from figs. 3(k) and 3(l) that in the initial range, the values of amplitude ratios $|Z_i|$ (i=3,4)for both MVS(NFS) and MS(NFS)are greater than other boundary stiffnesses. As the angle of incidence increases further $|Z_3|$ and $|Z_4|$ for MVS(MSS)and MS(MSS) are greater than other boundary stiffnesses in the range $54^0 \leq \theta_0 \leq 90^0$ except in the certain pockets all curves are close to each other which shows the effect of stiffness is more prominent incomparable to effect of viscosity of medium.

Figures 4(m) and 4(n) look as mirror image of each other. As the disturbances travels through different constituents of the medium, it suffers sudden changes, resulting in an inconsistent/non-uniform pattern of curves. Therefore, trend of curves exhibits the properties of of the medium.

The variations of $|Z_7|$ and $|Z_8|$ for both media in case of NFS are greater than TCS and MSS in $0^0 \leq \theta_0 \leq 21^0$ and attain peak value for MVS(NFS),reveals the effect of viscosity along with the stiffness effect.In the range $45^0 \leq \theta_0 \leq 90^0$, the values of $|Z_7|$ are increasing whereas decreasing for $|Z_8|$ and are shown in figures 4(o,p)

### 8.3 Incident CD I-wave

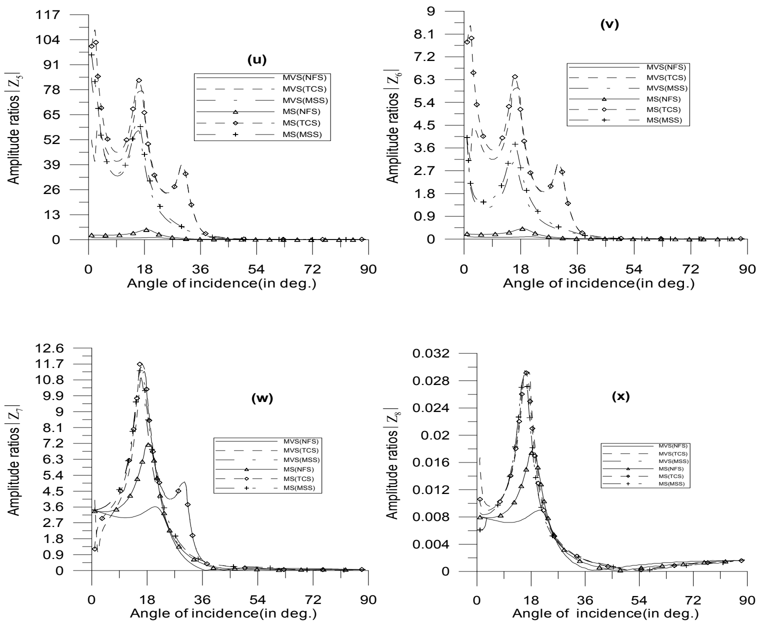


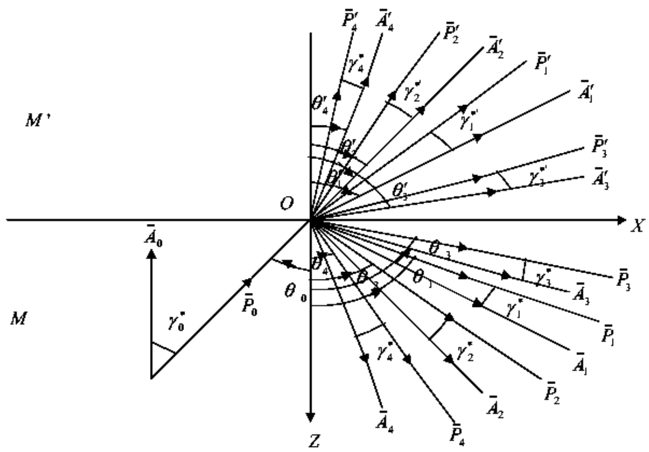

Figure (6) shows the angle of incidence with (u) Amplitude Ratio$|Z_5|$, (v) Amplitude Ratio$|Z_6|$, (w) Amplitude Ratio$|Z_7|$, (x) Amplitude Ratio $|Z_8|$ for CD-I Wave

Figures 5(q,r,s,t) and 6(u,v,w,x) shows the variations of amplitude ratios $|Z_i|$ (i=1,...,8)with angle of incidence for CD I-wave. The impact of TCS for both microstretch viscoelatic and microstrecth media is more than NFS and MSS from normal incidence to $\theta_0 = 40^0$. For CD I-wave, the values of all distribution curves from $\theta_0 = 45^0$ to the grazing incidence seems constant (very small)near the zero value. This inturn shows that with increase in angle of incidence the behavior of variations of $|Z_i|$ (i=1,...,8) are almost stable depicting almost negligible effect of stiffnesses and viscosity on the modulus of reflection and transmission coefficients in the range $45^0 \leq \theta_0 \leq 90^0$ .

## 9 Conclusion

It is observed that the viscosity and stiffness is appreciable on reflection and transmission coefficients.The behavior and trend of variations for both microstretch viscoelastic and microstretch media is almost same for all boundary stiffnesses.Near the grazing incidence,the variations seems to be almost stable for all waves i.e.LD-wave,LMS-wave and CD I-wave.The research work is supposed to be useful in further studies,both theoretical and observational of wave propagation in more realistic models of the microstretch viscoelastic solids present in the earth's interior. The problem is of geophysical interest ,particularly investigations concerned with earthquake and other phenomenon in seismology.

**Appendix A**




System's geometry

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