

An Application of PFS in selecting the best research that deserves the state award

Mohamed Shokry and Manar Omran*

Department of Engineering Physics and Mathematics, Faculty of Engineering, Tanta University, Tanta, Egypt

Received: 19 May 2020, Revised: 1 Aug. 2020, Accepted: 9 Aug. 2020

Published online: 1 Sep. 2020

Abstract: Most of real life problems are characterized by uncertainty, the theory of fuzzy set and its generalization are important for analyzing for real life data. One of the important generalizations of fuzzy set is picture fuzzy set (PFS) which is useful in providing a flexible model for handling uncertainty and vagueness in information systems. In this current paper, Sanchez presented a formulation of models that involve fuzzy measures, which represent knowledge. The concept of Picture Fuzzy Sets (PFS) is applied to make the suitable decision and take the decision with degree. An approach depending on PFS is suggested to determine the best research that deserves the state award. Using given data, we give a method for calculating membership and non-membership degrees. the suggested approach can be applied for several similar real-life cases.

Keywords: PFS, Archimedean t-Norm and t-Conorm in PFS, the best research that deserves the state award.

1 Introduction

Life problems are characterized by uncertainty and vagueness, and mathematical theories have appeared to address the uncertainty, such as Probability theory, Fuzzy Set, Intuitionistic Fuzzy Set, Neutrosophic Set and Soft Set. Fuzziness plays an essential role in human life because most of the classes encountered in the real physical world are fuzzy. In 1965, Zadeh [1] introduced the idea of a fuzzy set as an extension of the classical set theory. The idea of intuitionistic fuzzy set was first published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature [3,4]. Established by Smarandache [5,6,7] in 1980, neutrosophy was presented as the study of the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The main Idea was to consider an entity, "A" in relation to its opposite "Non-A", and to that which is neither "A" nor "Non-A", denoted by "Neut-A" and from on, neutrosophy became the basis of neutrosophic set theory, neutrosophic logic, neutrosophic probability, and neutrosophic statistics. In 2012 neutrosophic crisp sets have been investigated by Salma et.al [8,9,10,11].

On the other hand, scientists have started to color most domains of classical mathematics such as: topology,

algebraic structures, relation theory, and differential measure theory .etc. In mathematics, topology (from the Greek, place and study) is concerned with the properties of space that are preserved under continuous deformations, such as stretching, crumpling and bending, but not tearing or gluing.

We show the concept of PFS. Furthermore, some properties of this concept are investigated. We recall that some definitions essential concept of PFS and its operations, which were introduced by Sanchez [12]. PFS represent the direct expansions of the fuzzy sets and intuitionistic fuzzy sets. Accordingly, with some features, some operations on PFS are taken into consideration. In the following sections of the current paper, we will address the picture fuzzy relations and Zadeh Extension Principle. Then, PFS theory's fundamental preliminaries introduced. Primarily, the models which are based on PFS can be sufficient in situations where we encounter human opinions including more answers of kinds: no, abstain and yes, refusal. A good example of such situation is voting as it is possible to divide human voters into four categories including those who: abstain, vote for, refusal of the voting or vote against [13,14]. One of the direct expansions of the fuzzy set is PFS. In this paper, we would like to discuss how picture fuzzy set theory can be used for developing the calculations to take the suitable

* Corresponding author e-mail: manar.omran@f-eng.tanta.edu.eg

decision making and take the decision with degree and suggested its application in the best research that deserves the state award.

2 Concepts of PFS

Definition 2.1. [1]: Let A be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x) \rangle : x \in X\}$, where $\mu_A(x) : X \rightarrow [0, 1]$ is the membership function of the fuzzy set A .

Definition 2.2. [2]: Let X is a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$, Where the functions $\mu_A(x), \nu_A(x) : X \rightarrow [0, 1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A , which is a subset of X , and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

3 Basic Relations and Operation on PFS

IF A, B be PFS in X , then:

- 1.[inclusion] $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \eta_A(x) \leq \eta_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \forall x \in X$
- 2.[equality] $A = B \Leftrightarrow \mu_A(x) = \mu_B(x), \eta_A(x) = \eta_B(x) \text{ and } \nu_A(x) = \nu_B(x) \forall x \in X$
- 3.[union] $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$
- 4.[intersection] $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$

4 Archimedean t-Norm and t-Conorm in PFS

We use t-norm in the fuzzy set when we want to talk about the minimum between two memberships in intersection operation and use t-conorm to talk about the minimum between two memberships in union operation. Therefore, the t-norm and t-conorm [15, 16, 17, 18, 19] can be utilized in PFS where it is useful to the intersection or the union operations that we need the minimum and maximum in the same operation in PFS $P = (x, \mu(x), \eta(x), \nu(x))$ where the intersection operation, the positive and neutral membership degrees can use t-norm and the negative membership degree can use t-conorm and where the union operation, the positive and neutral membership degrees can use t-conorm and the negative membership degree can use t-norm. Consider that i is a t-norm and u is a t-conorm. When $x \in [0, 1]$, the following is defined:

$$\begin{aligned} x_i^{(n)} &= i(x, \dots, x) = i(x_i^{(n-1)}, x); n \geq 2 \\ &\quad \text{\small } n\text{-times} \\ x_u^{(n)} &= u(x, \dots, x) = u(x_u^{(n-1)}, x); n \geq 2 \\ &\quad \text{\small } n\text{-times} \end{aligned} \quad (1)$$

Definition 4.1. Consider that i, u are a t-norm and a t-conorm from $[0, 1]^n$ to $[0, 1]$. Then:

- (i) A t-norm i and a t-conorm u respectively is considered to be continuing when it is continues to be a function on the unit interval.
- (ii) A t-norm i and a t-conorm u respectively, of PFS $P = (x, \mu(x), \eta(x), \nu(x))$ if we talk about intersection operation: μ, η tends to minimum, ν tends to maximum is considered as Archimedean when $\lim_{n \rightarrow \infty} (\mu_i(x))^n = 0, \lim_{n \rightarrow \infty} (\eta_i(x))^n = 0$ and respectively $\lim_{n \rightarrow \infty} (\nu_u(x))^n = 1$ for any $x \in (0, 1)$.
- (iii) A t-norm i and a t-conorm u respectively, of PFS $P = (x, \mu(x), \eta(x), \nu(x))$ if we talk about union operation: μ tends to maximum, ν, η tends to minimum is considered as Archimedean when $\lim_{n \rightarrow \infty} (\mu_i(x))^n = 1$ and respectively $\lim_{n \rightarrow \infty} (\nu_u(x))^n = 0, \lim_{n \rightarrow \infty} (\eta_u(x))^n = 0$ for any $x \in (0, 1)$.

Proposition 4.1. Suppose that θ is a set of all PFS, i, u are a t-norm and a t-conorm [20, 21]. Then:

1. In intersection operation. If i and u are Archimedean [22] then

$$i(\mu(x), \mu(x)) < \mu(x), i(\eta(x), \eta(x)) < \eta(x) \quad \text{and} \\ u(\nu(x), \nu(x)) > \nu(x) \text{ for all}$$

$$P = (x, \mu(x), \eta(x), \nu(x)).$$

1. In union operation. If i and u are Archimedean then $u(\mu(x), \mu(x)) > \mu(x)$ and

$$i(\nu(x), \nu(x)) < \nu(x), i(\eta(x), \eta(x)) < \eta(x) \quad \text{for all} \\ P = (x, \mu(x), \eta(x), \nu(x)).$$

Proposition 4.2. In case of a continuing function $i, u : [0, 1]^2 \rightarrow [0, 1]$, the following statements are considered to be equal for any picture fuzzy set:

1. i represents a continuing Archimedean t-norm and u represents a continuous Archimedean t-conorm.
2. There exists a continuing additive generator for any PFS $P = (x, \mu(x), \nu(x))$ if we talk about intersection operation μ, η tends to minimum, ν tends to maximum, that's to say that there exists a continuing strictly decreasing $i(\mu(x)) : [0, 1] \rightarrow [0, \infty]$, $i(1) = 0$, which can be defined uniquely up to a multiplicative constant such that for all $x, y \in [0, 1]$, we have:

$$i(\mu(x), \mu(y)) = i^{-1}(\min\{i(\mu(x)) + i(\mu(y)), i(0)\}),$$

$$i(\eta(x), \eta(y)) = i^{-1}(\min\{i(\eta(x)) + i(\eta(y)), i(0)\})$$

and strictly increasing $u(\nu(x)) : [0, 1] \rightarrow [0, \infty]$, $u(0) = 0$, which is uniquely determined up to a multiplicative constant such that for all $x, y \in [0, 1]$, we have:

$$u(\nu(x), \nu(y)) = u^{-1}(\max\{u(\nu(x)) + u(\nu(y)), u(0)\}).$$

1. There are a continuous additive generator for any picture fuzzy set (PFS) $P = (x, \mu(x), \nu(x))$ if we talk about union operation μ tends to maximum, ν, η tends to minimum i.e., there are a continuous strictly decreasing [23]
 $i(\eta(x)) : [0, 1] \rightarrow [0, \infty], i(\nu(x)) : [0, 1] \rightarrow [0, \infty]$,
 $i(1) = 0$, which is defined uniquely up to a multiplicative constant such that for all $x, y \in [0, 1]$, we will have:

$$i(\nu(x), \nu(y)) = i^{-1}(\min\{i(\nu(x)) + i(\nu(y)), i(0)\}),$$

$i(\eta(x), \eta(y)) = i^{-1}(\min\{i(\eta(x)) + i(\eta(y)), i(0)\})$ and strictly increasing $u(\mu(x)) : [0, 1] \rightarrow [0, \infty]$, $u(0) = 0$, which is defined uniquely up to a multiplicative constant such that for all $x, y \in [0, 1]$, we have:

$$u(\mu(x), \mu(y)) = u^{-1}(\max\{u(\mu(x)) + u(\mu(y)), u(0)\}).$$

Proof. Let $i(x, y) = i^{-1}(\min\{i(x) + i(y), i(0)\})$ and $u(x, y) = u^{-1}(\max\{u(x) + u(y), u(0)\})$. We have to prove that they are a continuous Archimedean t-norm and t-conorm [24, 25, 26, 27].

$$\begin{aligned} I_1 : i(\nu(x), 1) &= i^{-1}(\min\{i(\nu(x)) + i(1), i(0)\}) \\ &= i^{-1}(\min\{i(\nu(x)) + 0, i(0)\}) \\ &= i^{-1}(\min\{i(\nu(x)), i(0)\}) \\ &= i^{-1}(i(\nu(x))) \\ &= \nu(x), \end{aligned}$$

$$\begin{aligned} i(\eta(x), 1) &= i^{-1}(\min\{i(\eta(x)) + i(1), i(0)\}) \\ &= i^{-1}(\min\{i(\eta(x)) + 0, i(0)\}) \\ &= i^{-1}(\min\{i(\eta(x)), i(0)\}) \\ &= i^{-1}(i(\eta(x))) \\ &= \eta(x) \end{aligned}$$

And

$$\begin{aligned} u(\mu(x), 0) &= u^{-1}(\max\{u(\mu(x)) + u(0), u(0)\}) \\ &= u^{-1}(\max\{u(\mu(x)) + 0, u(0)\}) \\ &= u^{-1}(\max\{u(\mu(x)), u(0)\}) \\ &= u^{-1}(u(\mu(x))) \\ &= \mu(x) \end{aligned}$$

(Boundary)

I_2 : To prove that i is increasing we consider $\nu(x) < \nu(w)$ and $\nu(y) < \nu(z)$. Then $i(\nu(x)) > i(\nu(w))$ and $i(\nu(y)) >$

$i(\nu(z))$ and $i(\nu(x)) + i(\nu(y)) > i(\nu(w)) + i(\nu(z))$. Since i^{-1} is decreasing we obtain:

$$\begin{aligned} i(\nu(x), \nu(y)) &= i^{-1}(\min\{i(\nu(x)) + i(\nu(y)), i(0)\}) \\ &\leq i^{-1}(\min\{i(\nu(w)) + i(\nu(z)), i(0)\}) \\ &\leq i(\nu(w), \nu(z)) \end{aligned}$$

For proving that i is increasing, we suppose that $\eta(x) < \eta(w)$ and $\eta(y) < \eta(z)$. As a result, $i(\eta(x)) > i(\eta(w))$ and $i(\eta(y)) > i(\eta(z))$ and $i(\eta(x)) + i(\eta(y)) > i(\eta(w)) + i(\eta(z))$. Since i^{-1} is decreasing we obtain:

$$\begin{aligned} i(\eta(x), \eta(y)) &= i^{-1}(\min\{i(\eta(x)) + i(\eta(y)), i(0)\}) \\ &\leq i^{-1}(\min\{i(\eta(w)) + i(\eta(z)), i(0)\}) \\ &\leq i(\eta(w), \eta(z)) \end{aligned}$$

And

For proving that i is increasing, we suppose that $\mu(x) < \mu(w)$ and $\mu(y) < \mu(z)$. As result, $u(\mu(x)) > u(\mu(w))$ and $u(\mu(y)) > u(\mu(z))$ and $u(\mu(x)) + u(\mu(y)) > u(\mu(w)) + u(\mu(z))$. Since u^{-1} is increasing we obtain:

$$\begin{aligned} u(\mu(x), \mu(y)) &= u^{-1}(\max\{u(\mu(x)) + u(\mu(y)), u(0)\}) \\ &\leq u^{-1}(\max\{u(\mu(w)) + u(\mu(z)), u(0)\}) \\ &\leq u(\mu(w), \mu(z)) \end{aligned}$$

(Monotonicity)

$$\begin{aligned} I_3 : i(\nu(x), \nu(y)) &= i(\nu(y), \nu(x)) \quad \text{is obvious} \\ i(\eta(x), \eta(y)) &= i(\eta(y), \eta(x)) \quad \text{is obvious} \\ u(\mu(x), \mu(y)) &= u(\mu(y), \mu(x)) \quad \text{is obvious} \end{aligned}$$

(Commutative). I_4 : We have:

$$\begin{aligned} i(i(\nu(x), \nu(y)), \nu(z)) &= i^{-1}(i^{-1}(\min\{i(\nu(x)) + u(\nu(y)), u(0), \nu(z)\})) \\ &= i^{-1}(i(\min\{i(i^{-1} \min\{u(\nu(x)) + u(\nu(y)), u(0)\}) \\ &\quad + u(\nu(z)), u(0)\})) \end{aligned}$$

$$\therefore i(\nu(x)) + i(\nu(y)) < i(0).$$

Since i^{-1} is strictly decreasing. We obtain:

$$\begin{aligned}
 & i(i(v(x), v(y)), v(z)) \\
 &= i^{-1}(\min\{i(i^{-1}(i(v(x)) + i(v(y)))) + i(v(z)), i(0)\}) \\
 &= i^{-1}(\min\{i(v(x)) + i(v(y)) + i(v(z)), i(0)\}), i(0) = 1 \\
 &= i^{-1}(i(v(x)) + i(v(y)) + i(v(z)))
 \end{aligned}$$

And

$$\begin{aligned}
 & i(v(x), i(v(y), v(z))) \\
 &= i^{-1}(v(x), i^{-1}(\min\{i(v(y)) + i(v(z)), i(0)\})) \\
 &= i^{-1}(\min\{i(u(x)) + i(i^{-1}(\min\{i(u(y)) \\
 &\quad + i(u(z)), i(0)\})), i(0)\})
 \end{aligned}$$

$$\therefore i(v(y)) + i(v(z)) < i(0).$$

Since i^{-1} is strictly increasing. We obtain:

$$\begin{aligned}
 & i(v(x), i(v(y), v(z))) \\
 &= i^{-1}(\min\{i(v(x)) + i(i^{-1}(i(v(y)) + i(v(z))))\}, i(0)\}) \\
 &= i^{-1}(\min\{i(v(x)) + i(v(y)) + i(v(z)), i(0)\}), i(0) = 1 \\
 &= i^{-1}(i(v(x)) + i(v(y)) + i(v(z)))
 \end{aligned}$$

$$\therefore i(i(v(x), v(y)), v(z)) = i(v(x), i(v(y), v(z)))$$

We have:

$$\begin{aligned}
 & i(i(\eta(x), \eta(y)), \eta(z)) \\
 &= i^{-1}(i^{-1}(\min\{u(\eta(x)) + u(\eta(y)), u(0)\}), \eta(z)) \\
 &= i^{-1}(\min\{i(i^{-1}(\min\{u(\eta(x)) + u(\eta(y)), u(0)\})), u(0)\}) \\
 &\quad + u(\eta(z)), u(0)\})
 \end{aligned}$$

$\therefore i(\eta(x)) + i(\eta(y)) < i(0)$. Since i^{-1} is strictly decreasing. We obtain:

$$\begin{aligned}
 & i(i(\eta(x), \eta(y)), \eta(z)) \\
 &= i^{-1}(\min\{i(i^{-1}(i(\eta(x)) + i(\eta(y)))) + i(\eta(z)), i(0)\}) \\
 &= i^{-1}(\min\{(i(\eta(x)) + i(\eta(y)) + i(\eta(z))), i(0)\}), i(0) = 1 \\
 &= i^{-1}(i(\eta(x)) + i(\eta(y)) + i(\eta(z)))
 \end{aligned}$$

And

$$\begin{aligned}
 & i(\eta(x), i(\eta(y), \eta(z))) \\
 &= i^{-1}(\eta(x), i^{-1}(\min\{i(\eta(y)) + i(\eta(z)), i(0)\})) \\
 &= i^{-1}(\min\{i(\eta(x)) + i(i^{-1}(\min\{i(\eta(y)) \\
 &\quad + i(\eta(z)), i(0)\})), i(0)\})
 \end{aligned}$$

$\therefore i(\eta(y)) + i(\eta(z)) < i(0)$. Since i^{-1} is strictly increasing. We obtain:

$$\begin{aligned}
 & i(\eta(x), i(\eta(y), \eta(z))) \\
 &= i^{-1}(\min\{i(\eta(x)) + i(i^{-1}(i(\eta(y)) + i(\eta(z))))\}, i(0)\}) \\
 &= i^{-1}(\min\{i(\eta(x)) + i(\eta(y)) + i(\eta(z)), i(0)\}), i(0) = 1 \\
 &= i^{-1}(i(\eta(x)) + i(\eta(y)) + i(\eta(z)))
 \end{aligned}$$

$$\therefore i(i(\eta(x), \eta(y)), \eta(z)) = i(\eta(x), i(\eta(y), \eta(z)))$$

And we have:

$$\begin{aligned}
 & u(u(\mu(x), \mu(y)), \mu(z)) \\
 &= u^{-1}(u^{-1}(\max\{u(\mu(x)) + u(\mu(y)), u(0)\}), \mu(z)) \\
 &= u^{-1}(\max\{u(u^{-1}(\max\{u(\mu(x)) + u(\mu(y)), u(0)\})) \\
 &\quad + u(\mu(z)), u(0)\})
 \end{aligned}$$

$\therefore u(\mu(x)) + u(\mu(y)) > u(0)$. Since u^{-1} is strictly decreasing. We obtain:

$$\begin{aligned}
 & u(u(\mu(x), \mu(y)), \mu(z)) \\
 &= u^{-1}(\max\{u(u^{-1}(u(\mu(x)) + u(\mu(y)))) + u(\mu(z)), u(0)\}) \\
 &= u^{-1}(\max\{(u(\mu(x)) + u(\mu(y)) \\
 &\quad + u(\mu(z))), u(0)\}), u(0) = 1 \\
 &= u^{-1}(u(\mu(x)) + u(\mu(y)) + u(\mu(z)))
 \end{aligned}$$

And

$$\begin{aligned}
 & u(\mu(x), u(\mu(y), \mu(z))) \\
 &= u^{-1}(\mu(x), u^{-1}(\max\{u(\mu(y)) + u(\mu(z)), u(0)\})) \\
 &= u^{-1}(\max\{u(\mu(x)) + u(u^{-1}(\max\{u(\mu(y)) \\
 &\quad + u(\mu(z)), u(0)\})), u(0)\})
 \end{aligned}$$

$$\therefore u(\mu(y)) + u(\mu(z)) > u(0).$$

Since u^{-1} is strictly decreasing. We obtain:

$$\begin{aligned}
 & u(\mu(x), u(\mu(y), \mu(z))) \\
 &= u^{-1}(\max\{u(\mu(x)) + u(u^{-1}(u(\mu(y)) \\
 &\quad + u(\mu(z))))\}, u(0)\}) \\
 &= u^{-1}(\max\{u(\mu(x)) + u(\mu(y)) \\
 &\quad + u(\mu(z)), u(0)\}), u(0) = 1 \\
 &= u^{-1}(u(\mu(x)) + u(\mu(y)) + u(\mu(z)))
 \end{aligned}$$

$$\therefore u(u(\mu(x), \mu(y)), \mu(z)) = u(\mu(x), u(\mu(y), \mu(z)))$$

[28,29,30]. (Associative).

Definition 4.2. [31]: The Normalized Hamming distance $d_{n-H}(A, B)$ between two PFS A and B is defined as: $d_{n-H}(A, B) = \frac{1}{2n} \sum_{i=1}^n |\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|$, $X = \{x_1, x_2, \dots, x_n\}$ for $i = 1, 2, \dots, n$.

5 Application of PFS in selecting the best research that deserves the state award

We will apply PFS to can select the best research that deserves the state award which has uncertainty. Let $S = \{S_1, S_2, S_3, S_4\}$ be the set of researches, $X = \{\text{language accuracy, modernity of references, safety results, scientific secretariat, innovation}\}$ be the set of selection criteria. The table 1 shows selection criteria.

Table 1: The selection criteria

Language accuracy	(0.8,0.05,0.05)
Modernity of references	(0.7,0.2,0.05)
Safety results	(0.9,0.05,0.0)
Scientific secretariat	(0.9,0.05,0.0)
Innovation	(0.6,0.3,0.1)

Using three numbers including negative membership, positive membership, and neural membership degrees, each performance can be described. The researches revealed the following findings as depicted in the Table 2.

Using Def. 4.2. of the distance's normalized Hamming law for calculating the distance between the researchers and the selection criteria, we get the Table 3.

Based on the Table 3 above, the best is given by the shortest distance. We can but that the result less than 0.5, it is the best and deserves the state award. So, S_3 and S_4 are the best and deserves the state award.

6 Conclusion

We deduce that this case study presented in this mark can be applied in many real life applications, for example: medical, political and social case [32,33,34,35,36,37,38].

Conflict of Interest

The authors declare that they have no conflict of interest.

Table 2: Researches V.S The selection criteria

S_1	Language accuracy	(0.9,0.1,0.0)
	Modernity of references	(0.9,0.1,0.0)
	Safety results	(0.6,0.2,0.2)
	Scientific secretariat	(0.9,0.1,0.0)
	Innovation	(0.5,0.5,0.0)
S_2	Language accuracy	(0.5,0.3,0.2)
	Modernity of references	(0.6,0.2,0.2)
	Safety results	(0.5,0.3,0.2)
	Scientific secretariat	(0.7,0.2,0.1)
	Innovation	(0.5,0.5,0.0)
S_3	Language accuracy	(0.7,0.1,0.2)
	Modernity of references	(0.6,0.3,0.1)
	Safety results	(0.7,0.1,0.2)
	Scientific secretariat	(0.5,0.4,0.1)
	Innovation	(0.4,0.5,0.1)
S_4	Language accuracy	(0.6,0.4,0.0)
	Modernity of references	(0.8,0.1,0.1)
	Safety results	(0.6,0.0,0.4)
	Scientific secretariat	(0.6,0.3,0.1)
	Innovation	(0.5,0.3,0.2)

Table 3: The results

S_i	The result
S_1	0.51
S_2	0.61
S_3	0.45
S_4	0.49

References

- [1] L. A. Zadeh, Fuzzy sets, Information and control, **8** (1965) 338-353.
- [2] K. T. Atanassov, Intuitionistic Fuzzy Sets. Proc. of Polish Symp. On Interval and Fuzzy mathematics, Poznan, 23-26, (1983).
- [3] K. T. Atanassov, Intuitionistic fuzzy set, Fuzzy sets and Systems, **20** (1986) 87-96.
- [4] K. T. Atanassov, More on Intuitionistic fuzzy sets, Fuzzy sets and Systems, **33** (1989) 37-46.
- [5] K. T. Atanassov, Operators Over Interval-valued Intuitionistic Fuzzy Sets, Fuzzy sets and Systems, **64** (1994) 159-174.
- [6] A. A. Salama, Neutrosophic crisp point & Neutrosophic crisp ideals, Neutrosophic sets and Systems, **1** (2013) 50-54.
- [7] F. Smarandache, M. Ali, M. Khan, *Arithmetic Operations of Neutrosophic Sets, Interval Neutrosophic Sets and Rough Neutrosophic Sets*. In: Kahraman C., Otay İ. (eds) Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets. Studies in Fuzziness and Soft Computing, vol 369, (2019). Springer, Cham. https://doi.org/10.1007/978-3-030-00045-5_2
- [8] A. A. Salama et,al, Generalized neutrosophic set and generalized neutrosophic spaces, Journal Computer Sci. Engineering, **2** (2012) 129-132.
- [9] A. A. Salama et,al, F. Smarandache, Neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp

- ideals, I. J. Information Engineering and Electrical Business, **3** (2014) 1-8.
- [10] A. A. Salama et.al, *Neutrosophic crisp set theory*, Education publisher Columbus, 2015.
- [11] A. A. Salama et.al, "Neutrosophic crisp sets & Neutrosophic crisp topology spaces", *Neutrosophic Sets and Systems*, **2** (2014) 25-30.
- [12] B. C. Cuong, concept of picture fuzzy sets and its operations, *Journal of Computer Science and Cybernetics*, **30** (2014) 409-420.
- [13] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Fuzzy Soft Semi Connected Properties in Fuzzy Soft Topological Spaces, *Journal of Computer Science and Cybernetics, Math. Sci. Lett.*, **4** (2015) 171-179.
- [14] P. Liu, K. Wong, Revealed homothetic preference and technology, *Journal of mathematical Economics*, **34** (2000) 287-314.
- [15] M. Omran et.al, *Applications on Some Uncertainty Theories*, Master of science in Physics and Engineering Mathematics, Physics and Engineering Mathematics Department, Faculty of Engineering, Tanta University, 2016.
- [16] A. Bede, *Mathematics of Fuzzy Sets and Fuzzy Logic*, Springer Nature, 2013.
- [17] M. Thorup, Undirected single-source shortest paths with positive integer weights in linear time, *Journal of the ACM*, **3** (1999) 362-394.
- [18] J. He, X. Wang, R. Zhang, L. Li, Some q-Rung Picture Fuzzy Dombi Hamy Mean Operators with Their Application to Project Assessment, *Mathematics*, **7** (2019) 468.
- [19] F. Qin, M. Baczynski, and A. Xie, Distributive Equations of Implications Based on Continuous Triangular Norms (I), *IEEE Transactions on Fuzzy Systems*, **20** (2012) 153-167.
- [20] Y. Shi, D. Ruana, E. E. Kerre, On the characterizations of fuzzy implications Satisfying $I(x,y) = I(x,I(x,y))$, *Information Sciences*, **177** (2007) 2954-2970.
- [21] A. Chakrabarti, M. Baczynski, Fuzzy Implications from Fuzzy Logic Operations, *Studies in Fuzziness and Soft Computing*, 39-107, 2008.
- [22] B. Mulansky, J. W. Schmidt, Convex interval interpolation using a three-term staircase algorithm", *Numerische Mathematik*, **82** (1999) 313-337.
- [23] C. Kahraman, I. Ota, Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets, Springer Science and Business Media LLC, 2019.
- [24] X. Wu, S. J. Leib, M. E. Goldstein. On the nonlinear evolution of a pair of oblique Tollmien Schlichting waves in boundary layers, *Journal of Fluid Mechanics*, **340** (1997) 361-394.
- [25] D. Zamfirescu, P. Grathwohl, Occurrence and attenuation of specific organic compounds in the groundwater plume at a former gasworks site", *Journal of Contaminant Hydrology*, **53** (2001) 407-427.
- [26] S. Sandhiy, K. Selvakumari, Decision Making Problem for Medical Diagnosis Using Hexagonal Fuzzy Number Matrix, *International Journal of Engineering & Technology*, **7** (2018) 660.
- [27] G. Cerami, R. Molle, Multiple positive solutions for singularly perturbed elliptic problems in exterior domains, *Annales de l'Institut Henri Poincaré (C) Non Linear Analysis*, **20** (2003) 759-777.
- [28] K. Q. Pu, A. O. Mendelzon, Concise descriptions of subsets of structured sets, *ACM Transactions on Database Systems*, **30** (2005) 211-248.
- [29] Y. Yu. "Public goods provision: Unit-by-unit costsharing rules and the core", *Review of Economic Design*, **9** (2005) 363-376.
- [30] M. Baczynski, B. Jayaram, On the Distributivity of Fuzzy Implications Over Nilpotent or Strict Triangular Conorms, *IEEE Transactions on Fuzzy Systems*, **17** (2009) 590-603.
- [31] E. Szmidt, *Distances and similarities in intuitionistic fuzzy sets*, Springer 2014.
- [32] J. Liu, Y. Wang, Z.-Q. Wang. "Solutions for Quasilinear Schrödinger Equations via the Nehari Method", *Communications in Partial Differential Equations*, **29** (2004) 879-901.
- [33] A. R. Meenakshi, M. Kaliraja "An application of interval valued fuzzy matrices in medical diagnosis," *International Journal of Mathematical Analysis*, **5** (2011) 1791-1802.
- [34] S. N. Bhatt, G. Bilardi, G. Pucci "Area-universal circuits with constant slowdown", *Proceedings 20th Anniversary Conference on Advanced Research in VLSI*, 1999.
- [35] A. M. Abd El-latif, "Fuzzy Soft α -Connectedness in Fuzzy Soft Topological Spaces, *Journal of Computer Science and Cybernetics*", *Math. Sci. Lett.*, **5** (2016) 85-91.
- [36] S. Díaz, P. Gil, J. Jiménez, S. Montes "Compatibility of t-norms with the concept of η -partition", *Information Sciences*, **177** (2007) 2925-2944.
- [37] J. Hohenegger, A. Bufardi, P. Xirouchakis "Fuzzy compatibility structures in new product development", *Advanced Engineering Informatics*, **21** (2007) 41-54.
- [38] T. Calvo, G. Mayor, R. Mesiar, "Aggregation Operators: New Trends and Applications", Springer Science and Business Media LLC, 2002.