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The Interaction between M-Configuration Five-Level Atom and a Single Mode Cavity Field

N. H. Abd El-Wahab, R. M. Farghly and H. A. Hassan*

Mathematics Department, Faculty of Science, Minia University, Minia, Egypt

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Abstract: The present paper aims to clarify several aspects of M-configuration five-level atom interacting with a single mode cavity field. The considered model is characterized by including both the detuning parameter and the initial field phase. The full solution of this model is given for the cases of resonance, off-resonance and non-resonance interaction when the atom is initially prepared in super-position state. However, the input radiation field is taken in a coherent state. Also, the influence of the presence and absent of the detuning parameter and the relative phase on the temporal evaluation for the collapses-revivals, the anti-bunching of photons, the squeezing of radiation field and the coherence properties has been examined. The results have indicated that the presence of both the detuning and the relative phase have significantly affects on these phenomena.

Keywords: Jaynes-Cummings model, Collapses-revivals phenomenon, Poisonian statistics, Squeezing, Coherent properties.

1 Introduction

The Jaynes-Cummings model (JCM) [1] is an ideal model describing the interaction between a two-level atom and one-mode electromagnetic cavity field in the rotating wave approximation (RWA). In the last few years, a lot of generalizations of this model have been proposed to explore new quantum aspects. For example, the multi-photon processes [2] and the Kerr-like medium [3] are considered. Also, it extended to describe the interaction between N-level atom and a few number of modes cavity field. Moreover, the third level is adding to discuss the two-photon process. Thus, the three configurations ($\Lambda;V$ and Ξ) three-level atomic system have been studied [4]. Moreover, the enhanced absorption of the probe Laser is possible when there are at least four levels. Thus, a four-level atom interacting with electromagnetic field has been considered and some of its properties have been extensively explored [5-8]. The concept of photon switching for four-level atom has been proposed experimentally and theoretically [9-11]. Also, we explore the interaction between a moving four-level atom and a one-mode cavity field in the presence of the nonlinear Kerr-like medium [12]. Furthermore, we discuss some configurations of a four-level atomic systems [13]. Furthermore, the effect of the relative phase on the interaction between a four-level atom and a single mode cavity field has been investigated [14,15].

Recently, much of work has been conducted on the five-level atomic systems in different areas of quantum optics. In particular, a five-level Λ-type atom interacting with two coherent fields has been studied [16]. The optical switching by controlling the double dark resonance in an N-tripod five-level atom is illustrated [17]. Furthermore, the optical switching, bistability and pluse propagation of five-level Y-type atom have been explored [18]. Also, the optical properties of five-level K-type atoms interacting with coherent laser fields have been demonstrated [19]. Moreover, amplitude and phase controlled and dispersion of a coherently driven five-level atom have been studied [20]. The electromagnetically induced transparency in a five-level X-type system with wavelength mismatching effects has been introduced [21]. Also, a theoretically study for a five-level M-type atom has been proposed [22]. Moreover, we examine two types of the five-level system [23,24]. For these reasons, the five-level atomic system is a very interesting source of non-classical light for theoretical and experimental investigations in quantum optics field. Over the years and based on the JCM and its various generalizations, many interesting quantum phenomena of both the atom and the

* Corresponding author e-mail: nady_aa@yahoo.com

field have been extensively investigated. For example, the collapses-revivals phenomenon [25], bunching and anti-bunching of photons [26,27] as well as squeezing of the radiation field [28,29] are explored. These characteristics have found interesting applications in ion traps [30], quantum state teleporation [31], quantum computation [32] and quantum nondemolition measurements [34].

The present paper present a model describes an atomic system of a five-level M-type atom interacting with a single mode cavity field. This real system represents the transition between $^2S_{1/2}$ to $^2P_{1/2}$ in the ^{87}Rb D1-line ($F = 1,2 \rightarrow F' = 1$) [35]. Furthermore, the considered configuration describes a five-level system with three Zeeman ground states $|3\rangle$, $|4\rangle$ and $|5\rangle$, coupled to two excited states $|1\rangle$ and $|2\rangle$. Moreover, the model under assumption includes the detuning parameters and the phase of the initial single mode field. The wave function of this atomic system is presented by using Schrödinger equation when the atom and the input field are assumed in a super-position state and a coherent state, respectively. After obtaining the solution of this model, the expressions of the mean photon number, the normalized second-order correlation function, the normal squeezing and the coherent properties are calculated. Also, the effects of the detuning and relative phase on the properties of the field mode are investigated. The numerical computations show that these properties are effected by the presence of the detuning or the initial relative phase.

The paper begins with describing and solving the model of the considered atomic system. The mean photon number, the normalized second-order correlation function, the normal squeezing and the coherence properties of the field are calculated and investigated in sections 3-6, respectively. Also, the results and discussion are presented section 7. We end the paper by introducing the conclusions and some features.

2 The model and its solution

We consider a non-resonant interaction between a five-level M-type atom with two upper states $|1\rangle$ and $|2\rangle$ and three lower levels $|3\rangle$, $|4\rangle$ and $|5\rangle$ and a single mode cavity field with annihilation and creation operators $\widehat{a}$ and $\widehat{a}^{\dagger}$, respectively. We denote that, the five levels $|j\rangle, (j = 1,2,...,5)$ with energy ω_j and the filed mode with frequency Ω. The allowed transitions of this system are $|1\rangle \rightarrow |3\rangle(|4\rangle)$ and $|2\rangle \rightarrow |3\rangle(|5\rangle)$, while the reduced transitions are forbidden as shown in Fig.1. The Hamiltonian $\widehat{H}$ of this atomic system in RWA with $\hbar = 1$ is given as,

$$\widehat{H} = \lambda \Big[\widehat{R}_1\widehat{\sigma}_{13} + \widehat{R}_1^{\dagger}\widehat{\sigma}_{31} + \widehat{R}_2\widehat{\sigma}_{14} + \widehat{R}_2^{\dagger}\widehat{\sigma}_{41} + \widehat{R}_3\widehat{\sigma}_{23} + \widehat{R}_3^{\dagger}\widehat{\sigma}_{32} + \widehat{R}_4\widehat{\sigma}_{25} + \widehat{R}_4^{\dagger}\widehat{\sigma}_{52}\Big] \quad (1)$$

where

$$\widehat{R}_m = \widehat{a}e^{-i(\Delta_m t+\psi)}, \qquad \widehat{R}_m^{\dagger} = \widehat{a}^{\dagger}e^{i(\Delta_m t+\psi)},$$
$$\Delta_{1(2)} = \omega_{3(4)} - \omega_1 + \Omega, \qquad \Delta_{3(4)} = \omega_{3(5)} - \omega_1 + \Omega. \quad (2)$$

where λ refers to the coupling constant, $\widehat{\sigma}_{jk} = |j\rangle\langle k|(j,k = 1,2,...,5)$ are the population operators for $j = k$ and the raising (lowering) operators for $j \neq k$, $\Delta_m(m = 1,2,3,4)$ is the detuning parameter and ψ is the initial phase of a single mode field. It is important to mention that the definition of detuning depends on transitions form between levels. Based on this fact, the detuning here is different from the normal detuning of such considered system [36-38], where have considered that transition is $|m\rangle \rightarrow |m+1\rangle$ while we consider other forms [35] as shown in Fig.1.

It is easy to show that the fermonic and bosonic operators of the considered model have the following aspects:

$$\widehat{\sigma}_{jk}|k\rangle = |j\rangle, \quad \widehat{R}_m|n\rangle = \sqrt{n}\ e^{-i(\Delta_m t+\psi)}|n-1\rangle,$$
$$\widehat{R}_m^{\dagger}|n\rangle = \sqrt{n+1}\ e^{i(\Delta_m t+\psi)}|n+1\rangle. \quad (3)$$

where $|n\rangle$ is field Fock states.

Now, we turn our attention to solve the model of the considered atomic system. It is clear, the atom absorbs and emits one-photon in ideal cavity, the basis vectors for the wave function are $|1,n\rangle$, $|2,n\rangle$, $|3,n+1\rangle$, $|4,n+1\rangle$ and $|5,n+1\rangle$. Accordingly, the explicit form of the wave function of the atomic system at any time t can be written as,

$$|\Psi(t)\rangle = \sum_n q_n\Bigg[\Big(A_1(t)|1\rangle + A_2(t)|2\rangle\Big)\otimes|n\rangle + \Big(A_3(t)|3\rangle + A_4(t)|4\rangle + A_5(t)|5\rangle\Big)\otimes|n+1\rangle\Bigg] \quad (4)$$

where q_n describes the amplitude of state $|n\rangle$ and the coefficients $A_j(t) = A_j(n,t)$ is the probability amplitude which determines the initial state $|\Psi(0)\rangle$. Based on the well known time dependent Schrodinger equation $i\frac{d}{dt}|\Psi(t)\rangle = \widehat{H}|\Psi(t)\rangle$, we have the following system of differential equations for coefficients probability amplitudes:

$$i\tfrac{d}{dt}A_1(n,t) = f\ e^{-i\psi}\ \Big(e^{-i\Delta_1 t}A_3(n,t) + e^{-i\Delta_2 t}A_4(n,t)\Big),$$
$$i\tfrac{d}{dt}A_2(n,t) = f\ e^{-i\psi}\ \Big(e^{-i\Delta_3 t}A_3(n,t) + e^{-i\Delta_4 t}A_5(n,t)\Big),$$
$$i\tfrac{d}{dt}A_3(n,t) = f\ e^{i\psi}\ \Big(e^{i\Delta_1 t}A_1(n,t) + e^{i\Delta_3 t}A_2(n,t)\Big),$$
$$i\tfrac{d}{dt}A_4(n,t) = f\ e^{i\psi}\ e^{i\Delta_2 t}A_1(n,t),$$
$$i\tfrac{d}{dt}A_5(n,t) = f\ e^{i\psi}\ e^{i\Delta_4 t}A_2(n,t), \quad (5)$$

where

$$f = \lambda \sqrt{n+1}. \tag{6}$$

The solution of the previous coupled system of differential equations depends on the initial conditions of the considered atomic system. Now, let us consider that the atom is initially in the superposition states (excited state $|1\rangle$ and ground state $|3\rangle$) while the field is initially a coherent state $|\alpha\rangle$. These initial conditions mean that, we can write

$$|\Psi(0)\rangle = \sum_{n=0} q_n \left[\cos(\theta)|1,n\rangle + \sin(\theta)e^{-i\phi}|3,n+1\rangle\right]. \tag{7}$$

where $q_n = e^{\frac{-\overline{n}}{2}} \frac{\alpha^n}{\sqrt{(n!)}}$, $\overline{n} = |\alpha|^2$ is the initial mean photon number, θ is arbitrary parameter and ϕ is the corresponding phase of two states. Now, we focus on solving the coupled system (5) in different cases.

2.1 The resonant case

In case of resonant interaction the detuning parameter is absent in equation (5) ($\Delta_m = 0$). In this case, one has the probability amplitudes in the following form

$$\begin{aligned}
A_1(n,t) &= \tfrac{A_1(0)}{2}\left[\cos(\sqrt{3}\ ft) + \cos(\ ft)\right] - \tfrac{iA_3(0)}{\sqrt{3}} e^{-i\psi} \sin(\sqrt{3}\ ft), \\
A_2(n,t) &= \tfrac{A_1(0)}{2}\left[\cos(\sqrt{3}\ ft) - \cos(\ ft)\right] - \tfrac{iA_3(0)}{\sqrt{3}} e^{-i\psi} \sin(\sqrt{3}\ ft), \\
A_3(n,t) &= \tfrac{A_3(0)}{3}\left[1 + 2\cos(\sqrt{3}\ ft)\right] - \tfrac{iA_1(0)}{\sqrt{3}} e^{i\psi} \sin(\sqrt{3}\ ft), \\
A_4(n,t) &= \tfrac{-iA_1(0)}{2} e^{i\psi}\left[\tfrac{1}{\sqrt{3}}\sin(\sqrt{3}\ ft) + \sin(\ ft)\right] + \tfrac{A_3(0)}{3}\cos(\sqrt{3}\ ft) - \tfrac{A_3(0)}{3}, \\
A_5(n,t) &= \tfrac{-iA_1(0)}{2} e^{i\psi}\left[\tfrac{1}{\sqrt{3}}\sin(\sqrt{3}\ ft) - \sin(\ ft)\right] + \tfrac{A_3(0)}{3}\cos(\sqrt{3}\ ft) - \tfrac{A_3(0)}{3},
\end{aligned} \tag{8}$$

where

$$A_1(0) = \cos(\theta), \qquad A_3(0) = \sin(\theta)\ e^{i\phi}. \tag{9}$$

In what follows, we turn to obtain the solution of (5) in another case.

2.2 The off-resonant case

In the off-resonance interaction the detuning parameter $\Delta_m = \Delta$ and the probability amplitudes are:

$$\begin{aligned}
A_1(n,t) &= e^{-i\frac{\Delta}{2}t}\Big[\tfrac{A_1(0)}{2}\left(C_- + i\tfrac{\Delta}{2}S_-\right) + \tfrac{iA_3(0)}{3f}e^{-i\psi}\left(i\tfrac{\Delta}{2}C_+ - \delta_1^2 S_+\right) + \tfrac{\delta_3}{2f}e^{-i\psi}\left(C_+ + i\tfrac{\Delta}{2}S_+\right)\Big], \\
A_2(n,t) &= e^{-i\frac{\Delta}{2}t}\Big[\tfrac{-A_1(0)}{2}\left(C_- + i\tfrac{\Delta}{2}S_-\right) + \tfrac{iA_3(0)}{3f}e^{-i\psi}\left(i\tfrac{\Delta}{2}C_+ - \delta_1^2 S_+\right) + \tfrac{\delta_3}{2f}e^{-i\psi}\left(C_+ + i\tfrac{\Delta}{2}S_+\right)\Big],
\end{aligned} \tag{10}$$

$$A_3(n,t) = \tfrac{A_3(0)}{3}\left[1 + 2e^{i\frac{\Delta}{2}t}C_+\right] - i\delta_3 e^{i\frac{\Delta}{2}t}S_+,$$

$$A_4(n,t) = -e^{i\frac{\Delta}{2}t}\left[\tfrac{ifA_1(0)}{2}e^{i\psi}S_- - \tfrac{A_3(0)}{3}C_+ + i\tfrac{\delta_3}{2}S_+\right] - \tfrac{A_3(0)}{3},$$

$$A_5(n,t) = -e^{i\frac{\Delta}{2}t}\left[\tfrac{-ifA_1(0)}{2}e^{i\psi}S_- - \tfrac{A_3(0)}{3}C_+ + i\tfrac{\delta_3}{2}S_+\right] - \tfrac{A_3(0)}{3},$$

where

$$\begin{aligned}
&C_+ = \cos(\delta_1 t), \quad C_- = \cos(\delta_2 t), \qquad S_+ = \tfrac{\sin(\delta_1 t)}{\delta_1}, \quad S_- = \tfrac{\sin(\delta_2 t)}{\delta_2}, \\
&\delta_1 = \sqrt{\tfrac{\Delta^2}{4} + 3f^2}, \qquad \delta_2 = \sqrt{\tfrac{\Delta^2}{4} + f^2}, \quad \delta_3 = \tfrac{\Delta A_3(0)}{3} + fA_1(0)e^{i\psi}.
\end{aligned} \tag{11}$$

Next, we turn our attention to find the general solution of (5) when the non-resonant interaction is considered.

2.3 The non-resonant case

In this case, $\Delta_1 \neq \Delta_2 \neq \Delta_3 \neq \Delta_4 \neq 0$ and the general solution of (5) can be obtained as follows:

$$\begin{aligned}
A_1(n,t) &= -\tfrac{1}{f^3}\textstyle\sum_j \ c_j[\mu_j^3 + \gamma_1\mu_j^2 + \gamma_2\mu_j - \Delta_{34}f^2] \times e^{i[(\mu_j+\gamma_3)t-\psi]}, \\
A_2(n,t) &= -\tfrac{1}{f}\textstyle\sum_j \ c_j\mu_j \ e^{i[(\mu_j-\Delta_4)t-\psi]}, \\
A_3(n,t) &= \tfrac{1}{f^2}\textstyle\sum_j \ c_j[\mu_j^2 - \Delta_4\mu_j - f^2] \ e^{i[\mu_j+\Delta_{34}]t}, \\
A_4(n,t) &= -\tfrac{1}{f^4}\textstyle\sum_j \ c_j[\mu_j^4 + \gamma_4\mu_j^3 + \gamma_5\mu_j^2 + \gamma_6\mu_j + \gamma_7] \times e^{i[\mu_j+\gamma_3+\Delta_2]t}, \\
A_5(n,t) &= \textstyle\sum_j \ c_j \ e^{i\mu_j t},
\end{aligned} \tag{12}$$

with

$$\begin{aligned}
&c_j = \tfrac{\beta_1+\beta_2\Re_1+\beta_3\Re_2}{(\mu_j-\mu_\eta)(\mu_j-\mu_\zeta)(\mu_j-\mu_\xi)(\mu_j-\mu_\nu)}, && \Delta_{34} = \Delta_3 - \Delta_4, \\
&\gamma_1 = \Delta_{34} - \Delta_4, && \gamma_2 = -\Delta_{34}\Delta_4 - 2f^2, \\
&\gamma_3 = \Delta_{34} - \Delta_1, && \gamma_4 = \gamma_1 + \gamma_2, \\
&\gamma_5 = \gamma_1\gamma_3 + \gamma_2 - f^2, && \gamma_6 = \gamma_2\gamma_3 + f^2(\Delta_{34} + \Delta_4), \\
&\gamma_7 = f^2(\gamma_3\Delta_{34} + f^2), && \beta_1 = f^2 A_3(0), \\
&\beta_2 = f^2(fA_1(0) + \gamma_1 A_3(0)), && \beta_3 = -\beta_1\gamma_5 - \beta_2\gamma_4, \\
&\Re_1 = \mu_\eta + \mu_\zeta + \mu_\xi + \mu_\nu, \\
&\Re_2 = \mu_\eta\mu_\zeta + \mu_\eta\mu_\xi + \mu_\eta\mu_\nu + \mu_\zeta\mu_\xi + \mu_\zeta\mu_\nu + \mu_\xi\mu_\nu.
\end{aligned} \tag{13}$$

where $(j \neq \eta \neq \zeta \neq \xi \neq \nu = 1,2,...,5)$ and μ satisfy the fifth-order equation

$$\mu^5 + \Gamma_1\mu^4 + \Gamma_2\mu^3 + \Gamma_3\mu^2 + \Gamma_4\mu + \Gamma_5 = 0, \quad (14)$$

where

$$\Gamma_1 = \gamma_3 + \gamma_4 + \Delta_2, \qquad \Gamma_2 = \gamma_5 + \gamma_4(\gamma_3 + \Delta_2) - f^2,$$
$$\Gamma_3 = \gamma_6 + \gamma_5(\gamma_3 + \Delta_2) - \gamma_1 f^2, \; \Gamma_4 = \gamma_7 + \gamma_6(\gamma_3 + \Delta_2) - \gamma_2 f^2,$$
$$\Gamma_5 = \gamma_7(\gamma_3 + \Delta_2) + \Delta_{34} f^4. \quad (15)$$

One can be obtain the five roots of (14) using MATHLAB or MATHIMATICA program for initial values of the atomic system parameters. Now, having obtained the state vector $|\Psi(t)\rangle$, we are in a position to calculate and investigate the time-dependence aspects of the atomic system under consideration. In particular, in the next sections, we shall investigate the effects of the detunig parameter and the relative phase on the dynamical evolution of the mean photon number, the second-order correlation, the normal squeezing and the coherent properties of the field.

3 Mean photon number

In this section, we focus on finding some statistical properties for our atomic system. For example, we calculate the expressions of both the atomic populations (occupation numbers) and the mean photon number. Once the wave function $|\Psi(t)\rangle$ is obtained, the time evolution of any operator $\widehat{O}(t)$ can be easily calculated through the well known formula $\langle\widehat{O}(t)\rangle = \langle\Psi(t)|\widehat{O}(t)|\Psi(t)\rangle$. According to the wave function (4), we find that the occupation numbers $\langle\widehat{\sigma}_{jj}(t)\rangle$ are,

$$\langle\widehat{\sigma}_{jj}(t)\rangle = \sum_n P_n |A_j(n,t)|^2 \quad (16)$$

where $P_n = |q_n|^2$ is the initial distribution function for the field-mode. Accordingly, we consider the initial field is in a coherent state, i.e. it has the characteristic of Poisson distribution, i.e $P_n = e^{-\overline{n}} \; \frac{\overline{n}}{n!}$, where $\overline{n}$ is the initial mean photon number.
Similarly, we find that the mean photon number $\langle\widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle$ is given as

$$\langle\widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle = \sum_n P_n \Big[n(|A_1(n,t)|^2 + |A_2(n,t)|^2) + (n+1)(|A_3(n,t)|^2 + |A_4(n,t)|^2 + |A_5(n,t)|^2) \Big], \quad (17)$$

$$\langle\widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle = \sum_n P_n \Big[|A_3(n,t)|^2 + |A_4(n,t)|^2 + |A_5(n,t)|^2 \Big] + \sum_n n P_n.$$

From the properties of Poisson distribution ($\sum_n nP_n = \overline{n}$) using (16), we have

$$\langle\widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle = \overline{n} + \langle\widehat{\sigma}_{33}(t)\rangle + \langle\widehat{\sigma}_{44}(t)\rangle + \langle\widehat{\sigma}_{55}(t)\rangle. \quad (18)$$

Investigation of the mean photon number $\langle\widehat{a}^{\dagger}(t)\,\widehat{a}(t)\rangle$, is important where this function provides information about the collapses and revivals phenomenon.
Fig.2, illustrates the behavior of $\langle\widehat{a}^{\dagger}(t)\,\widehat{a}(t)\rangle$ against the scaled time λt for different values of the system parameters when the atom is initially prepared in super-position state; for example we take $\theta = \pi/4$. Also, we take a constant value of the initial mean photon number $\overline{n} = 10$, because Poisson distribution tends to the normal distribution when $\overline{n} \approx 10$ far a single mode. Furthermore, we investigate this function in the resonant and off-resonant interaction cases $\Delta_m = 0$ and $\Delta_m = \Delta = 10$ in Fig.2(a, b) (left plots), respectively. On the other hand, the relative phase $(\psi - \phi)$ is defined as the difference between the phase of the initial single mode field (ψ) and phase of the atomic levels (ϕ). It is also used in determining the phase mismatch. Thus, to realize the influence of the relative phase $(\phi - \psi)$, we set $(\phi - \psi) = \pi/6$ and $\pi/4$ as shown in Fig.2(c, d) (right plots), respectively.

This figure reveals that the collapses and revivals phenomenon is evident in all cases. On the other hand, the presence of the detuning parameter maximizes the collapse time as shown in Fig.2(b). The mechanism of this result is established as follows: The solutions of the model show that the Rabi frequency couples to the single mode field and gives the form $\sqrt{\frac{\Delta_m^2}{4} + f(n)}$, that is obvious in (11). Based on this fact, the collapses occur by the dephasing of the various terms in the sum of equation (18), where the revival time is directly proportional to $\sqrt{\frac{\Delta_m^2}{4} + f(n)}$. On the other hand, it is noticeable that the amplitude of the oscillations increases as the relative phase increases as shown in Fig.2(c,d). In what follows, we turn to discuss the Poissonian statistics through the behavior of the second-order correlation function.

4 Second-order correlation

This section addresses the photon statistics for our quantum system, where we examine the normalized second-order correlation function $g^2(t)$ [39]. If $g^2(t) = 1$ the system is Poissonian. Whereas, for $g^2(t) < (>)1$ it exhibits sub(super)-Poissonian statistics.
This function is defined by

$$g^2(t) = \frac{\langle(\widehat{a}^{\dagger}(t)\widehat{a}(t))^2\rangle - \langle\widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle}{\langle\widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle^2}. \quad (19)$$

With the same manner as in equations (17) and (18), the expectation value of $(\widehat{a}^{\dagger}\widehat{a})^2$ is given by

$$\langle(\widehat{a}^{\dagger}(t)\widehat{a}(t))^2\rangle = \sum_n P_n \Big[n^2(|A_1(n,t)|^2+|A_2(n,t)|^2) + (n+1)^2(|A_3(n,t)|^2+|A_4(n,t)|^2+|A_5(n,t)|^2)\Big], \tag{20}$$

$$\langle(\widehat{a}^{\dagger}(t)\widehat{a}(t))^2\rangle = \sum_n n^2 P_n + \sum_n (2n+1)P_n \times \Big[|A_3(n,t)|^2+|A_4(n,t)|^2+|A_5(n,t)|^2\Big].$$

According to one of the well known properties of Poisson distribution ($\sum_n n^2 P_n = \overline{n}^2+\overline{n}$) and the normalization condition ($\sum_j |A_j(n,t)|^2 = 1$), we have

$$\langle(\widehat{a}^{\dagger}(t)\widehat{a}(t))^2\rangle = \overline{n}^2+3\ \overline{n}+1-\langle\widehat{\sigma}_{11}(t)\rangle-\langle\widehat{\sigma}_{22}(t)\rangle -2\sum_n nP_n\Big[|A_1(n,t)|^2+|A_2(n,t)|^2\Big]. \tag{21}$$

Based on the above calculations and equation (18), the second-order correlation function can be obtained. Now, we concentrate on the effects of the detuning parameter and the relative phase on the behavior of the correlation function $g^2(t)$. Fig.3, visualizes the behavior of this function, where we plot $g^2(t)$ against the scaled time λt with the same initial data as in Fig.2. We see that the quantum system is oscillating between sub-Poisonian and super-Poisonian in the onset, while the system is exactly sub-Poisonian as the time progressed. However, the system is sub-poisonian when the detuning is considered as in Fig.3(b). Moreover, it is remarkable that as the relative phase increases the amplitude of the oscillations increases and the system is more super-Poisonian compared to the absent relative phase case, as shown in Figs.3(c,d). In the next section, we explore the normal squeezing phenomenon.

5 Normal squeezing

This section is to critically analyze the effects of the detuning and the relative phase on the normal squeezing phenomenon. Squeezing is one of the most important phenomena in quantum optics because of its applications in different areas, such as optics communication, quantum information theory, etc [40,41]. Therefore, we discuss here the normal squeezing. To explore the phenomenon of the considered atomic system, we define the two hermitian operators $\widehat{X}$ and $\widehat{P}$ given by [42],

$$\widehat{X} = \tfrac{1}{2}(\widehat{a}+\widehat{a}^{\dagger}),\ \widehat{P} = \tfrac{1}{2i}(\widehat{a}-\widehat{a}^{\dagger}). \tag{22}$$

It is easy to show that these two operators satisfy the commutation relation $[\widehat{X},\widehat{P}] = \frac{i}{2}$, which implies the following uncertainly relation,

$$(\Delta\widehat{X})^2(\Delta\widehat{P})^2 \geq \frac{1}{16}. \tag{23}$$

where $(\Delta\widehat{X})^2 = \langle\widehat{X}^2\rangle - \langle\widehat{X}\rangle^2$ and $(\Delta\widehat{P})^2 = \langle\widehat{P}^2\rangle - \langle\widehat{P}\rangle^2$ are the variances of $\widehat{X}$ and $\widehat{P}$, respectively. According to the above relation, a state is said to be squeezed in the $\widehat{X}$ or $\widehat{P}$ if

$$(\Delta\widehat{X})^2 < \tfrac{1}{4}\ \textbf{or}\quad (\Delta\widehat{P})^2 < \tfrac{1}{4}. \tag{24}$$

The expressions of $\Delta\widehat{X}^2$ and $\Delta\widehat{P}^2$ are given in the following forms:

$$(\Delta\widehat{X})^2 = \tfrac{1}{4}\Big[1+2\langle\widehat{a}^{\dagger}\widehat{a}\rangle+2\textbf{Re}\langle\widehat{a}^2\rangle-(\textbf{Re}\langle\widehat{a}\rangle)^2\Big],$$
$$(\Delta\widehat{P})^2 = \tfrac{1}{4}\Big[1+2\langle\widehat{a}^{\dagger}\widehat{a}\rangle-2\textbf{Re}\langle\widehat{a}^2\rangle-(\textbf{Im}\langle\widehat{a}\rangle)^2\Big]. \tag{25}$$

The last two expressions of this equation can be calculated directly from the following general form:

$$\langle\widehat{a}^s\ \rangle = |\alpha|^s\sum_n\ P_n\Big[A_1^*(n,t)A_1(n+s,t)+A_2^*(n,t)A_2(n+s,t) + \sqrt{\frac{(n+s+1)}{(n+1)}}\Big(A_3^*(n,t)A_3(n+s,t) + A_4^*(n,t)A_4(n+s,t)+A_5^*(n,t)A_5(n+s,t)\Big)\Big] \tag{26}$$

where $A_j^*(n,t)$ is the conjugate of $A_j(n,t)$. Now, we examine the effects of the detuning Δ and the relative phase $\phi-\psi$ on the normal squeezing through the quantity $(\Delta\widehat{X})^2$. Fig.4 shows these effects, which are also investigated when the atom is initially prepared in its excited state (solid line) and in super-position state (dot line), through putting $\theta=0$ and $\pi/4$, respectively. Also, the values of Δ and $\phi-\psi$ are the same as in the previous figures. This figure indicates that squeezing occurs for a short time in all considered cases. However, it only occurs in the onset when the atom is initially in a super-position state as seen in this figure. For the off-resonance case $\Delta_m=\Delta$, the amount of squeezing increases when the atom is initially prepared in a super-position ($\theta=\pi/4$) while the order is reversed for excited state ($\theta=0$). On the other hand, the relative phase effect is weak when the atom is in its excited state while it decreases the time of squeezing when the atom is in its super-position state. Also, it is clear that the presence of the relative phase increases the minimum value of squeezing, as seen in Fig.4(c,d). The following section is devoted to investigate the coherence properties of the field.

6 Coherence properties

This section is devoted to discuss the coherence aspects of the field [43,44]. The electric field E is defined by,

$$E = \varepsilon\Big(E_+ + E_-\Big), \tag{27}$$

where ε is a complex number with the dimension of the electric field and $E_{+(-)}$ is the positive (negative) frequency. It is well known that the positive (negative) frequency is proportional to the creation (annihilation) operator $\widehat{a}^\dagger$ ($\widehat{a}$). This means that

$$\frac{\langle E_+E_-\rangle}{\varepsilon^2} = \langle \widehat{a}^\dagger(t)\widehat{a}(t)\rangle, \ \frac{\langle E_+\rangle\langle E_-\rangle}{\varepsilon^2} = \langle \widehat{a}^\dagger(t)\rangle\langle \widehat{a}(t)\rangle. \quad (28)$$

where $\langle \widehat{a}^\dagger(t)\rangle$ is the conjugate of $\langle \widehat{a}(t)\rangle$. The electric field is in a coherent state if $\langle \widehat{a}^\dagger(t)\widehat{a}(t)\rangle$ and $\langle \widehat{a}^\dagger\rangle\langle \widehat{a}\rangle$ are coincident. The function $\langle E_+E_-\rangle$ is given in (18) while $\langle E_-\rangle$ and its conjugate $\langle E_+\rangle$ are given in (26) when $s = 1$. In Figs.5 and 6, we plot $\langle \widehat{a}^\dagger(t)\widehat{a}(t)\rangle$ (solid curve) and $\langle \widehat{a}^\dagger(t)\rangle\langle \widehat{a}(t)\rangle$ (dot curves) versus scaled time λt when $\theta = 0$ and $\pi/4$, respectively. Also, the same initial values of the detuning and the relative phase as in the previous figures are considered. These figures show that the two curves are completely coincident in the onset. Also, the presence of the detuning increases the coincident time for $\theta = 0$ and $\pi/4$ as shown in Fig.5(b) and Fig.6(b), respectively. Moreover, the existence of the relative phase has negligible effects when $\theta = 0$ as shown in Fig.5(c,d) but it increases the coincident time when $\theta = \pi/4$ as clear from Fig.6(c,d) compared with Fig.6(a). These results are consistent with those of [14,43].

7 Results and Discussion

Based on the previous calculations, the interaction between a five-level atom M-configuration and one-mode cavity field is investigated. The wave function of the model under consideration for the resonance, off-resonance and non-resonant interaction is explicitly obtained. Some of phenomena related to the field such as the collapses-revivals, bunching and anti-bunching of photons, the squeezing and the coherence properties are investigated. These investigations are presented when the atom is initially prepared in its super-position while the input field is in a coherent state. The influence of both the detuning and relative phase on the behavior of these phenomena are explored. The numerical investigations for the mean photon number and second-order correlation function, the normal squeezing and coherence properties are analyzed. These investigations are realized when the atom is in a super-position state ($\theta = \pi/4$). In addition, both the normal squeezing and the coherence properties are also examined when the atom is in upper state ($\theta = 0$).

In Fig.2, investigations of the mean photon number $\langle \widehat{a}^\dagger(t)\ \widehat{a}(t)\rangle$, for different values of the system parameters when the atom is initially prepared in super-position state, show that the collapses and revivals phenomenon are evident in all cases. Also, the presence of the detuning parameter increases the collapse time. However, the amplitude of the oscillations increases in the existence of the relative phase. Moreover, Fig.3 illustrates the effect of the detuning parameter and the relative phase on the behavior of the correlation function $g^2(t)$. It is noticeable that, the system is oscillating between sub-Poisonian and super-Poisonian in the onset and is exactly sub-Poisonian as the time progressed. In the off-resonant interaction the system is sub-poisonian. Also, the presence of the relative phase increases the super-Poisonian statistics.

Exploring the effects of the detuning Δ and the relative phase $\phi - \psi$ on the normal squeezing shows that squeezing occurs for a short time but occurs in the onset when the atom is in a super-position state. Also, we observe that the presence of the detuning increases the squeezing for a super-position state and the opposite occurs for excited state. Moreover, the relative phase effect is weak when the atom is in its excited state while it is clear in super-position state. Finally, numerical computations of the coherence properties show that the coherence occurs in the onset where the two curves are completely coincident. Also, the presence of the detuning increases this coherence while the existence of the relative phase has no effects when $\theta = 0$ but it increases the coherence time when $\theta = \pi/4$.

8 Conclusion

The present study is one of the first investigations that focus on the M-type five-level atom interacting with one-mode cavity field. The exact solution of the model describing this atomic system is obtained. The wave function for the considered system is given when the atom and field are initially prepared in a super-position state and coherent state, respectively. Significant aspects of the field such as the mean value of the photon number, the normalized second-order correlation function, the normal squeezing and the coherence properties have been calculated and numerically investigated. We examined the effects of both the detuning and relative phase on the behavior of these quantities.

Moreover, the presence of the detuning and relative phase changes the behavior of the collapses-revivals, bunching and anti-bunching of photons, normal squeezing and coherence properties of the considered system. The present study can be in the same manner when the atom and the field are considered initially in other states. Future studies should focus on studying the model proposed in this paper when the coupling constants are various. Also, the considered model will be extended to include the Kerr-like medium or/and the intensity dependent coupling.

Competing interests: The authors declare that they have no competing interests.

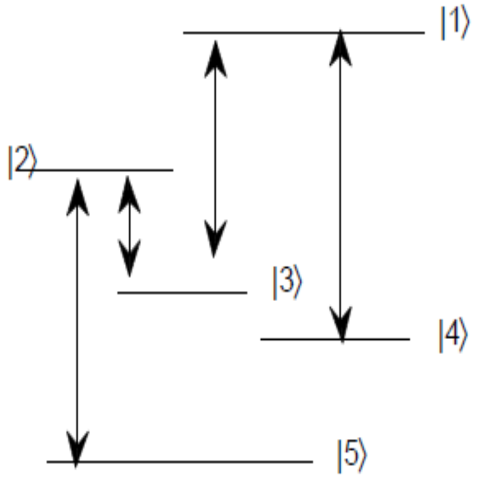

Fig. 1: Five-level atomic structure for M-type atom.

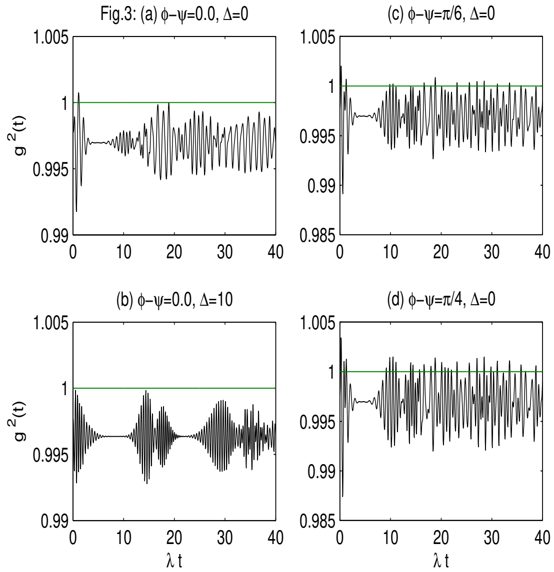

Fig. 3: The same as in Fig.2 but for the second-order correlation function $g^2(t)$.

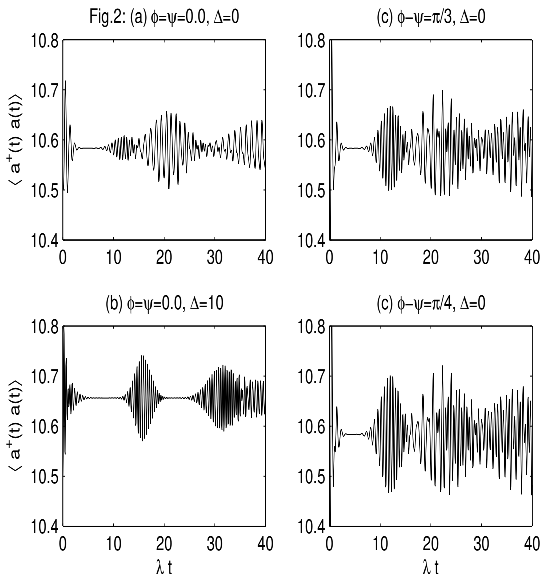

Fig. 2: Evolution of the mean photon number $\langle \hat{a}^{\dagger}\hat{a} \rangle$ versus the scaled time λt, for $\overline{n} = 10$ and $\theta = \pi/4$, and different values of Δ (left plots) and different values of $\phi - \psi$ (right plots).

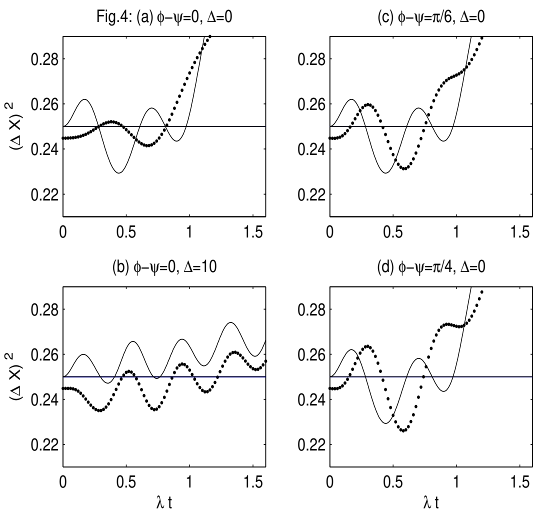

Fig. 4: The time evolutions for the normal squeezing through the parameter $(\Delta\widehat{X})^2$, with the same initial data as in Fig.2 but solid curves for $\theta = 0$ and dot curves for $\theta = \pi/4$.

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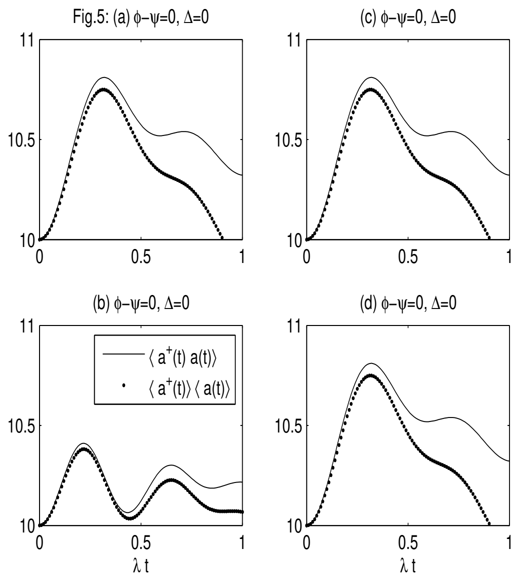

Fig. 5: The same as in Fig.2 but for the coherent properties when $\theta = 0$, solid curves for $\langle \hat{a}^{\dagger}\hat{a} \rangle$ and dot curves for $\langle \hat{a}^{\dagger} \rangle \langle \hat{a} \rangle$.

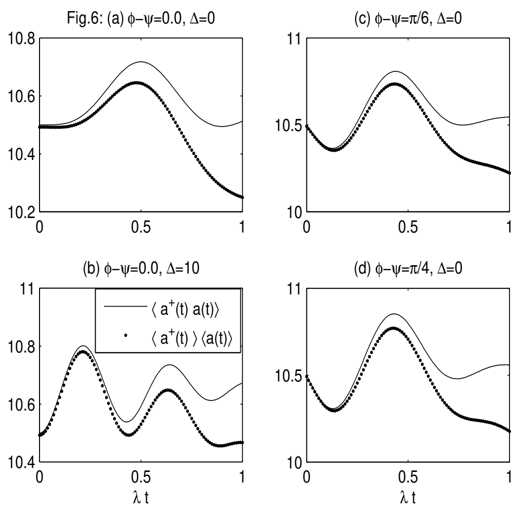

Fig. 6: The same as in Fig.5 but for $\theta = \pi/4$.

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N. H. Abdel-Wahab is presently employed as a professor of Applied Mathematics in Department of Mathematics, Faculty of science, Minia University, Egypt. His research interests Quantum Mechanics. He published and Co-authored more than 40 papers in reputed international journals. He is a reviewer for many papers and theses and also supervisor for some master's and doctoral theses.

R. M. Farghly is a lecturer of Pure Mathematics at Department of Mathematics, Faculty of science, Minia University, Egypt. Her research interests Quantum Algebra.

H. A. Hassan is Demonstrator of Applied Mathematics at Mathematics Department, Faculty of Science, Minia University, Minia, Egypt. Her research interests Quantum Mechanics.