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A Retrial Inventory System with Impatient Customers

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Abstract: This article presents a stochastic inventory system under continuous review at a service facility consisting of a finite waiting room and a single server, in which two types of customers arrive in Poisson processes with arrival rates λ_1 for high priority and λ_2 for low priority customers. The low priority customers arrive only for repair. The inventory is replenished according to an (s, S) policy and the replenishing times are assumed to be exponential. The service times follow exponential distributions with parameters μ_1 and μ_2 for high and low priority customers respectively. Retrial and impatience are introduced for low priority customers only. The orbiting customers independently renege the system after an exponentially distributed time with parameter $\alpha > 0$. The orbiting customers compete for service by sending signals that are exponentially distributed. The joint probability distribution of the number of customers in the waiting area, the number of customers in the orbit and the inventory level is obtained for the steady state case. Some important system performance measures in the steady state are derived. Several numerical examples are presented to illustrate the effect of the system parameters and costs on these measures.

Keywords: (s,S) policy, Continuous review, Inventory with service time, Markov process, Priority customers, Retrial, Impatient customers.

1 Introduction

Most inventory models in the literature make the assumption that inter demand times are independently and identically distributed random variables ([3], [5]). Some models make a more general assumption and consider the case of state-dependent demand rates. In all these models, the major assumption is that all customers are treated alike and if there is stock on-hand, the demand by a customer is met. However, in certain cases some customers may have financial resources to pay higher prices than others and hence are treated as preferential customers by retailers. The retailers would not like to turn away a preferential customer and as such even when there is stock on-hand, they may inform an ordinary customer that the product is sold out thus hoarding items for preferential customers. In assembly manufacturing system customers with long-term supply contracts have been given higher priority than the other ordinary customers. In multi-specialty hospitals patients with serious illness are given higher priority than the other patients opting for routine checks or else.

The concept of retrial demands in inventory was introduced by Artalejo et al. [1]. They assumed Poisson demand, exponentially distributed lead time and retrial time. In their work, the authors proceeded with an algorithmic analysis of the system. Ushakumari [11] considered a retrial inventory system with a classical retrial policy. The author has assumed a Markovian setup for the time between consecutive arrivals, replenishment, and retrials. Krishnamoorthy and Jose [4] analyzed and compared three (s,S) inventory systems with positive service time and retrial of customers.

An important issue in the queueing - inventory system with two classes of customers is the priority assignment problem. Ning Zhao and Zhaotong Lian [6] analyzed a queueing - inventory system with two classes of customers. The authors have assumed Poisson arrival and exponential service times. Each service using one item in the attached inventory supplied by an outside supplier with exponentially distributed lead time. Choi and Chang [2] analyzed single server retrial queues with priority calls. Sapna [10] analyzed a continuous review (s,S) inventory system with priority customers and arbitrarily

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distributed lead times. The author has assumed that the arrival of the two-types of customers form independent Poisson processes. Real life problems stimulate one to study the queueing - inventory system with two types of customers.

In all the above models, the authors assumed that at the service completion epoch the customer receives an item and hence the inventory level was decreased by one. But in real life situations, an arriving customer may demand an item or alternatively may overhaul the item. For example, in a car sales and service shop, a customer may buy a new car or demand repair only.

Reneging is an important feature in many real - world queueing contexts. In fact, Palm [7] introduced reneging as a means of modelling the behaviour of telephone switchboard customers more than 70 years ago. However, due to the explosive growth of the call centre industry, there has been renewed interest in such models in recent years. In the call centre setting, customer impatience (amplified by large customer loads) leads naturally to large amounts of reneging. Ignoring the presence of reneging can lead to inappropriate sizing of the system and poor staffing allocation. Models in which reneging is present are also potentially valuable in problem contexts, Ward and Glynn [12]. Paul Manual et al. [9] analyzed a service facility inventory system with impatient customers. The authors have assumed the arrival time points of customers form a Poisson process. The service time, life time of items in stock and the lead time of orders are all assumed to be independently distributed as exponential. The waiting customer independently reneges the system after an exponentially distributed amount of time. An item demanded by a customer is issued after performing service on the item. Patrick Wiichner et al. [8] analyzed a finite source M/M/S retrial queue with search for balking and impatient customers from the orbit. The authors have assumed that the arriving customers may either join the queue or go to the orbit. Moreover, the requests becomes impatient and abandon the buffer after a random time and enter the orbit, too.

This paper considers the two types of customers arriving in Poisson fashion and the service times follow exponential distributions. Retrial and impatience are introduced for low priority customers only. A non-pre-emptive priority service rule is assumed. The joint probability distribution of the number of customers in the waiting area, the number of customers in the orbit and the inventory level is obtained for the steady state case. Various measures of system performance are computed in the steady state case and the results are illustrated with numerical examples.

The rest of the paper is organized as follows: the problem formulation and the notations used in this paper are described in the next section. Analysis of the model and the steady state solutions of the model are proposed in section 3. Some key system performance measures are derived in section 4. In section 5, the results are illustrated with numerical examples. The last section is meant for conclusion.

2 Problem formulation

Consider an inventory system with a maximum capacity of S for stocking units at a service facility. Customers arriving at the service station belong to any one of the two types such that the high and the low priority customers and their arrivals belong to independent Poisson processes with parameters λ_1 and λ_2 respectively. The low priority customers request overhaul of the item only and the high priority customers demand unit item, which is delivered after performing service on the item. The high and low priority customers receive their service one-by-one. The service times follow exponential distributions with parameters μ_1 and μ_2 for high and low priority customers respectively. The waiting area is limited to accommodate a maximum number N of high priority customers including the one at the service point. The retrial and impatience are introduced for low priority customers only. Whenever the server is idle, an arriving low priority customer is immediately taken for service by the server. Otherwise, that customer enters into the orbit of finite size M. The orbiting customers compete for service by sending out signals that are exponentially distributed. The customers in the orbit may either retry or may leave the orbit. In this article the latter type of customers are described as impatient (reneging) customers. An impatient customer in the orbit leaves the orbit independently after a random time which is distributed as negative exponential with parameter $\alpha(>0)$.

When the server is busy (high/low) and there are spaces available in the waiting area, the arriving high priority customer can enter into the service station and waits for service. If the server is idle and the inventory level is positive then the arriving high priority customer can enter into the service station and taken for service immediately by the server. When the server is idle, the inventory level is zero and there is space available in the waiting area, then the arriving high priority customer can enter into the service station and waits for his turn. When the inventory level is zero and server is idle, the arriving low priority customer (primary/retrial) will be taken for service. If the server is busy with a low priority customer, the inventory level is zero with no high priority customers in the waiting area, then after the service completion the server becomes idle. Otherwise, if the server is busy with a low priority customer, the inventory level is positive with at least one high priority customer in the waiting area, then after the completion of service, the server will immediately become busy with a high priority customer. Any arriving high priority customer, who finds the waiting room full is considered to be lost. It is also assumed that any arriving low priority customer, who finds that the server is busy and there is no space in the orbit, is considered to be lost. The reorder level for the

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commodity is fixed as s and an order is placed when the inventory level reaches the reorder level s. The ordering quantity for the commodity is $Q(= S - s > s + 1)$ items. The requirement $S - s > s + 1$ ensures that after a replenishment the inventory level will be always above the reorder level. Otherwise it may not be possible to place reorders which leads to a perpetual shortage. The lead time is assumed to be distributed as negative exponential with parameter $\beta(> 0)$.

In this article the classical retrial policy is followed. More explicitly, when there are $i \geq 1$ low priority customers in the orbit, a signal is sent out according to an exponential distribution with parameter $\theta > 0$. That is, the probability of a repeated attempt in an interval $(t, t + dt)$ given that i customers are in the orbit at time t is $i\theta + o(dt)$. Also here a non-preemptive priority service rule is assumed, that is, when the server is engaging with a low priority customer and at that time a high priority customer arrives the high priority customer will get service only after completion of the service of the low priority customer who is in service. Further, it is assumed that the inter arrival times between high and low priority customers, intervals between repeated attempts of the retrial times, service times of high and low priority customers, and the lead times are mutually independent exponential distributions.

Notations:

I : Identity matrix

$\mathbf{e}^T$: $(1, 1, \ldots, 1)$

$\mathbf{0}$: Zero matrix

$[A]_{ij}$: entry at $(i, j)^{th}$ position of a matrix A

$$\delta_{ij} : \begin{cases} 1 & \text{if} \quad j = i \\ 0 & \text{otherwise} \end{cases}$$

$\bar{\delta_{ij}}$: $1 - \delta_{ij}$

$$H(x) : \begin{cases} 1, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$k \in V_i^j$: $k = i, i+1, \ldots j$

$$Y(t) : \begin{cases} 2, & \text{if the server is busy with a low priority customer at time t} \\ 1, & \text{if the server is busy with a high priority customer at time t} \\ 0, & \text{if the server is idle at time t} \end{cases}$$

3 Analysis

Let $L(t)$, $Y(t)$, $X_1(t)$ and $X_2(t)$, respectively, denote the inventory level, the server status, the number of high priority customers (waiting and being served) in the waiting area and the number of low priority customers in the orbit at time t. From the assumptions made on the input and output processes, it can be shown that the quadruplet $\{(L(t), Y(t), X_1(t), X_2(t), t \geq 0)\}$ is a continuous time Markov chain with state space given by $E = E_1 \cup E_2 \cup E_3 \cup E_4$ with

$$E_1 : \{(0, 0, i_3, i_4) \mid i_3 = 0, 1, 2, \ldots, N,\ i_4 = 0, 1, 2, \ldots, M, \}$$
$$E_2 : \{(i_1, 0, 0, i_4) \mid i_1 = 1, 2, \ldots, S,\ i_4 = 0, 1, 2, \ldots, M, \}$$
$$E_3 : \{(i_1, 1, i_3, i_4) \mid i_1 = 1, 2, \ldots, S,\ i_3 = 1, 2, \ldots, N,\ i_4 = 0, 1, 2, \ldots, M, \}$$
$$E_4 : \{(i_1, 2, i_3, i_4) \mid i_1 = 0, 1, 2, \ldots, S,\ i_3 = 0, 1, 2, \ldots, N,\ i_4 = 0, 1, 2, \ldots, M, \}$$

By ordering the set of states of E lexicographically, the infinitesimal generator A=$(A((i_1, i_2, i_3, i_4), (j_1, j_2, j_3, j_4)))$, $(i_1, i_2, i_3, i_4), (j_1, j_2, j_3, j_4) \in E$, can be conveniently expressed in a block partitioned matrix with entries.

$$[A]_{i_1 j_1} = \begin{cases} A_2, & j_1 = i_1 - 1, \ i_1 = 1 \\ A_1, & j_1 = i_1 - 1, \ i_1 \in V_2^S \\ C, & j_1 = i_1 + Q, \ i_1 \in V_1^s \\ C_1, & j_1 = i_1 + Q, \ i_1 = 0, \\ B_0, & j_1 = i_1, \qquad i_1 = 0 \\ B_1, & j_1 = i_1, \qquad i_1 \in V_1^s \\ B_2, & j_1 = i_1, \qquad i_1 \in V_{s+1}^S \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

with

$$[A_2]_{i_2 j_2} = \begin{cases} A_{10}^{(2)}, & j_2 = 0, \qquad i_2 = 1, \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[A_{10}^{(2)}]_{i_3 j_3} = \begin{cases} F_1, & j_3 = i_3 - 1, \ i_3 \in V_1^N, \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[F_1]_{i_4 j_4} = \begin{cases} \mu_1, & j_4 = i_4, \qquad i_4 \in V_0^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[A_1]_{i_2 j_2} = \begin{cases} A_{10}^{(1)}, & j_2 = 0, \qquad i_2 = 1, \\ A_{11}^{(1)}, & j_2 = i_2, \qquad i_2 = 1, \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[A_{10}^{(1)}]_{i_3 j_3} = \begin{cases} F_1, & j_3 = i_3 - 1, \ i_3 = 1 \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[A_{11}^{(1)}]_{i_3 j_3} = \begin{cases} F_1, & j_3 = i_3 - 1, \ i_3 \in V_2^N, \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[C]_{i_2 j_2} = \begin{cases} C_{00}, & j_2 = i_2, \qquad i_2 = 0, \\ C_{11}, & j_2 = i_2, \qquad i_2 = 1, \\ C_{22}, & j_2 = i_2, \qquad i_2 = 2, \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[C_{00}]_{i_3 j_3} = \begin{cases} W_1, & j_3 = 0, \qquad i_3 = 0 \\ \mathbf{0}, & \text{otherwise}, \end{cases}$$

$$[W_1]_{i_4 j_4} = \begin{cases} \beta, & j_4 = i_4, \qquad i_4 \in V_0^M, \\ 0, & \text{otherwise}, \end{cases}$$

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$$[C11]_{i_3 j_3} = \begin{cases} W_1, \; j_3 = i_3, & i_3 \in V_1^N, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[C_{22}]_{i_3 j_3} = \begin{cases} W_1, \; j_3 = i_3, & i_3 \in V_0^N, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[C_1]_{i_2 j_2} = \begin{cases} C_{00}^{(1)}, \; j_2 = i_2, & i_2 = 0, \\ C_{01}^{(1)}, \; j_2 = 1, & i_2 = 0, \\ C_{22}^{(1)}, \; j_2 = 2, & i_2 = 2, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[C_{00}^{(1)}]_{i_3 j_3} = \begin{cases} W_1, \; j_3 = i_3, & i_3 = 0, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[C_{01}^{(1)}]_{i_3 j_3} = \begin{cases} W_1, \; j_3 = i_3, & i_3 \in V_1^N, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[C_{22}^{(1)}]_{i_3 j_3} = \begin{cases} W_1, \; j_3 = i_3, & i_3 \in V_0^N, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[B_0]_{i_2 j_2} = \begin{cases} B_{00}^{(0)}, \; j_2 = i_2, & i_2 = 0, \\ B_{02}^{(0)}, \; j_2 = 2, & i_2 = 0, \\ B_{20}^{(0)}, \; j_2 = 0, & i_2 = 2, \\ B_{22}^{(0)}, \; j_2 = i_2, & i_2 = 2, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[B_{00}^{(0)}]_{i_3 j_3} = \begin{cases} G, \quad j_3 = i_3, & i_3 \in V_0^{N-1}, \\ G_1, \; j_3 = i_3, & i_3 = N, \\ F, \quad j_3 = i_3 + 1, & i_3 \in V_0^{N-1}, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[G]_{i_4 j_4} = \begin{cases} -(\lambda_1 + \lambda_2 + \beta + i_4\theta + i_4\alpha), \; j_4 = i_4, & i_4 \in V_0^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[G_1]_{i_4 j_4} = \begin{cases} -(\lambda_2 + \beta + i_4\theta + i_4\alpha), \; j_4 = i_4, & i_4 \in V_0^M, \\ i_4\alpha, \quad j_4 = i_4 - 1, & i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[F]_{i_4 j_4} = \begin{cases} \lambda_1, \; j_4 = i_4, & i_4 \in V_0^M, \\ 0, \quad \text{otherwise}, \end{cases}$$

$$[B_{02}^{(0)}]_{i_3 j_3} = \begin{cases} G_0, \; j_3 = i_3, & i_3 \in V_0^N, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[G_0]_{i_4 j_4} = \begin{cases} \lambda_2, \quad j_4 = i_4, & i_4 \in V_0^M, \\ i_4\theta, \; j_4 = i_4 - 1, & i_4 \in V_1^M, \\ 0, \quad \text{otherwise}, \end{cases}$$

$$[B_{20}^{(0)}]_{i_3 j_3} = \begin{cases} F_0, \; j_3 = i_3, & i_3 \in V_0^N, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[F_0]_{i_4 j_4} = \begin{cases} \mu_2, \; j_4 = i_4, & i_4 \in V_0^M, \\ 0, \quad \text{otherwise}, \end{cases}$$

$$[B_{22}^{(0)}]_{i_3 j_3} = \begin{cases} G_2, \; j_3 = i_3, & i_3 \in V_0^{N-1}, \\ G_3, \; j_3 = i_3, & i_3 = N, \\ F, \quad j_3 = i_3 + 1, & i_3 \in V_0^{N-1}, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[G_2]_{i_4 j_4} = \begin{cases} -(\lambda_1 + \lambda_2\bar{\delta}_{i_4 M} + \beta + \mu_2 + i_4\alpha), & j_4 = i_4, \; i_4 \in V_0^M, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ 0, & \text{otherwise}, \end{cases}$$

$$[G_3]_{i_4 j_4} = \begin{cases} -(\lambda_2\bar{\delta}_{i_4 M} + \beta + \mu_2 + i_4\alpha), & j_4 = i_4, \; i_4 \in V_0^M, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ 0, & \text{otherwise}, \end{cases}$$

$$[B_1]_{i_2 j_2} = \begin{cases} B_{00}^{(1)}, \; j_2 = i_2, & i_2 = 0, \\ B_{01}^{(1)}, \; j_2 = 1, & i_2 = 0, \\ B_{02}^{(1)}, \; j_2 = 2, & i_2 = 0, \\ B_{11}^{(1)}, \; j_2 = i_2, & i_2 = 1, \\ B_{20}^{(1)}, \; j_2 = 0, & i_2 = 2, \\ B_{21}^{(1)}, \; j_2 = 1, & i_2 = 2, \\ B_{22}^{(1)}, \; j_2 = i_2, & i_2 = 2, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[B_{00}^{(1)}]_{i_3 j_3} = \begin{cases} G, \; j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{01}^{(1)}]_{i_3 j_3} = \begin{cases} F, \; j_3 = 1, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{02}^{(1)}]_{i_3 j_3} = \begin{cases} G_0, \; j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[B_{11}^{(1)}]_{i_3 j_3} = \begin{cases} D, \quad j_3 = i_3, & i_3 \in V_1^{N-1}, \\ D_0, \; j_3 = i_3, & i_3 = N, \\ F, \quad j_3 = i_3 + 1, & i_3 \in V_1^{N-1}, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[D]_{i_4 j_4} = \begin{cases} -(\lambda_1 + \lambda_2\bar{\delta}_{i_4 M} + \beta + \mu_1 + i_4\alpha), & j_4 = i_4, \; i_4 \in V_0^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[D_0]_{i_4 j_4} = \begin{cases} -(\lambda_2\bar{\delta}_{i_4 M} + \beta + \mu_1 + i_4\alpha), & j_4 = i_4, \; i_4 \in V_0^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

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$$[B_{20}^{(1)}]_{i_3 j_3} = \begin{cases} F_0, \; j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{21}^{(1)}]_{i_3 j_3} = \begin{cases} F_0, \; j_3 = i_3, & i_3 \in V_1^N, \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{22}^{(1)}]_{i_3 j_3} = \begin{cases} G_2, \; j_3 = i_3, & i_3 \in V_0^{N-1} \\ G_3, \; j_3 = i_3, & i_3 = N \\ F, \; j_3 = i_3 + 1, & i_3 \in V_0^{N-1} \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_2]_{i_2 j_2} = \begin{cases} B_{00}^{(2)}, \; j_2 = i_2, & i_2 = 0, \\ B_{01}^{(2)}, \; j_2 = 1, & i_2 = 0, \\ B_{02}^{(2)}, \; j_2 = 2, & i_2 = 0, \\ B_{11}^{(2)}, \; j_2 = i_2, & i_2 = 1, \\ B_{20}^{(2)}, \; j_2 = 0, & i_2 = 2, \\ B_{21}^{(2)}, \; j_2 = 1, & i_2 = 2, \\ B_{22}^{(2)}, \; j_2 = i_2, & i_2 = 2, \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{00}^{(2)}]_{i_3 j_3} = \begin{cases} W, \; j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[W]_{i_4 j_4} = \begin{cases} -(\lambda_1 + \lambda_2 + i_4\theta + i_4\alpha), & j_4 = i_4, \quad i_4 \in V_0^M, \\ i_4\alpha, & j_4 = i_4, \quad i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[B_{01}^{(2)}]_{i_3 j_3} = \begin{cases} F, \; j_3 = 1, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{02}^{(2)}]_{i_3 j_3} = \begin{cases} G_0, \; j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{11}^{(2)}]_{i_3 j_3} = \begin{cases} G_4, \; j_3 = i_3, & i_3 \in V_1^{N-1}, \\ G_5, \; j_3 = i_3, & i_3 = N, \\ F, \; j_3 = i_3 + 1, & i_3 \in V_1^{N-1}, \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[G_4]_{i_4 j_4} = \begin{cases} -(\lambda_1 + \lambda_2 \bar{\delta}_{i_4 M} + \mu_1 + i_4\alpha), & j_4 = i_4, \quad i_4 \in V_0^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[G_5]_{i_4 j_4} = \begin{cases} -(\lambda_2 \bar{\delta}_{i_4 M} + i_4\alpha + \mu_1), & j_4 = i_4, \quad i_4 \in V_0^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[B_{20}^{(2)}]_{i_3 j_3} = \begin{cases} F_0, \; j_3 = i_3, & i_3 = 0 \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{21}^{(2)}]_{i_3 j_3} = \begin{cases} F_0, \; j_3 = i_3, & i_3 \in V_1^N, \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[B_{22}^{(2)}]_{i_3 j_3} = \begin{cases} G_6, \; j_3 = i_3, & i_3 \in V_0^{N-1}, \\ G_7, \; j_3 = i_3, & i_3 = N, \\ F, \; j_3 = i_3 + 1, & i_4 \in V_0^{N-1}, \\ \mathbf{0}, \; \text{otherwise}, \end{cases}$$

$$[G_6]_{i_4 j_4} = \begin{cases} -(\lambda_1 + \lambda_2 \bar{\delta}_{i_4 M} + \mu_2 + i_4\alpha), & j_4 = i_4, \quad i_4 \in V_0^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

$$[G_7]_{i_4 j_4} = \begin{cases} -(\lambda_2 \bar{\delta}_{i_4 M} + i_4\alpha + \mu_2), & j_4 = i_4, \quad i_4 \in V_0^M, \\ \lambda_2, & j_4 = i_4 + 1, \; i_4 \in V_0^{M-1}, \\ i_4\alpha, & j_4 = i_4 - 1, \; i_4 \in V_1^M, \\ 0, & \text{otherwise}, \end{cases}$$

It may be noted that the matrices A_1, A_2, B_0, B_1, B_2, C and C_1 are square matrices of size $2(N+1)(M+1)$. B_{00}^0, B_{02}^0, B_{20}^0, B_{22}^0, B_{22}^1, B_{22}^2, C_{22} and C_{22}^1 are square matrices of size $(N+1)(M+1)$. A_{11}^1, B_{11}^1, B_{11}^2 and C_{11} are square matrices of size $N(M+1)$. D, D_0, F, F_0, F_1, G, $G_i, i \in V_0^7$, W, W_1, B_{00}^1, B_{00}^2 and C_{00} are square matrices of size $(M+1)$. B_{21}^1, B_{21}^2 and C_{01}^1 are matrices of size $(N+1)(M+1) \times N(M+1)$. B_{20}^2, B_{20}^1 and C_{00}^1 are matrices of size $(N+1)(M+1) \times (M+1)$. B_{01}^1 and B_{01}^2 are matrices of size $(M+1) \times N(M+1)$. B_{02}^1 and B_{02}^2 are matrices of size $(M+1) \times (N+1)(M+1)$. A_{10}^1 is of size $N(M+1) \times (M+1)$. A_{10}^2 is of size $N(M+1) \times (N+1)(M+1)$.

3.1 Steady state analysis

It can be seen from the structure of A that the homogeneous Markov process $\{L(t), Y(t), X_1(t), X_2(t) : t \geq 0\}$ on the finite space E is irreducible, aperiodic and persistent non-null. Hence the limiting distribution
$\pi^{(i_1,i_2,i_3,i_4)} = \lim_{t\to\infty} Pr[L(t) = i_1, Y(t) = i_2, X_1(t) = i_3, X_2(t) = i_4 | L(0), Y(0), X_1(0), X_2(0)]$

exists. Let
$\Pi = (\Pi^{(0)}, \Pi^{(1)}, \ldots, \Pi^{(S)})$

$$\begin{aligned} \Pi^{(0)} &= (\Pi^{(0,0)}, \Pi^{(0,2)}), \\ \Pi^{(i_1)} &= (\Pi^{(i_1,0)}, \Pi^{(i_1,1)}, \Pi^{(i_1,2)}, \qquad i_1 = 1, \ldots, S; \end{aligned}$$

which is partitioned as follows:

$$\begin{aligned} \Pi^{(0,0)} &= (\Pi^{(0,0,0)}, \Pi^{(0,0,1)}, \ldots, \Pi^{(0,0,N)}), \\ \Pi^{(0,2)} &= (\Pi^{(0,2,0)}, \Pi^{(0,2,1)}, \ldots, \Pi^{(0,2,N)}), \\ \Pi^{(i_1,0)} &= \Pi^{(i_1,0,0)}, & i_1 = 1, 2, \ldots, S; \\ \Pi^{(i_1,1)} &= (\Pi^{(i_1,1,1)}, \Pi^{(i_1,1,2)}, \ldots, \Pi^{(i_1,1,N)}), & i_1 = 1, 2, \ldots, S; \\ \Pi^{(i_1,2)} &= (\Pi^{(i_1,2,0)}, \Pi^{(i_1,2,1)}, \ldots, \Pi^{(i_1,2,N)}), & i_1 = 1, 2, \ldots, S; \end{aligned}$$

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Further the above vectors are also partitioned as follows:

$$\begin{aligned}
\Pi^{(0,0,i_3)} &= (\Pi^{(0,0,i_3,0)}, \Pi^{(0,0,i_3,1)}, \ldots, \Pi^{(0,0,i_3,M)}), \\
& \quad i_3 = 0,1,2,\ldots,N; \\
\Pi^{(0,2,i_3)} &= (\Pi^{(0,2,i_3,0)}, \Pi^{(0,2,i_3,1)}, \ldots, \Pi^{(0,2,i_3,M)}), \\
& \quad i_3 = 0,1,2,\ldots,N; \\
\Pi^{(i_1,0,0)} &= (\Pi^{(i_1,0,0,0)}, \Pi^{(i_1,0,0,1)}, \ldots, \Pi^{(i_1,0,0,M)}), \\
& \quad i_1 = 1,2,\ldots,S; \\
\Pi^{(i_1,1,i_3)} &= (\Pi^{(i_1,1,i_3,0)}, \Pi^{(i_1,1,i_3,1)}, ,\ldots, \Pi^{(i_1,1,i_3,M)}), \\
& \quad i_1 = 1,2,\ldots,S;\ i_3 = 1,2\ldots N; \\
\Pi^{(i_1,2,i_3)} &= (\Pi^{(i_1,2,i_3,0)}, \Pi^{(i_1,2,i_3,1)}, ,\ldots, \Pi^{(i_1,2,i_3,M)}), \\
& \quad i_1 = 1,2,\ldots,S;\ i_3 = 0,1,2\ldots N;
\end{aligned}$$

The vector of limiting probabilities Π then satisfies

$$\Pi A = \mathbf{0} \text{ and } \Pi \mathbf{e} = 1. \tag{1}$$

The first equation of the above yields the following set of equations:

$$\begin{aligned}
\Pi^{(i_1)} B_0 + \Pi^{(i_1+1)} A_2 &= \mathbf{0},\ i_1 = 0, \\
\Pi(i_1) B_1 + \Pi^{(i_1+1)} A_1 &= \mathbf{0},\ i_1 = 1,2,\ldots,s, \\
\Pi^{(i_1)} B_2 + \Pi^{(i_1+1)} A_1 &= \mathbf{0},\ i_1 = s+1,\ldots,Q-1, \\
\Pi^{(0)} C_1 + \Pi^{(i_1)} B_2 + \Pi^{(i_1+1)} A_1 &= \mathbf{0},\ i_1 = Q, (*) \\
\Pi^{(i_1-Q)} C + \Pi^{(i_1)} B_2 + \Pi^{(i_1+1)} A_1 &= \mathbf{0},\ i_1 = Q+1,\ldots,S-1, \\
\Pi^{(i_1-Q)} C + \Pi^{(i_1)} B_2 &= \mathbf{0},\ i_1 = S
\end{aligned}$$

After lengthy simplifications, the above equations, (except (*)), yield

$$\Pi^{(i_1)} = \Pi^{(Q)} \Omega_{i_1},\ i_1 = 0,1,\ldots,S.$$

where

$$\Omega_{i_1} = \begin{cases}
(-1)^{Q-i_1} (A_1 B_2^{-1})^{(Q-(s+1))} (A_1 B_1^{-1})^{s} (A_2 B_0^{-1}), i_1 = 0, \\
(-1)^{Q-i_1} (A_1 B_2^{-1})^{(Q-(s+1))} (A_1 B_1^{-1})^{((s+1)-i_1)}, i_1 = 1,\ldots,s, \\
(-1)^{Q-i_1} (A_1 B_2^{-1})^{(Q-i_1)}, i_1 = s+1,\ldots,Q-1, \\
I, \qquad i_1 = Q, \\
\sum_{j=0}^{S-i_1} (-1)^{((2Q+1)-i_1)} (A_1 B_2^{-1})^{((S+s)-(i_1+j+1))} \\
\times (A_1 B_1^{-1})^{(j+1)} (C B_2^{-1}), i_1 = Q+1,\ldots,S.
\end{cases}$$

$\Pi^{(Q)}$ can be obtained by solving equation $(*)$ and $\Pi \mathbf{e} = 1$. That is,

$$\Pi^{(Q)} \Big((-1)^Q (A_1 B_2^{-1})^{(Q-(s+1))} (A_1 B_1^{-1})^{s} (A_2 B_0^{-1}) C_1 + B_2 + \sum_{j=0}^{s-1} (-1)^Q (A_1 B_2^{-1})^{(2(s-1)-j)} (A_1 B_1^{-1})^{(j+1)} (C B_2^{-1}) A_1 \Big) = \mathbf{0},$$

and

$$\begin{aligned}
& \Pi^{(Q)} (-1)^Q (A_1 B_2^{-1})^{(Q-(s+1))} (A_1 B_1^{-1})^{s} (A_2 B_0^{-1}) + \\
& \Pi^{(Q)} \sum_{i_1=1}^{s} (-1)^{Q-i_1} (A_1 B_2^{-1})^{(Q-(s+1))} (A_1 B_1^{-1})^{((s+1)-i_1)} \\
& + \Pi^{(Q)} \sum_{i_1=s+1}^{Q-1} (-1)^{Q-i_1} (A_1 B_2^{-1})^{(Q-i_1)} + \Pi^{(Q)} I + \\
& \Pi^{(Q)} \sum_{i_1=Q+1}^{S} \sum_{j=0}^{S-i_1} (-1)^{((2Q+1)-i_1)} (A_1 B_2^{-1})^{((S+s)-(i_1+j+1))} \\
& \qquad \times (A_1 B_1^{-1})^{(j+1)} (C B_2^{-1}) \mathbf{e} = 1.
\end{aligned}$$

4 System performance measures

In this section some performance measures of the system under consideration in the steady state are derived.

4.1 Expected inventory level

Let η_i denote the average inventory level in the steady state. Then

$$\eta_i = \sum_{i_1=1}^{S} \sum_{i_4=0}^{M} i_1 \left[\pi^{(i_1,0,0,i_4)} + \sum_{i_3=1}^{N} \pi^{(i_1,1,i_3,i_4)} + \sum_{i_3=0}^{N} \pi^{(i_1,2,i_3,i_4)} \right]$$

4.2 Expected reorder rate

Let η_r denote the expected reorder rate in the steady state. Then

$$\eta_r = \sum_{i_3=1}^{N} \sum_{i_4=0}^{M} \mu_1 \pi^{(s+1,1,i_3,i_4)}$$

4.3 Expected loss rate for high priority customers

Let η_{bh} denote the expected loss rate for high priority customers in the waiting area. Then

$$\eta_{bh} = \sum_{i_4=0}^{M} \lambda_1 \left[\pi^{(0,0,N,i_4)} + \sum_{i_1=1}^{S} \pi^{(i_1,1,N,i_4)} + \sum_{i_1=0}^{S} \pi^{(i_1,2,N,i_4)} \right]$$

4.4 Expected loss rate for primary low priority customers

Let η_{bl} denote the expected loss rate for primary low priority customers in the orbit. Then

$$\eta_{bl} = \lambda_2 \sum_{i_1=1}^{S} \sum_{i_3=1}^{N} \pi^{(i_1,1,i_3,M)} + \lambda_2 \sum_{i_1=0}^{S} \sum_{i_3=0}^{N} \pi^{(i_1,2,i_3,M)}$$

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4.5 Expected number of high priority customers in the waiting area

Let η_{qh} denote the expected number of high priority customers in the waiting area. Then

$$\eta_{qh} = \sum_{i_4=0}^{M}\sum_{i_3=1}^{N} i_3\pi^{(0,0,i_3,i_4)} + \sum_{i_4=0}^{M}\sum_{i_3=1}^{N}\sum_{i_1=1}^{S}(i_3-1)\pi^{(i_1,1,i_3,i_4)} + \sum_{i_4=0}^{M}\sum_{i_3=1}^{N}\sum_{i_1=0}^{S} i_3\pi^{(i_1,2,i_3,i_4)}$$

4.6 Expected number of low priority customers in the orbit

Let η_{ql} denote the expected number of low priority customers in the orbit. Then

$$\eta_{ql} = \sum_{i_4=1}^{M}\sum_{i_1=1}^{S} i_4\pi^{(i_1,0,0,i_4)} + \sum_{i_4=1}^{M}\sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_4\pi^{(i_1,1,i_3,i_4)}] + \sum_{i_4=1}^{M}\sum_{i_3=0}^{N} i_4\pi^{(0,0,i_3,i_4)} + \sum_{i_3=0}^{N}\sum_{i_1=0}^{S} i_4\pi^{(i_1,2,i_3,i_4)}$$

4.7 Effective arrival rate for high priority customers

Let η_{ah} denote the effective arrival rate for high priority customers in the waiting area. Then

$$\eta_{ah} = \sum_{i_4=0}^{M}\sum_{i_3=0}^{N-1} \lambda_1\pi^{(0,0,i_3,i_4)} + \sum_{i_4=0}^{M}\sum_{i_3=0}^{N-1}\sum_{i_1=0}^{S} \lambda_1\pi^{(i_1,2,i_3,i_4)} + \sum_{i_4=0}^{M}\sum_{i_1=1}^{S}\sum_{i_3=1}^{N-1} \lambda_1\pi^{(i_1,1,i_3,i_4)}$$

4.8 Effective arrival rate for primary low priority customers

Let η_{al} denote the effective arrival rate for primary low priority customers in the orbit. Then

$$\eta_{al} = \sum_{i_4=0}^{M-1}\lambda_2\left[\sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\pi^{(i_1,1,i_3,i_4)} + \sum_{i_1=0}^{S}\sum_{i_3=0}^{N}\pi^{(i_1,2,i_3,i_4)}\right]$$

4.9 Effective reneging rate for orbiting customers

Let η_{ro} denote the effective reneging rate for orbiting customers in the steady state. Then

$$\eta_{ro} = \sum_{i_4=1}^{M}\sum_{i_1=1}^{S} i_4\alpha\pi^{(i_1,0,0,i_4)} + \sum_{i_4=1}^{M} i_4\alpha\sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\pi^{(i_1,1,i_3,i_4)} + \sum_{i_4=1}^{M} i_4\alpha\sum_{i_3=0}^{N}\pi^{(0,0,i_3,i_4)} + \sum_{i_4=1}^{M} i_4\alpha\sum_{i_3=0}^{N}\sum_{i_1=0}^{S}\pi^{(i_1,2,i_3,i_4)}$$

4.10 Expected waiting time for a high priority customer

Let η_{wh} denote the expected waiting time for a high priority customer in the waiting area. Then

$$\eta_{wh} = \frac{\eta_{qh}}{\eta_{ah}}$$

4.11 Expected waiting time for a low priority customer in the orbit

Let η_{wl} denote the expected waiting time for a low priority customer in the orbit. Then

$$\eta_{wl} = \frac{\eta_{ql}}{\eta_{al}}$$

4.12 The overall rate of retrials

Let η_{or} denote the overall rate of retrials in the steady state. Then

$$\eta_{or} = \sum_{i_3=0}^{N}\sum_{i_4=1}^{M} i_4\theta\pi^{(0,0,i_3,i_4)} + \sum_{i_3=0}^{N}\sum_{i_1=0}^{S}\sum_{i_4=1}^{M} i_4\theta\pi^{(i_1,2,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} i_4\theta\pi^{(i_1,0,0,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\sum_{i_4=1}^{M} i_4\theta\pi^{(i_1,1,i_3,i_4)}$$

4.13 The successful retrial rate

Let η_{sr} denote the successful retrial rate in the steady state. Then

$$\eta_{sr} = \sum_{i_4=1}^{M} i_4\theta\left(\sum_{i_3=0}^{N}\pi^{(0,0,i_3,i_4)} + \sum_{i_1=1}^{S}\pi^{(i_1,0,0,i_4)}\right)$$

4.14 The fraction of successful rate of retrial

Let η_{fr} denote the fraction of successful retrial rate in the steady state. Then

$$\eta_{fr} = \frac{\eta_{sr}}{\eta_{or}}$$

5 Cost Analysis

To compute the total expected cost per unit time (total expected cost rate), the following costs are considered,

c_h : The inventory carrying cost per unit item per unit time.
c_s : Setup cost per order.
c_{bh} : Cost per high priority customer lost.
c_{bl} : Cost per low priority customer lost.
c_{wh} : Waiting cost of a high priority customer per unit time.
c_{wl} : Waiting cost of a low priority customer per unit time.
c_r : Reneging cost per orbiting customer per unit time.

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The long run total expected cost rate is given by

$$TC(s,S) = c_h\eta_i + c_s\eta_r + c_{bh}\eta_{bh} + c_{bl}\eta_{bl} + c_{wh}\eta_{wh} + c_{wl}\eta_{wl} + c_r\eta_{ro},$$

where $\eta's$ are as given in $(4.1)-(4.11)$.

Due to the complex form of the limiting distribution, it is difficult to discuss the properties of the cost function analytically. Hence, a detailed computational study of the cost function is carried out.

5.1 Numerical Illustrations

To study the behaviour of the model developed in this work, several examples were dealt with and a set of representative results are shown. Although not showing the convexity of $TC(s,S)$ analytically, experience with considerable numerical examples indicates the function $TC(s,S)$, to be convex. Some of the results are presented in tables 1 and 2. Simple numerical search procedures are used to obtain the optimal values of TC, s and S (say TC^*, s^* and S^*). A typical three dimensional plot of the expected cost function is given in figure 1. The effect of varying the system parameters and costs on the optimal values have been studied and the results agreed with as expected. Some of the results are presented in tables 1 and 2 where the lower entry in each cell gives the optimal expected cost rate and the upper entries the corresponding S^* and s^*.

Example 1.

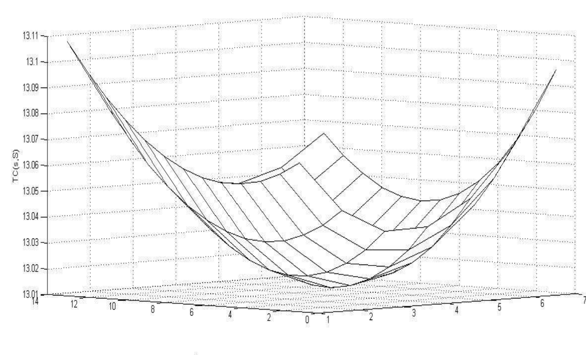
Here, we study the impact of arrival rates λ_1 and λ_2 of high and low priority customers respectively, service rates μ_1 and μ_2 of high and low priority customers respectively; the lead time parameter, β, the reneging rate, α and the retrial rate, θ, on the optimal values (s^*,S^*) and the corresponding total expected cost rate TC^* towards this end, first by fixing parameter $\alpha = 2.4$ and the cost values as $c_r = 2$, $c_h = 1$, $c_s = 15$, $c_{bh} = 6$, $c_{bl} = 3$, $c_{wh} = 7$ and $c_{wl} = 1.9$ is carried out. Observe the following from table 1.

1. If λ_1 increases monotonically and other parameters are fixed then the total expected cost rate TC^*, S^* and s^* increase. From this one may observe that, to maintain the maximum inventory level and reorder level is substantial for avoiding frequent ordering.
2. If λ_2 increases monotonically and other parameters are fixed then the total expected cost rate TC^* increases.
3. If β increases monotonically and other parameters are fixed then the total expected cost rate TC^*, S^* and s^* decrease.
4. If any one of the parameters μ_1 and μ_2 increases monotonically and other parameters are fixed then the total expected cost rate decreases. Also when θ increases monotonically then TC^* increases.

Example 2.

The impact of the setup cost c_s, holding cost c_h, lost (waiting) cost c_{bh} (c_{wh}) and c_{bl} (c_{wl}) of high and low priority customers, respectively, on the optimal values (s^*,S^*) are studied and the corresponding total expected cost rate TC^* towards this end, first by fixing the parameter values as $\lambda_1 = 9$, $\lambda_2 = 7$, $\mu_1 = 8$, $\mu_2 = 11.9$, $\theta = 5$, $\alpha = 2.4$ and $\beta = 6$, and cost values $cr = 2$. Observe the following from table 2:

1. If any one of the parameters c_h, $c_s, c_{bh}, c_{bl}, c_{wh}$ and c_{wl} increases monotonically whereas other parameters are fixed then the total expected cost rate TC^* increases.
2. If any one of the parameters c_s, c_{wh} and c_{bh} increases monotonically whereas other parameters are fixed then S^* increases. This is because if the setup cost increases and one has to maintain high inventory to avoid frequent ordering. Similarly the waiting cost of high priority customers and balking cost of the high priority customers increase, one has to maintain high inventory to reduce the number of waiting customers.
3. If c_h increases monotonically then S^* and s^* decrease. This is to be expected since the holding cost increases, and one resorts to maintain low stock in the inventory. We also note that s^* monotonically increases when c_{wh} increases.



$\lambda_1 = 4.6,\ \lambda_2 = 1.9,\ \theta = 5.9,\ \mu_1 = 6.3,\ \mu_2 = 4.0,$
$\beta = 0.7,\ \alpha = 2.4\ c_r = 2,\ c_s = 15,\ c_h = 1,\ c_{b_l} = 3,$
$c_{b_h} = 6,\ cw_h = 7,\ cw_l = 1.9$

Fig. 1: A three dimensional plot of the cost function $TC(s,S)$

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Table 1: Deviation in the Total expected cost rate at $N = 15,\ M = 11$ and $\alpha = 2.4$

			θ	5.7			5.9			6.1		
			β	.5	.7	.9	.5	.7	.9	.5	.7	.9
λ_1	λ_2	μ_1	μ_2									
4.5	1.7	6	3.8	28 7	26 6	24 5	28 7	26 6	24 5	28 7	26 6	24 5
				47.0223	45.6275	44.5636	47.0266	45.6360	44.5747	47.0305	45.6443	44.5855
			4.0	29 7	26 6	24 5	29 7	26 6	24 5	28 7	26 6	24 5
				46.9352	45.4814	44.3891	46.9392	45.4895	44.3998	46.9429	45.4974	44.4100
			4.2	29 7	26 6	24 5	29 7	26 6	24 5	29 7	26 6	24 5
				46.8516	45.3482	44.2294	46.8553	45.3560	44.2396	46.8589	45.3634	44.2494
		6.3	3.8	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				45.4887	44.0149	42.8934	45.4938	44.0247	42.9058	45.4988	44.0341	42.9177
			4.0	29 7	27 6	25 5	29 7	27 6	25 5	26 7	27 6	25 5
				45.3758	43.8426	42.6928	45.3806	43.8519	42.7046	45.3852	43.8608	42.7159
			4.2	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				45.2742	43.6854	42.5091	45.2787	43.6942	42.5203	45.2830	43.7027	42.5311
		6.6	3.8	30 7	27 5	25 4	30 7	27 5	25 4	30 7	27 5	25 4
				44.1105	42.5707	41.4044	44.1166	42.5798	41.4157	44.1223	42.5885	41.4267
			4.0	30 7	26 6	25 4	30 7	27 6	25 4	30 7	27 6	25 4
				43.9744	42.3822	41.1918	43.9800	42.3922	41.2025	43.9854	42.4019	41.2128
			4.2	30 7	27 6	25 5	30 7	27 6	25 5	30 7	27 6	25 5
				43.8516	42.2073	40.9931	43.8567	42.2169	41.0051	43.8617	42.2260	41.0167
	1.9	6	3.8	28 7	26 6	24 5	28 7	26 6	24 5	28 7	26 6	24 5
				47.1469	45.9006	44.9094	47.1508	45.9093	44.9208	47.1545	45.9177	44.9318
			4.0	28 7	26 6	24 5	28 7	26 6	24 5	28 7	26 6	24 5
				47.0577	45.7506	44.7302	47.0613	45.7590	44.7412	47.0648	45.7669	44.7517
			4.2	28 7	26 6	24 5	28 7	26 6	24 5	28 7	26 6	24 5
				46.9778	45.6136	44.5658	46.9811	45.6216	44.5763	46.9844	45.6292	44.5864
		6.3	3.8	29 7	26 5	24 4	29 7	26 5	24 4	29 7	26 5	24 4
				45.6697	44.3243	43.2822	45.6747	44.3326	43.2930	45.6796	44.3407	43.3035
			4.0	29 7	26 5	24 4	29 7	26 5	24 4	29 7	26 5	24 4
				45.5537	44.1593	43.0869	45.5584	44.1672	43.0972	45.5629	44.1748	43.1071
			4.2	29 7	26 6	25 5	29 7	26 6	25 5	29 7	26 5	25 5
				45.4491	44.0029	42.8986	45.4534	44.0118	42.9103	45.4576	44.0205	42.9215
		6.6	3.8	29 7	26 5	25 4	29 7	26 5	25 4	29 7	26 5	25 4
				44.3336	42.9082	41.8150	44.3393	42.9174	41.8268	44.3449	42.9262	41.8381
			4.0	29 7	27 5	25 4	29 7	27 5	25 4	29 7	27 5	25 4
				44.1997	42.7222	41.5954	44.2050	42.7310	41.6065	44.2101	42.7395	41.6173
			4.2	30 7	27 5	25 4	30 7	27 5	25 4	30 7	27 5	25 4
				44.0734	42.5493	41.3945	44.0786	42.5576	41.4050	44.0836	42.5656	41.4152
	2.1	6	3.8	27 6	25 5	23 4	27 6	25 5	23 4	27 6	25 5	23 4
				47.2571	46.1244	45.2246	47.2592	46.1314	45.2342	47.2613	46.1382	45.2434
			4.0	28 7	25 5	24 5	28 7	25 5	24 5	28 7	25 5	24 5
				47.1723	45.9846	45.0495	47.1756	45.9912	45.0606	47.1788	45.9977	45.0714
			4.2	28 7	25 5	24 5	28 7	25 5	24 5	28 7	25 5	24 5
				47.0909	45.8571	44.8816	47.0939	45.8635	44.8923	47.0969	45.8696	44.9026
		6.3	3.8	28 6	26 5	24 4	28 6	26 5	24 4	28 6	26 5	24 4
				45.8251	44.5984	43.6227	45.8285	44.6069	43.6338	45.8318	44.6149	43.6444
			4.0	28 7	26 5	24 4	28 7	26 5	24 4	28 6	26 5	24 4
				45.7188	44.4297	43.4229	45.7230	44.4377	43.4334	45.7263	44.4454	43.4435
			4.2	29 7	26 5	24 4	29 7	26 5	25 4	29 7	26 5	24 4
				45.6144	44.2757	43.2397	45.6186	44.2832	43.2497	45.6226	44.2905	43.2593
		6.6	3.8	29 6	26 5	24 4	29 6	26 5	24 4	28 6	26 5	24 4
				44.5214	43.2187	42.1959	44.5259	43.2280	42.2079	44.5299	43.2370	42.2194
			4.0	29 6	26 5	24 4	29 6	26 5	24 4	29 6	26 5	24 4
				44.3909	43.0293	41.9744	44.3950	43.0381	41.9859	44.3989	43.0466	41.9968
			4.2	29 6	26 5	25 4	29 6	26 5	25 4	29 6	26 5	25 4
				44.2732	42.8562	41.7700	44.2769	42.8645	41.7808	44.2805	42.8726	41.7912
4.6	1.7	6	3.8	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				47.7272	46.4101	45.3906	47.7306	46.4179	45.4009	47.7338	46.4254	45.4109
			4.0	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				47.6534	46.2775	45.2296	47.6565	46.2849	45.2394	47.6596	46.2921	45.2489
			4.2	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				47.5878	46.1568	45.0822	47.5907	46.1639	45.0916	47.5934	46.1707	45.1006
		6.3	3.8	30 7	27 6	26 5	30 7	27 6	26 5	30 7	27 6	26 5
				46.1892	44.7729	43.6919	46.1937	44.7819	43.7035	46.1981	44.7906	43.7148
			4.0	30 7	27 6	26 5	30 7	27 6	26 5	30 7	27 6	26 5
				46.0891	44.6175	43.5033	46.0933	44.6260	43.5144	46.0974	44.6342	43.5251
			4.2	30 7	28 6	26 5	30 7	28 6	26 5	30 7	28 6	26 5
				45.9993	44.4726	43.3306	46.0032	44.4808	43.3412	46.0069	44.4887	43.3515
		6.6	3.8	31 7	28 6	26 5	31 7	28 6	26 5	31 7	28 6	26 5
				44.7920	43.3006	42.1706	44.7974	43.3106	42.1832	44.8027	43.3202	42.1955
			4.0	31 7	28 6	26 5	31 7	28 6	26 5	31 7	28 6	26 5
				44.6678	43.1206	41.9609	44.6728	43.1301	41.9730	44.6778	43.1392	41.9846
			4.2	31 7	28 6	26 5	31 7	28 6	26 5	31 7	28 6	26 5
				44.5558	42.9563	41.7688	44.5605	42.9653	41.7804	44.5651	42.9740	41.7915
	1.9	6	3.8	28 7	26 6	25 5	28 7	26 6	25 5	28 7	26 6	25 5
				47.8193	46.6442	45.7045	47.8221	46.6519	45.7150	47.8247	46.6593	45.7251
			4.0	29 7	26 6	25 5	29 7	26 6	25 5	29 7	26 6	25 5
				47.7445	46.5133	45.5391	47.7474	46.5206	45.5492	47.7501	46.5277	45.5590
			4.2	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				47.6773	46.3911	45.3875	47.6799	46.3982	45.3972	47.6825	46.4051	45.4065
		6.3	3.8	29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 6	25 5
				46.3377	45.0619	44.0584	46.3417	45.0711	44.0704	46.3457	45.0799	44.0820
			4.0	30 7	27 6	25 5	30 7	27 6	25 5	30 7	27 6	25 5
				46.2378	44.9023	43.8679	46.2418	44.9110	43.8794	46.2458	44.9194	43.8905
			4.2	30 7	27 6	25 5	30 7	27 6	25 5	30 7	27 6	26 5
				46.1454	44.7564	43.6930	46.1492	44.7647	43.7040	46.1528	44.7727	43.7146

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θ				5.7			5.9			6.1		
β				.5	.7	.9	.5	.7	.9	.5	.7	.9
λ_1	λ_2	μ_1	μ_2									
4.6		6.6		30 7	27 5	26 4	30 7	27 5	26 4	30 7	27 5	26 4
			3.8	44.9823	43.6230	42.5705	44.9875	43.6316	42.5817	44.9924	43.6399	42.5925
				30 7	27 5	26 4	30 7	27 5	26 4	30 7	27 5	26 4
			4.0	44.8610	43.4502	42.3623	44.8657	43.4584	42.3729	44.8704	43.4662	42.3831
				30 7	28 6	26 4	30 7	28 6	26 4	30 7	28 5	26 4
			4.2	44.7515	43.2863	42.1716	44.7559	43.2955	42.1819	44.7602	43.3045	42.1915
	2.1	6		28 7	26 5	24 4	28 7	26 5	24 4	28 7	26 5	24 4
			3.8	47.8882	46.8426	45.9947	47.8906	46.8488	46.0034	47.8929	46.8548	46.0118
				28 7	26 5	25 5	28 7	26 5	25 5	28 7	26 5	25 5
			4.0	47.8190	46.7172	45.8286	47.8211	46.7231	45.8388	47.8234	46.7287	45.8487
				28 7	26 6	25 5	28 7	26 5	25 5	28 7	26 5	25 5
			4.2	47.7576	46.6018	45.6738	47.7596	46.6086	45.6836	47.7615	46.6140	45.6931
		6.3		29 7	26 5	25 4	29 7	26 5	25 4	29 7	26 5	25 4
			3.8	46.4632	45.3085	44.3824	46.4670	45.3159	44.3927	46.4706	45.3231	44.4025
				29 7	27 5	25 4	29 7	27 5	25 4	29 7	27 5	25 4
			4.0	46.3657	45.1574	44.1959	46.3692	45.1646	44.2056	46.3725	45.1716	44.2150
				29 7	27 5	25 4	29 7	27 5	25 4	29 7	27 5	25 4
			4.2	46.2780	45.0159	44.0249	46.2813	45.0227	44.0342	46.2844	45.0294	44.0432
		6.6		29 6	27 5	25 4	29 6	27 5	25 4	29 6	27 5	25 4
			3.8	45.1481	43.9079	42.9252	45.1516	43.9166	42.9366	45.1550	43.9249	42.9475
				30 7	27 5	25 4	30 7	27 5	25 4	30 7	27 5	25 4
			4.0	45.0346	43.7312	42.7159	45.0392	43.7394	42.7267	45.0436	43.7473	42.7371
				30 7	27 5	25 4	30 7	27 5	25 4	30 7	27 5	25 4
			4.2	44.9226	43.5698	42.5239	44.9269	43.5775	42.5342	44.9310	43.5850	42.5441
4.7	1.7	6		30 7	27 6	26 5	30 7	27 6	26 5	30 7	27 6	26 5
			3.8	48.4086	47.1677	46.2009	48.4113	47.1745	46.2103	48.4139	47.1811	46.2194
				30 7	27 6	26 5	30 7	27 6	26 5	30 7	27 6	26 5
			4.0	48.3479	47.0536	46.0534	48.3503	47.0601	46.0624	48.3527	47.0663	46.0711
				30 7	28 6	26 5	30 7	28 6	26 5	30 7	28 6	26 5
			4.2	48.2943	46.9457	45.9185	48.2966	46.9520	45.9271	48.2987	46.9581	45.9354
		6.3		31 7	28 6	26 5	31 7	28 6	26 5	31 7	28 6	26 5
			3.8	46.8749	45.5199	44.4813	46.8789	45.5282	44.4922	46.8826	45.5361	44.5027
				31 7	28 6	26 5	31 7	28 6	26 5	31 7	28 6	26 5
			4.0	46.7876	45.3774	44.3085	46.7913	45.3853	44.3190	46.7947	45.3928	44.3290
				31 7	28 6	27 5	31 7	28 6	27 5	30 7	28 6	27 5
			4.2	46.7096	45.2476	44.1474	46.7129	45.2550	44.1573	46.7161	45.2622	44.1669
		6.6		31 7	29 6	27 5	31 7	29 6	27 5	31 7	29 6	27 5
			3.8	45.4624	44.0245	42.9267	45.4671	44.0336	42.9388	45.4716	44.0430	42.9504
				32 7	29 6	27 5	32 7	29 6	27 5	32 7	29 6	27 5
			4.0	45.3536	43.8564	42.7283	45.3581	43.8654	42.7399	45.3626	43.8740	42.7510
				32 7	29 6	27 5	32 7	29 6	27 5	32 7	29 6	27 5
			4.2	45.2528	43.7031	42.5467	45.2570	43.7115	42.5577	45.2611	43.7197	42.5683
	1.9	6		29 7	27 6	25 5	29 7	27 6	25 5	29 7	27 5	25 5
			3.8	48.4556	47.3633	46.4816	48.4576	47.3700	46.4911	48.4595	47.3766	46.5003
				29 7	27 6	26 5	29 7	27 56	26 5	27 6	27 6	26 5
			4.0	48.4009	47.2465	46.3308	48.4027	47.2529	46.3400	48.4044	47.2592	46.3489
				29 7	27 6	26 5	29 7	27 6	26 5	29 7	27 6	26 5
			4.2	48.3529	47.1401	46.1921	48.3545	47.1463	46.2009	48.3561	47.1522	46.2094
		6.3		30 7	28 6	26 5	30 7	28 6	26 5	30 7	28 6	26 5
			3.8	46.9833	45.7786	44.8165	46.9867	45.7870	44.8276	46.9899	45.7950	44.8384
				30 7	28 6	26 5	30 7	28 6	26 5	30 7	28 6	26 5
			4.0	46.9008	45.6324	44.6391	46.9039	45.6404	44.6498	46.9069	45.6481	44.6601
				31 7	28 6	26 5	31 7	28 6	26 5	30 7	28 6	26 5
			4.2	46.8267	45.4989	44.4764	46.8299	45.5065	44.4866	46.8328	45.5138	44.4965
		6.6		31 7	28 5	27 5	31 7	28 5	27 5	31 7	28 5	27 5
			3.8	45.6216	44.3299	43.3157	45.6261	44.3379	43.3282	45.6305	44.3455	43.3402
				31 7	29 6	27 5	31 7	29 6	27 5	31 7	29 6	27 5
			4.0	45.5130	44.1649	43.1117	45.5172	44.1741	43.1236	45.5212	44.1828	43.1351
				31 7	29 6	27 5	29 7	29 6	27 5	29 7	29 6	27 5
			4.2	45.4153	44.0069	42.9245	45.4192	44.0156	42.9358	45.4229	44.0239	42.9468
	2.1	6		28 7	26 6	25 6	28 7	27 6	25 6	28 7	26 6	25 6
			3.8	48.4866	47.5254	46.7332	48.4868	47.5304	46.7428	48.4870	47.5353	46.7520
				29 8	26 6	25 6	29 8	26 6	25 6	29 8	26 6	25 6
			4.0	48.4394	47.4203	46.5835	48.4408	47.4251	46.5927	48.4422	47.4296	46.6016
				29 8	27 7	25 6	29 8	27 7	25 6	29 8	27 7	25 6
			4.2	48.3909	47.3178	46.4460	48.3921	47.3239	46.4548	48.3934	47.3298	46.4633
		6.3		29 7	27 6	26 6	29 7	27 6	26 5	29 7	27 6	26 5
			3.8	47.0789	45.9981	45.1290	47.0805	46.0047	45.1385	47.0821	46.0111	45.1476
				30 8	27 6	26 6	30 8	27 6	26 6	30 8	27 6	26 6
			4.0	46.9963	45.8625	44.9483	46.9991	45.8688	44.9592	47.0018	45.8748	44.9699
				30 8	28 7	26 6	30 8	28 7	26 6	30 8	28 7	26 6
			4.2	46.9214	45.7348	44.7822	46.9239	45.7424	44.7926	46.9264	45.7498	44.8027
		6.6		30 7	28 6	26 5	30 7	28 6	26 5	30 7	28 6	26 5
			3.8	45.7617	44.5885	43.6487	45.7645	44.5965	43.6593	45.7673	44.6042	43.6696
				31 8	28 6	26 5	31 8	28 6	26 5	31 8	28 6	26 5
			4.0	45.6602	44.4247	43.4519	45.6641	44.4323	43.4621	45.6680	44.4396	43.4719
				31 8	28 6	26 5	31 8	28 6	26 5	31 8	28 6	26 5
			4.2	45.5604	44.2753	43.2715	45.5640	44.2824	43.2812	45.5675	44.2893	43.2905

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Table 2: Deviation in the Total expected cost rate at $N = 15$, $M = 11$ and $c_r = 2$

c_{bh}				5			6			7		
c_{wl}				1.7	1.9	2.1	1.7	1.9	2.1	1.7	1.9	2.1
c_h	c_s	c_{bl}	c_{wh}									
.8	13	2		28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6
			5	39.4740	39.5566	39.6393	39.7058	39.7884	39.8711	39.9375	40.0202	40.1028
				29 6	29 6	29 6	30 7	30 7	30 7	30 7	30 7	30 7
			7	41.8341	41.9166	41.9990	42.0435	42.1265	42.2094	42.2473	42.3302	42.4131
				31 7	31 7	31 7	31 7	31 7	31 7	32 7	32 7	32 7
			9	44.0708	44.1535	44.2361	44.2656	44.3483	44.4310	44.4546	44.5370	44.6194
		3		28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6
			5	39.4747	39.5574	39.6400	39.7065	39.7891	39.8718	39.9382	40.0209	40.1035
				29 6	29 6	29 6	30 7	30 7	30 7	30 7	30 7	30 6
			7	41.8348	41.9172	41.9997	42.0443	42.1273	42.2102	42.2481	42.3310	42.4139
				31 7	31 7	31 7	31 7	31 7	31 7	32 7	32 7	32 7
			9	44.0716	44.1543	44.23697	44.2664	44.3491	44.4318	44.4553	44.5378	44.6202
		4		28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6
			5	39.4754	39.5581	39.6407	39.7072	39.7898	39.8725	39.9390	40.0216	40.1042
				29 6	29 6	29 6	30 7	30 7	30 7	30 7	30 7	30 7
			7	41.8355	41.9179	42.0004	42.0451	42.1281	42.2110	42.2489	42.3318	42.4147
				31 7	31 7	31 7	31 7	31 7	31 7	32 7	32 7	32 7
			9	44.0724	44.1550	44.2377	44.2672	44.3499	44.4325	44.4561	44.5385	44.6210
	15	2		28 6	28 6	28 6	29 6	29 6	29 6	29 6	29 6	29 6
			5	39.8807	39.9634	40.0460	40.1094	40.1918	40.2742	40.3302	40.4127	40.4950
				30 6	30 6	30 6	30 6	30 6	30 6	31 7	31 6	31 6
			7	42.2122	42.2943	42.3765	42.4231	42.5053	42.5875	42.6319	42.7144	42.7963
				32 7	32 7	32 7	32 7	32 7	32 7	32 7	32 7	32 7
			9	44.4454	44.5278	44.6102	44.6321	44.7145	44.7969	44.8188	44.9012	44.9837
		3		28 6	28 6	28 6	29 6	29 6	29 6	29 6	29 6	29 6
			5	39.8814	39.9641	40.0467	40.1101	40.1925	40.2749	40.3309	40.4133	40.4957
				30 6	30 6	30 6	30 6	30 6	30 6	31 7	31 6	31 6
			7	42.2128	42.2950	42.3772	42.4238	42.5060	42.5882	42.6327	42.7151	42.7970
				32 7	32 7	32 7	32 6	32 7	32 7	32 7	32 7	32 7
			9	44.4461	44.5286	44.6120	44.6328	44.7153	44.7977	44.8196	44.9020	44.9844
		4		28 6	28 6	28 6	29 6	29 6	29 6	29 6	29 6	29 6
			5	39.8822	39.9648	40.0475	40.1108	40.1932	40.2756	40.3316	40.4140	40.4964
				30 6	30 6	30 6	30 6	30 6	30 6	31 7	31 6	31 6
			7	42.2135	42.2957	42.3779	42.4245	42.5067	42.5888	42.6335	42.7157	42.7977
				32 7	32 7	32 7	32 7	32 7	32 7	32 7	32 7	32 7
			9	44.4469	44.5293	44.6117	44.6336	44.7160	44.7985	44.8203	44.9028	44.9852
	17	2		28 5	28 5	28 5	29 6	29 6	29 6	30 6	30 6	30 6
			5	40.2694	40.3512	40.4331	40.4995	40.5819	40.6643	40.7203	40.8024	40.8847
				30 6	30 6	30 6	31 6	31 6	31 6	31 6	31 6	31 6
			7	42.5869	42.6690	42.7512	42.7909	42.8729	42.9548	42.9929	43.0748	43.1568
				32 7	32 7	32 7	32 7	32 7	32 7	33 7	33 7	33 7
			9	44.8096	44.8920	44.9745	44.9963	45.0787	45.1612	45.1784	45.2606	45.3428
		3		28 5	28 5	28 5	29 6	29 6	29 6	30 6	30 6	30 6
			5	40.2700	40.3518	40.4337	40.5002	40.5826	40.6650	40.7210	40.8032	40.8854
				30 6	30 6	30 6	31 6	31 6	31 6	31 6	31 6	31 6
			7	42.5875	42.6697	42.7519	42.7916	42.8736	42.9555	42.9935	43.0755	43.1575
				32 7	32 7	32 7	32 7	32 7	32 7	33 7	33 7	33 7
			9	44.8104	44.8928	44.9752	44.9971	45.0795	45.1619	45.1791	45.2613	45.3435
		4		28 5	28 5	28 5	29 6	29 6	29 6	30 6	30 6	30 6
			5	40.2706	40.3524	40.4343	40.5009	40.5833	40.6657	40.7217	40.8039	40.8860
				30 6	30 6	30 6	31 6	31 6	31 6	31 6	31 6	31 6
			7	42.5882	42.6704	42.7526	42.7922	42.8742	42.9562	42.9942	43.0762	43.1582
				32 7	32 7	32 7	32 7	32 7	32 7	33 7	33 7	33 7
			9	44.8111	44.8936	44.9760	44.9978	45.0803	45.1627	45.1799	45.2621	45.3443
1	13	2		24 5	24 5	24 5	25 5	25 5	25 5	25 5	25 5	25 5
			5	41.5582	41.6409	41.7237	41.8515	41.9341	42.0166	42.1330	42.2155	42.2980
				26 6	26 6	26 6	27 6	27 6	27 6	27 6	27 6	27 6
			7	44.1537	44.2368	44.3200	44.4026	44.4855	44.5684	44.6465	44.7294	44.8123
				28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6
			9	46.5883	46.6709	46.7536	46.8201	46.9027	46.9854	47.0518	47.1345	47.2171
		3		24 5	24 5	24 5	25 5	25 5	25 5	25 5	25 5	25 5
			5	41.5588	41.6416	41.7243	41.8521	41.9347	42.0172	42.1336	42.2161	42.2986
				26 6	26 6	26 6	27 6	27 6	27 6	27 6	27 6	27 6
			7	44.1544	44.2376	44.3207	44.4033	44.4862	44.5691	44.6472	44.7301	44.8130
				28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6
			9	46.5890	46.6717	46.7543	46.8208	46.9034	46.9861	47.0525	47.1352	47.2178
		4		24 5	24 5	24 5	25 5	25 5	25 5	25 5	25 5	25 5
			5	41.5594	41.6422	41.7250	41.8528	41.9353	42.0178	42.1342	42.2167	42.2993
				26 6	26 6	26 6	27 6	27 6	27 6	27 6	27 6	27 6
			7	44.1551	44.2383	44.3214	44.4040	44.4869	44.5698	44.6479	44.7308	44.8137
				28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6	28 6
			9	46.5897	46.6724	46.7550	46.8215	46.9041	46.9868	47.0532	47.1359	47.2185
	15	2		25 5	25 5	25 5	25 5	25 5	25 5	26 5	26 5	26 5
			5	42.0082	42.0907	42.1732	42.2896	42.37222	42.4547	42.5688	42.6511	42.7334
				27 6	27 6	27 6	27 6	27 6	27 6	27 6	28 6	28 6
			7	44.5835	44.6664	44.7493	44.8274	44.9103	44.9932	45.0713	45.1540	45.2366
				28 6	28 6	28 6	29 6	29 6	29 6	29 6	29 6	29 6
			9	46.9950	47.0777	47.1603	47.2236	47.3060	47.3884	47.4444	47.5268	47.6092
		3		25 5	25 5	25 5	25 6	25 5	25 5	26 5	26 5	26 5
			5	42.0088	42.0913	42.1738	42.2902	42.3728	42.4553	42.5694	42.6517	42.7340
				27 6	27 6	27 6	27 6	27 6	27 6	28 6	28 6	28 6
			7	44.5842	44.6671	44.7500	44.8281	44.9110	44.9939	45.0720	45.1547	45.2373
				28 6	28 6	28 6	29 6	29 6	29 6	29 6	29 6	29 6
			9	46.9957	47.0784	47.1610	47.2243	47.3067	47.3891	47.4451	47.5275	47.6099

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c_{bh}				5			6			7		
c_{wl}				1.7	1.9	2.1	1.7	1.9	2.1	1.7	1.9	2.1
c_h	c_s	c_{bl}	c_{wh}									
1		4	5	25 5	25 5	25 5	25 5	25 5	25 5	26 5	26 5	26 5
				42.0094	42.0919	42.1745	42.2909	42.3734	42.4559	42.5700	42.6523	42.7346
			7	27 6	27 6	27 6	27 6	27 6	27 6	28 6	28 6	28 6
				44.5849	44.6678	44.7507	44.8289	44.9118	44.9947	45.0728	45.1554	45.2380
			9	28 6	28 6	28 6	29 6	29 6	29 6	29 6	29 6	29 6
				46.9965	47.0791	47.1617	47.2250	47.3074	47.3898	47.4458	47.5282	47.6106
	17	2	5	25 5	25 5	25 5	26 5	26 5	26 5	26 5	26 5	26 5
				42.4463	42.5288	42.6113	42.7215	42.8038	42.8861	42.9879	43.0701	43.1524
			7	27 5	27 5	27 5	27 5	27 5	27 5	28 6	28 6	28 6
				44.9878	45.0698	45.1519	45.2405	45.3226	45.4047	45.4781	45.5607	45.6434
			9	29 6	29 6	29 6	29 6	29 6	29 6	29 6	29 6	29 6
				47.3929	47.4753	47.5577	47.6137	47.6961	47.7785	47.8345	47.9169	47.9993
		3	5	25 5	25 5	25 5	26 5	26 5	26 5	26 5	26 5	26 5
				42.4469	42.5294	42.6119	42.7221	42.8044	42.8867	42.9885	43.0708	43.1530
			7	27 5	27 5	27 5	27 5	27 5	27 5	28 6	28 6	28 6
				44.9884	45.0704	45.1525	45.2411	45.3232	45.4053	45.4788	45.5614	45.6441
			9	29 6	29 6	29 6	29 6	29 6	29 6	29 6	29 6	29 6
				47.3936	47.4760	47.5584	47.6144	47.6968	47.7792	47.8352	47.9176	48.0000
		4		25 5	25 5	25 5	26 5	26 5	26 5	26 5	26 5	26 5
				42.4475	42.5300	42.6126	42.7228	42.8051	42.8873	42.9891	43.0714	43.1537
			7	27 5	27 5	27 5	27 5	27 5	27 5	28 6	28 6	28 6
				44.9890	45.0710	45.1531	45.2417	45.3238	45.4059	45.4795	45.5621	45.6448
			19	29 6	29 6	29 6	29 6	29 6	29 6	29 6	29 6	29 6
				47.3942	47.4767	47.5591	47.6151	47.6975	47.7799	47.8359	47.9183	48.0007
1.2	13	2	5	21 4	21 4	21 4	22 4	22 4	22 4	23 5	23 5	23 5
				43.3191	43.4016	43.4842	43.6821	43.7644	43.8467	44.0135	44.0966	44.1796
			7	24 5	24 5	24 5	24 5	24 5	24 5	24 5	24 5	24 5
				46.1482	46.2310	46.3137	46.4466	46.5294	46.6122	46.7451	46.8278	46.9106
			9	26 6	26 6	26 6	26 6	26 6	26 6	26 6	26 6	26 6
				48.7924	48.8756	48.9587	49.0499	49.1330	49.2162	49.3074	49.3905	49.4737
		3	5	21 4	21 4	21 4	22 4	22 4	22 4	23 5	23 5	23 5
				43.3196	43.4021	43.4847	43.6826	43.7649	43.8472	44.0142	44.0972	44.1802
			7	24 5	24 5	24 5	24 5	24 5	24 5	24 5	24 5	24 5
				46.1488	46.2316	46.3144	46.4472	46.5300	46.6128	46.7457	46.8285	46.9112
			9	26 6	26 6	26 6	26 6	26 6	26 6	26 6	26 6	26 6
				48.7931	48.8763	48.9595	49.0506	49.1338	49.2169	49.3081	49.3912	49.4744
		4	5	21 4	21 4	21 4	22 4	22 4	22 4	23 5	23 5	23 5
				43.3201	43.4026	43.4852	43.6831	43.7654	43.8477	44.0148	44.0978	44.1808
			7	24 5	24 5	24 5	24 5	24 5	24 5	24 5	24 5	24 5
				46.1494	46.2322	46.3150	46.4479	46.5306	46.6134	46.7463	46.8291	46.9118
			9	26 6	26 6	26 6	26 6	26 6	26 6	26 6	26 6	26 6
				48.7939	48.8770	48.9602	49.0513	49.1345	49.2177	49.3088	49.3920	49.4751
	15	2	5	22 4	22 4	22 4	22 4	22 4	22 4	23 4	23 4	23 4
				43.8055	43.8878	43.9701	44.1553	44.2376	44.3199	44.4945	44.5766	44.6586
			7	24 5	24 5	24 5	24 5	24 5	24 5	25 5	25 5	25 5
				46.6071	46.6899	46.7726	46.9056	46.9883	47.0711	47.1885	47.2710	47.3536
			9	26 6	26 6	26 5	26 6	26 6	26 6	27 6	27 6	27 6
				49.2369	49.3201	49.4029	49.4944	49.5776	49.6607	49.7516	49.8345	49.9174
		3	5	22 4	22 4	22 4	22 4	22 4	22 4	23 4	23 4	23 4
				43.8060	43.8883	43.9706	44.1558	44.2381	44.3204	44.4950	44.5771	44.6591
			7	24 5	24 5	24 5	24 5	24 5	24 5	25 5	25 5	25 5
				46.6077	46.6905	46.7733	46.9062	46.9889	47.0717	47.1891	47.2716	47.3542
			9	26 6	26 6	26 5	26 6	26 6	26 6	27 6	27 6	27 6
				49.2377	49.3208	49.4035	49.4951	49.5783	49.6615	49.7523	49.8352	49.9181
		4	5	22 4	22 4	22 4	22 4	22 4	22 4	23 4	23 4	23 4
				43.8065	43.8888	43.9711	44.1563	44.2386	44.3209	44.4955	44.5776	44.6596
			7	24 5	24 5	24 5	24 5	24 5	24 5	25 5	25 5	25 5
				46.6083	46.6911	46.7739	46.9068	46.9895	47.0723	47.1897	47.2723	47.3548
			9	26 6	26 6	26 5	26 6	26 6	26 6	27 6	27 6	27 6
				49.2384	49.3216	49.4041	49.4959	49.5790	49.6622	49.7530	49.8359	49.9188
	17	2	5	22 4	22 4	22 4	23 4	23 4	23 4	23 4	23 4	23 4
				44.2787	44.3610	44.4433	44.6175	44.6996	44.7817	44.9458	45.0279	45.1099
			7	25 5	25 5	25 5	25 5	25 5	25 5	25 5	25 5	25 5
				47.0637	47.1462	47.2288	47.3452	47.4277	47.5102	47.6266	47.7091	47.7917
			9	26 5	26 5	26 5	26 5	26 5	26 5	27 6	27 6	27 6
				49.6574	49.7396	49.8219	49.9237	50.0060	50.0883	50.1764	50.2593	50.3422
		3	5	22 4	22 4	22 4	23 4	23 4	23 4	23 4	23 4	23 4
				44.2792	44.3615	44.4438	44.6180	44.7001	44.7822	44.9463	45.0284	45.1104
			7	25 5	25 5	25 5	25 5	25 5	25 5	25 5	25 5	25 5
				47.0643	47.1468	47.2294	47.3458	47.4283	47.5108	47.6272	47.7097	47.7923
			9	26 5	26 5	26 5	26 5	26 5	26 5	27 6	27 6	27 6
				49.6580	49.7403	49.8226	49.9243	50.0067	50.0889	50.1771	50.2600	50.3429
		4	5	22 4	22 4	22 4	23 4	23 4	23 4	23 4	23 4	23 4
				44.2796	44.3620	44.4443	44.6185	44.7006	44.7826	44.9468	45.0289	45.1109
			7	25 5	25 5	25 5	25 5	25 5	25 5	25 5	25 5	25 5
				47.0649	47.1475	47.2300	47.3464	47.4289	47.5114	47.6278	47.7104	47.7929
			9	26 5	26 5	26 5	26 5	26 5	26 5	27 6	27 6	27 6
				49.6586	49.7409	49.8232	49.9249	50.0072	50.0895	50.1778	50.2607	50.3436

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6 Conclusion

The stochastic model discussed here is useful in studying a retrial inventory system with impatient customers in which two types of customers arrive say high priority and low priority customers. The joint probability distribution of the number of customers in the waiting area, the number of customers in the orbit and the inventory level are derived in the steady state and the stationary measures of system performances have been computed. Illustration has also been provided to the existence of local optimums when the total expected cost function is treated as a function of two variables s and S. The authors are working in the direction of MAP arrival for the two types of customers.

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