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An Advanced Count Data Model with Applications in Genetics

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Abstract: The present paper introduces an advanced count model which is obtained by compounding generalized negative binomial distribution with Kumaraswamy distribution. The proposed model has several properties such as it can be nested to different existing compound distributions on specific parameter setting. Similarity of the proposed model with existing compound distribution has been shown by means of reparameterization. The properties of the new model are discussed and explicit expressions are derived for the factorial moments. Further method of moments and maximum likelihood estimation is used to evaluate the moments. The potentiality of the proposed model has been tested by chi-square goodness of fit test by modeling the real world count data sets from genetics.

Keywords: Generalized negative binomial distribution, Kumaraswamy distribution, compound distribution, factorial moment, count data.

1 Introduction

Statistical distributions are commonly applied to describe real world phenomenon. Dueto the usefulness of statistical distributions their theory is widely studied and new distributions are developed. There has been an increased interest in developing generalized or generated families of new continuous distributions by introducing one or more additional shape parameters. The popularity and the use of these new families of distributions have attracted the attention of statisticians, engineers, economists, demographers and other applied researchers. The reason might be that this additional parameter has proved to be helpful in improving the goodness-of-fit of the proposed family of continuous distributions and they have the ability to fit skewed data better than existing distributions. There exists many generalized family of continuous distributions that have received a great deal of attention in recent years. We discuss below some generalized continuous distributions.

From the last few decades researchers are busy to obtain new probability distributions by using different techniques such as compounding, T-X family, transmutation etc. but compounding of probability distribution has received maximum attention which is an innovative and sound technique to obtain new generalized probability distributions. The compounding of probability distributions enables us to obtain both discrete as well as continuous distribution.

In several research papers it has been found that compound Distributions are very flexible and can be used efficiently to model different types of data sets. With this in mind many compound probability distributions have been constructed. Sankaran (1970) obtained a compound of Poisson distribution with that of Lindley distribution, Zamani and Ismail (2010) constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. The researchers like Adil Rashid and Jan obtained several compound distributions for instance, (2013) a compound of zero truncated generalized negative binomial distribution with generalized beta distribution, (2014a) they obtained compound of Geeta distribution with generalized beta distribution and (2014b) compound of Consul distribution with generalized beta distribution. Most recently AdilRashid and Jan (2014c) explored a mixture of generalized negative binomial distribution with that of generalized exponential distribution which contains several compound distributions as its sub cases and proved that this particular model is better in comparison to others when it comes to fit observed count data set.

In this paper we propose a new count data model by compounding generalized negative binomial distribution (NBD) with Kumaraswamy distribution (KSD) and some similarities of the proposed model will be shown with some

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already existing compound distribution.

2 Material and Methods

A discrete random variable is X said to have a generalized negative binomial distribution (GNBD) with parameters $m, p\, and\, \eta$ if its probability mass function is given by

$$f_1(x;m,p,\eta)=\frac{m}{m+\eta x}\binom{m+\eta x}{x}(1-p)^x p^{m+\eta x-x}, \qquad x=0,1,2,... \quad (1)$$

Where

$0<p<1,\ \ m>0,\ \ p\eta<1;\eta\geq 1,$

$0<p<1,\ \ m\in N,\ \ \eta=0.$

For $\eta=1$, equation (1) reduces to the negative binomial distribution (NBD). If $m\in N$, $for\, \eta=0$, one obtains from (1) the binomial distribution (BD) and for $\eta=1$, the Pascal distribution N.Johnson, S.Kotz and A. Kemp (1992,p.200).

A random variable X is said to have a Kumaraswamy distribution (KSD) if its pdf is given by

$$f_2(X;\alpha,\beta)=\alpha\beta\, x^{\alpha-1}\left(1-x^{\alpha}\right)^{\beta-1},\ 0<x<1 \quad (2)$$

Where $\alpha,\beta>0$ are shape parameters The raw moments of Kumaraswamy distribution are given by

$$E(X^r)=\int_0^1 x^r f_2(X;\alpha,\beta)dx$$

$$E(X^r)=\frac{\Gamma(\beta+1)\Gamma\left(1+\frac{r}{\alpha}\right)}{\Gamma\left(1+\beta+\frac{r}{\alpha}\right)} \quad (3)$$

Kumaraswamy distribution is a two parameter continuous probability distribution that has obtained by Kumaraswamy (1980) but unfortunately this distribution is not very popular among statisticians because researchers have not analyzed and investigated it systematically in much detail. Kumaraswamy distribution is similar to the beta distribution but unlike beta distribution it has a closed form of cumulative distribution function which makes it very simple to deal with. For more detailed properties one can see references Kumaraswamy (1980) and Jones (2009).

Usually the parameters $m, p, and\, \eta$ in GNBD are fixed constants but here we have considered a problem in which the probability parameter p is itself a random variable following KSD with p.d.f (2).

3 Definition of Proposed Model

If $X\mid p\sim$ GNBD(m,p,η) where p is itself a random variable following Kumaraswamy distribution KSD(α,β) then determining the distribution that results from marginalizing over p will be known as a compound of generalized negative binomial distribution with that of Kumaraswamy distribution which is denoted by GNBKSD (m,η,α,β) It may be noted that proposed model will be a discrete since the parent distribution GNBD is discrete.

Theorem 3.1: The probability mass function of a compound of GNBD(p,λ,m) with KSD(α,β) is given by

Proof: Using the definition (3), the p.m.f of a compound of GNBD(m,p,η) with KSD(α,β) can be obtained as

$$f_{GNBKSD}(X;m,\eta,\alpha,\beta)=\int_0^1 f_1(x\mid p)\, f_2(p)\, dp$$

$$f_{GNBKSD}(X;m,\eta,\alpha,\beta)=\frac{m\alpha\beta}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\int_0^1 p^{m+j+\eta x-x+\alpha-1}\left(1-p^{\alpha}\right)^{\beta-1}dp$$

substituting $1-p^{\alpha}=y$, we get

$$f_{GNBKSD}(X;m,\eta,\alpha,\beta)=\frac{m\beta}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\int_0^1 y^{\beta-1}(1-y)^{\frac{m+j+\eta x-x}{\alpha}}\, dy$$

$$f_{GNBKSD}(X;m,\eta,\alpha,\beta)=\frac{m\beta}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\ B\left(\beta,\frac{m+j+\eta x-x}{\alpha}+1\right)$$

$$f_{GNBKSD}(X;m,\eta,\alpha,\beta)=\frac{m}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{m+j+\eta x-x}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{m+j+\eta x-x}{\alpha}+1\right)} \quad (4)$$

where $x=0,1,2,...,m,\eta,\alpha,\beta>0$. From here a random X variable following a compound of GNBD with KSD will be symbolized by GNBKSD (m,η,α,β)

4 ReparameterizationTechniques

There are very few continuous probability distributions in statistics whose support lies between 0 and 1 so in order to ascribe a suitable distribution to a probability parameter p we have a limited choice, to remove this limitation researchers try to reparameterize the probability parameter by equating p to $e^{-\lambda}$ where $\lambda>0$ is a random variable. So instead of ascribing a suitable probability distribution to parameter p researchers ascribe a suitable distribution to the parameter λ by treating it as a random variable with support $\lambda>0$ and there are numerous probability distributions in statistics whose support lies $(0,\infty)$. In this section a similarity will be shown between a proposed model and a model which is obtained by compounding generalized negative binomial distribution with generalized exponential distribution through reparameterization technique.

Proposition 4.1: The probability function of the proposed model gets coincide with the compound of GNBD with GED obtained through reparameterization.

Proof: If $(X\mid\lambda)\sim GNBD(m,p=e^{-\lambda},\eta)$ where λ is itself a random variable following a generalized exponential distribution (GED) with probability density function

$$f_3(\lambda;\beta,\alpha)=\beta\alpha\left(1-e^{\alpha\lambda}\right)^{\beta-1}e^{-\alpha\lambda},\ \lambda>0 \text{ for } \beta,\alpha>0 \quad (5)$$

then determining the distribution that results from marginalizing over λ will give us a compound of GNBD with GED

$$f_{NBGED}(X;m,\eta,\beta,\alpha)=\frac{m}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\int_{0}^{\infty}e^{-\lambda(m+j+\eta x-x)}f_3(\lambda;\beta,\alpha)d\lambda$$

$$f_{NBGED}(X;m,\eta,\beta,\alpha)=\frac{m}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\,\Gamma\left(\frac{m+j+\eta x-x}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{m+j+\eta x-x}{\alpha}+1\right)} \quad (6)$$

Where $x=0,1,2,...,m,\eta,\alpha,\beta>0$. Interestingly, this gives rise to the same probability function as the probability function (4) of the proposed model. The probability

Hence our model can be treated as a simple and easy alternative since it has been obtained without reparameterization.function defined in (6) was obtained by Adil Rashid and Jan (2014)

5 Nested Distributions

In this particular section we show that the proposed model can be nested to different models under specific parameter setting.

Proposition 5.1: If $X \sim GNBKSD(m,\eta,\alpha,\beta)$ then by

$$f_{NBKSD}(X;m,\alpha,\beta)= \binom{x+m-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\,\Gamma\left(\frac{m+j}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{m+j}{\alpha}+1\right)} \quad \textbf{(7)}$$

$x=0,1,2,...,m,\alpha,\beta>0$. The compound distribution displayed in (7) was introduced by Adil and Jan (2014).

Proposition 5.2: If $X \sim GNBKSD(m,\eta,\alpha,\beta)$ then by setting $\eta=0$ and m as positive integer, we obtain a compound of binomial distribution with Kumaraswamy distribution.

Proof: For $\eta=0$ and $m\in N$ in (1), GNBD reduces to binomial distribution therefore a compound of binomial distribution with Kumaraswamy distribution is followed from (4) by simply putting $\eta=0$ and $m\in N$ in it.

$$f_{BKSD}(X;m,\alpha,\beta)=\binom{m}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{m+j-x}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{m+j-x}{\alpha}+1\right)}$$

For $x=0,1,2,...,\alpha,\beta>0, m\in N$ which is probability mass function of a compound of binomial distribution with Kumaraswamy distribution.

Proposition 5.3: If $X \sim GNBKSD(m,\eta,\alpha,\beta)$ then by setting $\eta=m=1$ we get a compound of geometric distribution with Kumaraswamy distribution.

Proof: For $\eta=m=1$ in (1) GNBD reduces to geometric distribution (GD) hence a compound of GD with KSD is followed from (4) by simply substituting $\eta=m=1$ in it.

$$f_{GKSD}(X;\alpha,\beta)=\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\,\Gamma\left(\frac{j+1}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j+1}{\alpha}+1\right)}$$

Setting $\eta=1$ we get a compound of negative binomial distribution with Kummarswamy distribution.

For $x=0,1,2,...,\alpha,\beta>0$

which is the probability mass function of a compound of GD with KSD.

Proposition 5.4: If $X \sim GNBKSD(m,\eta,\alpha,\beta)$ then by setting $\alpha=\beta=1$ we obtain a compound of GNBD distribution with uniform distribution.

Proof: Putting $\alpha=\beta=1$ in KSD reduces to uniform (0,1) distribution therefore a compound GNBD with uniform distribution is followed from (4) by simply putting $\alpha=\beta=1$ in it.

$$f_{GNBUD}(X;m,\eta)=\frac{m}{m+\eta x}\binom{m+\eta x}{x}\sum_{j=0}^{x}\binom{x}{j}\frac{(-1)^{j}}{(m+j+\eta x-x+1)}$$

For $x=0,1,2,...,m,\eta>0$

Which is probability mass function of a compound of NBD with uniform distribution.

Proposition 5.5: If $X \sim GNBKSD(m,\eta,\alpha,\beta)$ then by setting $\eta=1$ and $\alpha=\beta=1$ we get a compound of negative binomial distribution with uniform distribution.

Proof: For $\eta=1$ in (1), GNBD reduces to negative binomial distribution and for $\alpha=\beta=1$ in (2), KSD reduces to uniform (0,1) distribution. Therefore a compound of NBD with uniform distribution is followed from (4) by simply putting $\eta=\alpha=\beta=1$ in it.

$$f_{NBUD}(X;m)=\binom{x+m-1}{x}\sum_{j=0}^{x}\binom{x}{j}\frac{(-1)^{j}}{r+j+1}$$

For $x=0,1,2,\ldots;m>0$

Which is probability mass function of a compound of NBD with uniform distribution.

Proposition 5.6*:* If $X \sim GNBKSD\left(m,\eta,\alpha,\beta\right)$ then by setting $\eta=0, m\in N$ and $\alpha=\beta=1$ we get a compound of binomial distribution with uniform distribution.

Proof: For $\eta=0$ and $m\in N$ in (1), GNBD reduces to binomial distribution and for $\alpha=\beta=1$ in (2), KSD reduces to uniform (0,1) distribution. Thus a compound ofbinomial distribution with uniform (0,1)distribution is obtained from (4) by letting $\eta=0, m\in N$ and $\alpha=\beta=1$ in it.

$$f_{BUD}\left(X;m\right)=\binom{m}{x}\sum_{j=0}^{x}\binom{x}{j}\frac{(-1)^j}{r+j+1}$$

For $x=0,1,2,\ldots;m\in N$

Which is probability mass function of a mixture of binomial distribution with uniform distribution.

Proposition 5.7*:* If $X \sim GNBKSD\left(m,\eta,\alpha,\beta\right)$ then by setting $\eta=m=1$ and $\alpha=\beta=1$ we obtain a compound of geometric distribution with uniform distribution.

Proof: For $\eta=m=1$ in (1), GNBD reduces to geometric distribution and for $\alpha=\beta=1$ Kumaraswamy distribution reduces to U(0,1) distribution. Hence a compound of geometric distribution with uniform distribution can be obtained from (5) by substituting $\eta=m=1$ and $\alpha=\beta=1$ in it.

$$f_{GUD}(X)=\sum_{j=0}^{x}\binom{x}{j}\frac{(-1)^j}{j+2}\text{ For } x=0,1,2,..$$

Which is the probability function of GD with U(0,1) distribution.

6 Mean and Variance of the Proposed Model

In order to obtain the l^{th} moment of the proposed model $GNBKSD\left(m,\eta,\alpha,\beta\right)$ About origin we need to apply the well-known results of probability theory viz

(i) Conditional expectation identity $E(X^l)=E_p(X^l\mid P)$ and

(ii) Conditional variance identity $V(X)=E_p(Var(X\mid P))+Var_p(E(X\mid P)$

Since $X\mid p\sim GNBD\left(m,p,\eta\right)$ where p is itself a random variable following KSD $\left(\alpha,\beta\right)$, therefore we have

$$E(X^l)=E_p(X^l\mid P)$$

$$E(X^l)=mE_p\left(\frac{1-\eta p}{p}\right)^l$$

$$E(X^l)=m\sum_{j=0}^{l}\binom{l}{j}(-1)^j\eta^j\int_0^1 p^{j-l}f_2(p;\alpha,\beta)\,dp$$

Using argument (4) we get

$$E(X^l)=m\sum_{j=0}^{l}\binom{l}{j}(-\eta)^j\frac{\Gamma(\beta+1)\Gamma\left(1+\frac{j-l}{\alpha}\right)}{\Gamma\left(1+\beta+\frac{j-l}{\alpha}\right)}$$

For $l=1$ we get the mean of NBKSD

$$E(X)=r\left(\frac{\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(1+\beta-\frac{1}{\alpha}\right)}-\eta\right)=m_1$$

Similarly we can find variance of NBKSD using conditional variance identity (ii)

$$V(X)=E_p\left(\frac{m(1-\eta p)}{p^2}\right)+Var_p\left(\frac{r(1-\eta p)}{p}\right)$$

$$V(X)=\frac{(r+r^2)\Gamma(\beta+1)\Gamma\left(1-\frac{2}{\alpha}\right)}{\Gamma\left(\beta-\frac{2}{\alpha}+1\right)}-\frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\left(\eta+\frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\right)$$

7 Factorial Moments of a Compound of Negative Binomial Distribution with Kumarswamy Distribution

Theorem 7.1: The factorial moment of order k of a compound of negative binomial distribution with Kumarswamy distribution is given by the expression

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\sum_{j=0}^{k}(-1)^j\binom{k}{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

Where $x=0,1,2,...,r,\alpha,\beta>0$

Proof: The factorial moment of order k of NBD is

$$m_k(X\,|\,p)=\frac{\Gamma(r+k)}{\Gamma(r)}\quad\frac{(1-p)^k}{p^k}$$

since p itself is random variable following KSD, therefore one obtains factorial moment of the proposed model by using the definition

$$\mu_{[k]}(x)=E_p\left(m_k(X\,|\,p)\right)$$

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\;E_p\left(\frac{1-p}{p}\right)^k$$

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\,\alpha\beta\sum_{j=0}^{k}(-1)^j\binom{k}{j}\int_0^1 p^{\alpha+j-k-1}\left(1-p^\alpha\right)^{\beta-1}dp$$

Substituting $1-p^\alpha=z$, we get

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\,\beta\sum_{j=0}^{k}(-1)^j\binom{k}{j}\int_0^1 z^{\beta-1}(1-z)^{\frac{j-k}{\alpha}}dz$$

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\,\beta\sum_{j=0}^{k}(-1)^j\binom{k}{j}B\left(\beta,\frac{j-k}{\alpha}+1\right)$$

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\sum_{j=0}^{k}(-1)^j\binom{k}{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

(8)

where $x=0,1,2,...,r,\alpha,\beta>0$. For $k=1$ we get mean of NBKSD (r,α,β)

$$\mu_{[1]}(X)=r\left(\frac{\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(1+\beta-\frac{1}{\alpha}\right)}-1\right)=m_1$$

$$\mu_{[2]}(X)=r(r+1)\left(\frac{\Gamma(\beta+1)\Gamma\left(1-\frac{2}{\alpha}\right)}{\Gamma\left(\beta-\frac{2}{\alpha}+2\right)}-2\frac{\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+2\right)}+\frac{1}{\beta+1}\right)=\upsilon_2$$

$E(X^2)=\mu_{[2]}(X)+\mu_{[1]}(X)=\upsilon_2+m_1=m_2$, similarly we can find third moment $E(X^3)$

$$V(X)=m_2-m_1^2$$

$$V(X)=\frac{(r+r^2)\Gamma(\beta+1)\Gamma\left(1-\frac{2}{\alpha}\right)}{\Gamma\left(\beta-\frac{2}{\alpha}+1\right)}-\frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\left(1+\frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\right)$$

Standard deviation of NBKSD (r,α,β) model is $Sd=\sqrt{V(X)}$

Corollary 7.2: The factorial moment of order k of a compound of NBD with uniform distribution is given by

Where $x=0,1,2,...,r,\alpha,\beta>0$

Proof: Since KSD(α,β) reduces to Uniform$(0,1)$ distribution for $\alpha=\beta=1$, therefore factorial moment of order k of a compound of NBD with uniform$(0,1)$ distribution can be obtained from (8) by simply substituting $\alpha=\beta=1$ in it.

$$\mu_{[k]}(x)=\frac{\Gamma(r+k)}{\Gamma(r)}\sum_{j=0}^{k}(-1)^{j}\left(\frac{\binom{k}{j}}{j-k+2}\right)$$

Corollary 7.3: The factorial moment of a compound of GD with KSD is given by

$$\mu_{[k]}(x)=k\sum_{j=0}^{k}(-1)^{j}\binom{k}{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

Where $x=0,1,2,...,\alpha,\beta>0$

Proof: For $r=1$ NBD reduces to GD and a factorial moment of order k of a compound of geometric distribution with KSD is obtained from (8) by putting $r=1$ in it.

$$\mu_{[k]}(x)=k\sum_{j=0}^{k}(-1)^{j}\binom{k}{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

Corollary 7.4: The factorial moment of a compound of geometric distribution with uniform distribution is given by

$$\mu_{[k]}(x)=k\sum_{j=0}^{k}(-1)^{j}\left(\frac{\binom{k}{j}}{j-k+2}\right)$$

Where $x=0,1,2,...$

Proof: Substituting $r=1$ *and* $\alpha=\beta=1$ in (8) we get the result.

8 Parameter Estimation

In this section the estimation of parameters of $GNBKSD(m,\eta,\alpha,\beta)$ model will be discussed through method of moments and maximum likelihood estimation.

8.1 Method of Moments

In order estimate four unknown parameters of $GNBKSD(m,\eta,\alpha,\beta)$ model by the method of moments we need to equate first four sample moments with their corresponding population moments.

$$m_1=\gamma_1;\ m_2=\gamma_2;\text{and } m_3=\gamma_3\ m_3=\gamma_3 \text{ and } m_4=\gamma_4$$

Where $\gamma_i=\frac{1}{n}\sum_{i=1}^{n}x_i$ is the i[th] sample moment and m_i is the i[th] corresponding population moment and the solution for $\hat{m},\hat{\eta},\hat{\alpha}$ and $\hat{\beta}$ may be obtained by solving above equations simultaneously.

8.2 Maximum Likelihood Estimation

The estimation of parameters of $GNBKSD(m,\eta,\alpha,\beta)$ model via maximum likelihood estimation method requires the log likelihood function of $GNBKSD(m,\eta,\alpha,\beta)$

$$\pounds(X;m,\eta,\alpha,\beta)=\log L(X;m,\eta,\alpha,\beta)=n\log(m\beta)+\sum_{i=}^{n}\left[\log\binom{m+\eta x_i}{x_i}-\log(m+\eta x_i)\right]+\sum_{i=1}^{n}\log\left(\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\ B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)\right)$$

The maximum likelihood estimate of $\Theta=\left(\hat{m},\hat{\eta},\hat{\alpha},\hat{\beta}\right)^{T}$ can be obtained by differentiating it with respect unknown parameters m,η,α and β respectively and then equating them to zero.

$$\frac{\partial}{\partial m}\pounds(X;m,\eta,\alpha,\beta)=\frac{n}{m}+\sum_{i=1}^{n}\left[\frac{\frac{\partial}{\partial m}\binom{m+\eta x_i}{x_i}}{\binom{m+\eta x_i}{x_i}}-\frac{1}{m+\eta x_i}\right]+\sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j\frac{\partial}{\partial m}B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}\right)$$

$$\frac{\partial}{\partial \eta}\pounds(X;m,\eta,\alpha,\beta)=\sum_{i=1}^{n}\left[\frac{\frac{\partial}{\partial \eta}\binom{m+\eta x_i}{x_i}}{\binom{m+\eta x_i}{x_i}}-\frac{x_i}{m+\eta x_i}\right]+\sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j\frac{\partial}{\partial \eta}B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}\right)$$

$$\frac{\partial}{\partial \alpha}\pounds(X;m,\eta,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j\frac{\partial}{\partial \alpha}B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}\right)$$

$$\frac{\partial}{\partial \beta}\pounds(X;m,\eta,\alpha,\beta)=\frac{n}{\beta}+\sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j\frac{\partial}{\partial \beta}B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}{\sum_{j=0}^{x_i}\binom{x_i}{j}(-1)^j B\left(\beta,\frac{m+j+\eta x_i-x_i}{\alpha}+1\right)}\right)$$

These four derivative equations cannot be solved analytically, therefore $\hat{m},\hat{\eta},\hat{\alpha}$ and $\hat{\beta}$ will be obtained by maximizing the log likelihood function numerically using Newton-Raphson method which is a powerful technique for solving equations iteratively and numerically.

9 Application of the Proposed NBKSD Model

Classical Poisson distribution plays an important role in modeling count data processes but it requires a strong condition of independent successive events which may not be the characteristic of the count data under consideration such as a data given in table (1-3). Furthermore the equality of mean and variance of the observed counts is hardly satisfied in practice, negative binomial distribution can be used in such cases but negative binomial distribution is better for over dispersed count data that are not necessarily heavy tailed in such situations compound distribution models serves efficiently well.

9.1 Application in Genetics

Genetics is the science which deals with the mechanisms responsible for similarities and differences among closely related species. The term 'genetic' is derived from the Greek word'genesis' meaning grow into or to become. So, genetic is the study of heredity and hereditary variations it is the study of transmission of body features that is similarities and difference, from parents to offspring's and the laws related to thistransmission**,** any difference between individual organisms or groups of organisms of any species, caused either by genetic difference or by the effect of environmental factors, is called variation. Variation can be shown in physical appearance, metabolism, behavior, learning and mental ability, and other obvious characters. In this section the potentiality of proposed model will be justified by fitting it to the reported genetics count data sets of Catcheside et al.

10 Conclusions

In this paper we have proposed a new model by compounding GNBD with KSD and it has been shown that proposed model can be nested to different compound distributions. Furthermore, we have derived several properties of proposed model such as factorial moments, mean, variance. The similarity of the proposed model with some compound distribution has also been discussed. In addition to this parameter estimation of the proposed model has been discussed by means of method of moments and MLE. Finally, the application of the proposed model havebeen explored in genetics, based on the chi-square

Table 1. Distribution of number of Chromatid aberrations (0.2 g chinon 1, 24 hours)

Number of aberrations	Observed Frequency	Fitted Distribution			
		Poisson	NBD	NBKSD	GNBKSD
0	268	231.3	279.1	269.8	265.98
1	87	126.1	71.1	82.5	86.34
2	26	34.7	27.5	27.8	14.36
3	9	6.3	11.8	10.6	7.07
4	4	0.8	5.3	4..5	4.81
5	2	0.1	2.5	2.1	3.2
6	1	0.1	1.1	1.6	2.39
7+	3	0.1	0.5	0.5	1.81
Total	400	400		400	
Parameter Estimation		$\hat{\theta} = 0.54$	$\hat{r} = 0.49$ $\hat{p} = 0.48$	$\hat{r} = 1.74$ $\hat{\alpha} = 5.94$ $\hat{\beta} = 2$	$\hat{m} = 1.45$ $\hat{\alpha} = 6.01$ $\hat{\beta} = 3.01$ $\hat{\eta} = 3.79$
p value		0.00	0.09	0.37	0.64

goodness of fit test and it has been shown that $GNBKSD(m,\eta,\alpha,\beta)$ model offers a better fit as compared to classical Poisson distribution and NBKSD.
We hope that the proposed model will serve as an alternative to various models available in literature.

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