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Energy Minimization by Optimally Relocating Mobile Relays in Multi-hop Wireless Ad hoc Networks

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Abstract: This paper considers the energy minimization problem in a multi-hop wireless ad hoc network that consists of regular nodes and mobile relays. In such a network, mobile relays may relocate to optimal positions such that energy consumption for data transmission is minimized. In order to find the optimal positions for mobile relays, the mobile relay optimal relocation (MROR) problem is first defined. Since the cost in terms of data transmission energy can be modeled using a convex function, the problem is formulated as a convex optimization problem over mobile relay position variables. Using dual decomposition and subgradient methods enables mobile relays to cooperate in a distributed manner to attain the optimal relocations. Numerical results confirm that the proposed method can result in minimal energy consumption for data transmission. This study appears to be the first attempt to relocate mobile relays to globally minimize data transmission energy consumption in a multi-hop wireless network where multiple data flows exist.

Keywords: wireless mobile relay, energy efficiency, convex optimization, dual decomposition, distributed algorithm

1 Introduction

Recently, the use of controllable mobile nodes (aerial or ground vehicles) has been considered in wireless networks. For example, mobile nodes can be used as moving sensors and can form mobile sensor networks [1, 2]. Mobile nodes can also be used as a data ferry or mobile elements that collect or carry data in a sparse wireless network [3,4].

This paper considers a multi-hop wireless ad hoc network that consists of regular (stationary or mobile) nodes and mobile relays (e.g., autonomous aerial vehicles) for which movement can be controlled. By exploiting controllable mobility of mobile relays, the network can achieve better connectivity and network performance. One major concern in an ad hoc network is energy consumption, which relates to network lifetime.

Each node in the network transmits frames using transmission power with which an acceptable signal-to-noise ratio (SNR) can be obtained. The required transmission power mainly depends on the distance between transmitter and receiver. Therefore, if mobile relay nodes relocate to a certain position, the consumed energy may increase or decrease.

There have been several studies that use mobile nodes for relaying data in a wireless ad hoc network [5,6,7,8]. For example, Goldenberg et al. [5] showed that transmission energy consumption can be minimized by evenly spacing relay nodes along a line segment between source and destination nodes. Hamouda et al. [7] also used controllable mobility of mobile nodes in order to reduce energy consumption while nodes are maintaining network connectivity. Liu et al. [6] first determined the optimal number of hops to minimize transmission energy consumption between a source and destination pair, and then mobile nodes are placed based on the obtained number of hops.

Those studies in [5,6,7,8], however, only considered one source and destination connection pair to obtain the minimum energy consumption. Therefore, when there are multiple data connections that share mobile relays, the global optimal value in terms of energy consumption may not be achieved.

Le et al. [9] considered mobile relays in an ad hoc network. However, they also only added a mobile relay on the route between a single source and destination pair without taking the effect on the entire network into consideration. Moreover, their objective was to maximize

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data throughput rather than minimize energy consumption.

In this paper, in order to find mobile relays' globally optimal positions that minimize total energy consumption for data transmission in a network that has multiple data source-destination pairs, the mobile relay optimal relocation (MROR) problem is defined and formulated as a convex optimization problem such that a global optimum can be attained. In addition, for practical usage in a wireless ad hoc network, using a distributed method based on dual decomposition and a subgradient algorithm enables mobile relays to cooperate to obtain optimal positions. Due to the convex formulation of the MROR problem and strong duality holding, the proposed distributed method achieves the global optimal solution even when multiple data connections share mobile relays. The proposed method is validated by numerical results and analysis.

The rest of the paper is organized as follows. Section 2 presents the network model and problem definition. The proposed distributed method is described in Section 3. In Section 4, numerical results and analysis are presented. Finally, Section 5 concludes the paper.

2 Network Model and Problem Definition

The network is modeled as a directed graph $G=(V,E)$ where $E=\{(u,v):u,v\in V,u\neq v\}$. If an edge (u,v) exists, a communication link can be established and node u can transmit data to node v. This work focuses on the case where each link uses an allocated communication channel by using frequency-division multiple access (FDMA), code-division multiple access (CDMA), or time-division multiple access (TDMA). For example, a lot of tactical networks use TDMA for multiple access [10,11], and the proposed network scheme is particulary useful in such tactical networks where ground vehicles and aerial vehicles coexist.

The network includes the set of mobile relay (MR) nodes, M. MR nodes are labeled $1,\dots,M$. Note $M\subset V$. A subset of regular nodes are source nodes that send the data to the destination nodes, which are also regular nodes. The path between source and destination nodes may have multiple intermediate nodes (regular or MR nodes). This work assumes that the paths between source-destination pairs are given using ad hoc routing protocols.

Each MR node is associated with the position vector $x_u\in R^2$, $u\in M$. Define $\mathbf{x}$ as the vector for positions of the MR nodes, $\mathbf{x}\in R^{|M|\times 2}$. The position of regular node n is given by p_n, $n\in V\backslash M$. The traffic amount carried by link (u,v) is denoted as $f_{u,v}$. $D(u)$ denotes a set of regular nodes that transmit data to MR node u. Similarly, $U(u)$ denotes a set of regular nodes that receive data from MR node u. $R(u)$ denotes a set of MR nodes that receive data from another MR node u.

The function C_{uv} denotes the transmission power cost to transmit one bit at link (u,v), which is modeled based on the distance between two communicating peers. Also, denote α_{amp} as an amplification constant for the transmitter to obtain an acceptable SNR. Then, the transmission energy E_t to transmit l bits over distance d can be modeled as done by Kumar et al. [12] and Perumal et al. [7], which is

$$E_t^1(l,d)=\alpha_{\mathrm{amp}}\times l\times d^w \tag{1}$$

where exponent $w\geq 2$ is given by the path loss model [12].

Accordingly, given distance d, or the length of link (u,v), $C_{uv}(d)\triangleq\alpha_{\mathrm{amp}}\times d^w$. Note that α_{amp}, and w are constants.

It is worth while to note that, in some energy models [13], the energy to keep the transmitter circuitry powered up, denoted as E_c, is also considered i.e., the transmission energy consumption can be modeled as $E_t^2(l,d)=E_c\times l+\alpha_{\mathrm{amp}}\times l\times d^w$. Note that the use of this model does not affect the problem formulation, because E_c is a constant, and hence, it does not affect the optimal solution.

The objective is to select the optimal positions of MR nodes such that total transmission energy consumption is minimized. Note that the positions of regular nodes, data paths, and traffic amounts on the paths are given (i.e., the consumed data transmission energy at a link between two regular nodes is given). Therefore, the problem can be regarded as one to minimize the sum of transmission energy consumption over links where at least one end is an MR node. This problem is defined as the mobile relay optimal relocation problem, which is formulated as

$$\begin{aligned}\text{minimize}\quad &\sum_{u\in M}\Big(\sum_{v\in G(u)} f_{uv}C(\|x_u-p_v\|_2)\\ &+\sum_{v\in R(u)} f_{uv}C(\|x_u-x_v\|_2)\Big)\end{aligned} \tag{2}$$

where $G(u)$ represents $D(u)\cup U(u)$.

Here, the optimization variables are $\mathbf{x}$ (i.e., the positions of MR nodes x_u, $u\in M$). The term $\sum_{v\in G(u)} f_{uv}C(\|x_u-p_v\|_2)$ represents the energy cost for transmissions between regular nodes and MR node x_u. Similarly, the term $\sum_{v\in R(u)} f_{uv}C(\|x_u-x_v\|_2)$ represents the energy cost for transmissions between MR nodes.

This problem is a convex optimization problem because the objective function is the sum of norms, which implies that it can be solved efficiently by using descent methods. However, a distributed algorithm for solving the problem is desirable in a wireless ad hoc network scenario.

3 Distributed Algorithm

This paper considers the dual decomposition and subgradient method to solve the problem in a distributed manner.

3.1 Dual Problem Formulation

The object function in equation (2) is not separable due to the coupled variables, i.e., x_u and x_v. Therefore, in order to decompose and make it solvable in a distributed way, auxiliary variables $\mathbf{x}_{uv}$ for coupled arguments in the objective function are introduced, with equality constraints added to enforce consistency. In order to simplify the notations, define $P(u,v) \triangleq f_{uv}C(\|x_u - p_v\|_2)$. Then, the problem formulation becomes

$$\begin{aligned} \text{minimize} \quad & \sum_{u \in M} \Big(\sum_{v \in G(u)} P(u,v) + \sum_{v \in R(u)} f_{uv}C(\|x_u - x_{uv}\|_2) \Big) \\ \text{subject to} \quad & \mathrm{x}_{uv} = \mathrm{x}_v \qquad \forall u, v \in R(u) \\ & \mathrm{x}_{\mathrm{bl}} \preceq \mathrm{x} \preceq \mathrm{x}_{\mathrm{ur}} \end{aligned} \tag{3}$$

Now, the coupling in the objective function is transferred to constraints coupling. Coupled constraints can be decoupled via dual decomposition and solved by using *consistency pricing*.

Therefore, the dual problem is defined by introducing Lagrange multipliers $\mathbf{p} \in R$. The partial Lagrangian is

$$\begin{aligned} L(\mathbf{x}, \mathbf{x}_{uv}, \mathbf{p}) = & \sum_{u \in M} \Big(\sum_{v \in G(u)} P(u,v) + \sum_{v \in R(u)} f_{uv}C(\|\mathbf{x}_u - \mathbf{x}_{uv}\|_2) \Big) \\ & + \sum_{u \in M} \sum_{v \in R(u)} \mathbf{p}_{uv}^T (\mathbf{x}_v - \mathbf{x}_{uv}) \end{aligned} \tag{4}$$

Then, consider the term $\sum_{u \in M} \sum_{v \in R(u)} \mathbf{p}_{uv}^T \mathbf{x}_v$. This term can be re-arranged as follows because of its additive structure:

$$\sum_{u \in M} \sum_{v \in R(u)} \mathbf{p}_{uv}^T \mathbf{x}_v = \sum_{u \in M} \Big(\sum_{v : u \in R(v)} \mathbf{p}_{vu} \Big)^T \mathbf{x}_u \tag{5}$$

Therefore, equation (4) is equivalent to

$$\begin{aligned} L(\mathbf{x}, \mathbf{x}_{uv}, \mathbf{p}) = & \sum_{u \in M} \Big(\sum_{v \in G(u)} P(u,v) + \sum_{v \in R(u)} f_{uv}C(\|\mathbf{x}_u - \mathbf{x}_{uv}\|_2) \\ & + \Big(\sum_{v : u \in R(v)} \mathbf{p}_{vu} \Big)^T \mathbf{x}_u - \sum_{v \in R(u)} \mathbf{p}_{uv}^T \mathbf{x}_{uv} \Big) \end{aligned} \tag{6}$$

The objective function of the dual problem is thus

$$g(\mathbf{P}) = \inf_{\substack{\mathbf{x}, \mathbf{x}_{uv} \\ \mathrm{x}_{\mathrm{bl}} \preceq \mathrm{x} \preceq \mathrm{x}_{\mathrm{ur}}}} \Big\{ L(\mathbf{x}, \mathbf{x}_{uv}, \mathbf{p}) \Big\} \tag{7}$$

Due to the additivity structure of the Lagrangian in equation (6), the objective function in equation (7) can be separated into multiple sub-problems, in which minimization is done only using local variables (note the uth subproblem uses only variables with the first subscript index u). The uth node, for all u, locally solves the problem

$$\begin{aligned} \text{minimize} \quad & \sum_{v \in G(u)} P(u,v) + \sum_{v \in R(u)} f_{uv}C(\|x_u - x_{uv}\|_2) \\ & + \Big(\sum_{v : u \in D(v)} \mathbf{p}_{vu} \Big)^T \mathbf{x}_u - \sum_{v \in R(u)} \mathbf{p}_{uv}^T \mathbf{x}_{uv} \end{aligned} \tag{8}$$

Finally, the dual problem is

$$\text{maximize} \quad g(\mathbf{P}) \tag{9}$$

The dual problem in equation (9) is a convex optimization problem since the dual function (i.e., $g(\mathbf{P})$) is always convex [14,15]. In addition, Slater's condition for constraint qualification is satisfied because all constraints are linear in equation(3) [16]. Therefore, strong duality holds and the optimal values of the dual problem are equal to the optimal values of the primal problem.

3.2 Solving the Problem via Subgradient Method

Since the objective function in equation (2) is not strictly concave in $\mathbf{x}_{uv}$, the dual function may be piecewise differentiable i.e., the dual problem in equation (9) is a non-differentiable convex optimization problem. Therefore, this paper considers a subgradient method [15, Ch.6] [17].

A subgradient of a nondifferentiation convex function g at p is a vector h such that

$$g(q) \geq g(p) + h^T(q - p), \qquad \forall q \tag{10}$$

Given a dual variable $\mathbf{p}$, let $\mathrm{x}^\star$, $\mathrm{x}_{uv}^\star$ be an optimal solution to the problem in equation (8). From the definition of the dual function in equations (6) and (7), a negative dual problem's subgradient h is given by

$$\mathrm{h}_{uv} = \mathrm{x}_{uv}^\star - \mathrm{x}_v^\star \qquad v \in R(u) \tag{11}$$

In the subgradient algorithm, start with an initial point $\mathrm{p}_{uv}^{(1)}$. At each iteration step $t = 1, 2, \ldots$, compute the dual function $g(\mathrm{p}_{uv}^{(t)})$ and a subgradient $\mathrm{h}(t)$. Then, update the dual variable by

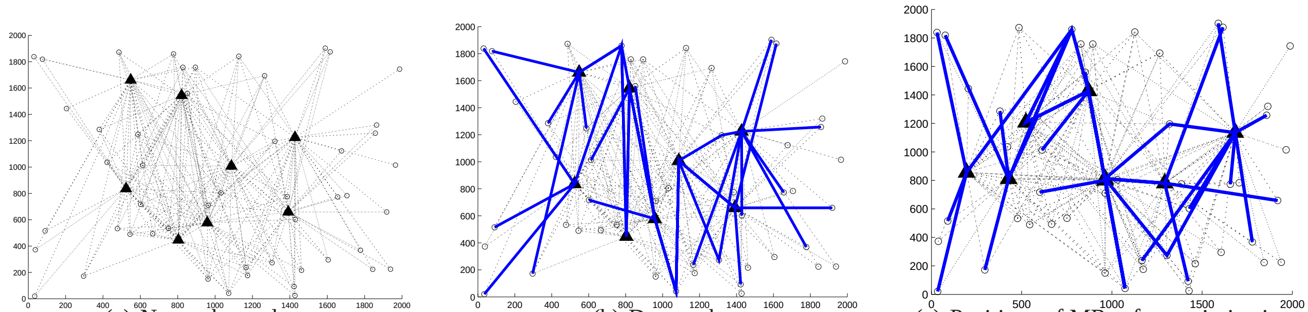
$$\begin{aligned} \mathrm{p}_{uv}^{(t+1)} & = \big[\mathrm{p}_{uv}^{(t)} - \alpha_k \mathrm{h}_{uv}^{(t)}\big] \\ & = \big[\mathrm{p}_{uv}^{(t)} - \alpha_k (\mathrm{x}_{uv}^{(t)} - \mathrm{x}_v^{(t)})\big], \qquad v \in R(u) \end{aligned} \tag{12}$$

where α_t denotes a positive scalar size.

A convergence condition requires that the stepsize sequence satisfies $\alpha_t \to 0$ and $\sum_{t=1}^{\infty} \alpha_t = \infty$ (i.e., the subgradient algorithm converges with a stepsize that satisfies those properties [18]). Therefore, a simple stepsize can be determined by

$$\alpha_t = \frac{c}{t} \tag{13}$$

where c is a constant.



(a) Network topology (b) Data path (c) Positions of MRs after optimization

Fig. 1: A wireless network with regular nodes and MR nodes

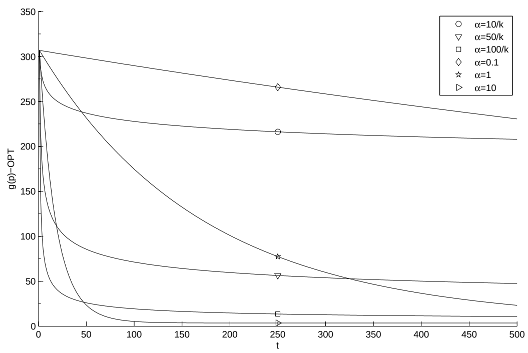


Fig. 2: Convergence of the subgradient method

4 Numerical Results

In order to validate the problem formulation and the algorithm, a wireless network that consists of 50 regular nodes and 8 MR nodes is considered. Those nodes are randomly distributed in an area 2000×2000 meters. The transmission range is 800meters. The resulting network topology is shown in Fig. 1 (a) where dark triangles, circles, and dotted lines represent MR nodes, regular nodes, and communication links, respectively.

Assume there are 10 communicating pairs. The source and destination nodes are randomly selected. The data rate of each of 10 connections is selected from the range [50, 550] kbps. Dijkstra's algorithm is used to find the path between source and destination pairs. Figure 1 (b) shows the data path with bold blue lines.

The transmission energy model defined in equation (1) is used. The value of α_{amp} is set to 100pJ/bit/m^2, which was introduced in [13]. The value of the path loss exponent w is set to 2.

Figure 2 shows the difference between the optimal solution and the dual objective function value in milliwatts over the number of iterations. This paper uses two types of step size. First is $\frac{c}{t}$, where c and t are a constant and the iteration step number, respectively. For the values of c, $\{10, 50, 100\}$ are used for the experiments. In addition, the algorithm with constant step sizes is also evaluated. The numbers $\{0.1, 1, 10\}$ are used as constant step sizes. As shown in Fig. 2, the algorithm with a different step size converges at a different rate. As expected, the algorithm converges quickly with step sizes $\frac{100}{k}$ and 10. With $\frac{100}{k}$, the algorithm shows the highest rate of convergence in the beginning phase up to around 60 rounds, after which the algorithm with the step size 10 takes first place.

Table 1: The optimal set and optimal value

	Before optimization		After optimization	
MR	x1	x2	x1	x2
m1	523.36	836.02	194.78	855.79
m2	1391.60	658.83	1292.70	780.20
m3	548.45	1660.90	427.87	810.76
m4	1427.40	1224.80	1683.80	1135.40
m5	1087.30	1007.40	962.05	811.86
m6	958.10	577.23	962.27	800.58
m7	803.46	446.14	523.31	1207.50
m8	820.48	1543.80	869.97	1427.30
Tx Power (mW)	3.78×10^3		2.73×10^3	

Table 1 shows the optimal set (i.e., $\mathbf{x}^*$) obtained from 500 iteration steps and step size $\frac{100}{t}$. The second and third columns indicate the coordinates of MR nodes before optimization, and the fourth and fifth columns are the coordinates after optimization. As shown in the table, all eight MR nodes' positions have been updated. Note that MR node 5 and MR node 6 have similar positions. Those positions are depicted in Fig. 1(c). The last row indicates the total required transmission power before and after optimization. As can be seen from that row, applying the algorithm can reduce transmission power by about 27%.

5 Concluding Remarks

This work has used controlled mobility of mobile relays (MRs) in order to minimize data transmission energy. In order to find the optimal positions for MRs, the mobile relay optimal relocation (MROR) problem has been defined and formulated as a convex optimization problem over MRs' position variables. Then, dual decomposition and subgradient methods have been used to find the solution and enable mobile relays to cooperate in a distributed manner to attain the optimal relocations. Numerical results show that the proposed method can achieve minimal energy consumption for data transmission.

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