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On Stochastic Multi-Level Multi-Objective Fractional Programming Problems

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Abstract: This paper proposes an approach where it can be applied to the optimization decisions making problems under uncertainties and solves a multi-level multi-objective fractional programming problems involving stochastic parameters coefficient in objective functions (SMLMOFPP). In this work, the first phase of the solution approach, we convert the probabilistic nature (stochastic) of this problem in objective functions into a multi-level multi-objective fractional programming problems (MLMOFPP).At the second phase, we use a computer-oriented technique to convert (MLMOFPP) into a multi-level multi-objective linear programming problems (MLMOLPP). Then a fuzzy approach solves (MLMOLPP) using the concept of tolerance membership function to develop a Tchebycheff problem for generating a compromise solution for this problem. In addition, a numerical example is provided to demonstrate the correctness of the proposed solution.

Keywords: Multi-level programming; Fractional programming; Stochastic programming; Chance Constraints.
MSC 2000: 90C15; 90C32; 90C99.

1 Introduction

Stochastic programming provides a suitable framework to model decisions making problems under uncertainty [8,10]. In recent years methods of multi-objective stochastic optimization have become increasingly important in scientifically based on decisions making involved in real life problems arising in economic, industry, health care, transportation, agriculture, military purposes and technology [1].

In the real world, there are two or more decision makers in an organization with a hierarchical structure, and they make decision in turn or at the same time to optimize their objective functions. Such situations are formulated as multi-level programming problems [2,7].

Fractional programming is a generalization of linear fractional programming. The objective function in a fractional program is a ratio of two functions that are in general nonlinear. The ratio to be optimized often describes some kind of efficiency of a system. Fractional programming problems are useful tools in production planning, financial and corporate planning [3,5,6,15].

In [11], Saad and Emam suggested a solution of stochastic multi objective integer linear programming problems with a parametric study. This study proposed to investigate a stability set of the efficient solution for this problem.

In literature there are many researchers have focused to solve multi-level linear or nonlinear multi-objective programming problems [12,14].

In [7], Osman, et al. provided a solution method for solving multi-level non-linear multi-objective under fuzziness. This solution method uses the concepts of tolerance membership functions and multi-objective optimization at every level to develop a fuzzy max-min decision model till generating optimal solution.

In [4], Emam proposed an algorithm for solving bi-level integer multi-objective fractional programming problem using cutting plan algorithm.

In [13], Saraj and Safaei, proposed solution method for fuzzy linear fractional bi-level multi-objective programming problems based on Taylor series and Kuhn-Tucker conditions.

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This paper is organized as follows: we start in section 2 by formulating the model of stochastic multi-level multi-objective fractional programming problems (SMLMOFPP) along with the solution concept. Section 3, the transformation stochastic parameters in the objective functions (SMLMOFPP) into multi-level multi-objective fractional programming problems (MLMOFPP) is presented. In section 4, a computer-oriented technique to solve (MLMOFPP) is described. In section 5, a fuzzy approach to solve the equivalent problems (MLMOPP). In addition, a numerical example is provided to illustrate the developed results in section 6. Finally, conclusion and future works are reported in Section 7.

2 Problem formulation and solution concept

Let $x_i \in R^n, (i=1,2,3)$ be a vector variables indicating the first decision level's choice, the second decision level's choice and the third decision level's choice. $F_i : R^n \to R^{N_t}, (i=1,2,3)$ be the first level objective function, the second level objective function and the third level objective function, respectively.

Assume that the first level decision maker (FLDM), second level decision maker (SLDM) and third level decision maker (TLDM) have N_1, N_2 and N_3 objective functions, respectively, M be the set of feasible choices $\{(x_1,x_2,x_3)\}$. Therefore, the SMLMOFPP may be formulated as follows:

[FLDM]

$$\underset{x_1}{Max}\ F_1(x,\theta^1) = \frac{\theta_i^1 c_{1i}^T x + \alpha_{1i}}{d_{1i}^T x + \beta_1}, (i=1,2,\ldots,N_1), \tag{1}$$

Where x_2, x_3 solve,

[SLDM]

$$\underset{x_2}{Max}\ F_2(x,\theta^2) = \frac{\theta_j^2 c_{2j}^T x + \alpha_{2j}}{d_{2j}^T x + \beta_2}, (j=1,2,\ldots,N_2), \tag{2}$$

Where x_3 solves,

[TLDM]

$$\underset{x_3}{Max}\ F_3(x,\theta^3) = \frac{\theta_r^3 c_{3r}^T x + \alpha_{3r}}{d_{3r}^T x + \beta_3}, (r=1,2,\ldots,N_3), \tag{3}$$

Subject to

$$M\{(x_1,x_2,x_3) | m_i(x_1,x_2,x_3) \le 0, i=1,2,\ldots,n.\}, \tag{4}$$

Where the functions $F_i(x,\theta^i)$ are stochastic fractional objective functions defined on FLDM, SLDM and TLDM.

Definition 1.

Let M_1, M_2, M_3 be the feasible regions of FLDM, SLDM and TLDM, respectively. For any $(x_1 \in M_1 = \{x_1 | (x_1,x_2,x_3) \in M_1\})$ given by FLDM, and $(x_2 \in M_2 = \{x_2 | (x_1,x_2,x_3) \in M_2\})$ given by SLDM, if the decision-making variable $(x_3 \in M_3 = \{x_3 | (x_1,x_2,x_3) \in M_3\})$ is the optimal solution of the TLDM, then (x_1,x_2,x_3) is a feasible solution of (SMLMOFPP).

Definition 2.

If (x_1^*, x_2^*, x_3^*) is a feasible solution of the SMLMOFP (1)-(4); no other feasible solution $(x_1,x_2,x_3) \in M$ exists, such that $f_1 i(x_1^*,x_2^*,x_3^*) \le f_1 i(x_1,x_2,x_3)$, with at least one $(i=1,2,\ldots,k_i)$; so (x_1^*,x_2^*,x_3^*) is the optimal solutions of the (SMLMOFPP).

3 Stochastic transformation for solving (SMLMOFPP)

The basic idea in treating (SMLMOFPP) is to convert the probabilistic nature of this problem into an equivalent deterministic. In this case, the set of objective functions can be written as [9]:

$$S_r(x) = \frac{\sum_{j=1}^{n} c_{rj} x_j + k_1^r \sum_{j=1}^{n} E(\theta_j^r) x_j + k_2^r \sqrt{\sum_{j=1}^{n} \sigma^2(\theta_j^r) x_j^2} + \alpha_r}{d_{rj}^T x + \beta_r}, (r=1,2,\ldots,k). \tag{5}$$

Where $E(\theta_j^r)$ = mean of θ_j^r and $\sigma^2(\theta_j^r)$ = variance of θ_j^r, and k_1^r, k_2^r are non-negative constants whose values indicate the relative importance of the mean and the standard deviation of the variable θ for maximization.

Therefore, the (MLMOFPP) equivalent to (SMLMOFPP) may be formulated as follows:

[FLDM]

$$\underset{x_1}{Max}\ S_1(x) = \frac{\sum_{j=1}^{n} c_{rj}x_j + k_1^r \sum_{j=1}^{n} E(\theta_j^r)x_j + k_2^r \sqrt{\sum_{j=1}^{n} \sigma^2(\theta_j^r)x_j^2} + \alpha_r}{d_{rj}^T x + \beta_r}, (r = 1, 2, \ldots, N_1) \tag{6}$$

Where x_2, x_3 solve,

[SLDM]

$$\underset{x_2}{Max}\ S_2(x) = \frac{\sum_{j=1}^{n} c_{rj}x_j + k_1^r \sum_{j=1}^{n} E(\theta_j^r)x_j + k_2^r \sqrt{\sum_{j=1}^{n} \sigma^2(\theta_j^r)x_j^2} + \alpha_r}{d_{rj}^T x + \beta_r}, (r = 1, 2, \ldots, N_2) \tag{7}$$

Where x_3 solves,

[TLDM]

$$\underset{x_3}{Max}\ S_3(x) = \frac{\sum_{j=1}^{n} c_{rj}x_j + k_1^r \sum_{j=1}^{n} E(\theta_j^r)x_j + k_2^r \sqrt{\sum_{j=1}^{n} \sigma^2(\theta_j^r)x_j^2} + \alpha_r}{d_{rj}^T x + \beta_r}, (r = 1, 2, \ldots, N_3) \tag{8}$$

Subject to

$$M = \{(x_1, x_2, x_3) | m_i(x_1, x_2, x_3) \le 0, i = 1, 2, \ldots, n.\} \tag{9}$$

4 A computer-oriented technique for solving (MLMOFPP)

In multi-level multi-objective fractional programming problems (MLMOFPP), the objective functions are transformed by using a computer-oriented technique [5], the main idea of this technique is to convert (MLMOFPP) into a multi-level multi-objective linear programming problems (MLMOLPP) for the FLDM, SLDM and TLDM in the following form as follows:

$$Z = py + g \tag{10}$$

where $p\left(c - d\frac{\alpha}{\beta}\right)$, $y = \frac{x}{dx+\beta}$ and $g = \frac{\alpha}{\beta}$

And the transformation of the constraints (4) can be written as follows:

$$\left(A + \frac{b}{\beta}d\right)\frac{x}{dx+\beta} \le \frac{b}{\beta}, \tag{11}$$
$$Gy \le h$$

where $A + \frac{b}{\beta}d = G$, $\frac{x}{dx+\beta} = y$, $\frac{b}{\beta} = h$

Now the equivalent MLMOLPP of problem (1)-(4) can be written as follows:

[FLDM]

$$\underset{y_1}{Max}\ Z_1(y_1, y_2, y_3) = \underset{y_2}{Max}\ (z_{11}(y_1, y_2, y_3), \ldots, z_{1N_1}(y_1, y_2, y_3)), \tag{12}$$

Where y_2, y_3 solve,

[SLDM]

$$\underset{y_2}{Max}\ Z_2(y_1, y_2, y_3) = \underset{y_2}{Max}\ (z_{21}(y_1, y_2, y_3), \ldots, z_{2N_2}(y_1, y_2, y_3)), \tag{13}$$

Where y_3 solves,

[TLDM]
$$\underset{y_3}{Max}\ \ Z_3(y_1,y_2,y_3) = \underset{y_3}{Max}\ \ (z_{31}(y_1,y_2,y_3),\ldots,z_{3N_3}(y_1,y_2,y_3)), \tag{14}$$

Subject to
$$G = \{(y_1,y_2,y_3)|g_i(y_1,y_2,y_3) \leq 0, i = 1,2,\ldots,n\} \tag{15}$$

Where y_1,y_2,y_3, represent decision variables under the control of FLDM, SLDM and TLDM respectively, G is the set of linear constrains.

From the above (MLMOLP), we get $y = \frac{x}{dx+\beta}$. Using this definition we can get:

$$x = \beta \frac{y}{1-dy} \tag{16}$$

Which is our required optimal solution. Then put this value of x in the original objective function, we can obtain the optimal value.

5 Fuzzy approach for solving (MLMOPP)

To solve the MLMOLPP by using fuzzy approach, first gets the satisfactory solution that is acceptable to the FLDM, and then give the FLDM decision variables and goals with some leeway to the SLDM for him/her to seek the satisfactory solution, then the SLDM give the decision variables and goals with some leeway to the TLDM for him/her to seek the satisfactory solution and to arrive at the solution which is closest to the optimal solution of the FLDM.

The FLDM solves his/her problem as follows:
1. Find individual optimal solution of problem FLDM by obtaining the best and the worst solutions of the FLDM problem are $(Z_{11}^*,\ldots,Z_{1N}^*),(Z_{11}^-,Z_{1N}^-)$. 2. Using this value of (Z_{1k}^*,Z_{1k}^*) to build the membership functions as follows:

$$\mu_{z_{1k}}[z_{1k}(y)] = \begin{cases} 1 & \text{if } z_{1k}(y) > z_{1k}^*, \\ \frac{z_{1k}(y)-z_{1k}^-}{z_{1k}^*-z_{1k}^-} & \text{if } z_{1k}^- \leq z_{1k}(y) \leq z_{1k}^*, \\ 0 & \text{if } z_{1k}^- \geq z_{1k}(y), k = 1,2,\ldots,N_1. \end{cases} \tag{17}$$

Now, we can get the solution of the FLDM problem by solving the following Tchebycheff problem

$$Max\ \lambda, \tag{18}$$

Subject to
$y \in G$
$\mu_{z_{1k}}[z_{1k}(y)] \geq \lambda, k = 1,2,\ldots,N_1,$
$\lambda \in [0,1]$
Whose solution is assumed to be

$$[y_1^F, y_2^F, y_3^F, Z_{1k}^F, K = 1,2,\ldots,N, \lambda^F(\text{Satisfactory level})]$$

The SLDM do the same action like the FLDM till he obtains his solution to be $[y_1^S,y_2^S,y_3^S,Z_{2q}^S,q = 1,2,\ldots,N,\beta^S$ (Satisfactory level)], then SLDM transform the value of (y_1^S,y_2^S,y_3^S) to obtain x_1^S,x_2^S,x_3^S using equation (15).

The TLDM do the same action like the SLDM till he obtains his solution is assumed to be $[y_1^T,y_2^T,y_3^T,Z_{2r}^S,r = 1,2,\ldots,N,\gamma^T$(Satisfactory level)], and then TLDM transform the value of (y_1^T,y_2^T,y_3^T) to obtain x_1^T,x_2^T,x_3^T using equation (15).

Now the solution of the three level decision makers is disclosed. However, three solutions are usually different because of nature between three levels objective functions.

The FLDM knows that using the optimal decisions x_1^F as a control factors for the SLDM are not practical. It is more reasonable to have some tolerance that gives the SLDM an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions, also the SLDM do the same action with the TLDM.

In this way, the range of decision variable x_1, x_2 should be around x_1^F, x_2^S with maximum tolerance t_1, t_2 and the following membership function specify x_1^F, x_2^S as:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_1)}{t_1} & x_1^F - t_1 \le x_1 \le x_1^F, \\ \frac{(X_1^F+t_1)-x_1}{t_1} & x_1^F \le x_1 \le x_1^F + t_1, \end{cases} \tag{19}$$

$$\mu(x_2) = \begin{cases} \frac{x_2-(x_2^S-t_2)}{t_2} & x_2^S - t_2 \le x_2 \le x_2^S, \\ \frac{(X_2^S+t_2)-x_2}{t_2} & x_2^S \le x_2 \le x_2^S + t_2, \end{cases} \tag{20}$$

First, the FLDM goals may reasonably consider all $Z_{1k} \ge Z_{1k}^F, k = 1,2,\ldots,N_1$ are absolutely acceptable and all $Z_{1k} < Z'_{1k} = Z_{1k}(X_1^S, X_2^S, X_3^S), k = 1,2,\ldots,N_1$ are absolutely unacceptable, and that the preference with $[Z'_{1k}, Z_{1k}^F, k = 1,2,\ldots,N_1]$ is linearly increasing. Is due to the fact that the SLDM obtained the optimum at (X_1^S, X_2^S, X_3^S), which in turn provides the FLDM the objective function values Z'_{1k}, makes any $Z_{1k} \le Z'_{1k}, k = 1,2,\ldots,N_1$ unattractive in practice. The membership functions of the FLDM can be stated as:

$$\mu'_{z_{1k}}[z_{1k}(x)] = \begin{cases} 1 & \text{if } z_{1k} > z_{1k}^F(x), \\ \frac{z_{1k}(x)-z'_{1k}}{z_{1k}^F-z'_{1k}} & \text{if } z'_{1k} \le z_{1k}(x) \le z_{1k}^F, \\ 0 & \text{if } z_{1k}(x) \ge z'_{1k}, k = 1,2,\ldots,N_1. \end{cases} \tag{21}$$

Second, the SLDM goals may reasonably consider all $Z_{2r} \ge Z_{2r}^S, r = 1,2,\ldots,N_2$ are absolutely acceptable and all $Z_{2r} < Z'_{2r} = Z_{2r}(X_1^F, X_2^F, X_3^F), r = 1,2,\ldots,N_2$ are absolutely unacceptable, and that the preference with $[Z'_{2r}, Z_{2r}^S, r = 1,2,\ldots,N_2$ is linearly increasing. Is due to the fact that the TLDM obtained the optimum at (X_1^F, X_2^F, X_3^F), which in turn provides the SLDM the objective function values Z'_{2r}, makes any $Z_{2r} \le Z'_{2r}, r = 1,2,\ldots,N_2$ unattractive in practice.

$$\mu'_{z_{2r}}[z_{2r}(x)] = \begin{cases} 1 & \text{if } z_{2r} > z_{2r}^S(x), \\ \frac{z_{2r}(x)-z'_{2r}}{z_{2r}^S-z'_{2r}} & \text{if } z'_{2r} \le z_{2r}(x) \le z_{2r}^S, \\ 0 & \text{if } z_{2r}(x) \ge z'_{2r}, r = 1,2,\ldots,N_2. \end{cases} \tag{22}$$

Third, the TLDM may be willing to build a membership function for his/her objective functions, so that he/she can rate the satisfaction of each potential solution. In this way, the TLDM has the following membership functions for his/her goals:

$$\mu'_{z_{3q}}[z_{3q}(x)] = \begin{cases} 1 & \text{if } z_{3q} > z_{3q}^T(x), \\ \frac{z_{3q}(x)-z'_{3q}}{z_{3q}^T-z'_{3q}} & \text{if } z'_{3q} \le z_{3q}(x) \le z_{3q}^T, \\ 0 & \text{if } z_{3q}(x) \ge z'_{3q}, q = 1,2,\ldots,N_3. \end{cases} \tag{23}$$

Where $Z'_{3q} = Z_{3q}[X_1^S, X_2^S, X_3^S]$.

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal (satisfactory) solution with overall satisfaction for all DMs, we can solve the following Tchebycheff problem.

$$Max\ \delta, \tag{24}$$

Subject to

$$\frac{\lfloor (x_1^F + t_{11}) - x_1 \rfloor}{t_{11}} \geq \delta I,$$
$$\frac{\lfloor x_1 - (x_1^F - t_{11}) \rfloor}{t_{11}} \geq \delta I,$$
$$\frac{\lfloor (x_2^S + t_{11}) - x_2 \rfloor}{t_{12}} \geq \delta I,$$
$$\frac{\lfloor x_2 - (x_2^S - t_{21}) \rfloor}{t_{21}} \geq \delta I,$$
$$\mu[Z_1(x)] \geq \delta,$$
$$\mu[Z_2(x)] \geq \delta,$$
$$\mu[Z_3(x)] \geq \delta,$$
$$(x_1, x_2, x_3) \in G,$$
$$t_{11}, t_{12} > 0,$$
$$\delta \in [0,1].$$

6 An illustrative example

To demonstrate the solution of (SMLMOFPP), let us consider the following example:

[FLDM]

$$\underset{x_1}{Max} \quad F_1(x,\theta^1) = [\frac{2\theta_1^1 x_1 + 3\theta_2^1 x_2 + \theta_3^1 x_3}{x_1 + 2x_2 + x_3 + 1}, \frac{\theta_4^1 x_1 + 3\theta_5^1 x_2 + \theta_6^1 x_3}{x_1 + 2x_2 + x_3 + 1}],$$

Where x_2, x_3 solve,

[SLDM]

$$\underset{x_2}{Max} \quad F_2(x,\theta^2) = [\frac{\theta_1^2 x_1 + 2\theta_2^2 x_2 + \theta_3^2 x_3}{2x_1 + x_2 + x_3 + 1}, \frac{2\theta_4^2 x_1 + \theta_5^2 x_2 + \theta_6^2 x_3}{2x_1 + x_2 + x_3 + 1}],$$

Where x_3 solves,

[TLDM]

$$\underset{x_3}{Max} \quad F_3(x,\theta^3) = [\frac{\theta_1^3 x_1 + 2\theta_2^3 x_2 + \theta_3^3 x_3}{x_1 + x_2 + 2x_3 + 1}, \frac{\theta_4^3 x_1 + 2\theta_5^3 x_2 + \theta_6^3 x_3}{x_1 + x_2 + 2x_3 + 1}],$$

Subject to

$$M = \{x \in R^3 : (x_1 + x_2 + x_3)\},$$
$$x_1 + x_2 + x_3 \leq 15,$$
$$3x_1 + 2x_2 + x_3 \leq 10,$$
$$x_1 + 2x_2 + 3x_3 \leq 12,$$
$$x_1 + x_2 + x_3 \geq 0.$$

Suppose that $\theta_j^i, (i = 1, 2, \ldots, 6)$ are independent normal distributed random variable with the following means and variances:

Table1. The means and variances of (θ_j^i)

Random variables	θ_1^1	θ_2^1	θ_3^1	θ_4^1	θ_5^1	θ_6^1	θ_1^2	θ_2^2	θ_3^2	θ_4^2	θ_5^2	θ_6^2	θ_1^3	θ_2^3	θ_3^3	θ_4^3	θ_5^3	θ_6^3
Mean	3	2	4	1	2	1	2	1	2	2	3	2	1	2	1	2	2	2
Variance	4	16	9	4	9	25	4	9	4	25	9	9	25	4	9	4	25	36

The equivalent (MLMOFPP) of the (SMLMOFPP) can be written as:

[FLDM]

$$\underset{x_1}{Max} \quad S_1(x) = [\frac{10x_1 + 6x_2 + 3x_3}{x_1 + 2x_2 + x_3 + 1}, \frac{7x_1 + 9x_2 + 5x_3}{x_1 + 2x_2 + x_3 + 1}],$$

Where x_2, x_3 solve,

[SLDM]

$$\underset{x_2}{Max} \quad S_2(x) = [\tfrac{6x_1+8x_2+4x_3}{2x_1+x_2+x_3+1}, \tfrac{10x_1+6x_2+3x_3}{2x_1+x_2+x_3+1}],$$

Where x_3 solves,

[TLDM]

$$\underset{x_3}{Max} \quad S_3(x) = [\tfrac{4x_1+6x_2+9x_3}{x_1+x_2+2x_3+1}, \tfrac{6x_1+8x_2+10x_3}{x_1+x_2+2x_3+1}],$$

Subject to

$$x_1+x_2+x_3 \leq 15,$$
$$3x_1+2x_2+x_3 \leq 10,$$
$$x_1+2x_2+3x_3 \leq 12,$$
$$x_1+x_2+x_3 \geq 0.$$

Using a computer-oriented technique [5], (MLMOFPP) is converted into (MLMOLPP) for the FLDM, SLDM and TLDM in the following form as follows:

First, the FLDM solves his/her problem as follows:

$$\underset{y_1}{Max} \quad Z_1(y) = 10y_1+6y_2+3y_3, 7y_1+9y_2+5y_3$$

Subject to

$$16y_1+31y_2+16y_3 \leq 15,$$
$$13y_1+22y_2+11y_3 \leq 10,$$
$$13y_1+26y_2+15y_3 \leq 12,$$
$$y_1+y_2+y_3 \geq 0.$$

1. Find individual optimal solution by solving (13), we get:

$$(Z_{11}^*, Z_{12}^*) = (5.3, 4.8), (Z_{11}^-, Z_{12}^-) = (0, 0)$$

2. By using (13), build the membership functions then solve (14) as follows:

$$Max \quad \lambda,$$

Subject to

$$y \in G,,$$
$$10y_1+6y_2+3y_3-5.3\lambda \geq 0,$$
$$7y_1+9y_2+5y_3-4.8\lambda \geq 0,$$
$$\lambda \in [0,1].$$

Secondly, the SLDM defines his/her problem in view of the FLDM as follows:

$$\underset{y_2}{Max} \quad Z_2(y) = 6y_1+8y_2+4y_3, 10y_1+6y_2+3y_3$$

Subject to

$$31y_1+16y_2+16y_3 \leq 15,$$
$$23y_1+12y_2+11y_3 \leq 10,$$
$$25y_1+14y_2+15y_3 \leq 12,$$
$$y_1+y_2+y_3 \geq 0.$$

Whose solution for the SLDM does the same action like the FLDM

$$(y_1^S, y_2^S, y_3^S) = (0.1, 0.2, 0), (Z_{21}^S, Z_{22}^S) = (2.2, 2.2), \beta = 0.4, (x_1^S, x_2^S, x_3^S) = (0.125, 0.25, 0)$$

Third, the TLDM defines his/her problem in view of the SLDM as follows:

$$\underset{y_3}{Max} Z_3(y) = 4y_1+6y_2+9y_3, 6y_1+8y_2+10y_3$$

Subject to

$$\begin{aligned}
&16y_1 + 16y_2 + 31y_3 \leq 15, \\
&13y_1 + 12y_2 + 21y_3 \leq 10, \\
&13y_1 + 14y_2 + 27y_3 \leq 12, \\
&y_1 + y_2 + y_3 \geq 0.
\end{aligned}$$

Whose solution for the TLDM does the same action like the SLDM

$$(y_1^T, y_2^T, y_3^T) = (0.03, 0, 0.1), (Z_{31}^T, Z_{32}^T) = (1, 1.2), \gamma = 0.2, (x_1^T, x_2^T, x_3^T) = (0.03, 0, 0.125)$$

Finally, 1. We assume the FLDM control decision is around 0 with tolerance 1.
2. We assume the SLDM control decision is around 0 with the tolerance 1.
3. By using (17)-(23), the TLDM solves the following problem of (24) as follows:

$$Max \quad \delta$$

Subject to

$$\begin{aligned}
&x_1 + x_2 + x_3 \leq 15, \\
&3x_1 + 2x_2 + x_3 \leq 10, \\
&x_1 + 2x_2 + 3x_3 \leq 12, \\
&x_1 + \delta \leq 0.86, \\
&-x_1 + 1.14\delta \leq 1, \\
&x_2 + \delta \leq 0.75, \\
&-x_2 + 1.25\delta \leq 1, \\
&10x_1 + 6x_2 + 3x_3 + 1.15\delta \leq 2.75, \\
&7x_1 + 9x_2 + 5x_3 + 1.6\delta \leq 3.125, \\
&6x_1 + 8x_2 + 4x_3 - 0.7\delta \leq 1.5, \\
&10x_1 + 6x_2 + 3x_3 - 0.3\delta \leq 1.88, \\
&4x_1 + 6x_2 + 9x_3 - 2.3\delta \leq 2, \\
&6x_1 + 8x_2 + 10x_3 - 2.25\delta \leq 2.75, \\
&x_i \geq 0, i = 1, 2, 3, \\
&\delta \in [0, 1].
\end{aligned}$$

Whose compromise solution is

$$X^0 = (0.1, 0, 0.4), \delta = 0.75 and (F_{11}^0, F_{12}^0) = (2.2, 2.7), (F_{21}^0, F_{22}^0) = (2.2, 2.2), (F_{31}^0, F_{32}^0) = (4, 4.6) \ .$$

7 Summary and concluding remarks

This paper proposed an approach for solving a multi-level multi-objective fractional programming problems involving stochastic parameters coefficient in objective functions (SMLMOFPP). In this work, the first phase of the solution approach, we converted the probabilistic nature of this problem into a multi-level multi-objective fractional programming problems (MLMOFPP).At the second phase, we used a computer-oriented technique to converted (MLMOFPP) into a multi-level multi-objective linear programming problems (MLMOLPP). Then a fuzzy approach solved (MLMOLPP) using the concept of tolerance membership function to developed a Tchebycheff problem for generating a compromise solution for this problem. Finally, a numerical example is provided to demonstrate the correctness of the proposed solution.

However, there are many open points for discussion in future, which should be explored and studied in the area of stochastic multi-level fractional optimization such as:
1- A decomposition algorithm for solving stochastic multi-level large scale integer fractional programming problems in the objective functions.
2- A decomposition algorithm for solving stochastic multi-level large scale integer fractional programming problems in the constraints.
3- A decomposition algorithm for solving stochastic multi-level large scale integer fractional programming problems in both the objective functions and constraints.

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