Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/090613

Optimal Design of FSS- PALT for the Generalized Inverted Rayleigh Distribution using Type II Censoring

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Received: 13 Feb. 2015, Revised: 13 Apr. 2015, Accepted: 14 Apr. 2015
Published online: 1 Nov. 2015

Abstract: In this paper, failure step stress-partially accelerated life tests (FSS-PALT) are considered when the lifetime of a product follows a generalized inverted Rayleigh distribution (GIRD). Based on Type II censoring, the maximum likelihood estimates (MLEs) of model parameters are obtained. Also, the asymptotic variances and covariances matrix of the estimators is studied. The optimum proportion of test units failing at normal and accelerate condition according to a certain optimality criterion in optimum test plans are discussed. The performance of the estimators are compared through Monte Carlo simulation study. Finally, numerical examples and concluding remarks are provided.

Keywords: Generalized inverted Rayleigh distribution; Failure step-stress partially accelerated life test; Type II censoring; Maximum likelihood estimation; Generalized asymptotic variance; Optimal design; Monte Carlo simulation.

1 Introduction

Accelerated life test is often used for reliability prediction. Accelerated life testing is achieved by subjecting the test units to conditions that are more severe than the normal ones, such as higher levels of temperature, voltage, pressure, vibration, load, etc. Items are tested at high stress levels to induce early failures then the failure information is related to that at an operational stress level through a given stress-dependent model. When such model is unknown, the accelerated life test cannot be conducted and instead the PALT become suitable. The PALT combines both ordinary and accelerated life tests. The aim of such testing is to rapid obtaining data, which yield desired information on product life or performance under normal use. PALT can be carried out using constant-stress, step-stress, or progressive-stress. According to [1], the stress can be applied in various ways. One way to accelerate failure in step-stress by increasing the stress applied to the test product in a specified discrete sequence. The step stress tests can be divided into two main types which are time step stress (TSS) and failure step stress (FSS). In TSS test, a test unit is subjected to successively higher levels of stress. Thus, the stress is increased step by step at pre-specified times until the test time terminates or obtain the pre-determined number of failures. While, in FSS test, test units start to run at normal stress until the occurrence of a fixed number of failures n_1. Then, stress on them is raised until the test time terminates or obtain the pre-determined number of failures.

In the literature, there are few studies based on failure step-stress partially accelerated life test (FSS-PALT), among them [2]. They considered optimum plans for FSS-PALT with two stress levels assuming Weibull distribution as a lifetime model. Also, [3] discussed the optimal design in the case of type II censoring for inverted Weibull distribution. On the other hand, there are many authors have studied the time- step stress partially accelerated life tests TSS-PALT, for example, [4], [5] and [6]. Also, PALT was studied under type I censoring. For example, [7] studied the estimation in constant stress partially accelerated life tests for Rayleigh distribution using type-I censoring. Recently, [8] concerned on PALT for the Burr type XII model. Based on type II censoring, many authors interested in applying the step-stress method, among them, [9] obtained the MLE and developed optimum test plans for simple TSS- PALT under type II censored data for items having Pareto distribution. Moreover, [10] focused on the maximum likelihood method for estimating the acceleration factor and the parameters of Burr type III distribution. [11] and

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[12] discussed the point and interval estimations of two-parameter Gompertz and Weibull distributions, respectively, under PALT.

This paper concerns on the estimation and the optimal design problem for GIRD under FSS- PALT using type II censored data. The reminder of this paper is organized as follows. Section 2 the GIRD and test model are discussed. The maximum likelihood point and interval estimation of the model parameters are obtained in Section 3. Optimum test plans for FSS- PALT of GIRD are developed in Section 4. Simulation study and numerical experimental are presented in Section 5. Finally, numerical examples are given in Section 6 and concluding remarks are presented in section 7.

2 The model and test model

A generalized inverted scale family of distributions was introduced first by [13]. It is proposed by introducing a shape parameter to the scale family of distributions. GIRD is a member of this new family. It is a generalized version of inverted Rayleigh distribution (IRD). IRD has many applications in the area of reliability studies. [14] mentioned that the distribution of lifetimes of several types of experimental units can be approximated by the IRD. Many authors interested in IRD among them [15], [16], [17] and [18]. The GIRD is considered by this article. This model has not been investigated widely in the literature. Some inferential procedures of the generalized inverted scale family considered by [13]. Also, some characterizations of GIRD discussed by [19].

Let random variable T have a GIRD with parameters α and λ, where α is the shape parameter and λ is the scale parameter. The probability density function (pdf), cumulative distribution function (cdf) and reliability function $S(t)$ of T is given by

$$f_1(t) = 2\alpha\lambda^{-2}t^{-3}\exp\left[-(\lambda t)^{-2}\right] \times \left[1-\exp\left(-(\lambda t)^{-2}\right)\right]^{\alpha-1},\ t>0,\ \alpha,\lambda>0, \tag{1}$$

$$F_1(t) = 1-\left(1-\exp\left[-(\lambda t)^{-2}\right]\right)^{\alpha}, \tag{2}$$

$$S_1(t) = \left(1-\exp\left[-(\lambda t)^{-2}\right]\right)^{\alpha}. \tag{3}$$

In FSS- PALT, all of n units are tested first under normal condition until they reach the time Y_{n_1}, where $n_1 = n\pi_1$ such that π_1 is the proportion of test units to be observed at normal condition is pre-specified. After time Y_{n_1} the test units $(n-n_1)$ are subjected at accelerated condition until censoring time Y_r is reached, where Y_r is the time of failed r units which is predetermined, where n_1 and $r-n_1$ are the number of items failed at normal conditions and accelerated conditions, respectively, and $n_c = n-r$ the number of censoring units i.e. $n_c = n\pi_c$ such that π_c is the proportion of test units to be censored. So, in the experiment of FSS- PALT type II censoring, we pre-specified π_1 and r. This means that if the item has not failed by some pre-specified r units the test is switched to higher level of stress and it is continued until items fail. The effect of this switch is to multiply the remaining lifetime on the item by the inverse of the acceleration factor β, which is the ratio of the hazard rate at accelerated condition to that at normal use condition ($\beta > 1$). In this case, switching to the higher stress level will shorten the life of the test item. Thus the total lifetime of the test item, denoted by Y passes through two stages, which are the normal and accelerated conditions see, [11].

The lifetime of the unit in FSS- PALT is given by

$$Y = \begin{cases} t & if\ \ t \le y_{n_1} \\ y_{n_1} + \beta^{-1}(t-y_{n_1}) & if\ \ t > y_{n_1}, \end{cases} \tag{4}$$

where, t is the lifetime of an item at use condition and β is the acceleration factor. This model is called the tampered random variable (TRV) model. It was proposed by [5]. Assume that the lifetime of the test item follows GIRD with shape parameter α and scale parameter λ, then, the pdf of total lifetime Y of an item is given by

$$f(y) = \begin{cases} 0 & if\ \ y \le 0 \\ f_1(y) & if\ \ 0 < y \le y_{n_1} \\ f_2(y) & if\ \ y > y_{n_1} \end{cases} \tag{5}$$

where $f_1(y)$ was given by (1), and $f_2(y)$ is given by

$$\begin{aligned} f_2(y) = {} & 2\alpha\beta\lambda^{-2}\left[y_{n_1}+\beta\left(y-y_{n_1}\right)\right]^{-3} \\ & \times \exp\left[-\left(\lambda\left[y_{n_1}+\beta\left(y-y_{n_1}\right)\right]\right)^{-2}\right] \\ & \times \left[1-\exp\left(-\left(\lambda\left[y_{n_1}+\beta\left(y-y_{n_1}\right)\right]\right)^{-2}\right)\right]^{\alpha-1}, \\ & y \ge 0,\ \alpha,\ \beta > 1,\ \lambda > 0 \end{aligned} \tag{6}$$

is obtained by the transformation variable technique by using (1) and (4), Also

$$F_2(y) = 1-\left[1-\exp\left(-\left(\lambda\left[y_{n_1}+\beta\left(y-y_{n_1}\right)\right]\right)^{-2}\right)\right]^{\alpha}, \tag{7}$$

and

$$S_2(y) = \left[1-\exp\left(-\left(\lambda\left[y_{n_1}+\beta\left(y-y_{n_1}\right)\right]\right)^{-2}\right)\right]^{\alpha}. \tag{8}$$

3 Maximum Likelihood Estimation

In this Section, the point and interval estimations of the model parameters are introduced using the maximum likelihood method as well as Fisher information matrix.

3.1 Point estimation

In this subsection, the maximum likelihood estimation based on observed-data likelihood function is used to estimate the parameters of the GIRD and acceleration factor for FSS-PALT. The observed data likelihood function of GIRD in FSS-PALT based on type II censoring data $y_1, y_2, \ldots, y_{n_1}, y_{n_1+1}, \ldots, y_r$ where n_1 and $r-n_1$ are the number of items failed at normal conditions and accelerated conditions respectively. The likelihood function of these data is given by

$$L(\alpha,\beta,\lambda|\underline{y}) \propto \left(\prod_{i=1}^{n_1} f_1(y_i)\right)\left(\prod_{j=n_1+1}^{r} f_2(y_j)\right)[S_2(y_r)]^{n-r}. \tag{9}$$

By using (1), (6) and (8) in (9) we get

$$\begin{aligned}L(\alpha,\beta,\lambda|\underline{y}) \propto (\alpha\lambda^{-2})^r\beta^{r-n_1}\prod_{i=1}^{n_1} y_i^{-3}\exp\left[-(\lambda y_i)^{-2}\right]\\ \times\left[1-\exp\left(-(\lambda y_i)^{-2}\right)\right]^{\alpha-1}\\ \times\prod_{i=n_1+1}^{r}\left\{[y_{n_1}+\beta(y_i-y_{n_1})]^{-3}\right.\\ \times\exp\left[-(\lambda[y_{n_1}+\beta(y_i-y_{n_1})])^{-2}\right]\\ \times\left.\left[1-\exp\left(-(\lambda[y_{n_1}+\beta(y_i-y_{n_1})])^{-2}\right)\right]^{\alpha-1}\right\}\\ \times\left[1-\exp\left(-(\lambda[y_{n_1}+\beta(y_r-y_{n_1})])^{-2}\right)\right]^{\alpha(n-r)}.\end{aligned} \tag{10}$$

Therefore, the natural logarithm of the likelihood function can be written as:

$$\begin{aligned}\ell(\alpha,\beta,\lambda|\underline{y}) \propto r\log\alpha-2r\log\lambda+(r-n_1)\log\beta\\ -\lambda^{-2}\psi_1(\beta)-3\psi_2(\beta)+(\alpha-1)\psi_3(\lambda,\beta)\\ +\alpha(n-r)\psi_4(\lambda,\beta),\end{aligned} \tag{11}$$

where

$$\psi_1(\beta)=\sum_{i=1}^{n_1} y_i^{-2}+\sum_{i=n_1+1}^{r}[y_{n_1}+\beta(y_i-y_{n_1})]^{-2}, \tag{12}$$

$$\psi_2(\beta)=\sum_{i=1}^{n_1}\log y_i+\sum_{i=n_1+1}^{r}\log[y_{n_1}+\beta(y_i-y_{n_1})], \tag{13}$$

$$\begin{aligned}\psi_3(\lambda,\beta)=\sum_{i=1}^{n_1}\log\left[1-\exp\left(-(\lambda y_i)^{-2}\right)\right]\\ +\sum_{i=n_1+1}^{r}\log\left[1-\exp\left(-(\lambda[y_{n_1}+\beta(y_i-y_{n_1})])^{-2}\right)\right]\end{aligned} \tag{14}$$

and

$$\psi_4(\lambda,\beta)=\log\left[1-\exp\left(-(\lambda[y_{n_1}+\beta(y_r-y_{n_1})])^{-2}\right)\right]. \tag{15}$$

Maximum likelihood estimators of α, β and λ are the solutions of the system of equations obtained by letting the first partial derivatives of the total log likelihood with respect to α, β and λ respectively, to be zero. Hence, the system of equations is as follows

$$\frac{\partial\ell(\alpha,\beta,\lambda|\underline{y})}{\partial\alpha}=\frac{r}{\alpha}+\psi_3(\lambda,\beta)+(n-r)\psi_4(\lambda,\beta)=0. \tag{16}$$

Hence

$$\hat{\alpha}=\frac{-r}{\psi_3(\lambda,\beta)+(n-r)\psi_4(\lambda,\beta)}. \tag{17}$$

And

$$\frac{\partial\ell(\alpha,\beta,\lambda|\underline{y})}{\partial\lambda}=0$$

hence

$$\begin{aligned}\tfrac{-2r}{\lambda}+2\lambda^{-3}\psi_1(\beta)+(\alpha-1)\psi_3^{(\lambda)}(\lambda,\beta)\\ +\alpha(n-r)\psi_4^{(\lambda)}(\lambda,\beta)=0,\end{aligned} \tag{18}$$

Also

$$\frac{\partial\ell(\alpha,\beta,\lambda|\underline{y})}{\partial\beta}=0$$

here

$$\begin{aligned}\tfrac{(r-n_1)}{\beta}-\lambda^{-2}\psi_1^{(\beta)}(\beta)-3\psi_2^{(\beta)}(\beta)\\ +(\alpha-1)\psi_3^{(\beta)}(\lambda,\beta)+\alpha(n-r)\psi_4^{(\beta)}(\lambda,\beta)=0,\end{aligned} \tag{19}$$

where

$$\psi_N^{(q)}(.)=\frac{\partial\psi_N(.)}{\partial q},\; N=1,\,2,\,3,\,4,\; q=\{\alpha,\,\beta,\,\lambda\}. \tag{20}$$

Substituting for α from (17) in (18) and (19), we have two non-linear equations, then an iterative procedure is used to solve the non linear equations (18) and (19). Newton-Raphson method is conducted by using Mathematica 9. Since the non-linearity of $\hat{\lambda}$ and $\hat{\beta}$, it is impossible to find their exact marginal or joint distributions for exact inference. Therefore, the statistical inferences on the MLEs are based on the asymptotic distributional result.

3.2 Asymptotic variances and covariances matrix

The asymptotic variances and covariances of maximum likelihood estimates are given by the elements of the inverse of the Fisher information matrix

$$I_{ij}(\underline{\theta})=\mathrm{E}\left(\frac{-\partial\ell^2}{\partial\theta_i\partial\theta_j}\right). \tag{21}$$

Unfortunately, the exact mathematical expressions for the above expectation are very difficult to obtain. Therefore, the observed Fisher information matrix is given by

$$I_{ij}(\underline{\theta}) = \frac{-\partial \ell^2}{\partial \theta_i \partial \theta_j}, \tag{22}$$

which is obtained by dropping the expectation on operation E see [20]. When $\underline{\theta} = \{\alpha,\ \beta,\ \lambda\}$, the observed Fisher information matrix $I(\alpha,\beta,\lambda)$, for the MLEs ($\hat{\alpha}$, $\hat{\beta}$, and $\hat{\lambda}$), is the 3×3 symmetric matrix of negative second partial derivatives of the log-likelihood function with respect to (α, β, and λ). In practice , we usually estimate $I_0^{-1}(\alpha,\beta,\lambda)$ by $I^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})$, given by

$$\begin{bmatrix} -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \alpha 2} & -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \alpha \partial \beta} & -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \beta \partial \alpha} & -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \beta 2} & -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \beta \partial \lambda} \\ -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \lambda \partial \alpha} & -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \lambda \partial \beta} & -\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \lambda 2} \end{bmatrix}^{-1}_{(\hat{\alpha},\hat{\beta},\hat{\lambda})} \tag{23}$$

From the log-likelihood function in (11), we have the second partial derivatives of the maximum likelihood function are given as the following:

$$\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \alpha^2} = -\frac{r}{\alpha^2}, \tag{24}$$

$$\begin{aligned}\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \beta^2} &= \frac{-(r-n_1)}{\beta^2} - \lambda^{-2}\psi_1^{(\beta\beta)}(\beta) - 3\psi_2^{(\beta\beta)}(\beta) \\ &+(\alpha-1)\psi_3^{(\beta\beta)}(\lambda,\beta) + \alpha(n-r)\psi_4^{(\beta\beta)}(\lambda,\beta),\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \lambda^2} &= \frac{2r}{\lambda^2} - 6\lambda^{-4}\psi_1(\beta) \\ &+(\alpha-1)\psi_3^{(\lambda\lambda)}(\lambda,\beta) + \alpha(n-r)\psi_4^{(\lambda\lambda)}(\lambda,\beta),\end{aligned} \tag{26}$$

$$\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \alpha \partial \beta} = \frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \beta \partial \alpha} = \psi_3^{(\beta)}(\lambda,\beta) + (n-r)\psi_4^{(\beta)}(\lambda,\beta), \tag{27}$$

$$\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \alpha \partial \lambda} = \frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \lambda \partial \alpha} = \psi_3^{(\lambda)}(\lambda,\beta) + (n-r)\psi_4^{(\lambda)}(\lambda,\beta), \tag{28}$$

and

$$\begin{aligned}\frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \beta \partial \lambda} &= \frac{\partial^2 \ell(\alpha,\beta,\lambda|\underline{y})}{\partial \lambda \partial \beta} = 2\lambda^{-3}\psi_1^{(\beta)}(\beta) \\ &+(\alpha-1)\psi_3^{(\lambda\beta)}(\lambda,\beta) + \alpha(n-r)\psi_4^{(\lambda\beta)}(\lambda,\beta).\end{aligned} \tag{29}$$

where

$$\psi_N^{(pq)}(.) = \frac{\partial^2 \psi_N(.)}{\partial p \partial q},\ N = 1,\ 2, 3, 4,\ p,\ q = \{\alpha,\ \beta,\ \lambda\}. \tag{30}$$

Thus, the $100(1\text{-}\gamma)\%$ approximate confidence intervals for α,β and λ are

$$\hat{\alpha} \mp z_{\frac{\gamma}{2}}\sqrt{v_{11}},\ \hat{\beta} \mp z_{\frac{\gamma}{2}}\sqrt{v_{22}} \text{ and } \hat{\lambda} \mp z_{\frac{\gamma}{2}}\sqrt{v_{33}} \tag{31}$$

respectively, where v_{11}, v_{22} and v_{33} are the elements on the main diagonal of the covariance matrix $I_0^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})$ and $z_{\frac{\gamma}{2}}$ is the percentile of the standard normal distribution with right-tail probability $\frac{\gamma}{2}$.

4 Optimum Test Plan

In this section, we consider the problem of optimally designing of simple FSS-PALT, which terminates at a pre-specified number of failure r. The optimum criterion is to find the optimal stress change-number, π_1^* proportion of test units that must fail at normal stress such that the generalized asymptotic variance (GAV) of the MLE of the model parameters at normal use condition is minimized. The stress change-time Y_{n_1} is a pre-specified time by the pre-specified proportion π_1 for the stage of parameter estimation. But for the optimal design stage of the test π_1, is considered a switching parameter that to be optimally determined according to a certain optimally criterion. The problem that considered was of optimally designing a FSS-PALT, which terminated at a pre-specified number of failure. The optimum criterion is to find π_1^* such that the GAV of the MLEs of the model parameters at normal use condition is minimized. The GAV of the MLEs of the model parameters is the reciprocal of the determinant of F see [21].

$$GAV = \frac{1}{|F|} \tag{32}$$

where $|F|$ is determinant of the Fisher's information matrix. The minimization of the GAV is equivalent to maximization of $|F|$. Therefore, the optimal value π_1^* of maximized the determinant and minimized the GAV is reduced to

$$\frac{\partial |F|}{\partial \pi_1} = 0 \tag{33}$$

In general, the solution of (33) is not in a closed form and therefore requires a numerical method such as Newton-Raphson method. The Newton-Raphson method was applied to obtain the optimal π_1^* which minimize the GAV. Accordingly, the corresponding optimal numbers of items failed at normal use condition for switching to accelerated condition n_1^* is

$$n_1^* = n\pi_1^* \tag{34}$$

where n is the sample size and π_1^* is the the optimal proportion of test units that must fail at normal condition.

Table 1: The MSE, ARB and RE of the parameters under type II censoring sample.

n	(α,λ,β), π_1	(0.5, 2, 2), 0.5			(2, 2, 1.1), 0.3		
		MSE	ARB	RE	MSE	ARB	RE
30	α	0.40049	0.49918	1.26568	1.22803	0.03029	0.55408
	λ	0.12806	0.05842	0.17893	0.08165	0.02383	0.14287
	β	1.03209	0.01301	0.50796	0.85308	0.41936	0.83966
50	α	0.10434	0.24907	0.64604	1.09826	0.03575	0.52399
	λ	0.07889	0.03835	0.14043	0.05181	0.01103	0.11381
	β	0.90668	0.03462	0.47610	0.44507	0.26298	0.60649
100	α	0.03720	0.11265	0.38572	0.96612	0.08729	0.49146
	λ	0.04159	0.01679	0.10197	0.02928	0.00243	0.08556
	β	0.56994	0.02426	0.37747	0.21147	0.11642	0.41805
150	α	0.02435	0.07926	0.31212	0.81798	0.10416	0.45221
	λ	0.02884	0.01251	0.08491	0.01933	0.00203	0.06952
	β	0.40036	0.00824	0.31637	0.10097	0.05157	0.28887
200	α	0.01320	0.05843	0.22980	0.73297	0.11282	0.42807
	λ	0.02054	0.00971	0.07166	0.01666	0.00359	0.06454
	β	0.27915	0.01275	0.26417	0.09157	0.03345	0.27509

Table 2: Asymptotic variances and covariances of estimates.

n	(α,λ,β), π_1	(0.5, 2, 2), 0.5			(2, 2, 1.1), 0.3		
		$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\beta}$
30	$\hat{\alpha}$	0.01678	-0.00816	-0.02180	0.09668	-0.01618	-0.02974
	$\hat{\lambda}$		0.03884	-0.01776		0.03105	-0.02384
	$\hat{\beta}$			0.20310			0.09174
50	$\hat{\alpha}$	0.01132	-0.00784	-0.02150	0.08799	-0.01327	-0.02499
	$\hat{\lambda}$		0.03275	-0.00407		0.01809	-0.01212
	$\hat{\beta}$			0.17905			0.05151
100	$\hat{\alpha}$	0.00738	-0.00638	-0.02006	0.06697	-0.00935	-0.01780
	$\hat{\lambda}$		0.02208	0.00674		0.00902	-0.00472
	$\hat{\beta}$			0.14589			0.02410
150	$\hat{\alpha}$	0.00569	-0.00518	-0.01721	0.07268	-0.00973	-0.01946
	$\hat{\lambda}$		0.01633	0.00825		0.00636	-0.00208
	$\hat{\beta}$			0.11865			0.01789
200	$\hat{\alpha}$	0.00520	-0.00491	-0.01815	0.06066	-0.00798	-0.01616
	$\hat{\lambda}$		0.01370	0.01124		0.00482	-0.00134
	$\hat{\beta}$			0.12093			0.01366

5 Simulation Study

In this section. we adopt some numerical experiments performed to evaluate the behavior of our proposed methods for different sample sizes, different parameter values and different proportion π_1. All of the computations were performed by (Mathematica 9.0) using a Pentium IV processor. By considering Type II censored samples. The performance of the resulting estimators of the distribution parameters and acceleration factor has been considered in terms of their mean square error (MSE), absolute relative bias (ARB) and relative error (RE), given by

$$\text{MSE}(\hat{\theta}) = \text{E}(\theta-\hat{\theta})^2,\ \text{ARB}(\hat{\theta}) = |\tfrac{\theta-\hat{\theta}}{\theta}|,\ \text{and RE}=\frac{\sqrt{\text{MSE}(\hat{\theta})}}{\theta}. \tag{35}$$

Moreover, Fisher information matrix, the asymptotic variance and covariance matrix and confidence intervals of the distribution parameters and acceleration factor are obtained. Also, optimum test plans are developed numerically. The optimal GAV of the MLEs of the model parameters and optimal number of items failed at normal use condition are computed. In our computation , we used two sets of parameter $(\alpha,\ \lambda,\ \beta) = \{(0.5,\ 2,\ 2),\ (2,\ 2,\ 1.1)\}$, sample of sizes 30, 50(50)200, proportion of test units failing at normal condition $\pi_1 = \{0.5,\ 0.3\}$ and the effects sample size $r = 75\%$ of complete sample. The steps of simulation procedure can be described as follows:

Step 1.Generate a random sample of sizes n from GIRD (1), $(t_1, t_2, ..., t_n)$ using the transformation

$$T = \frac{1}{\lambda\sqrt{\log\left[1-(1-U)^{\frac{1}{\alpha}}\right]^{-1}}},$$

where U has a uniform (0, 1) random number.

Table 3. Confidence bounds of the estimates and the width of the intervals at confidence level 0.90.

n	(α,λ,β)	(0.5, 2, 2)			(2, 2, 1.1)		
		LCB	UCB	Width	LCB	UCB	Width
30	α	0.53713	0.96204	0.42490	1.42949	2.44937	1.01988
	λ	1.55993	2.20639	0.64646	1.75863	2.33669	0.57805
	β	1.28692	2.76513	1.47821	1.06455	2.05805	0.99349
50	α	0.45000	0.79906	0.34906	1.58501	2.55798	0.97296
	λ	1.62648	2.22014	0.59366	1.80143	2.24270	0.44127
	β	1.37528	2.76321	1.38793	1.01703	1.76152	0.74448
100	α	0.41537	0.69727	0.28190	1.75017	2.59901	0.84883
	λ	1.72271	2.21014	0.48743	1.84902	2.16069	0.31166
	β	1.42210	2.67493	1.25284	0.97344	1.48267	0.50921
150	α	0.41582	0.66343	0.24760	1.76619	2.65045	0.88426
	λ	1.76541	2.18457	0.41916	1.86507	2.12679	0.26172
	β	1.45156	2.58139	1.12983	0.93737	1.37608	0.438711
200	α	0.41093	0.64749	0.23656	1.82171	2.62959	0.80787
	λ	1.78857	2.17260	0.38403	1.87894	2.10669	0.22774
	β	1.45518	2.59581	1.14063	0.94509	1.32851	0.38341

Table 4. The results of optimal design of FFS- PALT under type II censoring.

(α,λ,β)	(0.5, 2, 2)			(2, 2, 1.1)		
n	π_1^*	n_1^*	GAV	π_1^*	n_1^*	GAV
30	0.22049	7	6.64349 x 10-4	0.13055	4	6.4842 x 10-4
50	0.26666	13	2.48517 x 10-4	0.12676	6	2.00057 x 10-4
100	0.34947	35	5.86175 x 10-5	0.15960	16	6.26542 x 10-5
150	0.27109	41	1.01820 x 10-5	0.18906	28	9.86235 x 10-6
200	0.28969	58	2.44024 x 10-6	0.15030	30	4.49904 x 10-6

Step 2. After ordering the sample and for given π_1 choose $y_{n_1} = t_{(n_1)}$, using equation (4) we have Type II censored samples $y_1, y_2, ..., y_{n_1}, y_{n_1+1}, ..., y_r$.

Step 3. Based on the original Type II censored samples $y_1, y_2, ..., y_{n_1}, y_{n_1+1}, ..., y_r$, obtain the point estimate of parameters α, β and λ say $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\lambda}$ from (17), (18) and (19) and interval estimation from (31).

Step 4. Repeating steps from 1 to 3, 1000 times.

Step 5. The MSE, ARB and RE of the estimators for the distribution parameters and the acceleration factor for all sample sizes and for the two sets of parameters computed from (35) and the results are presented in Table 1.

Step 6. The asymptotic variance and covariance matrix of the estimators for different sample sizes are presented in Table 2.

Step 7. The confidence limit with confidence level $\gamma = 90$ of the distribution parameters and the acceleration factor are presented in Table 3.

Step 8. The optimal proportion of units π_1^* that must fail at normal condition was obtained by using (33) are presented in Table 4.

6 Numerical examples

Table 4 shows that the values of the optimal proportion of units are in the range from 0.1 to 0.4, which means that less than half observations will fail under the normal conditions, while the others will fail at high condition. For instance, in the first set of parameters when n = 100, 35 observations will fail under use condition, 40 will fail under accelerated conditions and 25 will be censored. Also, in the second set of parameters when n = 200, 30 observations will fail under use condition, 120 will fail under accelerated conditions and 50 will be censored. These examples illustrate that the partial accelerating is very important to run the test.

7 Conclusion

One of the major reasons of using ALT are some of products having a high reliability, the test of product life under normal use often requires a long period of time. So PALT is used to facilitate estimating the reliability of the unit in a short period of time. In ALT test items are run only at accelerated conditions, while in PALT they are run at both normal and accelerated conditions. One way to accelerate failure is FSS-PALT, test units start to run at a design (normal) stress until the occurrence of a fixed number of failures. Then, stress on them is raised and fixed over a specified time to obtain the predetermined number of failures. In this study GIRD with FSS- PALT based on type II censoring was considered. The performance of the resulting estimators of the distribution parameters and acceleration factor has been considered in terms of their MSE, ARB and RE. It can be shown from the results that displayed in Tables 1-4 the following observations:

1. For the second set of parameters the maximum likelihood estimates of λ based on MSEs, ARB are better than the first set of parameters. On the other hand, the MLEs of α for the first set of parameters performs better than the corresponding of the second set based on MSE, ARB and RE (see Table 1).
2. As the sample size increases the MSE, ARB and RE of the estimated parameters and accelerating factor decrease (see Table 1). On the other hand, it is noted that the behavior of the ARB of α for the second set is the worst, but the ARB of β for the first set is the worst.
3. The asymptotic variances and the absolute value of the asymptotic covariances of the estimates usually decrease as the sample size increase. Also, the asymptotic variances of the estimates of λ and β for the second set of parameters are smaller than the corresponding for the first set. But the asymptotic variance of the estimates of α for the first set of parameters are smaller than the corresponding for the second set (see Table 2).
4. The width of the interval of the estimates decreases when the sample size increases (see Table 3). By comparing the two sets of parameters, the width of the intervals of the estimates of λ and β for the second set smaller than the corresponding for the first set. But the width of the intervals of the estimates of α for the first set smaller than the corresponding for the second set.
5. The GAV decreases as sample size increase, for both sets (see Table 4).

From the previous discussion, it can be said that the two sets of parameters have good statistical properties, but the second one performs better for all sample sizes. Maximum likelihood estimates are consistent and asymptotically normally distributed for the parameters and accelerating factor. As the sample size increases the asymptotic variance and covariance of estimators decrease. Regarding the interval of the estimators, it can be noted that as sample size increases the width of the interval of the estimators decreases for the confidence level and for both sets.

Acknowledgement

This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The author, therefore, acknowledge with thanks DSR technical and financial support.

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