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Intuitionistic Fuzzy π Generalized β Closed Mappings

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Abstract: In this paper we introduce intuitionistic fuzzy π generalized β closed mappings and intuitionistic fuzzy π generalizedβ open mappings. We investigate some of their properties.

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1 Introduction

Fuzzy set as proposed by Zadeh [11] in 1965 is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. By adding the degree of non-membership to Fuzzy set, Atanassov [1] proposed intuitionistic fuzzy set in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [2] introduced the concept of intuitionistic fuzzy topological space.

In this paper we introduce the notion of intuitionistic fuzzy π generalized β closed Mappings and intuitionistic fuzzy π generalized β open mappings and study some of their properties. We also introduce intuitionistic fuzzy Mπ generalized β closed mappings as well as intuitionistic fuzzy Mπ generalized β open mappings. We provide the relation between intuitionistic fuzzy Mπ generalized β closed mappings and intuitionistic fuzzy π generalized β closed mappings, and establish the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

2 Preliminaries

Definition 2.1.[1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2.[1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

(a)$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.
(b)$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c)$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$.
(d)$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
(e)$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3.[2] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(i)$0_\sim, 1_\sim \in \tau$

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(ii)$G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(iii)$\bigcup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

Definition 2.4.[2] Let (X,τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$\mathrm{Int}(A) = \bigcup\{G/G$ is an IFOS in X and $G \subseteq A\}$
$\mathrm{Cl}(A) = \bigcap\{K/K$ is an IFCS in X and $A \subseteq K\}$ Note that for any IFS A in (X,τ), we have $\mathrm{Cl}(A^c) = (\mathrm{Int}(A))^c$ and $\mathrm{Int}(A^c) = (\mathrm{Cl}(A))^c$ [2].

Definition 2.5.[3] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i)Intuitionistic fuzzy semi closed set(IFSCS in short) if $\mathrm{Int}(\mathrm{cl}(A)) \subseteq A$
(ii)Intuitionistic fuzzy pre closed set(IFPCS in short) if $\mathrm{cl}(\mathrm{Int}(A)) \subseteq A$
(iii)Intuitionistic fuzzy αclosed set (IFαCS in short) if $\mathrm{cl}(\mathrm{Int}(\mathrm{cl}(A))) \subseteq A$
(iv)Intuitionistic fuzzy β closed set (IFβCS in short) if $\mathrm{Int}(\mathrm{cl}(\mathrm{Int}(A))) \subseteq A$
(v)Intuitionistic fuzzy β closed set(IFβCS for short) $\mathrm{Int}(\mathrm{cl}(\mathrm{Int}(A))) \subseteq A$.
The respective complements of the above IFCSs are called their respective IFOSs.

Definition 2.6.[4] An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy gβ closed set (IFGβCS for short) if $\beta\mathrm{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X,τ).

Definition 2.7.[6] An IFS A in an IFTS (X,τ) is said to be an intuitioRemarknistic fuzzy $\pi g\beta$ closed set (IFGβCS for short) if $\beta\mathrm{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFπOS in (X,τ).

The family of all IFπGβCSs of an IFTS (X,τ) is denoted by IFπGβC(X).

Definition 2.8.[2] Let A be an IFS in an IFTS (X,τ). Then
$\beta\mathrm{Int}(A) = \bigcup\{G/G$is an IFSPOS in $X$and $G \subseteq A\}$.
$\beta\mathrm{Cl}(A) = \bigcap\{K/K$is an IFSPCS in X and$A \subseteq K\}$.
Note that for any IFS A in (X,τ), we have $\beta\mathrm{Cl}(A^c) = (\beta\mathrm{Iint}(A))^c$ and $\beta\mathrm{Int}(A^c) = (\beta\mathrm{Cl}(A))^c$ [3].

Definition 2.9.[6] The complement A^c of an IFπGβCS A in an IFTS (X,τ) is called an IFπGβOS in X.

Definition 2.10.[3]Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy continuous mapping (IFCM) for short if $f^{-1}(B) \in \mathrm{IFO}(X)$ for every $B \in \sigma$.

Definition 2.11.[5] Let a mapping $f : (X,\tau) \to (Y,\sigma)$. Then f is said to be an

(i)intuitionistic fuzzy semi-continuous mapping if $f^{-1}(B) \in \mathrm{IFSO}(X)$ for every $B \in \sigma$.
(ii)intuitionistic fuzzy α-continuous mapping if $f^{-1}(B) \in$ IFαO(X) for every $B \in \sigma$.
(iii)intuitionistic fuzzy pre-continuous mapping if $f^{-1}(B) \in \mathrm{IFPO}(X)$ for every $B \in \sigma$.
(iv)intuitionistic fuzzy β-continuous mapping if $f^{-1}(B) \in$ IFβO(X) for every $B \in \sigma$.
(v)intuitionistic fuzzy $g\beta$-continuous mapping if $f^{-1}(B) \in$ IFGβO(X) for every $B \in \sigma$.

Definition 2.12.[7] A mapping $f : (X,\tau) \to (Y,\sigma)$ is called an intuitionistic fuzzy $\pi g\beta$ continuous mapping if $f^{-1}(V) \in$ IFπGβCS in (X,τ) for every IFCS V of (Y,σ).

Definition 2.13.[8] An IFTS (X,τ) is said to be IFT$_{1/2}$ space if every IFGCS is an IFCS in (X,τ).

Definition 2.14.[7] If every IFπGβCS in (X,τ) is an IFβCS in (X,τ), then the space can be called as an intuitionistic fuzzy $\pi\beta T_{1/2}$ space.

Definition 2.15.[8] A map $f : X \to Y$ is called an intuitionistic fuzzy closed mapping (IFCM) if $f(A)$ is an IFCS in Y for each IFCS A in X.

Definition 2.16.[3] A map $f : X \to Y$ is called an

(i)intuitionistic fuzzy semi-open mapping (IFSOM for short) if $f(A)$ is an IFSOS in Y for each IFOS A in X.
(ii)intuitionistic fuzzy α-open mapping (IFαOM for short) if $f(A)$ is an IFOS in Y for each IFOS A in X.
(iii)intuitionistic fuzzy pre-open mapping (IFPOM for short) if $f(A)$ is an IFPOS in Y for each IFOS A in X.
(iv)intuitionistic fuzzy βopen mapping (IFβOM for short) if $f(A)$ is an IFβOS in Y for each IFOS A in X.

Definition 2.17.[5] A map $f : X \to Y$ is called an intuitionistic fuzzy generalized βopen mapping (IFGβOM for short) if $f(A)$ is an IFβOS in Y for each IFOS A in X.

Definition 2.18.[9] A mapping $f : (X,\tau) \to (Y,\sigma)$ is called an intuitionistic fuzzy pre-regular closed mapping (IFPRCM for short) if $f(V)$ is an IFRCS in (Y,σ) for every IFRCS V of (X,τ).

Definition 2.19.[8] The IFS $p(\alpha,\beta) = \langle x, p_\alpha, p_{1-\beta} \rangle$ where $\alpha \in (0,1,\ \beta \in [0,1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Definition 2.20.[8] Let $p(\alpha,\beta)$ be an IFP of an IFTS (X,τ). An IFS A of X is called an intuitionistic fuzzy neighborhood of $p(\alpha,\beta)$ if there exists an IFOS B in X such that $p(\alpha,\beta) \in B \subseteq A$.

Definition 2.21. Let $p(\alpha,\beta)$ be an IFP of an IFTS (X,τ). An IFS A of X is called an intuitionistic fuzzy β-neighborhood of $p(\alpha,\beta)$ if there exists an IFβOS B in X such that $p(\alpha,\beta) \in B \subseteq A$.

Remark 2.1. Let (X,τ) be an IFTS where X is an IFβT$_{1/2}$ space. An IFS A is an IFπGβOS in X if and only if A is an IFSN of $c(\alpha,\beta)$ for each IFP $c(\alpha,\beta) \in A$.
Necessity: Let $c(\alpha,\beta) \in A$. Let A be an IFπGβOS in X. Since X is an IFβT$_{1/2}$ space, A is an IFβOS in X. Then clearly A is an IFβN of $c(\alpha,\beta)$.
Sufficiency: Let $c(\alpha,\beta) \in A$. Since A is an IFβN of $c(\alpha,\beta)$, there is an IFβOS B in X such that $c(\alpha,\beta) \in B \subseteq A$. Now $A = \bigcup\{c(\alpha,\beta) | c(\alpha,\beta) \in A\} \subseteq \bigcup\{B_{c(\alpha,\beta)} | c(\alpha,\beta) \in A\} \subseteq A$.

This implies $A = \bigcup\{B_{c(\alpha,\beta)} | c(\alpha,\beta) \in A\}$. Since each B is an IFβOS, A is an IFβOS and hence an IFπGβOS in X.

Remark 2.2. For any IFS A in an IFTS(X,τ) where X is an IFβT$_{1/2}$ space, $A \in$ IFπGβO(X) if and only if for every IFP $c(\alpha,\beta) \in A$, there exists an IFπGβOS B in X such that $c(\alpha,\beta) \in B \subseteq A$.

Proof. Necessity: If $A \in$ IFπGβO(X), then we can take $B = A$ so that $c(\alpha,\beta) \in B \subseteq A$ for every IFP $c(\alpha,\beta) \in A$.

Sufficiency: Let A be an IFS in X and assume that there exists $B \in$ IFπGβO(X) such that $c(\alpha,\beta) \in B \subseteq A$.. Since X is an IFβT$_{1/2}$ space, B is an IFβOS of X.

Then $A = \bigcup_{c(\alpha,\beta)\in A} \{c(\alpha,\beta)\} \subseteq \bigcup_{c(\alpha,\beta)\in A} B \subseteq A$

Therefore $A = \bigcup_{c(\alpha,\beta)\in A} B$ is an IFβOS (X) and hence A is an IFπGβOS in X. Thus $A \in$ IFπGβO(X).

3 Intuitionistic fuzzy π generalized β closed mappings and intuitionistic fuzzy π generalized β open mappings

In this section we introduce intuitionistic fuzzy π generalized β closed mappings and intuitionistic fuzzy π generalized β open mappings. We study some of their properties.

Definition 3.1. A map $f : X \to Y$ is called an intuitionistic fuzzy π generalized β closed mapping (IFπGβCM for short) if $f(A)$ is an IFπGβCS in Y for each IFCS A in X.

Example 3.1. Let $X = \{a,b\}, Y = \{u,v\}$ and

$$G_1 = \langle x, (0.7_a, 0.7_b), (0.2_a, 0.3_b)\rangle,$$

$$G_2 = \langle y, (0.3_u, 0.4_v), (0.7_u, 0.5_v)\rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f ia na IFπGβCM.

Theorem 3.1. Every IFCM is an IFπGβCM but not conversely.

Proof. Let $f : X \to Y$ be an IFCM. Let A be an IFCS in X. Then $f(A)$ is an IFCS in Y. Since every IFCS is an IFπGβCS, $f(A)$ is an IFπGβCS in Y. Hence f is an IFπGβCM.□

Example 3.2. Let $X = \{a,b\}, Y = \{u,v\}$ and

$$G_1 = \langle x, (0.3_a, 0.4_b), (0.2_a, 0.3_b)\rangle,$$

$$G_2 = \langle y, (0.2_u, 0.3_v), (0.3_u, 0.4_v)\rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively, then f is an IFπGβCM but not an IFCM, since $G_1^c = \langle x, (0.2_a, 0.3_b), (0.3_a, 0.4_b)\rangle$ is an IFCS in X, but $f(G_1^c) = \langle y, (0.2_u, 0.3_v), (0.3_u, 0.4_v)\rangle$ is not an IFCS in Y.□

Theorem 3.2. Every IFαCM is an IFπGβCM but not conversely.

Proof. Let $f : X \to Y$ be an IFαCM. Let A be an IFCS in X. Then $f(A)$ is an IFαCS in Y. Since every IFαCS is an IFπGβCS, $f(A)$ is an IFπGβCS in Y. Hence f is an IFπGβCM. □

Example 3.3. Let $X = \{a,b\}, Y = \{u,v\}$ and

$$G_1 = \langle x, (0.6_a, 0.5_b), (0.3_a, 0.2_b)\rangle,$$

$$G_2 = \langle y, (0.2_u, 0.2_v), (0.6_u, 0.7_v)\rangle,$$

$$G_3 = \langle x, (0.4_a, 0.5_b), (0.5_a, 0.5_b)\rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFπGβCM but not an IFαCM.□

Theorem 3.3. Every IFSCM is an IFπGβCM but not conversely.

Proof. Let $f : X \to Y$ be an IFSCM. Let A be an IFCS in X. Then $f(A)$ is an IFSCS in Y. Since every IFSCS is an IFπGβCS, $f(A)$ is an IFπGβCS in Y. Hence f is an IFπGβCM.□

Example 3.4. Let $X = \{a,b\}, Y = \{u,v\}$ and

$$G_1 = \langle x, (0.5_a, 0.7_b), (0.2_a, 0.3_b)\rangle,$$

$$G_2 = \langle y, (0.3_u, 0.4_v), (0.4_u, 0.6_v)\rangle,$$

then f is an IFπGβCM but not an IFSCM, Since G_1^c is an IFCS in X, but $f(G_1^c) = \langle y, (0.2_u, 0.3_v), (0.5_u, 0.7_v)\rangle$ is not an IFSCS in Y.□

Theorem 3.5. Every IFPCM is an IFπGβCM but not conversely.

Proof. Let $f : X \to Y$ be an IFPCM. Let A be an IFCS in X. Then $f(A)$ is an IFPCS in Y. Since every IFPCS is an IFπGβCS, $f(A)$ is an IFπGβCS in Y. Hence f is an IFπGβCM.□

Example 3.5. Let $X = \{a,b\}, Y = \{u,v\}$ and

$$G_1 = \langle x, (0.7_a, 0.5_b), (0.3_a, 0.4_b)\rangle,$$

$$G_2 = \langle y, (0.3_u, 0.4_v), (0.7_u, 0.5_v)\}.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFπGβCM but not an IFPCM, since $f(G_1^c)$ is an IFCS in Y but not an IFPCS in Y.□

Definition 3.2. A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy M π generalized β closed mapping (IFMπGβCM) if $f(A)$ is an IFπGβCS in Y for every IFπGβCS A in X.

Theorem 3.5. Every IFMπGβCM is an IFπGβCM but not conversely.

Proof. Let $f : X \to Y$ be an IFMπGβCM. Let A be an IFCS in X. Then A is an IFπGβCS in X. By hypothesis $f(A)$ is an IFπGβCS in Y. Therefore f is an IFπGβCM.□

Example 3.6. Let $X = \{a,b\}, Y = \{u,v\}$ and

$$G_1 = \langle x, (0.1_a, 0.3_b), (0.5_a, 0.6_b)\rangle,$$

$$G_2 = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle,$$

$$G_3 = \langle y, (0.1_u, 0.3_v), (0.5_u, 0.6_v)\rangle,$$

$$G_4 = \langle x, (0.0_a, 0.3_b), (0.5_a, 0.6_b)\rangle.$$

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFπGβCM but not an IFMπGβCM. Since $A = \langle x, (0.0_a, 0.3_b), (0.5_a, 0.6_b)\rangle$ is IFπGβCS in X but $f(A) = \langle y, (0.0_u, 0.3_v), (0.5_u, 0.6_v)\rangle$ is not an IFπGβCS in Y.□

Theorem 3.6. Let $f : X \to Y$ be a mapping. Then the following are equivalent if Y is an IFβT$_{1/2}$ space. (i) f is an IFπGβCM (ii) $\beta\mathrm{Cl}(f(A)) \subseteq f(\mathrm{cl}(A))$ for each IFS A of X.

Proof. (i) $\to$ (ii): Let A be an IFS in X. Then $\mathrm{cl}(A)$ is an IFCS in X. (i) implies that $f(\mathrm{cl}(A))$ is an IFπGβCS in Y. Since Y is an IFβT$_{1/2}$ space, $f(\mathrm{cl}(A))$ is an IFβCS in Y. Therefore $\beta\mathrm{Cl}(f(\mathrm{cl}(A))) = f(\mathrm{cl}(A))$. Now $\beta\mathrm{Cl}(f(A)) \subseteq \beta\mathrm{Cl}(f(\mathrm{cl}(A))) = f(\mathrm{cl}(A))$. Hence $\beta\mathrm{Cl}(f(A)) \subseteq f(\mathrm{cl}(A))$ for each IFS A of X.

(ii) $\to$ (i): Let A be any IFCS in X. Then $\mathrm{cl}(A) = A$, (ii) implies that $\beta\mathrm{Cl}(f(A)) \subseteq f(\mathrm{cl}(A)) = f(A)$. But $f(A) \subseteq \beta\mathrm{Cl}(f(A))$, therefore $\mathrm{cl}(f(A)) = f(A)$. This implies $f(A)$ is an IFβCS in Y. Since every IFβCS is an IFπGβCS, $f(A)$ is an IFπGβCS in Y. Hence f is an IFπGβCM.□

Theorem 3.7. Let $f : X \to Y$ be a bijection. Then the following are equivalent if Y is an IFβT$_{1/2}$ space

(i) f is an FπGβCM
(ii) $\beta\mathrm{Cl}(f(A)) \subseteq f(\mathrm{cl}(A))$ for each IFS A of X
(iii) $f^{-1}(\beta\mathrm{Cl}(B)) \subseteq \mathrm{cl}(f^{-1}(B))$ for every IFS B of Y.

Proof. (i) implies (ii) is obvious by theorem 3.6.

(ii)$\to$(iii): Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. Since f is onto, $\beta\mathrm{Cl}(B) = \beta\mathrm{Cl}(f(f^{-1}(B)))$ and (ii) implies $\beta\mathrm{Cl}(f(f^{-1}(B))) \subseteq f(\mathrm{Cl}(f^{-1}(B)))$.
Therefore $\beta\mathrm{Cl}(B) \subseteq f(\mathrm{Cl}(f^{-1}(B)))$.
Now $f^{-1}(\beta\mathrm{Cl}(B)) \subseteq f^{-1}(f(\mathrm{Cl}(f^{-1}(B)))$. Since f is one to one, $f^{-1}(\beta\mathrm{Cl}(B)) \subseteq \mathrm{Cl}(f^{-1}(B))$.

(iii)$\to$ (ii): Let A be any IFS of X. Then $f(A)$ is an IFS of Y. Since f is one to one, (iii) implies that $f^{-1}(\beta\mathrm{Cl}(f(A)) \subseteq \mathrm{Cl}(f^{-1}(f(A)) = \mathrm{Cl}(A)$. Therefore $f(f^{-1}(\beta\mathrm{Cl}(f(A)))) \subseteq f(\mathrm{Cl}(A))$. Since f is onto $\beta\mathrm{Cl}(f(A)) = f(f^{-1}(\beta\mathrm{Cl}(f(A)))) \subseteq f(\mathrm{cl}(A))$.□

Theorem 3.8. Let $f : X \to Y$ be an IFπGβCM, then for every IFS A of X, $f(\mathrm{cl}(A))$ is an IFπGβCS in Y.

Proof. Let A be any IFS in X. Then $\mathrm{Cl}(A)$ is an IFCS in X. By hypothesis $f(\mathrm{Cl}(A))$ is an IFπGβCS in X.□

Theorem 3.9. Let $f : X \to Y$ be an IFπGβCM where Y is an IFβT$_{1/2}$ space, then f is an IFCM if every IFβCS is an IFCS in Y.

Proof. Let f be an IFπGβCM. Then for every IFCS A in X, $f(A)$ is an IFπGβCS in Y. Since Y is an IFβT$_{1/2}$ space, $f(A)$ is an IFβCS in Y and by hypothesis $f(A)$ is an IFCS in Y. Hence f is an IFCM.□

Theorem 3.10. Let $f : X \to Y$ be an IFπGβCM where Y is an IFβT$_{1/2}$ space. Then f is an IFPRCM if every IFβCS is an IFRCS in Y.

Proof. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By Hypothesis $f(A)$ is an IFπGβS in Y. Since Y is an IFβT$_{1/2}$ space, $f(A)$ is an IFβCS in Y and hence is an IFRCS in Y, by hypothesis. This implies that $f(A)$ is an IFPRCM.□

Theorem 3.11. If every IFS is an IFCS, then an IFπGβCM is an IFGβ continuous mapping.

Proof. Let A be an IFCS in Y. Therefore $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFπGβCS, $f^{-1}(A)$ is an IFπGβCS in X. This implies that f is an IFπGβ continuous mapping.□

Theorem 3.12. Let A be an IFπGβCS in X. An onto mapping $f : X \to Y$ is both IF continuous mapping and IFπGβCM, then $f(A)$ is an IFπGβCS in Y.

Proof. Let $f(A) \subseteq U$ where U is an IFπOS in Y, then $A \subseteq f^{-1}(U)$ where $f^{-1}(U)$ is an IFπOS in X, by hypothesis. Since A is an IFπGβCS, $\mathrm{Cl}(A) \subseteq f^{-1}(U)$ in X. Hence, $f(\mathrm{Cl}(A)) \subseteq f(f^{-1}(U)) = U$. But $f(\mathrm{Cl}(A))$ is an IFπGβCS in Y, since $\mathrm{Cl}(A)$ is an IFCS in X and f is an IFπGβCM., Therefore $\beta\mathrm{Cl}(f(\mathrm{Cl}(A))) \subseteq U$. Now $\beta\mathrm{Cl}(f(A)) \subseteq \beta\mathrm{Cl}(f(\mathrm{Cl}(A))) \subseteq U$. Hence $f(A)$ is an IFπGβCS in Y.□

Theorem 3.13. A mapping $f : X \to Y$ is an IFπGβCM if and only if for every IFS B of Y and for every IFπOS U containing $f^{-1}(B)$, there is an IFπGβOS A of Y such that $B \subset A$ and $f^{-1}(A) \subset U$.

Proof. Necessity: Let B be any IFS in Y. Let U be an IFπOS in X such that $f^{-1}(B) \subseteq U$. Then U^c is an IFπCS in X. By hypothesis $f(U^c)$ is an IFπGβCS in Y. Let $A = (f(U^c))^c$, then A is an IFπGβOS in Y and $B \subset A$. Now $f^{-1}(A) = f^{-1}(f(U^c))^c = (f^{-1}(f(U^c)))^c \subset U$.
Sufficiency: Let A be any IFCS in X, then A^c is an IFOS in X and $f^{-1}(f(A^C))^C \subseteq A^C$. By hypothesis there exists an IFπGβS B in Y such that $f(A^C) \subseteq B$ and $f^{-1}(B) \subseteq A^C$. Therefore, $A \subseteq f^{-1}(B))^C$.
Hence $B^C \subseteq f(A) \subseteq f(f^{-1}(B))^C \subseteq B^C$. This implies that $f(A) = B^C$. Since B^C is an IFπGβCS in Y, $f(A)$ is an IFπGβCS in Y. Hence f is an IFπGβCM.□

Theorem 3.14. If $f : X \to Y$ is an IFCM and $g : Y \to Z$ is an IFπGβCM, then $g \circ f$ is an IFπGβCM.

Proof. Let A be an IFCS in X, then $f(A)$ is an IFCS in Y, Since f is an IFCM. Since g is an IFπGβCM, $g(f(A))$ is an IFπGβCS in Z. Therefore $g \circ f$ is an IFπGβCM.□

Theorem 3.15. Let $f : X \to Y$ be a bijective map where Y is an IFβT$_{1/2}$ space. Then the following are equivalent.

(i) f is an IFπGβCM
(ii) $f(B)$ is an IFπGβOS in Y for every IFOS B in X.

(iii)$f(\mathrm{Int}(B)) \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f(B))))$ for every IFS B in X.

Proof. (i)$\to$ (ii) is obvious.
(ii)$\to$(iii): Let B be an IFS in X, then $\mathrm{Int}(B)$ is an IFOS in X. By hypothesis $f(\mathrm{Int}(B))$ is an IFGβOS in Y. Since Y is an IF$\beta\mathrm{T}_{1/2}$ space, $f(\mathrm{Int}(B))$ is an IFβOS in Y. Therefore

$$\begin{aligned} f(\mathrm{Int}(B)) &= \beta\mathrm{Int}(f(\mathrm{Int}(B))) \\ &= f(\mathrm{Int}(B)) \cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f(\mathrm{Int}(B))))) \\ &\subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f(\mathrm{Int}(B))))) \\ &\subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f(B)))). \end{aligned}$$

(iii)$\to$ (i): Let A be an IFCS in X. Then A^C is an IFOS in X. By hypothesis,

$$f(\mathrm{Int}(A^c)) = f(A^C) \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f(A^C))))).$$

That is $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f(A)))) \subseteq f(A)$. This implies $f(A)$ is an IFβCS in Y and hence an IFπGβCS in Y. Therefore f is an IFπGβCM.□

Theorem 3.16. Let $f : X \to Y$ be a bijective map where Y is an IF$\beta\mathrm{T}_{1/2}$ space. Then the following are equivalent.

(i)f is an IFπGβCM
(ii)$f(B)$ is an IFπGβCS in Y for every IFCS B in X.
(iii)$\mathrm{Int}(\mathrm{cl}(\mathrm{Int}(f(B)))) \subseteq f(\mathrm{cl}(B))$ for every IFS B in X.

Proof. (i)$\to$(ii) is obvious.
(ii)$\to$(iii): Let B be an IFS in X, then $\mathrm{cl}(B)$ is an IFCS in X. By hypothesis $f(\mathrm{Cl}(B))$ is an IFπGβCS in Y. Since Y is an IF$\beta\mathrm{T}_{1/2}$ space, $f(\mathrm{Cl}(B))$ is an IFβCS in Y. Therefore
$f(\mathrm{Cl}(B)) = \beta\mathrm{Cl}(f(\mathrm{Cl}(B)))$
$= f(\mathrm{Cl}(B)) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f(\mathrm{Cl}(B)))$
$\supseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f(\mathrm{Cl}(B))) \supseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f(B))).$
(iii)$\to$(i): Let A be an IFCS in X. By hypothesis, $f(\mathrm{Cl}(A)) = f(A) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(f(A))))$. This implies $f(A)$ is an IFβCS in Y and hence an IFπGβCS in Y. Therefore f is an IFπGβCM.□

Definition 3.3. A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy open mapping (IFOM for short) if $f(A)$ is an IFOS in Y for each IFOS A in X.

Definition 3.4. A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy π generalized β open mapping (IFπGβOM for short) if $f(A)$ is an IFπGβOS in Y for each IFOS in X.

Theorem 3.17. If $f : X \to Y$ is a mapping. Then the following are equivalent if Y is an IF$\beta\mathrm{T}_{1/2}$ Space

(i)f is an IFπGβOM .
(ii)$f(\mathrm{Int}(A)) \subseteq \beta\mathrm{Int}(f(A))$ for each IFS A of X
(iii)$\mathrm{Int}(f^{-1}(B)) \subseteq f^{-1}(\beta\mathrm{Int}(B))$ for every IFS B of Y.

Proof. (i)$\to$(ii): Let f be an IFπGβOM. Let A be any IFS in X. Then $\mathrm{Int}(A)$ is an IFOS in X. (i) implies that $f(\mathrm{Int}(A))$ is an IFπGβOS in Y. Since Y is an IF$\beta\mathrm{T}_{1/2}$ space, $f(\mathrm{Int}A))$ is an IFβOS in Y. Therefore $\beta\mathrm{Int}(f(\mathrm{Int}(A))) = f(\mathrm{Int}(A)) \subseteq f(A)$. Now $f(\mathrm{Int}(A)) = \beta\mathrm{Int}(f(\mathrm{Int}(A))) \subseteq \beta\mathrm{Int}(f(A))$.

(ii)$\to$(iii): Let B be any IFS in Y. Then $f^{-1}(B)$ is an IFS in X. (ii) implies that $f(\mathrm{Int}(f^{-1}(B)) \subseteq \beta\mathrm{Int}(f(f^{-1}(B)) = \beta\mathrm{Int}(B)$. Now $\mathrm{Int}(f^{-1}(B)) \subseteq f^{-1}(f(\mathrm{Int}(f^{-1}(B))) \subseteq f^{-1}(\beta\mathrm{Int}(B))$.

(iii)$\to$(i): Let A be an IFOS in X. Then $\mathrm{Int}(A) = A$ and $f(A)$ is an IFS in Y. (iii) implies that $\mathrm{Int}(f^{-1}(f(A))) \subseteq f^{-1}(\beta\mathrm{Int}(f(A)))$. Now $A = \mathrm{Int}(A) \subseteq \mathrm{Int}(f^{-1}(f(A))) \subseteq f^{-1}(\beta\mathrm{Int}(f(A)))$. Hence $f(A) \subseteq f(f^{-1}(\beta\mathrm{Int}(f(A)) = \beta\mathrm{Int}(f(A)) \subseteq f(A)$. This implies $\beta\mathrm{Int}(f(A)) = f(A)$. Hence $f(A)$ is an IFβOS in Y. Since every IFβOS is an IFπGβOS, $f(A)$ is an IFπGβOS in Y. Thus f is an IFπGβOM.□

Theorem 3.18. A mapping $f : X \to Y$ is an IFπGβOM if $f(\beta\mathrm{Int}(A)) \subseteq \beta\mathrm{Int}(f(A))$ for every $A \subseteq X$.

Proof. Let A be an IFOS in X. Then $\mathrm{Int}(A) = A$. Now $f(A) = f(\mathrm{Int}(A)) \subseteq f(\beta\mathrm{Int}(A)) \subseteq \beta\mathrm{Int}(f(A))$, by hypothesis. But $\beta\mathrm{Int}(f(A)) \subseteq f(A)$. Therefore $f(A)$ is an IFβOS in X. That is $f(A)$ is an IFπGβOS in X. Hence f is an IFπGβOM.□

Theorem 3.19. A mapping $f : X \to Y$ is an IFπGβOM if and only if $\mathrm{Int}(f^{-1}(B)) \subseteq f^{-1}(\beta\mathrm{Int}(B))$ for every $B \subseteq Y$, where Y is an IF$\beta\mathrm{T}_{1/2}$ space.

Proof. Necessity: Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$ and $\mathrm{Int}(f^{-1}(B))$ is an IFOS in X. By hypothesis, $f(\mathrm{Int}(f^{-1}(B)))$ is an IFπGβOS in Y. Since Y is an IF$\beta\mathrm{T}_{1/2}$ space, $f(\mathrm{Int}(f^{-1}(B)))$ is an IFβOS in Y. Hence $f\,(\mathrm{Int}(f^{-1}(B))) = \beta\mathrm{Int}(f(\mathrm{Int}(f^{-1}(B)))) \subseteq \beta\mathrm{Int}(B)$.
This implies $\mathrm{Int}(f^{-1}(B)) \subseteq f^{-1}(\beta\mathrm{Int}(B))$.
Sufficiency: Let A be an IFOS in X. Therefore $\mathrm{Int}(A) = A$. Then $f(A) \subseteq Y$. By hypothesis $\mathrm{Int}(f^{-1}(f(A))) \subseteq f^{-1}(\beta\mathrm{Int}(f(A)))$. That is $\mathrm{Int}(A) \subseteq \mathrm{Int}(f^{-1}(f(A))) \subseteq f^{-1}(\beta\mathrm{Int}(f(A)))$. Therefore $A \subseteq f^{-1}(\beta\mathrm{Int}(f(A)))$. This implies $f(A) \subseteq \beta\mathrm{Int}(f(A)) \subseteq f(A)$. Hence $f(A)$ is an IFβOS in Y and hence an IFπGβOS in Y. Thus f is an IFπGβOM.□

Theorem 3.20. Let $f : X \to Y$ be an onto mapping where Y is an IF$\beta\mathrm{T}_{1/2}$ space. Then f is an IFπGβOM if and only if for any IFP $c(\alpha,\beta) \subseteq Y$ and for any IFN B of $f^{-1}(c(\alpha,\beta))$, there is an IFβN A of $c(\alpha,\beta)$ such that $c(\alpha,\beta) \subseteq A$ and $f^{-1}(A) \subseteq B$.

Proof. Necessity: Let $c(\alpha,\beta) \subseteq Y$ and let B be an IFN of $f^{-1}(c(\alpha,\beta))$. Then there is an IFOS C in X such that $f^{-1}(c(\alpha,\beta)) \subseteq C \subseteq B$. Since f is an IFπGβCM, $f(C)$ is an IFπGβOS in Y. Since Y is an IF$\beta\mathrm{T}_{1/2}$ space, $f(C)$ is an IFβOS in Y and $c(\alpha,\beta) \subseteq f(f^{-1}(c(\alpha,\beta)))$
$\subseteq f(C) \subseteq f(B)$. Put $A = f(C)$. Then A is an IFβN of $c(\alpha,\beta)$ and $c(\alpha,\beta) \in A \subseteq f(B)$. Thus $c(\alpha,\beta) \in A$ and $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$. That is $f^{-1}(A) \subseteq B$.
Sufficiency: Suppose that $c(\alpha,\beta) \subseteq f(B)$. This implies $f^{-1}(c(\alpha,\beta)) \subseteq B$. Then B is an IFN of $f^{-1}(c(\alpha,\beta))$. By hypothesis there is an IFβN A of $c(\alpha,\beta)$ such that $c(\alpha,\beta) \subseteq A$ and $f^{-1}(A) \subseteq B$. Therefore there is an IFβOS C in Y such that $c(\alpha,\beta) \subseteq C \subseteq A$
$= f(f^{-1}(A)) \subseteq f(B)$. Hence $f(B) = \{c(\alpha,\beta) | c(\alpha,\beta) \subseteq f(B)\} \subseteq \{C_c(\alpha,\beta) | c(\alpha,\beta) \in f(B)\} \subseteq f(B)$. Thus

$f(B)=\{C_c(\alpha,\beta)|c(\alpha,\beta)\subseteq f(B)\}$. Since each C is an IFβOS, $f(B)$ is also an IFβOS and hence is an IFπGβOS in Y. Therefore f is an IFπGβOM.□

Theorem 3.21. If $f:X\to Y$ is a mapping, then the following are equivalent.

(i) f is an IFMπGβCM
(ii) $f(A)$ is an IFπGβCS in Y for every IFπGβCS A in X
(iii) $f(A)$ is an IFπGβOS in Y for every IFπGβOS A in X.

Proof. (i)→(ii)is obvious from the Definition 3.2.

(ii)→(iii): Let A be an IFπGβOS in X Then A^C is an IFπGβCS in X. By hypothesis, $f(A^C)$ is an IFπGβCS in Y. That is $f(A)^C$ is an IFπGβCS in Y and hence $f(A)$ is an IFπGβOS in Y.

(iii)→(i): Let A be an IFπGβCS in X. Then A^C is an IFπGβOS in X. By hypothesis, $f(A^C)$ is an IFπGβOS in Y. Hence $f(A)^C$ is an IFπGβOS in Y and hence $f(A)$ is an IFπGβCS in Y, and hence f is an IFMπGβCM.□

Theorem 3.22. Let $f:X\to Y$ be a bijective mapping, where X is an IFβT$_{1/2}$ space. Then the following are equivalent.

(i) f is an IFMπGβCM
(ii) For each IFP $c(\alpha,\beta)\in Y$ and every IFβN A of $f^{-1}(c(\alpha,\beta))$, there exists an IFπGβOS B in Y such that $c(\alpha,\beta)\in B\subseteq f(A)$.
(iii) For each IFP $c(\alpha,\beta)\in Y$ and every IFβN A of $f^{-1}(c(\alpha,\beta))$, there exists an IFπGβOS B in Y such that $c(\alpha,\beta)\in B$ and $f^{-1}(B)\subseteq A$.

Proof. (i) → (ii): Let $c(\alpha,\beta)\subseteq Y$ and A the IFβN of $f^{-1}(c(\alpha,\beta))$. Then there exists an IFβOS C in X such that $f^{-1}(c(\alpha,\beta))\subseteq C\subseteq A$. Since every IF$\beta$OS is an IF$\pi$G$\beta$OS, C is an IFπGβOS in X. Then by hypothesis, $f(C)$ is an IFπGβOS in Y. Now $c(\alpha,\beta)\subseteq f(C)\subseteq f(A)$. Put $B=f(C)$. This implies $c(\alpha,\beta)\subseteq B\subseteq f(A)$.

(ii)→ (iii): Let $c(\alpha,\beta)\subseteq Y$ and A the IFβN of $f^{-1}(c(\alpha,\beta))$. Then there exists an IFβOS C in X such that $f^{-1}(c(\alpha,\beta))\in C\subseteq A$. Since every IF$\beta$OS is an IF$\pi$G$\beta$OS, C is an IFπGβOS in X. Then by hypothesis, $f(C)$ is an IFπGβOS in Y. Now $c(\alpha,\beta)\in f(C)\subseteq f(A)$. Put $B=f(C)$. This implies $c(\alpha,\beta)\subseteq B\subseteq f(A)$. Now $f^{-1}(B)\subseteq f^{-1}(f(A))\subseteq A$. That is $f^{-1}(B)\subseteq A$.

(iii)→(i): Let A be an IFπGβOS in X. Since X is an IFβT$_{1/2}$ space, A is an IFβOS in X. Let $c(\alpha,\beta)\subseteq Y$ and $f^{-1}(c(\alpha,\beta))\subseteq A$. That is $c(\alpha,\beta)\subseteq f(A)$. This implies A is an IFβN of $f^{-1}(c(\alpha,\beta))$. Then by hypothesis, there exists an IFπGβOS B in Y such that $c(\alpha,\beta)\subseteq B$ and $f^{-1}(B)\subseteq A$. Hence by Remark 2.2, $f(A)$ is an IFπGβOS in Y. Therefore f is an IFMπGβCM.□

Theorem 3.23. If $f:X\to Y$ is a bijective mapping, then the following are equivalent.

(i) f is an IFMπGβCM
(ii) $f(A)$ is an IFπGβOS in Y for every IFπGβOS A in X.
(iii) for every IFP $c(\alpha,\beta)\in Y$ and for every IFGβOS B in X such that $f^{-1}(c(\alpha,\beta))\in B$, there exists an IF$\pi$G$\beta$OS A in Y such that $c(\alpha,\beta)\in A$ and $f^{-1}(A)\subseteq B$.

Proof. (i)→ (ii): is obvious by Theorem 3.21.

(ii)→(iii): Let $c(\alpha,\beta)\in Y$ and let B be an IFGβOS in X such that $f^{-1}(c(\alpha,\beta))\in B$. This implies $c(\alpha,\beta)\in f(B)$. By hypothesis, $f(B)$ is an IFπGβOS in Y. Let $A=f(B)$. Therefore $c(\alpha,\beta)\in f(B)=A$ and $f^{-1}(A)=f^{-1}(f(B))\subseteq B$.

(iii)→(i): Let B be an IFπGβCS in X. Then B^c is an IFπGβOS in X. Let $c(\alpha,\beta)\in Y$ And $f^{-1}(c(\alpha,\beta))\subseteq B^C$. This implies $c(\alpha,\beta)\subseteq f(B^C)$. By hypothesis there exists an IFπGβOS A in Y such that $c(\alpha,\beta)\in A$ and $f^{-1}(A)\subseteq B^C$. Put $A=f(B^C)$. Then $c(\alpha,\beta)\in f(B^c)$ and $A=f(f^{-1}(B^c))\subseteq f(B^c)$. Hence by Remark 2.2, $f(B^c)$ is an IFπGβOS in Y. Therefore $f(B)$ is an IFπGβCS in Y. Thus f is an IFMπGβCM.□

Theorem 3.24. If $f:X\to Y$ is a bijective mapping, then the following are equivalent.

(i) f is an IFMπGβCM
(ii) $f(A)$ is an IFπGβOS in Y for every IFπGβOS A in X
(iii) $f(\beta\mathrm{Int}(B))\subseteq\beta\mathrm{Int}(f(B))$ for every IFS B in X
(iv) $\beta\mathrm{Cl}(f(B))\subseteq f(\beta\mathrm{Cl}(B))$ for every IFS B in X.

Proof. (i)→ (ii) is obvious.

(ii)→ (iii): Let B be any IFS in X. Since $\beta\mathrm{Int}(B)$ is an IFβOS, it is an IFπGβOS in X. Then by hypothesis, $f(\beta\mathrm{Int}(B))$ is an IFπGβOS in Y. Since Y is an IFβT$_{1/2}$ space, $f(\beta\mathrm{Int}(B))$ is an IFβOS in Y. Therefore $f(\beta\mathrm{Int}(B))=\beta\mathrm{Int}(f(\beta\mathrm{Int}(B)))\subseteq\beta\mathrm{Int}(f(B))$.

(iii)→(iv) can easily proved by taking complement in (iii).

(iii)→(i): Let A be an IFπGβCS in X. By hypothesis, $\beta\mathrm{cl}(f(A))\subseteq f(\beta\mathrm{cl}(A))$.

Since X is an IFβT$_{1/2}$ space, A is an IFβCS in X. Therefore, $\beta\mathrm{cl}(f(A))\subseteq f(\beta\mathrm{Cl}(A))=f(A)\subseteq\beta\mathrm{cl}(f(A))$. Hence $f(A)$ is an IFβCS in Y and hence an IFπGβCS in Y. Thus f is an IFMπGβCM.□

Theorem 3.25. If $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ are both IFMπGβCM, then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an IFMπGβCM.

Proof. Let A be an IFπGβCS in X. Then $f(A)$ is an IFπGSCS in Y, by hypothesis. Since g is an IFMπGS closed mapping, $g(f(A))=(g\circ f)(A)$ is an IFπGβCS in Z. Hence $g\circ f$ is an IFπGβCM.□

Theorem 3.26. If $f:(X,\tau)\to(Y,\sigma)$ is an IF closed mapping and $g:(Y,\sigma)\to(Z,\delta)$ is an IFMπGβ closed mapping, then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an IFπGβ closed mapping.

Proof. Let A be an IFCS in X. Then $f(A)$ is an IFCS in Y, by hypothesis. Since every IFCS is an IFπGβCS, $f(A)$ is an IFπGβCS in Y. Since g is IFMπGβ closed mapping, $g(f(A))=(g\circ f)(A)$ is an IFπGβCS in Z. Hence $g\circ f$ is an IFπGβ closed mapping.□

Theorem 3.27. If $f:(X,\tau)\to(Y,\sigma)$ be an IFπGβ closed mapping and Y is an IFπGT$_{1/2}$ space, then f is an IFGβ closed mapping.

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be an IFπGβ closed mapping and let A be an IFCS in X. Then by hypothesis

$f(A)$ is an IFπGβCS in Y. Since Y is an IFπGT$_{1/2}$ space, $f(A)$ is an IFGβCS in Y. This implies f is an IFGβclosed mapping.□

4 Conclusion

In this paper Intuitionistic fuzzy πGβ closed mapping were introduced and studied with already existing sets in Intuitionistic fuzzy topological spaces. The idea of this paper will be helpful in the extension of bitopological spaces. The scope for further research can be focused on the applications.

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