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Hybrid Teleportation between Qutrit and Qubit

Wei Yang[1,2,*], *Liusheng Huang*[2] *and Lu Li*[1]

[1] School of Computer Science and Technology, University of Science and Technology of China, Hefei 230026, China
[2] Suzhou Institute for Advanced Study, USTC, Suzhou, 215123, China

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Abstract: Previous quantum teleportation schemes mainly focus on teleporting the unknown state via entanglement state with the same dimension. In this paper, we study the problem of hybrid teleportation. We present a protocol for faithful teleportation of an unknown qutrit state using a special kind of four-qubit entanglement state. The construction of the special four-qubit entanglement state is described and the number of the qubits consumed in the protocol is shown to be minimal.

Keywords: quantum information, hybrid teleportation, quantum entanglement, qutrit

1 Introduction

Quantum teleportation is an amazing technique for moving an unknown quantum state to a remote place via quantum entanglement [1]. During the teleportation process, the original quantum state is destroyed, while its perfect replica can be recovered with certain classical information from the sending site to the receiving site. Quantum teleportation does not result in the copying of quantum states, for the original unknown state disappeared after the teleportation, and hence it does not violate the no cloning theorem. Moreover, it does not allow communication of information at superluminal (faster than light) speed, for the speed of the transmission of classical information cannot exceed the speed of light.

In the first teleportation scheme [1], Bennett *et al.* employed a bipartite EPR state as the quantum channel to teleport an unknown qubit from one party (say, Alice) to another party (say, Bob). Four years later, in 1997, this scheme was experimentally demonstrated by Bouwmester *et al.* [2]. After Ref. [1], teleportation has been extensively studied theoretically and a great many schemes have been proposed [3,4,5,6,7,8,9,10,11,12, 13]. Among these proposals, the entanglement states used as the quantum channels have the same dimensions with the unknown states in Hilbert space. For example, they teleport a single qubit or qubits using N-qubit entanglement states [3,5,6,7,8,9,11], and teleport a single qudit (d-dimensional system) or qudits using N-qudit entanglement states [4,10,12]. Specially, A-H. M. Ahmed *et al.* discussed the problem of quantum teleportation in the presences of noise quantum operations and showed that the accuracy of the information transfer not only depends on the laboratory equipment, but also on nature of the information to be teleported [13]. However, few attention has been paid to the teleportation using entanglement state with a dimension different from that of the unknown state.

A natural question to ask is whether we can teleport an unknown state via entanglement state with different dimension. For example, can we teleport a qutrit using a N-qubit entanglement state? In this paper, we investigate the problem of this kind of “hybrid” teleportation. We propose a new teleportation protocol and show that a specially constructed four-qubit entanglement state is sufficient to realize the teleportation of an unknown qutrit. We also expound that the number of qubits used in our teleportation protocol is optimal.

The rest of this paper is organized as follows. In section 2, we briefly introduce how to prepare the special entanglement state that will be used in our protocol. Section 3 contains the concrete protocol for teleportation of an unknown qutrit via the special four-qubit entanglement state. In section 4 we conclude the present paper with some discussions.

* Corresponding author e-mail: qubit@ustc.edu.cn

2 Preparation of the special four-qubit state

The special four-qubit entanglement state that will be used in our teleportation scheme can be represented as follows

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|0010\rangle + |0100\rangle + |1001\rangle) \tag{1}$$

We can prepare this state by a three-qubit W state and a auxiliary qubit in an initial state $|0\rangle$. The W state is given as

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) \tag{2}$$

Now the whole system of the W state and the auxiliary qubit state is

$$|\psi_0\rangle = |W\rangle \otimes |0\rangle = \frac{1}{\sqrt{3}}(|0010\rangle + |0100\rangle + |1000\rangle) \tag{3}$$

We then send the first qubit (qubit 1) and the last qubit (qubit 4, the auxiliary qubit) though a Controlled-Not (CNOT) gate, with qubit 1 being the control qubit and qubit 4 being the target qubit. After the CNOT operation, we obtain

$$|\psi_1\rangle = \frac{1}{\sqrt{3}}(|0010\rangle + |0100\rangle + |1001\rangle) \tag{4}$$

It is clear that $|\psi_1\rangle$ is the same as $|\psi\rangle$. Thus we complete the preparation of the special state $|\psi\rangle$.

3 Teleportation of An Unknown Qutrit

Now we describe the quantum protocol for teleportation of an unknown qutrit using the four-qubit entanglement state introduced in Section 2. Suppose that the unknown qutrit (particle 0) Alice needs to teleport is

$$|\phi\rangle_0 = \alpha|a\rangle + \beta|b\rangle + \gamma|c\rangle \tag{5}$$

where $|a\rangle$, $|b\rangle$, $|c\rangle$ are three-dimensional column vectors with the values

$$|a\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, |b\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, |c\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

and α, β and γ are complex numbers that satisfy the normalization condition

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1 \tag{7}$$

Suppose further that Alice and Bob share the special four-qubit entanglement state $|\phi\rangle_{1234}$ as in Eq. (1)

$$|\phi\rangle_{1234} = \frac{1}{\sqrt{3}}(|0010\rangle + |0100\rangle + |1001\rangle)_{1234} \tag{8}$$

where qubits 1 and 2 are with Alice, and qubits 3 and 4 are at Bob's side. Thus the initial joint state is as below

$$\begin{aligned} |\Phi\rangle &= |\phi\rangle_0 \otimes |\phi\rangle_{1234} \\ &= \frac{1}{\sqrt{3}}(\alpha|a0010\rangle + \alpha|a0100\rangle + \alpha|a1001\rangle \\ &+ \beta|b0010\rangle + \beta|b0100\rangle + \beta|b1001\rangle \\ &+ \gamma|c0010\rangle + \gamma|c0100\rangle + \gamma|c1001\rangle)_{01234} \end{aligned} \tag{9}$$

Then Alice performs a joint measurement on her three particles (qutrit 0, qubit 1 and qubit 2) in the following basis. These basis can be divided into four groups.

Group 1:

$$|\Phi_1\rangle = \frac{1}{\sqrt{3}}(|a00\rangle + |b01\rangle + |c10\rangle) \tag{10}$$

$$\begin{aligned} |\Phi_2\rangle = \frac{1}{\sqrt{3}}[&|a00\rangle + (\cos\theta - i\sin\theta)|b01\rangle \\ &- (\cos\theta + i\sin\theta)|c10\rangle] \end{aligned} \tag{11}$$

In Eq. (11), θ is a constant. Here and henceforth, it takes the value $\theta = \frac{1}{3}\pi$.

$$\begin{aligned} |\Phi_3\rangle = \frac{1}{\sqrt{3}}[&|a00\rangle - (\cos\theta + i\sin\theta)|b01\rangle \\ &+ (\cos\theta - i\sin\theta)|c10\rangle] \end{aligned} \tag{12}$$

Group 2:

$$|\Phi_4\rangle = \frac{1}{\sqrt{3}}(|a01\rangle + |b10\rangle + |c00\rangle) \tag{13}$$

$$\begin{aligned} |\Phi_5\rangle = \frac{1}{\sqrt{3}}[&|a01\rangle + (\cos\theta - i\sin\theta)|b10\rangle \\ &- (\cos\theta + i\sin\theta)|c00\rangle] \end{aligned} \tag{14}$$

$$\begin{aligned} |\Phi_6\rangle = \frac{1}{\sqrt{3}}[&|a01\rangle - (\cos\theta + i\sin\theta)|b10\rangle \\ &+ (\cos\theta - i\sin\theta)|c00\rangle] \end{aligned} \tag{15}$$

Group 3:

$$|\Phi_7\rangle = \frac{1}{\sqrt{3}}(|a10\rangle + |b00\rangle + |c01\rangle) \tag{16}$$

$$\begin{aligned} |\Phi_8\rangle = \frac{1}{\sqrt{3}}[&|a10\rangle + (\cos\theta - i\sin\theta)|b00\rangle \\ &- (\cos\theta + i\sin\theta)|c01\rangle] \end{aligned} \tag{17}$$

$$\begin{aligned} |\Phi_9\rangle = \frac{1}{\sqrt{3}}[&|a10\rangle - (\cos\theta + i\sin\theta)|b00\rangle \\ &+ (\cos\theta - i\sin\theta)|c01\rangle] \end{aligned} \tag{18}$$

Group 4:

$$|\Phi_{10}\rangle = \frac{1}{\sqrt{3}}(|a11\rangle + |b11\rangle + |c11\rangle) \tag{19}$$

$$\begin{aligned}|\Phi_{11}\rangle = \frac{1}{\sqrt{3}}[&|a11\rangle + (\cos\theta - i\sin\theta)|b11\rangle \\ &- (\cos\theta + i\sin\theta)|c11\rangle]\end{aligned} \tag{20}$$

$$\begin{aligned}|\Phi_{12}\rangle = \frac{1}{\sqrt{3}}[&|a11\rangle - (\cos\theta + i\sin\theta)|b11\rangle \\ &+ (\cos\theta - i\sin\theta)|c11\rangle]\end{aligned} \tag{21}$$

These states form an orthonormal basis $\sum_{i=1}^{n}|\Phi_i\rangle\langle\Phi_i| = I$ and $\langle\Phi_j|\Phi_k\rangle = \delta_{jk}$. Using the basis in Group 1–4, we can rewrite Eq. (9) as

$$\begin{aligned}|\Phi\rangle = \frac{1}{3}\{&|\Phi_1\rangle_{012}(\alpha|10\rangle + \beta|00\rangle + \gamma|01\rangle)_{34} \\ &+ |\Phi_2\rangle_{012}[\alpha|10\rangle + (\cos\theta - i\sin\theta)\beta|00\rangle \\ &- (\cos\theta + i\sin\theta)\gamma|01\rangle]_{34} \\ &+ |\Phi_3\rangle_{012}[\alpha|10\rangle - (\cos\theta + i\sin\theta)\beta|00\rangle \\ &+ (\cos\theta - i\sin\theta)\gamma|01\rangle]_{34} \\ &+ |\Phi_4\rangle_{012}(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle)_{34} \\ &+ |\Phi_5\rangle_{012}[\alpha|00\rangle + (\cos\theta - i\sin\theta)\beta|01\rangle \\ &- (\cos\theta + i\sin\theta)\gamma|10\rangle]_{34} \\ &+ |\Phi_6\rangle_{012}[\alpha|00\rangle - (\cos\theta + i\sin\theta)\beta|01\rangle \\ &+ (\cos\theta - i\sin\theta)\gamma|10\rangle]_{34} \\ &+ |\Phi_7\rangle_{012}(\alpha|01\rangle + \beta|10\rangle + \gamma|00\rangle)_{34} \\ &+ |\Phi_8\rangle_{012}[\alpha|01\rangle + (\cos\theta - i\sin\theta)\beta|10\rangle \\ &- (\cos\theta + i\sin\theta)\gamma|00\rangle]_{34} \\ &+ |\Phi_9\rangle_{012}[\alpha|01\rangle - (\cos\theta + i\sin\theta)\beta|10\rangle \\ &+ (\cos\theta - i\sin\theta)\gamma|00\rangle]_{34}\}\end{aligned} \tag{22}$$

By calculation, we know that after Alice's joint measurement, she gets each of the state in Group 1–3 with equal probability $1/9$ and gets each of the state in Group 4 with probability 0. Then she sends Bob four bits classical information to inform him the measurement outcome. With this message, Bob now knows to what state his two particles (qubits 3 and 4) have collapsed. Thus he is aware of which operation he should perform on his two particles to faithfully recover the teleported quantum state. For instance, if Alice's measurement result is $|\Phi_5\rangle$, she tells Bob this outcome via a classical channel. On receiving this message Bob knows that qubits 3 and 4 are now projected to the state $\alpha|00\rangle + (\cos\theta - i\sin\theta)\beta|01\rangle - (\cos\theta + i\sin\theta)\gamma|10\rangle$, and he apply the following operation on his two particles: $|00\rangle \rightarrow |a\rangle$, $(\cos\theta - i\sin\theta)|01\rangle \rightarrow |b\rangle$, $(\cos\theta + i\sin\theta)|10\rangle \rightarrow -|c\rangle$. Till now Alice and Bob have succeeded in teleporting the qutrit using the special four-qubit entanglement channel. See Tab. 1 for all the cases. For each measurement outcome, the state that qubits 3 and 4 collapse to can be found in Eq. (22).

Table 1: List of Bob's operations, according to Alice's measurement outcome (AMO).

AMO	Bob's operation to recover $\lvert\phi\rangle_0$
$\lvert\Psi_1\rangle$	$\lvert 10\rangle \rightarrow \lvert a\rangle$, $\lvert 00\rangle \rightarrow \lvert b\rangle$, $\lvert 01\rangle \rightarrow \lvert c\rangle$
$\lvert\Psi_2\rangle$	$\lvert 10\rangle \rightarrow \lvert a\rangle$, $(\cos\theta - i\sin\theta)\lvert 00\rangle \rightarrow \lvert b\rangle$, $(\cos\theta + i\sin\theta)\lvert 01\rangle \rightarrow -\lvert c\rangle$
$\lvert\Psi_3\rangle$	$\lvert 10\rangle \rightarrow \lvert a\rangle$, $(\cos\theta + i\sin\theta)\lvert 00\rangle \rightarrow -\lvert b\rangle$, $(\cos\theta - i\sin\theta)\lvert 01\rangle \rightarrow \lvert c\rangle$
$\lvert\Psi_4\rangle$	$\lvert 00\rangle \rightarrow \lvert a\rangle$, $\lvert 01\rangle \rightarrow \lvert b\rangle$, $\lvert 10\rangle \rightarrow \lvert c\rangle$
$\lvert\Psi_5\rangle$	$\lvert 00\rangle \rightarrow \lvert a\rangle$, $(\cos\theta - i\sin\theta)\lvert 01\rangle \rightarrow \lvert b\rangle$, $(\cos\theta + i\sin\theta)\lvert 10\rangle \rightarrow -\lvert c\rangle$
$\lvert\Psi_6\rangle$	$\lvert 00\rangle \rightarrow \lvert a\rangle$, $(\cos\theta + i\sin\theta)\lvert 01\rangle \rightarrow -\lvert b\rangle$, $(\cos\theta - i\sin\theta)\lvert 10\rangle \rightarrow \lvert c\rangle$
$\lvert\Psi_7\rangle$	$\lvert 01\rangle \rightarrow \lvert a\rangle$, $\lvert 10\rangle \rightarrow \lvert b\rangle$, $\lvert 00\rangle \rightarrow \lvert c\rangle$
$\lvert\Psi_8\rangle$	$\lvert 01\rangle \rightarrow \lvert a\rangle$, $(\cos\theta - i\sin\theta)\lvert 10\rangle \rightarrow \lvert b\rangle$, $(\cos\theta + i\sin\theta)\lvert 00\rangle \rightarrow -\lvert c\rangle$
$\lvert\Psi_9\rangle$	$\lvert 01\rangle \rightarrow \lvert a\rangle$, $(\cos\theta + i\sin\theta)\lvert 10\rangle \rightarrow -\lvert b\rangle$, $(\cos\theta - i\sin\theta)\lvert 00\rangle \rightarrow \lvert c\rangle$

We should like to emphasize the fact that the number of qubits used in our teleportation scheme is optimal, that is to say, we cannot faithfully teleport a single qutrit with less than four qubits. Generally, a qutrit has three coefficients, thus at least two qubits are necessary to distinguish them. Or in other words, we need two qubits to "store" these coefficients, together with the basis they belong to (in fact, two qubits can "store" a maximum of four different items, though). On the other hand, a qutrit is a unit vector in three-dimensional Hilbert space, therefore we again need not less than two qubits to "carry" it during teleportation process. It is not hard to verify that a joint measurement on only one single qubit and the unknown qutrit cannot achieve the teleporting task.

4 Summary

In this paper, we have presented a protocol for teleportation of an unknown qutrit state via a special kind of four-qubit entanglement state. The preparation method of the special four-qubit entanglement state has also been described. Moreover, the number of qubits consumed in the protocol has been shown to be minimal. Namely, one cannot faithfully teleport an unknown qutrit with less than four qubits.

We have shown that "hybrid" teleportation is feasible with carefully constructed quantum channel and joint measurement basis. It is worth mentioning that the thought of "hybrid" has been an interesting and useful tool and has been widely used recently [14,15]. Particularly, in 2008 B. P. Lanyon *et al.* realized the first instance of qubit-qutrit entanglement [16], which provides the practicability of the quantum measurement used in our teleportation protocol.

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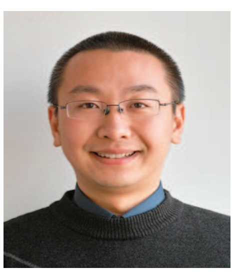

Wei Yang received the Ph.D degree from the Department of Computer Science and Technology, University of Science and Technology of China in 2007. He is currently a post doctorate in University of Science and Technology of China. His major research interest is information security and quantum information.

Liusheng Huang received the M.S. degree in computer science from University of Science and Technology of China in 1988. He is currently a professor and Ph.D supervisor of the Department of Computer Science and Technology at University of Science and Technology of China. He has published 6 books and more than 60 papers. His research interests are in the areas of information security, distributed computing and Wireless Sensor Networks.

Lu Li is a Ph.D. student in School of Computer Science and Technology at University of Science and Technology of China. His research interests include cryptography and information security.