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Enhanced Quadrupole Interaction by Co and Counter-Propagating Hermite-Gaussian Beams

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Abstract: Effective optical quadrupole traps and a planar array of optical wells based on the interference of two optical beams are proposed. Due to this interference a sort of variety distributions are created with intensity maxima and minima that can be used to trap atoms that have transition frequencies appropriately detuned from the frequency ω of the light. This technique assists to avoid some undesirable effects of single beam interaction such as destabilizing dissipative force as well as it enhances the magnitude of quadrupole force. In particular, the mutual coupling of two co and counter-propagating Hermite-Gaussian light beams is presented. Both configurations could be turn out to make quadrupole interaction more exploitable to manipulate the spatial position of trapped atoms..

Keywords: Quadrupole interaction, Planar array of optical wells, trapped atom.

1 Introduction

One of the primary aims, yet to be fully accomplished in the rapidly developing field of quantum optics is the trap small objects to well defined, and often predetermined, regions in space. The idea was first put forward by Ashkin in 1970. He showed the use of optical forces to capture and manipulate micrometer sized particles [1]. Since then, optical traps have become influential tools for the trapping and manipulating of different particles, such as micro-sized dielectric particles, neutral atoms, DNA molecules, living biological cells and metallic particles [2,3,4,5].

It is known that two types of optical forces are identified in the optical traps: gradient force and dissipative force. The gradient force is proportional to the gradient of the square of the electric field and is responsible to pull the particles towards the center of the focus. The dissipative force is due to the net momentum transfer caused by spontaneous-absorption of photons from the particles and tend to push the particles out of the focus, and destabilize the optical trap [6].

The optical traps have been the subject of numerous previous studies and formed a topic of considerable interest in the context of both theoretical and experimental works where are employed only as the electric dipole transition [7,8,9]. So, Higher-order processes such as magnetic dipole and electric quadrupole transitions also play an important part in the light emission from transition metal ions and semiconductor quantum dots. Nevertheless, most applications have overlooked the device implications of these electric-dipole-forbidden transitions throughout the visible and near-infrared regime, and their contributions to many important emitters have not been fully characterized [10,11].

Here we carefully analyze electric quadrupole effects in the interaction of atoms with planar array of optical wells that generated by Hermite-Gaussian beams. These classes of beam are a family of structurally stable laser beams which have rectangular symmetry along the propagation axis and are described by the product of Hermite polynomials and Gaussian functions. The aim is to explore the spatial dependence of the quadrupole forces and its magnitude, and the consequences of this on the atom trapping. These will be evaluated with recent experimental parameters in order to determine the possibility of using the quadrupole interaction in the sufficient atom trapping processes. We believe this topic is definitely interesting because now a lot of new devices based on atoms in lattices are under development.

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2 General Theory

2.1 Quadrupole interaction

The system consists of an atom , modeled here as a neutral two-particle hydrogenic atom possessing only two energy levels, a ground state, denoted $|g\rangle$ of energy E_g, and an excited state $|e\rangle$ of energy E_e, such that the resonance frequency is $\omega_0 = (E_e - E_g)/\hbar$. At what time the atom interacts with the electromagnetic field of frequency ω, the total Hamiltonian for the atom plus field can be written as the sum of three terms

$$\hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{int} \quad (1)$$

where H_A and H_F are the zero-order Hamiltonians for the atom and the Hermite-Gaussian field, respectively,

$$\hat{H}_A = \left(P^2/2M\right) + \hbar\omega_0\pi^\dagger\pi \quad (2)$$

$$\hat{H}_F = \hbar\omega a^\dagger a \quad (3)$$

Here π and $\pi^\dagger$ are the ladder operators for the two-level system; P is the center-of-mass momentum operator with M the mass and ω_0 the dipole transition frequency. The operators a and $a^\dagger$ entering $\hat{H}_F$ are the annihilation and creation operators of the field and ω is its frequency. The interaction Hamiltonian $\hat{H}_{int}$ involves the coupling of the electric polarization $p(r)$ to the electric field vector, as follows

$$\hat{H}_{int} = -\int d^3r\, p(r).E(r) \quad (4)$$

The explicit form of the polarization field $p(r)$is

$$p(r) = e\int_0^1 ds(q-R)\delta(r-R-s(q-R)) \quad (5)$$

where R is the center-of-mass coordinate and $(q-R)$ is the internal position variable relative to the center of mass. Expansion of the polarization field $p(r)$ in a multipolar series about the center of mass R followed by integration with respect to r, yields up to quadrupolar order,

$$\hat{H}_{int} = \hat{H}_d + \hat{H}_Q + ... \quad (6)$$

where $\hat{H}_d$ is the coupling of field to the electric dipole moment and $\hat{H}_Q$ is the coupling to the electric quadrupole moment. Explicitly we have

$$\hat{H}_d = -d.E(R) \quad (7)$$

where $d = ex$ is the electric dipole moment operator with $x = (q-R)$. The quadrupole term is then explicitly given by

$$\hat{H}_Q = -\frac{1}{2}ex_ix_j\nabla_iE_j(R) \quad (8)$$

where the Einstein summation convention is applicable. Here x_i are the components of the internal position vector $x = (x,y,z)$and ∇_i are components of the gradient operator that act only on the spatial coordinate of the transverse electric field vector E as function of the center-of-mass variable R.

2.2 Beam Structure

The Hermite-Gaussian beam characterized by the quantum numbers n and m propagating along the X direction with an axial wave vector k is such that its quantized electric field as a function of the center-of-mass coordinate $R = (X,Y,Z)$, has the form [12]

$$E_{knm}(R) = \hat{\imath}\xi_{k00}\cdot\mathscr{F}_{knm}(R)\cdot\hat{a}_{knm}\exp i\theta_{knm}(Z) + H.c. \quad (9)$$

where $\mathscr{F}_{knm}(R)$is given by

$$\mathscr{F}_{knm}(R) = C_{nm}\cdot\frac{w_0}{w(Z)}\exp\left[-ik\frac{(X^2+Y^2)}{2R(Z)}\right]\times\exp\left[\frac{-(X^2+Y^2)}{w^2(Z)}\right]\times H_n\left(\frac{\sqrt{2}X}{w(Z)}\right)\times H_m\left(\frac{\sqrt{2}Y}{w(Z)}\right) \quad (10)$$

where $C_{nm} = \left[2/\left(2^{n+m}n!m!\pi\right)\right]^{1/2}$is the normalization constant of the Hermite-Gaussian function. θ is the phase of the mode

$$\theta_{knm}(Z) = (n+m+1)\tan^{-1}(Z/Z_R) + kZ \quad (11)$$

Here $H_n(.) and H_m(.)$ are the Hermite polynomials and w_0 the radius at which the Hermite-Gaussian beam amplitude and intensity drop to $1/e$ and $1/e^2$ of their axial values, respectively,

$$w^2(Z) = (Z_R^2 + Z^2)/kZ_R \quad (12)$$

The position $Z = 0$, referred to as the Hermite-Gaussian beam waist, corresponds to the waist size w_0 of the Hermite-Gaussian beam, such that: $w_0^2 = 2Z_R/k$. ξ_{k00} is the constant amplitude of a plane wave of the same intensity and $\hat{a}_{knm}$ is the annihilation operator for the field beam, while $H.c.$ stands for Hermitian conjugate. In order not to obscure the main purpose of this investigation by avoiding cluttered formalism, we have assumed that the Hermite-Gaussian beam has a long Rayleigh range and we ignore all beam curvature effects.

2.3 Optical forces

The optical forces due to the dipole interaction equation (7) with the different classes of electromagnetic field have been extensively analyzed [7,8,9]. However, with any electric field polarized along the x direction, the quadrupole interaction Hamiltonian equation (8) now takes the form

$$H_Q = \frac{1}{2}\left\{\hat{Q}_{xx}\frac{\partial E_x}{\partial X} + \hat{Q}_{xy}\frac{\partial E_x}{\partial Y} + \hat{Q}_{xz}\frac{\partial E_x}{\partial Z}\right\} \quad (13)$$

where $Q_{ij} = -ex_ix_j$ are the elements of the quadrupole tensor operator, which for the two-level atom can be written as

$$\hat{Q}_{ij} = Q_{ij}(\pi + \pi^\dagger) \quad (14)$$

where $Q_{ij} = \left\langle 1\left|Q_{ij}\right|2\right\rangle$ are quadrupole matrix elements between the two atomic levels.

Substituting from Eq.(9) in Eq. (13) we can write the quadrupole interaction Hamiltonian in the form

$$H_Q = \hbar\Omega^Q_{knm}(X,Y)\exp i\theta_{knm}(Z)\hat{a}_{knm} + H.c. \tag{15}$$

Here $\Omega^Q_{knm}(X,Y)$ is the complex Rabi frequency defined as follows:

$$\hbar\Omega^Q_{knm}(X,Y) = \xi_{k00}\cdot\mathscr{F}_{knm}(R)\left\{\hat{Q}_{xx}\alpha + \hat{Q}_{xy}\beta + \hat{Q}_{xz}\gamma\right\} \tag{16}$$

where

$$\alpha = \frac{1}{H_n}\frac{\partial H_n}{\partial X} - ik\frac{X}{R(Z)} - \frac{2X}{w^2(Z)} \tag{17}$$

$$\beta = \frac{1}{H_m}\frac{\partial H_m}{\partial Y} - ik\frac{Y}{R(Z)} - \frac{2Y}{w^2(Z)} \tag{18}$$

$$\begin{aligned}\gamma = {} & \tfrac{1}{H_n}\tfrac{\partial H_n}{\partial Z} + \tfrac{1}{H_m}\tfrac{\partial H_m}{\partial Z} - \tfrac{1}{R(Z)} + ik + \tfrac{i(n+m+1)}{kw^2(Z)} \\ & + \tfrac{(X^2+Y^2)}{R(Z)}\left[\tfrac{ik}{R(Z)} - \tfrac{ik}{2Z} + \tfrac{2}{w^2(Z)}\right]\end{aligned} \tag{19}$$

With both the phase and the complex Rabi frequency defined, the steady state force on the moving atom due to the laser beam is written

$$\langle F\rangle^Q_{knm} = \left\langle F^Q_{diss}\right\rangle_{knm} + \left\langle F^Q_{quad}\right\rangle_{knm} \tag{20}$$

where $\left\langle F^Q_{diss}\right\rangle_{knm}$ is the dissipative force

$$\left\langle F^Q_{diss}\right\rangle_{knm} = 2\hbar\Gamma_Q\left|\Omega^Q_{knm}(X,Y)\right|^2\times \left(\frac{\nabla\theta_{knm}(Z)}{\Delta^2_{knm}(R,V) + \left|\Omega^Q_{knm}(X,Y)\right|^2 + \Gamma^2_Q}\right) \tag{21}$$

and $\left\langle F^Q_{quad}\right\rangle_{knm}$ is the quadrupole force

$$\left\langle F^Q_{quad}\right\rangle_{knm} = -2\hbar\nabla\left|\Omega^{eq}_{knm}(X,Y)\right|^2\times \left(\frac{\Delta_{knm}(R,V)}{\Delta^2_{knm}(R,V) + \left|\Omega^Q_{knm}(X,Y)\right|^2 + \Gamma^2_Q}\right) \tag{22}$$

where Γ_Q is the relaxation rate for the quadrupole decay emission. The variable $R(t)$ now denotes the position of the atom and $V = \dot{R}$ is the velocity, both at time t, while $\Delta_{knm}(R,V)$ is the position- and velocity-dependent detuning,

$$\Delta_{knm}(R,V) = \Delta_0 - V\cdot\nabla\theta_{knm}(Z) \tag{23}$$

In these expressions Δ_0 is the static detuning defined by $\Delta_0 = \omega - \omega_0$. The dissipative force can now be understood as a quadrupole absorption followed by decay emission of the light by the atom, while the quadrupole force, which is proportional to the gradient of the Rabi frequency, is responsible for confining the atom to the center of the focus, depending on the detuning Δ. The quadrupole force is derivable from a

$$U_{qurd}(X,Y) = -\frac{\hbar\Delta_0}{2}\ln\left(1 + \frac{2\left|\Omega^Q_{knm}(X,Y)\right|^2}{\Delta^2 + \Gamma^2_Q}\right) \tag{24}$$

In experimental situations where we have large detuning $|\Delta| >> \left|\Omega^Q\right|$; $|\Delta| >> \Gamma_Q$ the quadrupole potential can be written to a good approximation as follows:

$$U_{qurd}(X,Y) \approx -\frac{\hbar}{\Delta}\left|\Omega^Q_{knm}\right|^2 \tag{25}$$

It is clear from the above expressions responsible for the steady state atomic motion that the modulus squared Rabi frequency $\left|\Omega^Q_{knm}\right|^2$ is the key factor determining the dynamics of atoms in the electromagnetic field.

By assuming that the atom is constrained to move in the XY plane and the quadrupole transition is such that $Q_{xy} = 0 = Q_{xz}$. Under these circumstances the Rabi frequency Eq. (16) takes the following form

$$\hbar\Omega^Q_{knm}(X,Y) = \xi_{k00}\cdot\mathscr{F}_{knm}(R)\left\{\hat{Q}_{xx}\tfrac{1}{H_n}\tfrac{\partial H_n}{\partial X} - ik\tfrac{X}{R(Z)} - \tfrac{2X}{w^2(Z)}\right\} \tag{26}$$

3 Numerical results and discussion

3.1 Typical parameters

For a suitable detuned atom and for small velocities, equation (24) ought to give a quadrupole potential that traps the natural atoms in the area of the minimum of the quadrupole potential . In order to obtain this, it is fine to concentrate on typical parameters. Thus, we consider the $6^2S_{1/2} \rightarrow 5^2D_{5/2}$ transition in cesium atom ($\lambda = 675nm$). In this case the quadrupole emission rate will be $\Gamma^{eq} = 7.8\times 10^5 s^{-1}$ and a quadrupole matrix element is $Q_{xx} = 10ea^2_B$ where a_B is the Bohr radius. We assume that the beam intensity $I = 5.0\times 10^9 Wm^{-2}$. As well, we assume a large value for the detuning $\Delta_0 = 10^3\Gamma_{eq}$ and the input Gaussian waist size w_0 is taken to be $w_0 = 35\lambda$. Lastly, it is convenient to define a scaling potential U_0 as follow

$$U_0 = \frac{1}{2}\hbar\Gamma_{EQ} \tag{27}$$

The scaling potential U_0 as $4.1\times 10^{-29}J$ and this value is equivalent to about $61.9kHz$. In the figure below, quadrupole potential is measured in units of U_0. Most focus here will be on quadrupole potential, because it is clear that such light field distribution is limited to use the trapping atom as it is well-known. Consequently the dissipative force will be dealt with only in this context such as the undesirable interaction operator. Thus we will set all of our focus on how to reduce its role in the destruction of the trapping process. With this in mind, we will call it from now a destabilizing dissipative force.

3.2 Single beam

The spatial distribution shown in Figure 1 *(dots curve)* is the variation of the quadrupole potential due to the single beam of order $TEM_{0,2}$ propagating along the positive X axis. It is clear that the quadrupole potential possesses well-defined maxima and minima that can be used to trap atoms that have transition frequencies appropriately detuned from the frequency ω of light. The depth of this potential is seen to be of the following order

$$U_{\min} = 4.9 \times 10^{-3} K \tag{28}$$

which is definitely not sufficiently deep to trap cesium atoms because this magnitude of potential required that the atom velocity must be less than $V \approx 3.2 \times 10^{-14} m/s$ in order to considered stably trapped presses. This degree of velocity is difficult to achieve in such context where the at least velocity due to the destabilizing dissipative force under the same conditions will be much greater (without considering the original atomic velocity). In fact, even we arrange that the light beam and atom move in the opposite direction, this will not solve this serious obstacle and it just could be give temporally very short stable trapped presses.

However, the depth of potential can be easily increased with increasing the order of the using beam. For this reason, we should pay attention towards experiential researches, the more they have generated higher order beams, it means that get a lower depth of potential. Experimentally, it was demonstrated that it is possible to generate Hermite-Gaussian beams up to the $TEM_{0,80}$ [13, 14].

In fact, the using of a single high-order beams will increase both the destabilizing dissipative force and the quadrupole potential, thus the previous obstacle is still existed as well as, this way will give rise to the problem of the quantum tunneling effect due to create large number of potential wells within the spatial beam distribution. Besides, there are alternative techniques, can be used to enhance the depth of quadrupole potential, these are co-propagating or counter-propagating beams which will possess a variety of beams distribution forms whose interference effects are predictable to enhance the depth of potential. Co-propagating and counter propagating beams generally mean they have the same wavevector : for counter propagating beams the wavevector has opposite sign, for co propagating, the same sign. Accordingly, these ways could be turn out to make quadrupole potential more exploitable in numerous applications, such as those that have been used dipole-active transitions [15, 16, 17].

3.3 Co-propagating beams

We have seen above that an atom immersed in a Hermite-Gaussian beam will experience a destabilizing dissipative force that is predominantly in the direction of propagation and a quadrupole potential in the radial direction. If a second beam is added propagating in the same direction, we have a configuration that can be referred to as the one-dimensional co-propagating beams configuration. For independent one co-propagating beams we can write the mean force on the atom as a sum of forces due to individual beams

$$\left\langle F^{diss}_{kn_1m_1+kn_2m_2} \right\rangle = \left\langle F^{diss}_{kn_1m_1} \right\rangle + \left\langle F^{diss}_{kn_2m_2} \right\rangle \tag{29}$$

$$\left\langle U^{eq}_{kn_1m_1+kn_2m_2} \right\rangle = \left\langle U^{eq}_{kn_1m_1} \right\rangle + \left\langle U^{eq}_{kn_2m_2} \right\rangle \tag{30}$$

The spatial distribution shown in Figure 1 *(dashes curve)* is the variation of the optical quadrupole potential due to the co-propagating beams for same order $TEM_{0,2}$. From this Figure *(dashes curve)*, we see that the quadrupole potential is simply double depth that of a single-mode case *(dots curve)*. It is clear that, this configuration gives rise to a potential pushing the atom towards the center of the focus. At the same time the destabilizing dissipative force (not shown) coming from both co-propagating beams is doubled as well and pushes the atoms out of the focus which can be helped the atom to escape from the potential.

To trap an atom, the longitudinal component of the quadrupole potential has to balance the destabilizing dissipative force which is very difficult to achieve in standard situation interactions as we have mentioned before. Although the obvious disadvantageous of the co-propagating technique but we indicate that in some applications, this technique can play a significant role. For example, in recent experimental work, on the optical interference between different twisted light beams has revealed a rich variety of intensity distributions to produce a so-called 'optical Ferris wheel' [18].

3.4 Counter-propagating beams

In the remainder of this work, we examine the one-dimensional counter-propagating beams configuration. This technique can be easily achieved by making the second beam propagating in the opposite direction. This configuration is usually used to overcome the destabilizing dissipative forces by making the optical molasses forces that work to cool the atoms axially. For independent counter-propagating beams we can write the mean force on the atom as a sum of forces due to individual beams

$$\left\langle F^{diss}_{kn_1m_1-kn_2m_2} \right\rangle = \left\langle F^{diss}_{kn_1m_1} \right\rangle + \left\langle F^{diss}_{-kn_2m_2} \right\rangle \tag{31}$$

$$\left\langle U^{eq}_{kn_1m_1-kn_2m_2}\right\rangle = \left\langle U^{eq}_{kn_1m_1}\right\rangle + \left\langle U^{eq}_{-kn_2m_2}\right\rangle \tag{32}$$

Figure 1 *(solid curve)* depicts the spatial distribution of the optical quadrupole potential due to the counter-propagating beams for the same order $TEM_{0,2}$. From Figure 1, we see that the counter-propagating modes *(solid curve)*, have the advantage over a single beam *(dots curve)* and the co-propagating beams *(dashes curve)*, traps in that, they can trap strongly atoms. The destabilizing dissipative force on an atom is cancelled due to symmetry of the two beams along the optical axis which can be given stably confined. At the same time, the quadrupole potential is added resulting therefore in powerful confinement of the atom in all directions compared to a single-beam trap *(dots curve)*.

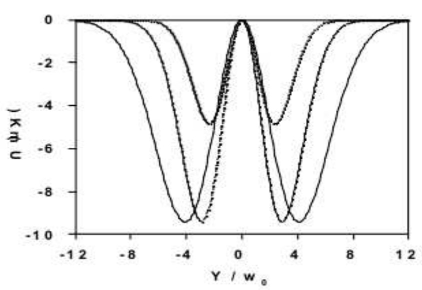

Figure 1 : *The quadrupole potential distribution in milli-degrees Kelvin (mK) for Hermite-Gaussian beam of order $n = 0, m = 2$. In the figure the different curves correspond to different configuration : single beam (dots curve); co-propagating beams (dashes curve) and counter-propagating beams (solid curve).See the text for the parameters used to generate these plots.*

Finally, we should mention that the distance between neighboring potential traps is bigger in the counter-propagating beams *(solid curve)* comparing to the co-propagating ones *(dashes curve)*. This is without doubt will give a much higher possibility of the atoms trapping process. On the other hand, we note that in Figure spaced of the potential minimum points is more in the counter-propagating beams *(solid curve)* comparing to the co-propagating ones *(dashes curve)* which means that the probability of the atoms tunnel effect will be less.

4 Conclusions

In conclusion, our analysis has shown that the interaction of Hermite-Gaussian light which possesses variety distributions can lead to significant trapping process on a two-level atoms characterized by a quadrupole-allowed transition. We have noted that in addition to the intrinsic importance of quadrupole-active transition as physical entities in their own right, some technical applications can be envisaged at this stage. In this context, we have introduced several techniques to raise the depth of quadrupole potential in order to ensure that the trapping process to be effective. In each of these techniques, we have reported the advantages and the disadvantage as well as we have clearly determined the obstacles those are appeared with each technique. In the end, it has become understandable that the counter-propagation technique was the best way to use. Mainly, it has given us a quadrupole potential multiplier along with it has eliminated the unwanted effects of the dissipative force. This has made that quadrupole interactions usually ignored for atom manipulation should to be accessible for utilization.

Here we have considered paraxial description of Hermite-Gaussian beam [19,20]. However this description is valid only for weakly focused beams, where gradients of fields are small and longitudinal electric field is zero. Consequently we are just concerned with the properties of Hermite-Gaussian beams in the transverse plane as a result of this description. For this reason, in our calculations of the quadrupole force, the longitudinal derivation of quadrupole Rabi frequency will be neglected, as it is too weak and cannot be utilized. Generally, these explain why quadrupole interaction potential is weak and is not enough for a high-quality trapping process of atoms. We have used higher order beams to overcome this problem. It is remarkable that the door is still open to search for more effective ways to support the quadrupole interactions. We think such a study might be provided an initial step towards a more comprehensive understanding of the nature of the eclectic-quadrupole interaction which could be led to further studies and tackled to investigate their influence on atoms and molecules.

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