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Transient Dynamics of Atoms Between Parallel Conducting Plates

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The purpose of this work is to investigate the cooling and trapping of two-level atom moving between two perfect conductor parallel plates. We first quantize the field. Then we explore the solutions of the Bloch equations in the dynamic regime, and we call special attention to the forces in terms of the distance between the plates. The case of Eu^{3+} is investigated in some details.

Keywords: Radiation forces, transient effects, atom-field interaction, laser cooling, trapping, micro-cavity, optical Bloch equations.

1 Introduction

The study of radiation forces on atoms is at the present time one of the most important developing fields of physics, and it is the basis for some of the most important applications of lasers. In order to investigate concretely the limits of the recent expanding research field, called laser cooling and trapping, several theoretical treatments have been proposed. The review of some of these approaches and the comparison of their advantages, difficulties and domains of validity, can be found in the literatures [1, 2]. Furthermore, the motion of atoms in spatially varying light field has also led to the emergence of atom optics as a new filed of research. The operation of a typical atom guide makes use of spatially varying light fields either in free space or confined within metallic or dielectric interfaces and relies on the principle that appropriately detuned atoms are either attracted to or repelled from regions of high intensity [3, 4]. On the other hand, planar waveguide structures have been considered. An evanescent mode parallel plate atom guide has been suggested [5] and the success was achieved in controlling atomic motion within a gold two-plate micro cavity of typical dimensions in the sub-wavelength regime and the potential on atoms between conducting plates was investigated experimentally [6].

In general the physical mechanisms of the atomic motion in light are governed by the optical Bloch equations and they are solved in the steady-state [7], to illustrate the mean

radiative forces for a two- and three-level atom. In this case the range of the validity of these solutions is assumed to be in the same order of the characteristic time required by the internal state of the atom to reach its steady state. It therefore seems appropriate to seek for an enlargement of the range of investigations in order to include more general types of problems, which are frequently met in practice. However, the dynamical aspects of the Bloch equations will be considered here. We consider in this work a two level atom in a particular cavity, namely a two infinite planar parallel perfectly conducting plates separated by a distance L in vacuum [7]. The motivation for such a structure reside essentially in the fact that this structure exhibits a possibilities of enhanced and inhibited decay rates. The cavity modes will be fundamentally quantized, to allow the position-dependent spontaneous emission rate and Rabi frequency to be evaluated.

The principal interest in such investigation arises from the theoretical difficulty in resolving the system of coupled differential equations, which is related to the Bloch equations. This difficulty can be overcome using a new separation approach of coupled differential equations [10, 11]. Using this new technique, the Bloch equations can be solved exactly for two-level atoms, and various physical quantities are investigated with the Eu^{3+} ion.

The outline of the paper is as follows: in section 2 we first introduce and review the mathematical structure of the two-level atom. Section 3 is devoted to the derivation of the different parameters which are related to the two-level atom, or a two-level atom, interacting with the vacuum fields that are now constrained by two infinite planar parallel perfectly conducting plates separated by a distance L in a vacuum. In section 4, the suggested separation approach of the original optical Bloch equations (OBE) is presented. Section 5, the emphasis is given to the case of the atomic motion in this structure. The essential features of the forces have been reported and discussed for the Eu^{3+} ion.

2 Dynamics of Bloch Equations for a Two-Level Atom

We are concerned here with the evolution of the average forces acting on a two-level atom, due to its interaction with light. We assume that the light is in the form of a coherent beam with complex amplitude α and has a plane wave distribution. The Hamiltonian of the system reads

$$H = H_{atom} + H_{field} + H_I, \tag{2.1}$$

where H_{atom} and H_{field} are the unperturbed Hamiltonian for the atom and field respectively and can be written as

$$H_{atom} = \frac{\mathbf{P}^2}{2M} + \hbar\omega_0 b^\dagger b, \tag{2.2}$$

$$H_{field} = \hbar\omega a^\dagger a, \tag{2.3}$$

where $\mathbf{P}$ is the center-of-mass momentum operator, M is the mass of the atom, ω_0 is the atomic transition frequency and $b^\dagger$ and b are the atomic raising and lowering operators. In equation (2.3), $a^\dagger$ and a are the creation and annihilation operators for a photon with energy $\hbar\omega$. The interaction Hamiltonian H_I in equation (2.1), describes the coupling of the atom to the electromagnetic field and is given in the electric dipole approximation by

$$H_I = -\mathbf{d} \cdot \mathbf{E}(\mathbf{r}). \tag{2.4}$$

At this point, it is convenient to deal with the field associated with an excited mode in the classical limit. In the rotating wave approximation, the interaction Hamiltonian may then be written as

$$H_I = -i\hbar \left[\tilde{b}^\dagger \alpha f(\mathbf{r}) - h.c.\right], \tag{2.5}$$

where $\tilde{b} = be^{i\omega t}$, $f(\mathbf{r}) = \Omega(\mathbf{r})e^{i\theta(\mathbf{r})}$ and the quantity $\Omega(\mathbf{r})$ is the Rabi frequency which represents the rate of the induced transition from the upper level to the lower and vice versa. It is given by

$$\hbar\Omega(\mathbf{r}) = \left|\alpha \left\langle \mathbf{d} \right\rangle_{12} \cdot \boldsymbol{\mathcal{F}}\right|, \tag{2.6}$$

where $\boldsymbol{\mathcal{F}}$ is the mode vector function. The forces acting on the atomic center-of-mass can be derived using the optical Bloch equations for the atomic density matrix elements once we make the semi-classical approximation which allows the gross motion of the atom to be treated classically, while maintaining a quantum treatment for the internal dynamics of the atom. Thus the position and momentum operators of the center of mass may be replaced by their expectation values $\mathbf{r}_0$ and $\mathbf{P}_0$. The density matrix associated with the internal motion of the atom $\rho(t)$ evolves with time according to the well-known relation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[H, \rho\right] + \Re\rho, \tag{2.7}$$

where $\Re\rho$ accounts for the relaxation dynamics of the atomic system. In the adiabatic approximation, the atomic velocity $\mathbf{v} = \mathbf{P}_0/M$ is assumed constant during the time taken for the dipole moment to reach its steady-state value. The position $\mathbf{r}_0$ of the atom at time t is then given by

$$\mathbf{r}_0 = \mathbf{r} + \mathbf{v}t, \tag{2.8}$$

where we have redefined $\mathbf{r}$ so that it now denotes the (updated) initial position of the atom. Thus we can write

$$f(r_0) = f(\mathbf{r} + \mathbf{v}t) \approx f(\mathbf{r})e^{i(\mathbf{v}.\nabla\theta(\mathbf{r}))t} \tag{2.9}$$

where we have assumed that the change in the field amplitude is negligible during this period of run.

Substituting the expression of H in the master equation and using the aforementioned properties of H_{atom} and H_I, we obtain the following optical Bloch equations for the atomic

density matrix elements

$$\frac{d\hat{\rho}_{11}}{dt} = 2\Gamma\rho_{22} + f(\mathbf{r})\hat{\rho}_{12} + f^*(\mathbf{r})\hat{\rho}_{21}, \tag{2.10}$$

$$\frac{d\hat{\rho}_{12}}{dt} = -(\Gamma + i\Delta(\mathbf{r},\mathbf{v}))\hat{\rho}_{12} + f^*(\mathbf{r})(\rho_{22} - \rho_{11}), \tag{2.11}$$

$$\frac{d\hat{\rho}_{21}}{dt} = -(\Gamma - i\Delta(\mathbf{r},\mathbf{v}))\hat{\rho}_{21} - f(\mathbf{r})(\rho_{11} - \rho_{22}), \tag{2.12}$$

where we use Γ for the upper-to-lower-level decay rate, $\Delta(\mathbf{r},\mathbf{V}) = \Delta_0 - \mathbf{v}.\nabla\theta(\mathbf{r})$ is the total detuning, $\Delta_0 = \omega - \omega_0$ is the detuning of the field frequency from atomic resonance, $\tilde{\rho}_{21} = \left\langle \tilde{b} \right\rangle$ and $\hat{\rho}_{21} = \tilde{\rho}_{21} e^{-it\mathbf{v}\cdot\nabla\theta(\mathbf{r})}$. Since the two levels are the only ones in the problem, the conservation of probability gives

$$\rho_{11} + \rho_{22} = 1. \tag{2.13}$$

The mean radiative force acting on the atom is defined as the average rate of change of the atomic momentum which yields

$$\langle \mathbf{F} \rangle = -\langle \nabla H_{int} \rangle . \tag{2.14}$$

After some simple algebra, the force reads

$$\langle \mathbf{F} \rangle = \langle \mathbf{F}_{reac.} \rangle + \langle \mathbf{F}_{spont.} \rangle . \tag{2.15}$$

This causes two types of forces to appear: a reactive force, proportional to the Rabi frequency gradient and to the in-phase component of the dipole

$$\langle \mathbf{F}_{react} \rangle = i\hbar \left\{ \hat{\rho}_{12}(t) f(\mathbf{r}) - \hat{\rho}_{21}(t) f^*(\mathbf{r}) \right\} \frac{\nabla\Omega(\mathbf{r})}{\Omega(\mathbf{r})}, \tag{2.16}$$

and a dissipative force, proportional to the phase gradient and to the quadrature component of the dipole

$$\langle \mathbf{F}_{dissip} \rangle = -\hbar \left\{ \hat{\rho}_{12}(t) f(\mathbf{r}) + \hat{\rho}_{21}(t) f^*(\mathbf{r}) \right\} \nabla\theta(\mathbf{r}). \tag{2.17}$$

For given initial conditions the solution of the optical Bloch equations (2.10), (2.11) and (2.12) leads formally to the determination of the forces by direct substitution in equations (2.16) and (2.17).

The steady state takes place when all time derivatives in the optical Bloch equations are set equal to zero and corresponds to the long-time limit. The steady-state solution of the optical Bloch equations can be written as

$$\langle \mathbf{F}_{dissip}(\mathbf{r},\mathbf{v}) \rangle = 2\hbar\Gamma(\mathbf{r})\Omega^2(\mathbf{r}) \left(\frac{\nabla\theta(\mathbf{r})}{\Delta^2(\mathbf{r},\mathbf{v}) + 2\Omega^2(\mathbf{r}) + \Gamma^2(\mathbf{r})} \right), \tag{2.18}$$

$$\langle F_{react}(\mathbf{r},\mathbf{v}) \rangle = -2\hbar\Omega(\mathbf{r})\nabla\Omega(\mathbf{r}) \left(\frac{\Delta(\mathbf{r},\mathbf{v})}{\Delta^2(\mathbf{r},\mathbf{v}) + 2\Omega^2(\mathbf{r}) + \Gamma^2(\mathbf{r})} \right). \tag{2.19}$$

In our recent paper [10] we have recognised that there were some special cases of interest where the original optical Bloch equations can be solved analytically. Then the transient effects can be incorporated using our treatment. We shall report and discus the results of these phenomena in the last section.

3 Two-Level Atom Between Conducting Plates

We consider a two-level atom, interacting with the vacuum fields that are now constrained by two infinite planar parallel perfectly conducting plates separated by a distance *L* in a vacuum (see figure 3.1). Due to rotational symmetry in the plane of the plates, we define a parallel coordinate specified by planar vector $\mathbf{r}_{\parallel}$($r_{\parallel}^2 = x^2 + y^2$), the parallel wave vector is described in the plane of the plates by the vector $\mathbf{k}_{\parallel}$ ($k_{\parallel}^2 = k_x^2 + k_y^2$) and the remaining component k_z describes the wave vector perpendicular to the plates, in the z-direction.

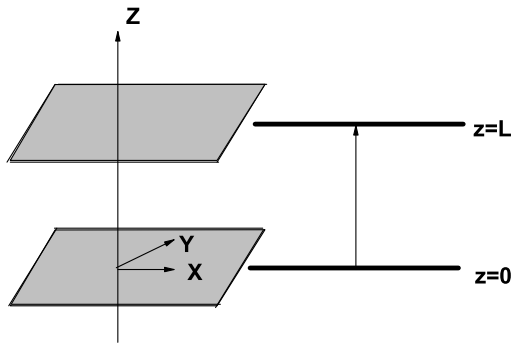

Figure 3.1: Schematic drawing of the two conducting plates separated by a distance L.

In this case, the structure leads to two types of physical effects. First, the spontaneous dipole emission rate of the atom is modified and, second, the atom experiences energy shifts to both levels [7]. In this paper we are concerned with the first one, for the details about the second effect can be found in [7]. Using Fermi's Golden Rule [8,9]with this symmetric structure, the spontaneous emission rate is independent on the coordinate $\mathbf{r}_{\parallel}$ and can be evaluated for the two possible orientations for a dipole situated at an arbitrary point (0, z),

$$\Gamma_{\parallel}(z) \quad = \quad \Gamma_o \sum_{n=0}^{[2L/\lambda]} \frac{3\lambda}{4L} \left[1 + \left(\frac{n\lambda}{2L}\right)^2\right] \sin^2\left(\frac{n\pi}{L} z\right), \tag{3.1}$$

$$\Gamma_{\perp}(z) = \Gamma_o \left\{ \frac{3\lambda}{4L} + \sum_{n=1}^{[2L/\lambda]} \frac{3\lambda}{2L} \left[1 - \left(\frac{n\lambda}{2L} \right)^2 \right] \cos^2 \left(\frac{n\pi}{L} z \right) \right\}, \tag{3.2}$$

where Γ_0 is the free-space decay rate, λ is the free space transition wavelength and $[2L/\lambda]$ stands for the integer part of the bracketed quantity. Both expressions for the spontaneous emission rate are functions of the ratio L/λ and the spatial position z within the structure (see figure 3.2).

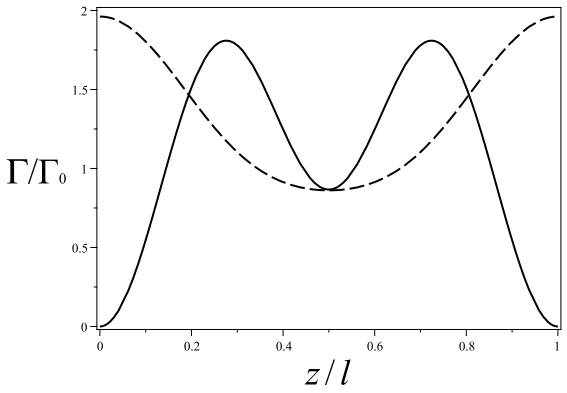

Figure 3.2: Distribution plots for the spontaneous emission rate for Eu^{3+} between infinite parallel plates. These plots show the variations of Γ/Γ_0 with the position of the ion within the structure. The solid curve is the rate for a dipole parallel to the plates. The dashed curve is the rate for a dipole normal to the plates.

On the other hand, the position-dependent Rabi frequency $\Omega(z)$, which characterizes the interaction of an atom with the electric fields and for the TM_1 excited mode, Ω for two possible orientations is given by [12]

$$\Omega_{\parallel}(z) = \sqrt{2}\Omega_0 \left(\frac{\lambda}{2L} \right) \sin \left(\frac{\pi}{L} z \right), \tag{3.3}$$

$$\Omega_{\perp}(z) = \sqrt{2}\Omega_0 \left(1 - \frac{\lambda^2}{4L^2} \right) \cos \left(\frac{\pi}{L} z \right), \tag{3.4}$$

where Ω_0 is the free-space decay rate.

The last factor which must be considered in this structure is the laser light detuning and for such system the dynamic detuning is given in [12] by the following expression

$$\Delta(k_{\parallel}, n, \mathbf{v}) = \Delta_0 - k_{\parallel} v_{\parallel}, \tag{3.5}$$

where the static detuning $\Delta_0 = \omega_0 - \omega(k_{||}, n)$**.**

These different factors will be introduced in the original optical Bloch equations which can be solved by using the treatment developed in [10, 11], we outline here the essential steps for the solution.

4 Treatment of the Original Optical Bloch Equation

It is possible to make the atom density matrix equations of motion (2.8, 2.9, 2.10) resemble those for a magnetic dipole undergoing precession in a magnetic field. This approach has value not only in solving the equations, but also in providing a physical picture of the density matrix in motion. The equations we derive here are equivalent to the Bloch equations treated in [10, 11]. On the other hand, for several useful situations (e.g. upper-to-ground-lower-level decay of (2.8, 2.9, 2.10), a single level decay constant is a good approximation so that the Bloch model is accurate and may be easier to use. The solutions obtained are used in the discussion of light forces in the considered structure.

We introduce the real quantities

$$U = \hat{\rho}_{21} + c.c, \tag{4.1}$$

$$V = i\hat{\rho}_{21} + c.c, \tag{4.2}$$

$$W = \rho_{22} - \rho_{11}, \tag{4.3}$$

in terms of which

$$\hat{\rho}_{21} = \frac{1}{2}(U - iV). \tag{4.4}$$

These quantities vary little in an optical period and are the components of the vector Bloch **B**. Taking derivatives of (4.1, 4.2, 4.3) and using (2.12, 2.13), we find the Bloch equations

$$\dot{U} = -\Gamma U + \Delta(\mathbf{r}, \mathbf{v})V + 2\Re(f(\mathbf{r}))W, \tag{4.5}$$

$$\dot{V} = -\Gamma V - \Delta(\mathbf{r}, \mathbf{v})U - 2\Im(f(\mathbf{r}))W, \tag{4.6}$$

$$\dot{W} = -2\Gamma(W + 1) - 2\Re(f(\mathbf{r}))U + 2\Im(f(\mathbf{r}))V. \tag{4.7}$$

Here $\dot{W}$ equation is written for the case of upper to lower level decay. This system can be written in matrix form as

$$\begin{pmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{pmatrix} = \begin{pmatrix} -\Gamma & \Delta & 2\Re(f) \\ -\Delta & -\Gamma & -2\Im(f) \\ -2\Re(f) & 2\Im(f) & -2\Gamma \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -2\Gamma \end{pmatrix}. \tag{4.8}$$

These equations are similar to those of Bloch studied in [10, 11]. The same treatment can be use here to obtain the solutions of this linear system.

This system can be written in the matrix form as

$$\dot{\mathbf{B}} = \mathbf{M}\mathbf{B} + \mathbf{C}, \tag{4.9}$$

where **B** is the Bloch vector, **M** is a non-diagonal (3×3) matrix and **C** is an inhomogeneous vector. The general solution of the linear coupled system (4.8) is

$$\mathbf{B} = \mathbf{B}_h + \mathbf{B}_p, \tag{4.10}$$

where $\mathbf{B}_h$ is the homogenous solution of the system without the term $\mathbf{C}$ and $\mathbf{B}_p$ is the particular solution. By choosing $\dot{\mathbf{B}} = 0$ in the system (4.8), the particular solution can be obtained $\mathbf{B}_p = \mathbf{M}^{-1}\mathbf{C}$ which is the steady-state solution of given Equations (2.8), (2.9) and (2.10).

Consider now the homogeneous system

$$\mathbf{B}_h = \mathbf{MB}. \tag{4.11}$$

The eigenvalues of the homogeneous system satisfy the characteristic equation

$$(\Gamma + \lambda)^2(2\Gamma + \lambda) + 4\left|f(\mathbf{r})\right|^2(\Gamma + \lambda) + \Delta(\mathbf{r}, \mathbf{v})^2(2\Gamma + \lambda) = 0, \tag{4.12}$$

where $\left|f(\mathbf{r})\right|^2 = \left|\Omega(\mathbf{r})\right|^2$is the Rabi frequency, as we have seen that the Rabi frequency depends only on the z-coordinate, then the solutions depend on the same component. In fact, the eigenvalues give us all the information we need to know about how the homogeneous solution behaves in time. It is possible that two of the eigenvalues can be complex numbers that if $\lambda_2 = \lambda_3^* = a \pm ib$ then the homogeneous solutions can be rearranged so that they are of the form

$$h(t) = A_{h1}e^{\lambda_1 t} + A_{h2}e^{at}\cos(bt) + A_{h3}e^{at}\sin(bt), \tag{4.13}$$

where $h(t)$ stands for either U, V or W, and the A's coefficients are constants to be determined from the initial conditions.

5 Results and Conclusion

Here we emphasize the sub-wavelength cavity dimensions in the parallel-plate context. Our aim is to describe the forces that act on the ion, the manner in which such forces change the state of motion in the steady state and the transient optical regime. The state of the motion of the ion between the plates can be altered by the excitation of one or more cavity modes. With one of the modes excited with sufficient intensity, the ion experiences radiation forces of the type familiar in the case of ions subject to spatially varying light in free space [10, 11]. On the other hands, the solutions of the optical Bloch equations show that the dipole moment and hence the forces acting on the ion relax to steady state values outside a time of the order $5\Gamma^{-1}$ from the instant at which the light field is switched on [10, 11]. Since this time is typically much greater than the observation times involved in the channeling of ions through the structure, we need to investigate the effects of the transient state forces.

To show the efficacy of our treatment and the effects of the transient regimes, we shall focus on the dissipative forces. The explicit form of the dissipative force is given in equation (2.17) which contains the factor $\boldsymbol{\nabla}\theta$ corresponding to the momentum imparted by light

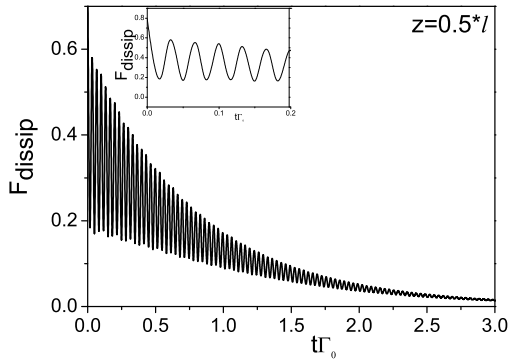

Figure 5.3: Dynamic dissipative force F_{dissip} (in units of $\hbar k\Omega_0$) at the center of the guide as a function of time, when the p-polarization cavity mode is excited and the electric dipole matrix is oriented parallel to the plates.

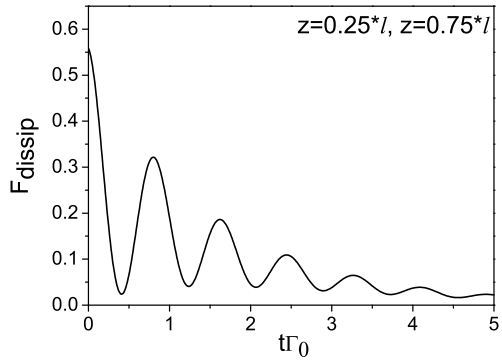

Figure 5.4: Dynamic dissipative force F_{dissip} (in units of $\hbar k\Omega_0$) at different position between the plates when the p-polarization cavity mode is excited and the electric dipole matrix is oriented parallel to the plates.

to the ion. However, in view of equations of the electric field inside this structure [7], the mode phase for all modes is given by $\theta = \mathbf{k}_{\|}.\mathbf{r}_{\|}$. Therefore, the gradient of the mode phase is $\vec{\nabla}\theta = k_{\|}$. This immediately fixes the direction of the dissipative force as parallel to the plates in the direction of the mode propagation

$$\langle F_{dissip} \rangle = -\hbar \left\{ \hat{\rho}_{12}(t) f(r) + \hat{\rho}_{21}(t) f^*(r) \right\} k_{\|}. \tag{5.1}$$

To point out the transient effects we need to consider the transitions with a long upper-

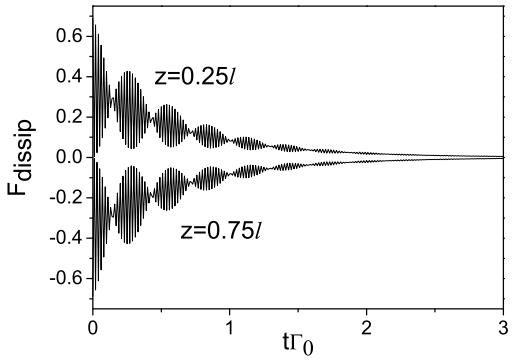

Figure 5.5: Dynamic dissipative force F_{dissip} (in units of $\hbar k\Omega_0$) at different position between the plates, when the p-polarization cavity mode is excited and the electric dipole matrix is oriented normal to the plates.

state lifetime, and rare-earth ions are good examples of this. In particular, we consider a Eu^{3+} ion in the calculations with the following parameters [11]: λ=614 nm, $\Gamma_0 = 1111.11\ \mathrm{s}^{-1}$, $\Omega_0 = 3.878 \times 10^7\ \mathrm{s}^{-1}$, $k = 1.023 \times 10^7\ \mathrm{m}^{-1}$, and in the case $\Delta(r, v) = 0$, with the initial condition V(0)=0. In figures 5.3 and 5.4 we display the dissipative force in terms of time at different positions of the ion between two parallel plates and for the case when the p-polarization cavity mode is excited and the electric dipole matrix is oriented parallel to the plates. The oscillatory behavior persists again and the time of the transient effects is about three time of the decay rate. Figure 5.3 shows the dynamic dissipative force acting on $Eu^{3+}$when the p-polarization cavity mode is excited and the electric dipole moment matrix is oriented normal to the plates for different position of the ion inside the cavity. The remarkable point in these cases is the decay of the force in an oscillatory manner, with manifestation of the beats phenomena. On the other hand, we mention here that the dissipative force between the plates is evaluated at some position. If the atom moves then it acquires velocity, which means its position changes with time and the dynamics detuning delta also changes with velocity, so the forces are evaluated at fixed positions as time passes, but the atom is not there, as it has moved on. This important point may be examined in details by solving the classical motion of the center of mass of the atom. The calculations are in progress and can be reported elsewhere.

In summary this approach can be extended to the particular cases of translational and rotational aspects of atomic motion in laser light and cold laser-driven molecules in different structure of cavity where the transient regime plays a crucial role and gives rise to new aspects in the subsequent dynamics.

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