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Operator-based Robust Nonlinear Tracking Control for A Human Multi-joint Arm-like Manipulator with Unknown Time-varying Delays

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Abstract: In this paper, an operator-based robust nonlinear tracking control for a human multi-joint arm-like manipulator with unknown time-varying delays is proposed by using robust right coprime factorization approach and a forward predictive operator. That is, first, considering the uncertainties of dynamic model consist of measurement error and disturbances, a nonlinear feedback control scheme is designed to eliminate effect of uncertainties. Second, an operator controller based on real measured data from human multi-joint arm viscoelasticity is presented to obtain desired motion mechanism of human multi-joint arm viscoelastic properties, the unknown time-varying delays are described by a delay operator, and the forward predictive operator is designed to compensate the effect of CNS during human multi-joint arm movements and to remove the effect of unknown time-varying delays while the real on-line measured data of human multi-joint arm viscoelasticity is fed to the designed operator controller. Finally, the effectiveness of the proposed design scheme is confirmed by the simulation results based on experimental data.

Keywords: Manipulator, human multi-joint arm viscoelasticity, robust right coprime factorization, time-varying delays, forward predictive operator.

1. Introduction

Compared to a manipulator, a human multi-joint arm can exhibit outstanding manipulability by adjusting dynamic characteristics of its musculoskeletal system in executing various tasks. Therefore, the development of dexterous manipulators which are as dexterous as possible like the human multi-joint arms based on the dynamic characteristics of human multi-joint arms has aroused people's growing interests [1-3]. However, till now, how to obtain and use the online dynamic characteristics of human multijoint arms is still a challenge issue in building the human multi-joint arm-like manipulators.

Many of the methods have been used to control the manipulators. The simplest controller for the manipulators is the Proportional Integral Derivative (PID) controller. In general, this type of controller is designed on the basis that the manipulator model is composed of independent coupled dynamic (differential) equations. While this controller is widely used in industrial manipulators, depending on the task to be carried out, they do not always result in the best performance. To improve performance, many of the concepts that are used to describe the motion mechanism of human multi-joint arm have been borrowed from the manipulators [4-7]. Among these concepts, equilibriumpoint control hypothesis, feedback and feedforward control, internal models are especially relevant for the present review. Based on above concepts, some existing methods such as learning control, optimal control and adaptive control have been used to design appropriate control rules [8-10]. However, the learning control rules based on a reinforcement learning algorithms are difficult due to the combinatorial explosion of the states and control action spaces, the optimal control theory also requires a precise model, but system identification against the complex robotic manipulator system having nonlinear dynamics and high dimensional state space is difficult. To obtain a precise model and use optimal control theory, several trajectory formation models have been proposed for human multi-joint arm movement control, such as minimum jerk model [11], min-

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imum torque change model [12], and minimum muscle tension-change model [13]. A variety of given trajectories can be generated based on the optimal criterions of these models in the presence of learned information pairs. However, to generate different speed movement trajectories for the same motion task, information pairs for different speed movements would need to be learned and saved in advance. Therefore, it is also difficult to achieve online control.

Although a human can control his multi-joint arm flexibly and robustly, controlling such complex system by existing control methods would be difficult because of its complexity. As an alternative to such theoretical methods, studies on control methods imitating biological system can be considered. A human-like robotic arm control method based on the attractor selection model by imitating the anatomy of bones and muscles in human arm was proposed [14], owing to not depending on the control target model, this method can respond to unpredictable disturbance or environmental change and control performance may not be optimal. Tsuji and Tanaka investigated human hand impedance in preparation for task operations, and discussed a bio-mimetic impedance control of robotic manipulators for contact tasks [15]. Moreover, due to not considering the motion mechanism of CNS, the manipulators would not move skillfully and smoothly like the human multi-joint arm, and movements would tend to be jerky and inaccurate. During human multi-joint arm movements, it is widely believed that CNS first plans for a desired response trajectory, which is converted into appropriate commands to be generated by the musculoskeletal system, mechanical properties of musculoskeletal can be mainly described as viscoelasticity of human multi-joint arm [16, 17]. The viscoelasticity of human multi-joint arm consists of joint stiffness and viscosity, which are adjusted by CNS to make the human multi-joint arm adapt to the external environment or moving objects. If the viscoelasticity properties during human multi-joint arm movements can be used effectively in designing human arm-like manipulator, there is a possibility that skillful strategies of human multi-joint arm can be integrated into the manipulator control. However, fast and coordinated arm movements cannot be executed solely having joint stiffness and viscosity [5, 18]. To coordinate movement, the CNS can be considered as a forward predictive model and a smith predictor [18], and usually imitated through a number of learning algorithms [19]. The CNS is not necessary for movement, but without it movements of the human multi-joint arms tend to jerky, tremulous, and inaccurate. Therefore, how to use the real data from viscoelastic properties during human multi-joint arm movements and compensate the effect of CNS are essential in building the dexterous human multijoint arm-like manipulators.

As a continuous research, several online measuring methods of human multi-joint arm viscoelasticity have been proposed in our former researches [20-22]. Moreover, based on the obtained experimental data of human multi-joint arm viscoelasticity, two control schemes to the manipulator have been studied [23, 24]. It is well known that muscle intrinsic mechanical properties produce stiffness and viscosity gains without delay. However, it produces unknown time-varying delays while the real measured data of human multi-joint arm viscoelasticity is fed to the designed controller based on human multi-joint arm viscoelastic properties. The unknown time-varying delays are not considered in [23, 24]. Moreover, the manipulator is a highly nonlinear and dynamically coupled system, which is subject to disturbances and model uncertainties. Many approaches have been proposed to develop controllers that are more robust so that their performance is not sensitive to modeling errors. Especially, robust right coprime factorization approach has attracted much attention due to its convenient in researching input-output stability problems of nonlinear system with uncertainties [25-30]. For the time-varying delay, many control methodologies have been proposed to deal with the control synthesis of delayed systems [31, 32], such as Smith predictor method, observer-based control method, Lyapunov function based approach and so on. However, most of the existing control methods are based on constant time delay or a known upper bound on it, or the designed system is linear. As a result, these solutions do not allow for the direct use in controller design process of nonlinear systems with time-varying delays. Therefore, in this paper, considering the model uncertainties and unknown time-varying delays, a robust stable tracking control based on robust right coprime factorization approach for the manipulator is investigated, the real measured data from viscoelasticity of human multi-joint arm is used in designing controller, the unknown time-varying delays are described by a delay operator, and a forward predictive operator is used to compensate the effect of CNS and remove the effect of unknown time-varying delays.

The outline of the paper is given as follows. Operator theorem and robust right coprime factorization, manipulator dynamic model, human multi-joint arm dynamic model and problem statement are described in Section 2. How to deal with model uncertainties and how to describe the property of time-varying delay are introduced, and robust stable tracking control scheme for the manipulator with unknown time-varying delays is proposed in Section 3. Simulation results based on experimental data are shown in Section 4, and Section 5 is the conclusions.

2. Preliminaries and problem statement

2.1. Operator theorem and robust right coprime factorization

Let *X* and *Y* be linear spaces over the field of real numbers, and let *X_s* and *Y_s* be normed linear subspaces, called the stable subspaces of *X* and *Y*, respectively, defined suitably by two normed linear spaces under certain norm $X_s = \{x \in X : ||x|| < \infty\}$ and $Y_s = \{y \in Y : ||y|| < \infty\}$. Generally, an operator $Q: X \to Y$ is said to be bounded input bounded

$$u(X,Y) = \{M : M \in S(X,Y)\}$$
 (1)

where, *M* is invertible with $M^{-1} \in S(Y,X)$. Elements of u(X,Y) are called unimodular operators.

Next, generalized Lipschitz operator is introduced, which is defined on extended linear space. Thus, extended normed linear space, or simply, extended linear space is noted firstly.

Let Z be the family of real-valued measurable functions defined on $[0,\infty)$, which is a linear space. For each constant $T \in [0,\infty)$, let P_T be the Projection operator mapping from Z to another linear space, Z_T , of measurable functions such that

$$f_T(t) := P_T(f)(t) = \begin{cases} f(t), \ t \le T \\ 0, \ t > T \end{cases}$$
(2)

where, $f_T(t) \in Z_T$ is called the truncation of f(t) with respect to *T*. Then, for any given Banach space *X* of measurable functions, set

$$X^e = \{ f \in \mathbb{Z} : \| f_T \|_X < \infty, \text{ for all } T < \infty \}$$
(3)

Obviously, X^e is a linear subspace of Z. The space so defined is called the extended linear space associated with the Banach space X.

We note that the extended linear space is not complete in norm in general, and hence not a Banach space (complete normed vector space), but it is determined by a relative Banach space. The reason of using extended linear space is that all the control signals are finite time-duration in practice, and many useful techniques and results can be carried over from the standard Banach space X to the extended space X^e if the norm is suitably defined.

Definition 2. Let X^e and Y^e be extended linear spaces associating respectively with two given Banach spaces Xand Y of measurable functions defined on the time domain $[0,\infty)$, and let D be a subset of X^e . A nonlinear operator $Q: D \to Y^e$ is called a generalized Lipschitz operator on Dif there exists a constant L such that

$$\left\| [Q(x)]_T - [Q(\tilde{x})]_T \right\|_Y \le L \|x_T - \tilde{x}_T\|_X$$
(4)

for all $x, \tilde{x} \in D$ and for all $T \in [0, \infty)$. Note that the least such constant *L* is given by the norm of *Q* with

$$\begin{aligned} \|Q\|_{Lip} &:= \|Q(x_0)\|_{Y} + \|Q\| \\ &= \|Q(x_0)\|_{Y} + \sup_{T \in [0,\infty)} \sup_{\substack{x, \bar{x} \in D \\ x_T \neq \bar{x}_T}} \frac{\|[Q(x)]_T - [Q(\bar{x})]_T\|_{Y}}{\|x_T - \bar{x}_T\|_{X}} \end{aligned}$$
(5)

for any fixed $x_0 \in D$.

We remark that the family of standard Lipschitz operator and generalized Lipschitz operator are not comparable since they have different domains and ranges. The definition of generalized Lipschitz operator has been proved more useful than standard Lipschitz operator for nonlinear system control and engineering in the considerations of stability, robustness, uniqueness of internal control signals. For any operators defined throughout the paper, they are always assumed to be generalized Lipschitz operators. For simplicity, Lipschitz operator is always mean the one defined in generalized case in this paper.

Based on the concept of Lipschitz operator, an operatorbased nonlinear feedback control system with uncertainty shown in Fig. 1 was considered in [26], where, Uand Y are used to denote the input and output spaces of a given plant operator P, i.e., $P: U \rightarrow Y$, r and y are the reference input and output of the system, respectively. The nominal plant and uncertainty are P and ΔP , respectively, and the real plant $\tilde{P} = P + \Delta P$. The right factorization of the nominal plant P and the real plant \tilde{P} are $P = ND^{-1}$, $P + \Delta P =$ $(N + \Delta N)D^{-1}$, where N, ΔN , and D are stable operators, D is invertible, ΔN is unknown but the upper and lower bounds are known. Moreover, the factorization is said to be coprime, or P is said to have a right coprime factorization, if there exist two stable operators $A: Y \rightarrow U$ and $B: U \rightarrow U$ satisfying the Bezout identity,

$$AN + BD = M \tag{6}$$

where, B is invertible, $M \in u(W, U)$ is unimodular operator. Under the condition of (6), if

$$\begin{cases} A(N + \Delta N) + BD = \tilde{M} \in u(W, U) \\ \|(A(N + \Delta N) - AN)M^{-1}\| < 1 \end{cases}$$
(7)

the BIBO stability of the nonlinear feedback control system with uncertainty can be guaranteed, that is, the system has the robust stability property, where, $\tilde{M} \in u(W, U)$ is unimodular operator, and $\|\cdot\|$ is Lipschitz operator norm.



Fig. 1 A nonlinear feedback control system with uncertainty

Similarly, operator theory based robust control for multiinput multi-output (MIMO) nonlinear systems with uncertainties shown in Fig. 2 was considered in [27], where, $P = (P_1, \dots, P_n)$ is nominal plant and $\Delta P = (\Delta P_1, \dots, \Delta P_n)$ is uncertainty. Suppose that the nominal plant P and real plant $\tilde{P} = P + \Delta P$ have right factorization as $P = ND^{-1}$ and $\tilde{P} = \tilde{N}D^{-1} = (N + \Delta N)D^{-1}$, namely

$$\begin{cases} P_i = N_i D_i^{-1}, & i = 1, 2, \cdots, n \\ P_i + \Delta P_i = (N_i + \Delta N_i) D_i^{-1}, & i = 1, 2, \cdots, n \end{cases}$$
(8)



where N_i , ΔN_i and D_i are stable such that D_i^{-1} is invertible, ΔN_i is unknown but the upper and lower bounds are known. Consequently, for the MIMO nonlinear system with uncertainties and coupling effects, the Bezout identity of the nominal plant and the real plant are $A_iN_i + B_iD_i =$ $M_i \in S(W,U)$, $A_i(N_i + \Delta N_i) + B_iD_i = \tilde{M}_i \in S(W,U)$, respectively, then the robust BIBO stability can be guaranteed provided that

$$\left\| \left[A_i(N_i + \Delta N_i) - A_i N_i \right] M_i^{-1} \right\| < 1, \ i = 1, 2, \cdots, n \quad (9)$$



Fig. 2 A MIMO nonlinear feedback system with uncertainties

It's worth to mention that the initial state should also be considered, that is, $AN(w_0, t_0) + BD(w_0, t_0) = M(w_0, t_0)$ should be satisfied. In this paper, $t_0 = 0$ and $w_0 = 0$ are selected.

2.2. Manipulator dynamic model

Two-link rigid manipulator dynamics in the horizontal plane can be generally modeled by the following second-order nonlinear differential equation [3]:

$$\mathbf{M}(\boldsymbol{\theta})\boldsymbol{\theta} + \mathbf{H}(\boldsymbol{\theta},\boldsymbol{\theta}) = \boldsymbol{\tau} \tag{10}$$

where, θ , $\dot{\theta}$, and $\ddot{\theta}$ denote angle position, velocity and



Fig. 3 Structure of the two-link manipulator

$$\mathbf{M} = \begin{bmatrix} Z_1 + 2Z_2 \cos \theta_2 & Z_3 + Z_2 \cos \theta_2 \\ Z_3 + Z_2 \cos \theta_2 & Z_3 \end{bmatrix}$$
(11)

$$\mathbf{H} = \begin{bmatrix} -Z_2 \sin \theta_2 (\dot{\theta_2}^2 + 2\dot{\theta_1} \dot{\theta_2}) \\ Z_2 \dot{\theta_1}^2 \sin \theta_2 \end{bmatrix}$$
(12)

where

$$\begin{cases} Z_1 = m_1 l_{g1}^2 + m_2 (l_1^2 + l_{g2}^2) + I_1 + I_2 \\ Z_2 = m_2 l_1 l_{g2} \\ Z_3 = m_2 l_{g2}^2 + I_2 \end{cases}$$
(13)

 m_i denotes the masses of link *i*, l_{gi} denotes the distance between the joint *i* and to the center of mass of link *i*, I_i denotes the moment of inertia of link *i* about the center of mass, and l_i denotes the length of link *i*. Z_1 , Z_2 and Z_3 are the structural parameters which denote physical feature of manipulator.

The manipulator dynamics given in (10) have a main property, that is, the inertia matrix is symmetric and positive definite, and satisfies the following inequalities:

$$\alpha_1 ||z||^2 \le z^T \mathbf{M}(\boldsymbol{\theta}) z \le \alpha_2 ||z||^2 \quad \forall z \in \mathbb{R}^n$$
(14)

where $\alpha_1, \alpha_2 \in R$ are positive constants and $\|\cdot\|$ denotes the standard Euclidean norm.

2.3. Human multi-joint arm dynamic model

Two-link rigid human arm dynamics in the horizontal plane can be generally modeled by the following second-order nonlinear differential equation [17, 20]:

$$\mathbf{M}_{A}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{H}_{A}(\dot{\mathbf{q}},\mathbf{q}) = \tau_{A}(\dot{\mathbf{q}},\mathbf{q},\mathbf{u})$$
(15)

where, **q**, **q̇**, and **q̈** denote angular position, velocity and acceleration vectors, respectively, and, $\mathbf{q} = [\theta_s(t), \theta_e(t)]^T$, $\theta_s(t)$ is shoulder joint angular and $\theta_e(t)$ is elbow joint angular, the subscripts *s* and *e* denote shoulder joint and elbow joint, respectively. $\tau_A = [\tau_s, \tau_e]^T$ is the multi-joint muscle generated torque, which can be represented as a function of angular position, velocity, and motor command **u** descending from CNS. **M**_A and **H**_A have the same structure as **M** and **H** in (10), respectively.

According to human arm dynamic model (15), the following relationship can be obtained by perturbation [17],

$$\delta \tau_A = -\mathbf{R}_A(t) \delta \dot{\mathbf{q}} - \mathbf{K}_A(t) \delta \mathbf{q} + \frac{\partial \tau_A}{\partial \mathbf{u}} \delta \mathbf{u} \qquad (16)$$



where $\mathbf{R}_A(t)$ and $\mathbf{K}_A(t)$ represent multi-joint viscosity matrix (2 × 2) and multi-joint stiffness matrix (2 × 2), respectively, and

$$-\frac{\partial \tau_A}{\partial \dot{\mathbf{q}}} \equiv \mathbf{R}_A(t) = \begin{pmatrix} R_{A-ss} & R_{A-se} \\ R_{A-es} & R_{A-ee} \end{pmatrix}$$
(17)

$$-\frac{\partial \tau_A}{\partial \mathbf{q}} \equiv \mathbf{K}_A(t) = \begin{pmatrix} K_{A-ss} & K_{A-se} \\ K_{A-es} & K_{A-ee} \end{pmatrix}$$
(18)

the subscript *ss* of $\mathbf{R}_A(t)$ and $\mathbf{K}_A(t)$ represent the shoulder single-joint effect on each coefficient, *se* and *es* denote cross-joint effects, *ee* denotes the elbow single-joint effect.

2.4. Problem statement

If the viscoelastic properties of human multi-joint arm can be used in the manipulator control, there is a possibility that human skillful strategies can be integrated into the manipulator moving. Form (16), we can find that the control input torque vector can be assumed to be a function of multi-joint viscosity matrix $\mathbf{R}_A(t)$, multi-joint stiffness matrix $\mathbf{K}_{A}(t)$, and motor command **u** descending from CNS during human multi-joint arm movements, and can be assumed to be divided into two parts $\tau_A(\dot{\mathbf{q}},\mathbf{q})$ and $\tau_A(\mathbf{u})$. Therefore, the motion mechanism of human multi-joint arm viscoelasticity and the dynamics of CNS can be considered in building human multi-joint arm-like manipulator which moves like the human multi-joint arm. If we design the same structural parameters as them of a human multi-joint arm in building human multi-joint arm-like manipulator, the real time-varying measured human multijoint arm viscoelasticity can be used to obtain desired motion mechanism of human multi-joint arm-like viscoelasticity. However, the term of $\frac{\partial \tau_A}{\partial u} \delta u$ related to the effect of CNS of (16) is not considered in measuring viscoelasticity of human multi-joint arm. To make the human armlike robot move accurately and smoothly, the control input torque vector related to the effect of CNS need also to be compensated. Moreover, it produces unknown timevarying delays while the real measured data of human multijoint arm viscoelasticity is fed to the designed controller based on motion mechanism of human multi-joint arm viscoelasticity, the unknown time-varying delays also need to be considered.

The objective of this paper is to design the human multijoint arm-like manipulator based on the motion mechanism of human multi-joint arm viscoelasticity and the dynamics of CNS and to make it move like the human multijoint arm. Based on robust right coprime factorization approach, an operator-based robust nonlinear tracking control scheme is proposed. The operator controller using the real measured data from human multi-joint arm viscoelasticity is designed to obtain desired motion mechanism, the unknown time-varying delays are described by a delay operator, and the forward predictive operator is designed to compensate the effect of CNS to remove the effect of unknown time-varying delays. The robust stability and output tracking performance of the present control system are discussed.

3. Proposed robust nonlinear tracking control scheme

In this section, an operator-based robust nonlinear output tracking control for the manipulator with human multijoint arm-like viscoelastic properties and unknown timevarying delays is considered by using robust right coprime factorization approach. Firstly, the control scheme design is presented.



Fig. 4 The proposed operator-based robust nonlinear tracking control system

According to the dynamic model of manipulator, we can see that the two-input two-output process is nonlinear system with coupling effect. Generally, it is difficult to obtain the real values in modeling the structural parameters of manipulator. Therefore, there exists the measurement error between the actual value of structural parameters and the value obtained by measurement. Moreover, there exist unpredictable disturbances in the control system. In this paper, the measurement error and disturbances are considered to be uncertainties of model. For the manipulator with uncertainties, an operator-based robust nonlinear output tracking control scheme is proposed by using robust right coprime factorization and forward predictive operator (see Figure 4), where, the overall plant is $\tilde{P} = (\tilde{P}_1, \tilde{P}_2)$, which includes two parts, the nominal plant $P = (P_1, P_2)$ and the uncertain plant $\Delta P = (\Delta P_1, \Delta P_2)$, namely, $\tilde{P} = P + \Delta P$. The nominal plant P and overall plant \tilde{P} are assumed to have right factorization as $P_i = N_i D_i^{-1}$ (i=1, 2) and $\tilde{P}_i = P_i + \Delta P_i = (N_i + \Delta N_i)D_i^{-1}$ (i=1, 2), respectively, N_i , ΔN_i , and D_i (i=1, 2) are stable operators, D_i is invertible, ΔN_i is unknown but the upper and lower bounds are known. For the manipulator, right factorizations N and D of manipulator dynamics are shown as the following forms,

$$\begin{cases} D_i(\boldsymbol{\omega}_i)(t) = \mathbf{M}_i(\boldsymbol{\omega}(t))\ddot{\boldsymbol{\omega}}_i(t) + \mathbf{H}_i(\dot{\boldsymbol{\omega}}(t), \boldsymbol{\omega}(t)) \\ N_i(\boldsymbol{\omega}_i)(t) = \boldsymbol{\omega}_i(t), \quad i = 1,2 \end{cases}$$
(19)



 $r = (\theta_{1d}, \theta_{2d})$ and $y = (\theta_1, \theta_2)$ are the reference input and plant output, respectively, $u = (u_1, u_2)$ is joint torque control input. $\hat{r} = (\hat{\theta}_1, \hat{\theta}_2)$ is an estimated value of r, and is represented as the rapid prediction of movement. In this paper, for brevity, we limit the problem on the case of $\hat{r} = r$. The operator controllers S and R are designed to eliminate effect of uncertainties, $u^* = (u_1^*, u_2^*)$ is the input of the equivalent plant P^* . The controller B is stable operator and invertible, and is designed to obtain desired motion mechanism of human multi-joint arm viscoelasticity using online time-varying measured human multi-joint arm viscoelasticity. While the real online measured data of human multi-joint arm viscoelasticity is fed to the designed operator controller B, the unknown time-varying delays will be produced. In the proposed control scheme, the delays are also considered. F is the forward predictive operator, and is designed to compensate the effect of CNS during human multi-joint arm movements and to remove the effect of unknown time-varying delays. In the following, how to design the operator controllers S and Rto eliminate effect of uncertainties, and how to design the operator controller *B* and the forward predictive operator F to guarantee the robust stable tracking conditions will be explained detailedly.

3.1. Eliminating effect of uncertainties

The fact that the uncertain plant ΔP is unknown generates difficulties in designing controllers to obtain the desired performance. In order to solve that, a nonlinear feedback control system based on operator approach as a part of the proposed robust nonlinear tracking control system is designed, and is shown in Figure 5. Then, concerning the nonlinear control system with uncertainties shown in Figure 5, it follows that

$$e^{*}(t) = u^{*}(t) - S(N + \Delta N)(\omega)(t) + SN(w_{0})(t)$$

= RD(w_{0})(t) (20)

That is,

$$u^{*}(t) - S(N + \Delta N)(\omega)(t) = (RD - SN)(w_{0})(t)$$
 (21)

thus the equivalent block diagram of Figure 5 is given as Figure 6. Based on the concept of Lipschitz operator and Contraction Mapping Theorem, the effect of uncertainties can be eliminated by designing the operator controllers S and R under the following conditions.

Lemma 1. The proposed nonlinear feedback control system with uncertainties shown in Figure 5, if

$$\begin{cases} SP = I \\ R = I \end{cases}$$
(22)

then the new equivalent plant $P^* = P$, and the effect of uncertainties can be eliminated, where, *I* is an identity operator.



Fig. 5 A nonlinear feedback control system based on operator approach



Fig. 6 Equivalent block diagram of Figure 5

Proof. For the nonlinear feedback system shown in Figure 6, if condition (22) is satisfied, it has that

$$u^{*}(t) - S(y)(t) + SND^{-1}(u)(t) = R(u)(t)$$
(23)

That is,

$$u^{*}(t) = S(y)(t) - SP(u)(t) + R(u)(t)$$

= P⁻¹(y)(t) - I(u)(t) + I(u)(t)
= P⁻¹(y)(t) (24)

Then, $y(t) = P(u^*)(t)$, and the new equivalent plant $P^* = P$, the effect of uncertainties can be eliminated, this completes the proof.

3.2. Property of Time-varying Delays

In the proposed control system, assume the output single of controller B^{-1} is $\tilde{u}(t)$, and the unknown time-varying delay is $\tau(t)$, then the output signal after the delay is

$$u^*(t) = \tilde{u}(t - \tau)(t) \tag{25}$$

which can be denoted by a delay operator Γ , that is $u^*(t) = \Gamma(\tilde{u})(t) = \tilde{u}(t-\tau)(t)$.

The existence of time-varying delays makes the control problem more difficult because the time translation is not reversible. However, the property of the control process ensures that the time-varying delay is bounded and will not increase as fast as the time, that is, the time-varying delay $\tau(t)$ has the following performance

$$0 \le \tau(t) \le \tau_{max}, \quad for \ all \ t \ge 0 \tag{26}$$



where $\tau_{max} \ge 0$ is an upper bound on the delay. Thus, the delay operator Γ is stable.

On the other hand, for $u^*(t)$, because the time delay is irreversible, we assume that the signal is sent before $\delta(t)$, and the process is denoted by an operator Φ , namely,

$$\tilde{u}(t) = u^*(t+\delta)(t) = \Phi(u^*)(t)$$
(27)

The operator Γ and Φ are indeed existing [30]. Also, based on the definition of BIBO stability, both of the operators Γ and Φ are stable. Based on the above discussion, it follows that the process from $\tilde{u}(t)$ to $u^*(t)$ and the process from $u^*(t)$ to $\tilde{u}(t)$ can be described by two stable operators, respectively.

3.3. Main results on conditions of robust stability and tracking

According the results of **Sections 3.1** and **3.2**, the equivalent block diagram of the proposed control system shown in Figure 4 can be shown by Figure 7.



Fig. 7 Equivalent block diagram of Figure 4

For the proposed control system shown in Figure 4 or the equivalent control system shown in Figure 7, the operator controller *B* is designed to satisfy that $M_i = N_i + BD_i$ are unimodulars and described by following form,

$$B^{-1}(e)(t) = -\mathbf{R}(t)\dot{e}(t) - \mathbf{K}(t)e(t)$$
(28)

where, $\mathbf{R}(t)$ and $\mathbf{K}(t)$ are assumed to be the desired timevarying two-joint viscosity and time-varying two-joint stiffness, respectively, and can be defined based on the dynamics of manipulator. According to model (10), the following equation can be obtained by perturbation [17, 24],

$$\mathbf{M}(\theta)\frac{d\dot{\theta}}{dt} + \frac{\partial \mathbf{H}(\dot{\theta},\theta)}{\partial \dot{\theta}}\frac{d\dot{\theta}}{dt} + \left[\frac{\partial \mathbf{M}(\theta)\dot{\theta}}{\partial \theta} + \frac{\partial \mathbf{H}(\dot{\theta},\theta)}{\partial \theta}\right]\frac{d\theta}{dt}$$
$$= \frac{\partial \tau_{in}}{\partial \dot{\theta}}\ddot{\theta} + \frac{\partial \tau_{in}}{\partial \theta}\dot{\theta} = -\mathbf{R}(t)\ddot{\theta} - \mathbf{K}(t)\dot{\theta}$$
(29)

where, the viscosity matrix $\mathbf{R}(t)$ and the stiffness matrix $\mathbf{K}(t)$ are defined, respectively.

$$-\frac{\partial \tau}{\partial \dot{\theta}} \equiv \mathbf{R}(t) = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$
(30)

$$-\frac{\partial \tau}{\partial \theta} \equiv \mathbf{K}(t) = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}$$
(31)

In this research, the assumed viscosity matrix $\mathbf{R}(t)$ and the stiffness matrix $\mathbf{K}(t)$ are obtained by using the online measured viscoelastic exprimental data $\mathbf{R}_A(t)$ and $\mathbf{K}_A(t)$ of human multi-joint arm with the approximate same structural parameters as the manipulator.

Based on the equations (10) and (29), the following equation is obtained,

$$\mathbf{M}(\theta)\ddot{\theta} + \mathbf{H}(\dot{\theta},\theta) = -\mathbf{R}(t)\dot{\theta} - \mathbf{K}(t)\theta + C$$
(32)

where C is an unknown constant. Based on the equations (28) and (32), the following relationship can be obtained,

$$B^{-1}(e)(t) = -\mathbf{R}(t)\dot{e}(t) - \mathbf{K}(t)e(t)$$

= $\mathbf{M}(e(t))\ddot{e}(t) + \mathbf{H}(\dot{e}(t), e(t))$ (33)

According to the designed right factorizations $N_i(\omega_i)(t)$, $D_i(\omega_i)(t)$, and $B^{-1}(e)(t)$, we can find that,

$$M_i = N_i + BD_i = 2\omega_i(t), \quad i = 1, 2$$
 (34)

are unimodulars.

Based on the designed operator controller B, then the tracking performance can be realized by designing the forward predictive operator F.

Theorem 1. The proposed nonlinear tracking control for the human multi-joint arm-like manipulator with unknown time-varying delays shown in Figure 4, if

$$\|(B\Phi D - BD)M^{-1}\| < 1 \tag{35}$$

the BIBO stability can be guaranteed, and the output y(t) tracks to the reference input r(t) provided that FP = I. **Proof.** Based on the proposed design scheme, the block di-

agram shown in Figure 8 is equivalent to Figure 4, where, $\tilde{M} = N + B\Phi D$, then,

$$y(t) = N\tilde{M}^{-1}((r)(t) + B\Phi F(\hat{r})(t))$$

= $N\tilde{M}^{-1}((r)(t) + B\Phi P^{-1}(\hat{r})(t))$
= $N\tilde{M}^{-1}((r)(t) + B\Phi DN^{-1}(\hat{r})(t))$
= $N\tilde{M}^{-1}(N + B\Phi D)N^{-1}(r)(t)$
= $N\tilde{M}^{-1}\tilde{M}N^{-1}(r)(t)$
= $r(t)$ (36)

This completes the proof.

Fig. 8 Equivalent block diagram of Fig. 4

From **Lemma 1** and **Theorem 1** we can see that based on the proposed design scheme, the effect caused by the uncertain plant can be eliminated by the designed operator controllers S and R in the nonlinear feedback control system, the BIBO stability can be guaranteed by the designed operator controller B. The output tracking performance can be realized by the designed operator controller Band the forward predictive operator F.

The main proposal of this paper is as follows. First, we extend a pioneering research of Kawato in [5] to operatorbased control design approach. That is, a design condition including information of time-varying delays in (35) is proposed. Second, an operator-based feedback control in (28) is presented by using motion mechanism of human multi-joint arm viscoelasticity obtained from experimental results. Third, considering the fact that the uncertain plant ΔP is unknown, generates limitations in obtaining the condition $F = (P + \Delta P)^{-1}$ in [27], in this paper, a realizable condition FP = I is proposed.

4. Simulation results based on experimental data

4.1. Experimental data of viscoelasticity

The experimental system of human multi-joint arm viscoelasticity estimation has two major parts (see Figs. 9 and 10): 1) Image measuring system; 2) Force measuring system. The image measuring system is comprised of a CCD camera (CIS corporation, VCC-8350CL, 648×492, RGB24bit, 60fps) and an image input board. The force measuring system is comprised of a force sensor and DSP board, a handle is connected with the force sensor (NITTA IFS-67M25A15-I40, Ser. No. 3038), the hand holding the handle moves together, the force sensor can measure force exerted by human arm. Computer receives angular position information θ_s , θ_e from the image measuring system and generated torque information τ_s , τ_e from the force measuring system during in human multi-joint arm movements.



Fig. 9 Experimental system schematic illustration



Fig. 10 Experimental system

By using the designed experimental system and the proposed estimating method in [20], the joint stiffness and joint viscosity of a human multi-joint arm are estimated online and shown in Figs. 11 and 12, respectively, where, the human multi-joint arm (The structure parameters, $Z_1 = 0.4507$, $Z_2 = 0.1575$ and $Z_3 = 0.1530$, the physical parameters $l_1=0.29$ (m), $l_2=0.32$ (m)) moves from the start position (x, y) = [-41.8397, 33.8566](cm) to the end position (x, y) = [20.3261, 42.4389](cm).



Fig. 11 Estimated human multi-joint arm stiffness by experiment

4.2. Simulation results based on experimental data

In the manipulator control system simulation with human multi-joint arm-like viscoelastic properties and time varying delays, the controlled manipulator is a human multijoint arm model, the desired trajectory is the experimental





Fig. 12 Estimated human multi-joint arm viscoesity by experiment

trajectory of measuring joint stiffness and joint viscosity of human multi-joint arm, and the estimated joint stiffness and joint viscosity are fed to the operator controller B. The controllers S and R are designed by conditions (22), namely, SP = I and R = I. The forward predictive operator F is designed based on conditions (35), namely, FP = I. The uncertainties of structural parameters of the manipulator is considered to be $Z_i = Z_i^* + \Delta Z_i$, $\Delta = 0.05$, where Z_i^* is assumed to be real value. Moreover, the disturbances is considered to be $\tau_d = 0.5 + 0.05 * sin(2\pi t)$, and the timevarying delays are considered to be $\tau(t) = 0.05 * cos(2\pi t)$. The effect of structural uncertainties and disturbances is summarized into ΔN . Based on the proposed method, the joint angles and endpoint position tracking simulation results are shown in Figs. 13 and 14, respectively. From Figs. 13 and 14, we can find that the simulation motion trajectory can track the desired trajectory, and the effectiveness of the proposed control system can be confirmed by simulation results based on experimental data.

5. Conclusions

In this paper, the operator-based robust nonlinear tracking control design for the nonlinear manipulator with uncertainties and time-varying delays was considered by using robust right coprime factorization approach. The motion mechanism of human multi-joint arm viscoelasticity was considered in designing operator controller, and the timereal estimated experimental data of human multi-joint arm viscoelasticity was used in simulation. The effect of uncertain plant was eliminated by the designed nonlinear feedback control system based on operator approach. The effect of CNS was compensated and the effect of timevaring elays is removed by the forward predictive operator. Based on the proposed design scheme, the sufficient conditions for the robust stable were derived, and the out-



Fig. 13 The joint angles tracking results by simulation



Fig. 14 The endpoint position tracking result by simulation

put tracking performance was realized. The effectiveness of the proposed design scheme was confirmed by the simulation results based on experimental data.

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