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# Joint Frequency Offset and Channel Estimation for Distributed MIMO in Time-Varying Channels

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**Abstract:** In this paper, the joint estimation of frequency offsets and channel for distributed MIMO system in time-varying channel is discussed. We assume that each pair of transmit and receive antennas has a different frequency offset. We promote the multi-parameter estimation based on expectation conditional maximization (ECM) and space-alternating generalized expectation-maximization (SAGE) for a MIMO system operating under a flat-fading environment to the multi-parameter estimation using EM algorithm for distributed MIMO systems in time-varying channel. Theoretical analysis and simulation results indicate that the improvement can well compensate for the performance loss caused by time-variability and achieve good performance for time-varying distributed MIMO systems

Keywords: Joint frequency offset and channel estimation;EM;distributed MIMO;time-varying

# **1** Introduction

Multiple-Input Multiple-Output (MIMO) which is widely used in Long Term Evolution (LTE) has been proved to be effective in combating multipath fading, as well as increasing the channel capacity [1,2,3]. It can be found that the prominent direction about MIMO technology is the distributed MIMO system [4, 5, 6, 7, 8] from the trend of research. It can be set up transmitting and receiving antennas according to the specific needs for its higher capacity. As the transmitting and receiving antennas may be located in different geographic locations, the signals transmit through different channels, so the distributed MIMO system puts up higher requirements for channel and frequency offset estimation [4,9]. This problem must be solved for its advantages. In addition, with the rapid development of high-speed mobile communications, there is an increasing demand for distributed MIMO systems operating in high mobility environment.

To date, related studies of multi-parameter estimation for distributed MIMO systems have been relatively mature [10, 11, 12, 13]. However, the research on multi-parameter estimation for time-varying distributed MIMO systems still requires some detailed work. In [9], the authors have researched on the frequency offsets and channel gains Maximum-Likelihood (ML) estimation for a MIMO flat-fading channel using a training sequence. In this work, they held that the ML estimation is a multi-dimensional minimization problem and thus has a very high computational complexity. Therefore, they proposed two computationally efficient algorithms. In [10], the authors pointed out that there exists numerical problems when the frequency offsets are estimated using the popular training sequences, which is due to the fact that the involved matrices are rank-deficient. For overcoming this drawback, a correlation-based algorithm for frequency offset estimation was proposed in [12], which brings an error floor in MSE performance caused by the existence of the interference in multi-antenna system. Compared with the method in [12], an iterative algorithm has been proposed in [10], which does not have the error floor.

In [11], the authors proposed two iterative algorithms to estimate the channel coefficients and frequency offsets in distributed MIMO flat-fading channels, which assumed that each pair of transmit-receiver antenna has a distinct frequency offset value. In addition, the rapid development of high-speed mobile communications brings an increasing demand for distributed MIMO systems operating in high mobility environment. Therefore, the

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research on joint channel and frequency offsets estimation for time-varying distributed MIMO systems has important theoretical value and practical significance.

Motivated by all of the above, this paper focuses on joint frequency offsets and channel estimation for distributed MIMO systems in time-varying channels. Notice that EM algorithm are widely used for iterative receivers, such as [14, 15, 16, 17]. We promote the multi-parameter estimation based on EM algorithm in distributed MIMO flat-fading channels [11] to the multi-parameter estimation using EM algorithm in distributed MIMO time-varying channels. Theoretical analysis and simulation results show that the proposed algorithm in this paper can well compensate for the performance loss caused by time-variability.

The rest of the paper is organized as follows. In Section 2, we describe system model and EM algorithm. The initialization of the proposed algorithm and the iterative algorithm are discussed in Section 3. Simulation results are presented in Section 4 to demonstrate the effectiveness of the proposed algorithm. Finally, Section 5 states the conclusion.

# 2 System model

In this section, system model and an overview of EM algorithms are presented, respectively.

#### 2.1 Distributed MIMO system model

Consider a distributed MIMO system with  $N_T$  transmitter antennas and  $N_R$  receiver antennas in time-varying channels. Therefore, each transmitter and receiver typically requires its own radio frequency- intermediate (RF-IF) chain. Consequently, each pair of transmit-receive antenna has a distinct frequency offset value. The received signal of the *k*-th receive antenna at time *t* can be expressed as

$$y_k(t) = \sum_{l=1}^{N_T} h_{k,l}(t) e^{jw_{k,l}t} s_l(t) + n_k(t), t = 1, 2, \cdots, N \quad (1)$$

where  $s_l(t), t = 1, 2, \dots, N$  is the sequence of symbols transmitted from the *l*-th transmit antenna;  $h_{k,l}(t)$  and  $w_{k,l}$ are the channel coefficient at time *t* and frequency offset between the *l*-th transmit antenna and the *k*-th receive antenna, respectively. In addition,  $n_k(t), t = 1, 2, \dots, N$ denotes a sequence of zero- mean, independent and identically distributed complex-valued Gaussian random variables with variance of  $\sigma^2$  [11]. Noise sequences at  $N_R$ receiver antennas are statistically independent [11].

Let we define

$$\mathbf{y}_k = [y_k(1), y_k(2), \cdots, y_k(N)]^T$$
 (2)

$$\mathbf{h}_{k} = [\mathbf{h}_{k,1}, \mathbf{h}_{k,2}, \cdots, \mathbf{h}_{k,N_{T}}]^{T}$$
(3)

$$\mathbf{h}_{k,l} = [h_{k,l}(1), h_{k,l}(2), \cdots, h_{k,l}(N)]^T$$
(4)

$$\mathbf{w}_k = [w_{k,1}, w_{k,2}, \cdots, w_{k,N_T}]^T$$
(5)

$$\mathbf{n}_{k} = [n_{k}(1), n_{k}(2), \cdots, n_{k}(N)]^{T}$$
 (6)

As is known that multi-parameter estimation for a  $N_T \times N_R$  distributed MIMO system can be equivalent to  $N_R$  independent multi-parameter estimation problems for MISO (Multi-Input Single-Output) systems, therefore, we consider an equivalent  $2 \times 1$  distributed MIMO system for simplicity. And then, the received signal at time *t* can be expressed as

$$y(t) = \sum_{l=1}^{2} h_l(t) e^{jw_l t} s_l(t) + n(t), t = 1, 2, \cdots, N$$
 (7)

Let we define

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T \tag{8}$$

$$\Phi(w_1) = diag(\left[e^{jw_1} \ e^{j2w_1} \ \cdots \ e^{jNw_1}\right]) \tag{9}$$

$$\Phi(w_2) = diag(\left[e^{jw_2} \ e^{j2w_2} \ \cdots \ e^{jNw_2}\right])$$
(10)

$$\mathbf{h}_1 = diag([h_1(1) \ h_1(2) \ \cdots \ h_1(N)])$$
(11)

$$\mathbf{h}_2 = diag([h_2(1) \ h_2(2) \ \cdots \ h_2(N)])$$
(12)

We assume that the first transmit antenna transmits  $\mathbf{s}_1 = [s_1(1) \ 0 \ s_1(3) \ \cdots \ s_1(N-1) \ 0]^T$ ; The second transmit antenna transmits  $\mathbf{s}_2 = [0 \ s_2(2) \ 0 \ \cdots \ 0 \ s_2(N)]^T$ . Then, the received signal is given by

$$\mathbf{y} = \begin{bmatrix} \sum_{l=1}^{2} h_l(1)e^{jw_l}s_l(1) + n(1) \\ \sum_{l=1}^{2} h_l(2)e^{j2w_l}s_l(2) + n(2) \\ \vdots \\ \sum_{l=1}^{2} h_l(N)e^{jNw_l}s_l(N) + n(N) \end{bmatrix}$$
  
$$= \mathbf{h}_1 \Phi(w_1)\mathbf{s}_1 + \mathbf{h}_2 \Phi(w_2)\mathbf{s}_2 + \mathbf{n} \\ \begin{bmatrix} h_1(1)e^{jw_1}s_1(1) \\ h_2(2)e^{j2w_2}s_2(2) \\ h_1(3)e^{j3w_1}s_1(3) \\ \vdots \\ h_1(N-1)e^{j(N-1)w_1}s_1(N-1) \\ h_2(N)e^{jNw_2}s_2(N) \end{bmatrix} + \mathbf{n} = \Phi_{\mathbf{s}}\mathbf{h} + \mathbf{n}$$
  
(13)



where 
$$\Phi_{\mathbf{s}} = diag([s_1(1)e^{jw_1} \ s_2(2)e^{j2w_2} \ s_1(3)e^{j3w_1} \cdots$$

$$s_1(N-1)e^{j(N-1)w_2} s_2(N)e^{jNw_2}]);$$

 $\mathbf{h} = \begin{bmatrix} h_1(1) & h_2(2) & h_1(3) & \cdots & h_1(N-1) & h_2(N) \end{bmatrix}^T.$ Therefore, Eq. (7) can be expressed as

$$\mathbf{y} = \boldsymbol{\Phi}_{\mathbf{s}} \mathbf{h} + \mathbf{n} \tag{14}$$

### 2.2 EM algorithm

EM algorithm, namely expectation maximization algorithm, is an effective method of seeking parameter ML estimation, which can perform parameter estimation from incomplete data space and significantly reduce computational complexity of ML estimation.

Let  $\theta$  denotes the parameter to be estimated from the observational datay. And the probability density function (PDF) of y is  $f(y|\theta)$ . Therefore, the ML estimation of  $\theta$  can be expressed as

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} f(\mathbf{y}|\,\boldsymbol{\theta}) \tag{15}$$

The ML estimation has a very high computational complexity and thus EM algorithm is proposed which uses an iterative approach to solving ML estimation problem.

The derivation of EM algorithm depends on the concept a hypothesis, so-called complete data space  $\mathbf{x}$ . The observed random variable  $\mathbf{y}$ , which is referred to as incomplete data space, is related to  $\mathbf{x}$  by a mapping $\mathbf{y} = g(\mathbf{x})$ . The function g is a many-to-one transformation. Since  $\mathbf{x}$  is not observable, at the *m*-th iteration, the EM algorithm computes its first step, called expectation step(E-step), [11] which is given by

$$J\left(\theta | \hat{\theta}^{[m]}\right) = \mathrm{E}\left\{\log f\left(\mathbf{x} | \theta\right) | \mathbf{y}, \hat{\theta}^{[m]}\right\}$$
(16)

In the second step, called maximization step (M-step), the parameter vector is updated according to

$$\hat{\theta}^{[m+1]} = \arg \max_{\theta} J\left(\left.\theta\right| \,\hat{\theta}^{[m]}\right) \tag{17}$$

In some cases, the expectation conditional maximization (ECM) can be adopted to simplify the computation when the M-step of EM algorithm is too complicated. The ECM algorithm [16] replaces the complicated M-step of EM algorithm by a series of smaller and less complicated steps. Specifically, if the parameter  $\theta$  can be divided into M groups of  $\theta_l, l = 1, 2, \dots, M$ , then the M-step of EM algorithm at the *m*-th iteration can be performed by M smaller steps in which  $\theta_l$  is updated at the *l*-th step,  $l = 1, 2, \dots, N$ , while  $\theta_v$ 's,  $v \neq l$  are fixed at their most updated values.[11]

The *l*-th step can be described as follows

Finding:

$$\theta_{l}^{[m+1]} = \arg \max_{\theta_{l}} J\left(\left.\theta\right| \hat{\theta}^{[m]}\right) \Big|_{\theta_{v} = \hat{\theta}_{v}^{[m]}, v \neq l}$$
(18)

Updating:

$$\hat{\theta}_{\mathbf{l}}^{[\mathbf{m}]} = \hat{\theta}_{\mathbf{l}}^{[\mathbf{m}+1]} \tag{19}$$

# **3** Joint frequency offset and channel estimation

In this section, we first describe the initialization of the proposed algorithm, and then the iterative algorithms which are applicable to time-varying channel are discussed in detail.

# 3.1 Initialization of frequency offsets and channel

After a careful analysis of Eq. (14), we can know that the ML estimation of frequency offsets and channel are achieved by minimizing of the following log-likelihood function [9]

$$\Lambda = \|\mathbf{y} - \boldsymbol{\Phi}_{\mathbf{s}} \mathbf{h}\|^2 \tag{20}$$

The initialization of frequency offsets can be obtained using the correlation algorithm proposed in [13].

For a given value of frequency offset  $\mathbf{w}$ , the initialization of  $\mathbf{h}$  can be expressed as

$$\mathbf{h}_0 = (\boldsymbol{\Phi}_{\mathbf{s}}^H \boldsymbol{\Phi}_{\mathbf{s}})^{-1} \boldsymbol{\Phi}_{\mathbf{s}}^H \mathbf{y}$$
(21)

Substituting Eq. (21) into Eq. (20), the frequency offsets are obtained by multi-dimensional optimization of the following equation

$$\mathbf{w} = \arg\max_{\mathbf{w}} \mathbf{y}^{H} \boldsymbol{\Phi}_{\mathbf{s}} \left( \boldsymbol{\Phi}_{\mathbf{s}}^{H} \boldsymbol{\Phi}_{\mathbf{s}} \right)^{-1} \boldsymbol{\Phi}_{\mathbf{s}}^{H} \mathbf{y}$$
(22)

In this paper, we adopt the method similar to [11]. However, the method in this paper is applicable to time-varying channel while the method in [11] is used in flat-fading channels. Therefore, Eq. (22) and the counterpart in [11] share similar form but have different nature.

### 3.2 The iterative algorithm

Specifically, we define the sequence of the symbols and the frequency offset of the l-th transmit antenna as follows [11]

$$\mathbf{s}_{l} = [s_{l}(1), s_{l}(2), \cdots, s_{l}(N)]^{T}$$
 (23)

$$\mathbf{w}_l = \left[e^{jw_l}, e^{j2w_l}, \cdots, e^{jNw_l}\right]$$
(24)

And then the received signal can be expressed as

$$\mathbf{y} = \sum_{l=1}^{2} \left( \mathbf{s}_{l} \odot \mathbf{w}_{l} \right) \odot \mathbf{h}_{l} + \mathbf{n}_{k}$$
(25)

where  $\mathbf{n} = [n(1), n(2), \dots, n(N)]^T$  and  $\mathbf{n}^{\tilde{c}} CN(0, \sigma^2 \mathbf{I}_N)$ ;  $\mathbf{h}_l = [h_l(1), h_l(2), \dots, h_l(N)] \ l = 1, 2$ , and  $\odot$  denotes the element-wise product of two vectors/matrices. The parameter to be estimated is  $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T]^T$ , where  $\boldsymbol{\theta}_l = [\mathbf{w}_l, \mathbf{h}_l]^T$  is the two parameters corresponding to the pair of the *l*-th transmit antenna and receive antenna. In the EM algorithm, the observed signal  $\mathbf{y}$  is the incomplete data space. Following [17], we define the complete data space as  $\mathbf{z} = [z_1, z_2]^T$ , where

$$z_l \stackrel{\Delta}{=} (\mathbf{s}_l \odot \mathbf{w}_l) \odot \mathbf{h}_l + \mathbf{n}_l, \quad l = 1, 2$$
(26)

Therefore, the relation between the complete data spacez and incomplete data y can be expressed as

$$\sum_{l=1}^{2} z_l = \mathbf{y} \tag{27}$$

Dividing the total noise  $\mathbf{n}$  into two components, we have

$$\sum_{l=1}^{2} \mathbf{n}_l = \mathbf{n} \tag{28}$$

where  $\mathbf{n}_l$ 's are statistically independent, zero-mean Gaussian random with covariance matrix of  $\beta_l \sigma^2 \mathbf{I}_N$ .  $\beta_l$ 's are determined by

$$\sum_{l=1}^{2} \beta_l = 1, \beta_l > 0 \tag{29}$$

We assume that  $\beta_l$ 's are equal, namely,  $\beta_l = 1/N_T = 1/2$ .

#### 3.2.1 ECM algorithm

The *m*-th iteration of ECM algorithm consists two steps, i.e., the E-step and M-step.

The first step is E-step. Assuming that the parameter  $\theta$  and conditioned upon the incomplete data and the current estimated value of  $\hat{\theta}^{[m]}$  are given, the expectation of the complete data space log-likelihood function can be expressed as [11]

$$Q\left(\theta \left| \hat{\theta}^{[m]} \right) \stackrel{\Delta}{=} E\left\{ \log f\left(\mathbf{z} \mid \theta \right) \mid \mathbf{y}, \hat{\theta}^{[m]} \right\}$$
(30)

Because of the statistical independence among  $\mathbf{n}_l$ 's, the probability density function of  $z_l$  as a function of  $\theta$  is given by

$$f(\mathbf{z}|\boldsymbol{\theta}) = \prod_{l=1}^{2} f(z_{l}|\boldsymbol{\theta}_{l}) = \prod_{l=1}^{2} \frac{1}{(\pi\beta_{l}\sigma^{2})^{N}} \exp \left\{-\frac{\|z_{l}-(\mathbf{s}_{l}\odot\mathbf{w}_{l})\odot\mathbf{h}_{l}\|^{2}}{\beta_{l}\sigma^{2}}\right\}$$
(31)

Substituting Eq. (31) into Eq. (30), we can have

$$Q\left(\theta \left| \hat{\theta}^{[m]} \right) = C_1 - E\left\{\sum_{l=1}^2 \frac{1}{\beta_l \sigma^2} \| z_l - (\mathbf{s}_l \odot \mathbf{w}_l) \odot \mathbf{h}_l \|^2 | \mathbf{y}, \hat{\theta}^{[m]} \right\}$$
(32)  
$$= C_2 - \sum_{l=1}^2 \frac{1}{\beta_l \sigma^2} \left\| \hat{z}_l^{[m]} - (\mathbf{s}_l \odot \mathbf{w}_l) \odot \mathbf{h}_l \right\|^2$$

where

$$\hat{z}_{l}^{[m]} = E\left\{z_{l} | \mathbf{y}, \hat{\boldsymbol{\theta}}^{[m]}\right\}$$
(33)

and  $C_1$  and  $C_2$  are two constants which are independent of  $\theta$ .

We can easily obtain the following equation for  $z_l$  and **y** are jointly Gaussian distributed and satisfy Eq. (27)

$$\hat{z}_{l}^{[m]} = \left(\mathbf{s}_{l} \odot \hat{\mathbf{w}}_{l}^{[m]}\right) \odot \hat{\mathbf{h}}_{l}^{[m]} + \beta_{l} \left(\mathbf{y} - \sum_{\nu=1}^{2} \left(\left(\mathbf{s}_{\nu} \odot \hat{\mathbf{w}}_{\nu}^{[m]}\right) \odot \hat{\mathbf{h}}_{\nu}^{[m]}\right)\right)$$
(34)

where

$$\hat{\mathbf{w}}_{\nu}^{[m]} = \left[e^{j\hat{w}_{\nu}^{[m]}}, e^{j\hat{\omega}_{\nu}^{[m]}}, \cdots, e^{jN\hat{w}_{\nu}^{[m]}}\right]^{T}$$
(35)

The second step is the M-step. In this step, the updated value of  $\theta$ ,  $\hat{\theta}^{[m+1]}$ , can be determined as

$$\hat{\boldsymbol{\theta}}^{[m+1]} = \arg \max_{\boldsymbol{\theta}} Q\left(\left.\boldsymbol{\theta}\right| \hat{\boldsymbol{\theta}}^{[m]}\right) 
= \arg \max_{\boldsymbol{\theta}} \left( C_2 - \sum_{l=1}^2 \frac{1}{\beta_l \sigma^2} \left\| \hat{\boldsymbol{z}}_l^{[m]} - (\mathbf{s}_l \odot \mathbf{w}_l) \odot \mathbf{h}_l \right\|^2 \right) 
= \arg \max_{\boldsymbol{\theta}} \left( -\sum_{l=1}^2 \left\| \hat{\boldsymbol{z}}_l^{[m]} - (\mathbf{s}_l \odot \mathbf{w}_l) \odot \mathbf{h}_l \right\|^2 \right) 
= \arg \min_{\boldsymbol{\theta}} \sum_{l=1}^2 \left\| \hat{\boldsymbol{z}}_l^{[m]} - (\mathbf{s}_l \odot \mathbf{w}_l) \odot \mathbf{h}_l \right\|^2$$
(36)

From the above equation, we can easily know that the updating process of  $\theta$  can be decoupled into two (i.e.  $N_T$ ) updating processes of  $\theta_l$  for  $l = 1, 2(N_T = 2)$  [11]. Therefore,  $\hat{\theta}^{[m+1]}$  can be determined by the following equation

$$\hat{\boldsymbol{\theta}}_{l}^{[m+1]} = \arg\min_{\boldsymbol{\theta}_{l}} \left\| \hat{z}_{l}^{[m]} - (\mathbf{s}_{l} \odot \mathbf{w}_{l}) \odot \mathbf{h}_{l} \right\|^{2}, l = 1, 2 \quad (37)$$

In ECM algorithm, the updating process of  $\hat{\theta}_l^{[m]} = \left[\hat{w}_l^{[m]}, \hat{\mathbf{h}}_l^{[m]}\right]$  consist two smaller steps. The Eq. (37) is minimized with respect to one of  $\left[\hat{w}_l^{[m]}, \hat{\mathbf{h}}_l^{[m]}\right]$  while the others are kept at their most updated values [9]. We denote  $\hat{\theta}_l^{[m+c/2]}$  as the estimate of  $\theta_l$  at *c*-th step of *m*-th iteration of the ECM algorithm, c = 1, 2.

Firstly, we determine the updated value of  $w_l$  while  $\mathbf{h}_l$  is fixed at  $\hat{\mathbf{h}}_l^{[m]}$ , i.e., we determine  $\hat{\theta}_l^{[m+1/2]} = \left[ \hat{w}_l^{[m+1]}, \hat{\mathbf{h}}_l^{[m]} \right]$ 

where

$$\hat{w}_{l}^{[m+1]} = \arg\min_{\omega_{l}} \left\| \hat{z}_{l}^{[m]} - (\mathbf{s}_{l} \odot \mathbf{w}_{l}) \odot \mathbf{h}_{l} \right\|^{2} |_{\mathbf{h}_{l} = \hat{\mathbf{h}}_{l}^{[m]}} 
= \arg\min_{\omega_{l}} \sum_{t=1}^{N} |\hat{z}_{l}^{[m]}(t) - s_{l}(t) e^{jw_{l}t} \hat{h}_{l}^{[m]}(t) |^{2} 
= \arg\max_{\omega_{l}} \sum_{t=1}^{N} \Re\left\{ \left( \hat{z}_{l}^{[m]}(t) \right)^{*} s_{l}(t) \hat{h}_{l}^{[m]}(t) e^{jw_{l}t} \right\}$$
(38)

where  $\hat{z}_{l}^{[m]}(t)$  is the *t*-th element of  $\hat{z}_{l}^{[m]}$ ,  $t = 1, 2, \cdots, N$ . We can resort to Taylor's series expansion of  $e^{jw_{l}t}$ 

around  $\hat{w}_i^{[m+1]}$  to the second-order term as the following equation for handling the nonlinearity of Eq. (38)

$$e^{jw_{l}t} \approx e^{j\hat{w}_{l}^{[m]}t} + \left(w_{l} - \hat{w}_{l}^{[m]}\right)(jt)e^{j\hat{w}_{l}^{[m]}t} + \frac{1}{2}\left(w_{l} - \hat{w}_{l}^{[m]}\right)^{2}(jt)^{2}e^{j\hat{w}_{l}^{[m]}t}$$
(39)

Simulations indicate that Eq. (39) is always convex. Therefore, substituting Eq. (39) into Eq. (40), we can obtain the updated value  $\hat{w}_l^{[m+1]}$  as

$$\hat{w}_{l}^{[m+1]} = \arg\max_{\omega_{l}} \sum_{t=1}^{N} \Re\{\left(\hat{z}_{l}^{[m]}(t)\right)^{*} s_{l}(t) \hat{h}_{l}^{[m]}(t) * (e^{j\hat{w}_{l}^{[m]}t} + \left(w_{l} - \hat{w}_{l}^{[m]}\right)(jt) e^{j\hat{w}_{l}^{[m]}t} + \frac{1}{2}\left(w_{l} - \hat{w}_{l}^{[m]}\right)^{2}(jt)^{2} e^{j\hat{w}_{l}^{[m]}t})\}$$
(40)

Differentiating the function inside  $\{\cdot\}$  of Eq. (40) with respect to  $w_l$  and equating the result to zero, we can obtain the updated value  $\hat{w}_l^{[m+1]}$  as

$$\hat{w}_{l}^{[m+1]} = \hat{w}_{l}^{[m]} - \frac{\sum_{t=1}^{N} t \Im\left\{\left(\hat{z}_{l}^{[m]}(t)\right)^{*} s_{l}(t) \hat{h}_{l}^{[m]}(t) e^{jw_{l}t}\right\}}{\sum_{t=1}^{N} t^{2} \Re\left\{\left(\hat{z}_{l}^{[m]}(t)\right)^{*} s_{l}(t) \hat{h}_{l}^{[m]}(t) e^{jw_{l}t}\right\}}$$
(41)

Secondly, the updated value of  $\mathbf{h}_l$  at time t,  $\hat{\mathbf{h}}_l^{[m]}$  is determined, where  $w_l$  is fixed at its newest value of  $\hat{w}_l^{[m+1]}$ . Therefore, we have  $\hat{\theta}_l^{[m+1]} = \left[\hat{w}_l^{[m+1]}, \hat{\mathbf{h}}_l^{[m+1]}\right]$  where

$$\hat{h}_{l}^{[m+1]}(t) = \arg\min_{h_{l}(t)} \left| \hat{z}_{l}^{[m]}(t) - s_{l}(t) e^{j\hat{w}_{l}^{[m+1]}t} h_{l}(t) \right|^{2}$$
(42)  
$$t = 1, 2, \cdots, N$$

After some algebra, we can have

$$\hat{h}_{l}^{[m+1]}(t) = \frac{1}{|s_{l}(t)|^{2}} * \frac{\hat{z}_{l}^{[m]}(t)s_{l}^{*}(t)}{e^{j\hat{w}_{l}^{[m+1]}t}} \quad t = 1, 2, \cdots, N$$
(43)

Thus,  $\hat{\theta}_l^{[m+1]} = \left[\hat{w}_l^{[m+1]}, \hat{\mathbf{h}}_l^{[m+1]}\right]^T$  and the *m*-th iteration is finished.

#### 3.2.2 SAGE algorithm

In the ECM algorithm, the noise variance is distributed over  $z_l$  for all value of l. Therefore, the Fisher information of  $z_l$  is relatively large. To improve the convergence rate, we use space-alternating generalized expectation-maximization (SAGE) algorithm where the parameter  $\theta$  is divided into two ( $N_T$ ) groups of  $\theta_l$ , l = 1, 2. The updated process of any group is taken place while the others are fixed at their latest updated values [11].

Similarly, for the group of  $\theta_l$ , *l* belongs to the set  $\{1,2\}$ . The hidden data space is defined as

$$\mathbf{x}_l \triangleq (\mathbf{s}_l \odot \mathbf{w}_l) \odot h_l + \mathbf{n} \tag{44}$$

The update process of  $\theta_l$  at the *m*-th iteration also consist of two steps, i.e., E-step and M-step. Given  $\theta$  and **y**, we can determine the expectation of the hidden data space log-likelihood function as

$$Q\left(\theta_{l}\left|\hat{\theta}^{[m]}\right.\right) = E\left\{\log f\left(\mathbf{x}_{l}\left|\theta_{l},\left\{\hat{\theta}_{v}^{[m]}\right\}_{v\neq l}\right)\right|\mathbf{y},\hat{\theta}^{[m]}\right\}\right\}$$
(45)

where

1

$$\begin{aligned} c\left(\mathbf{x}_{l} \left| \boldsymbol{\theta}_{l}, \left\{ \hat{\boldsymbol{\theta}}_{v}^{[m]} \right\}_{v \neq l} \right) &= f\left(\mathbf{x}_{l} \left| \boldsymbol{\theta}_{l} \right. \right) \\ &= \frac{1}{\left(\pi\sigma^{2}\right)^{N}} \exp\left\{ -\frac{\|\mathbf{x}_{l} - (\mathbf{s}_{l} \odot \mathbf{w}_{l}) \odot \mathbf{h}_{l}\|^{2}}{\sigma^{2}} \right\} \end{aligned}$$

$$(46)$$

Substituting Eq. (46) into Eq. (45), we can have

$$Q\left(\theta_{l}\left|\hat{\boldsymbol{\theta}}^{[m]}\right) = E\left\{\frac{1}{\left(\pi\sigma^{2}\right)^{N}}\exp\left\{-\frac{\|\mathbf{x}_{l}-(\mathbf{s}_{l}\odot\mathbf{w}_{l})\odot\mathbf{h}_{l}\|^{2}}{\sigma^{2}}\right\}\left|\mathbf{y},\hat{\boldsymbol{\theta}}^{[m]}\right\} = C_{3} - \frac{1}{\sigma^{2}}E\left\{\left\|\mathbf{x}_{l}-(\mathbf{s}_{l}\odot\mathbf{w}_{l})\odot\mathbf{h}_{l}\right\|^{2}\left|\mathbf{y},\hat{\boldsymbol{\theta}}^{[m]}\right.\right\} = C_{4} - \frac{1}{\sigma^{2}}\left\|\hat{\mathbf{x}}_{l}^{[m]}-(\mathbf{s}_{l}\odot\mathbf{w}_{l})h_{l}\right\|^{2}$$

$$(47)$$

where

as

$$\begin{aligned} \hat{\mathbf{x}}_{l}^{[m]} &\stackrel{\Delta}{=} E\left\{\mathbf{x}_{l} \mid \mathbf{y}, \hat{\boldsymbol{\theta}}^{[m]}\right\} \\ &= \left(\mathbf{s}_{l} \odot \hat{\mathbf{w}}_{l}^{[m]}\right) \odot \hat{\mathbf{h}}_{l}^{[m]} + \left(\mathbf{y} - \sum_{\nu=1}^{2} \left(\left(\mathbf{s}_{\nu} \odot \hat{\mathbf{w}}_{\nu}^{[m]}\right) \odot \hat{\mathbf{h}}_{\nu}^{[m]}\right)\right) \\ &= \mathbf{y} - \sum_{\nu=1, \nu \neq l}^{2} \left(\mathbf{s}_{\nu} \odot \hat{\mathbf{w}}_{\nu}^{[m]}\right) \hat{h}_{\nu}^{[m]} \end{aligned}$$

$$(48)$$

In this step, the update value of  $\theta_l$ ,  $\hat{\theta}_l^{[m+1]}$ , is expressed

$$\hat{\boldsymbol{\theta}}_{l}^{[m+1]} = \arg\max_{\boldsymbol{\theta}_{l}} Q\left(\boldsymbol{\theta}_{l} \left| \hat{\boldsymbol{\theta}}_{l}^{[m]} \right.\right)$$
$$= \arg\min_{\boldsymbol{\theta}_{l}} \left\| \hat{\mathbf{x}}_{l}^{[m]} - (\mathbf{s}_{l} \odot \mathbf{w}_{l}) \odot \mathbf{h}_{l} \right\|^{2}$$
(49)

The above equation can be solved like the M-step of the previous where ECM is deployed, i.e., it consists of two smaller steps and elements of  $\hat{\theta}_l^{[m]} = \left[\hat{w}_l^{[m]}, \hat{\mathbf{h}}_l^{[m]}\right]$  are updated sequentially.

Firstly, we determine the updated value of as the following equation while  $\mathbf{h}_l$  is fixed at  $\hat{\mathbf{h}}_l^{[m]}$ 

$$\hat{w}_{l}^{[m+1]} = \hat{w}_{l}^{[m]} - \frac{\sum_{t=1}^{N} t \Im\left\{\left(\hat{x}_{l}^{[m]}(t)\right)^{*} s_{l}(t) \hat{h}_{l}^{[m]}(t) e^{jw_{l}t}\right\}}{\sum_{t=1}^{N} t^{2} \Re\left\{\left(\hat{x}_{l}^{[m]}(t)\right)^{*} s_{l}(t) \hat{h}_{l}^{[m]}(t) e^{jw_{l}t}\right\}}$$
(50)

where  $\hat{x}_{l}^{[m]}(t)$  is the -th element of  $\hat{x}_{l}^{[m]}$  in Eq. (48),  $t = 1, 2, \cdots, N$ .

Secondly, we determine the updated value of time of  $\mathbf{h}_l$  as the following equation while  $w_l$  is fixed at its newest value of  $\hat{w}_l^{[m+1]}$ 

$$\hat{h}_{l}^{[m+1]}(t) = \frac{1}{|s_{l}(t)|^{2}} * \frac{\hat{x}_{l}^{[m]}(t)s_{l}^{*}(t)}{e^{j\hat{w}_{l}^{[m+1]}t}} \quad t = 1, 2, \cdots, N$$
(51)

Thus,  $\hat{\theta}_l^{[m+1]} = \left[\hat{w}_l^{[m+1]}, \hat{\mathbf{h}}_l^{[m+1]}\right]^T$  and the *m*-th iteration is finished.

# **4** Simulation results

In this section, we present the simulation results and analysis of the proposed method. At the same time, we compare the performance of the proposed algorithm in terms of MSE with the existing algorithm.

The simulation parameters are shown in Table 1.

<b>Fuble 1.</b> White system sinulation parameters	
Parameter	Value
Frequency offsets	$2\pi[0.01\ 0.011]^T$
Modulation	QPSK
Doppler shiftHz	260
Sampling time (s)	2.5e-6
Number of antenna	$2 \times 2$
Channel	independent Rayleigh, time-varying

Table 1: MIMO system simulation parameters

As is mentioned above, the algorithm in [11] is appropriate for distributed MIMO systems in the flat-fading channel. We promote the multi-parameter estimation algorithms in [11] to the multi-parameter estimation using EM algorithm in distributed MIMO time-varying channels. And then we compare the performances of the algorithm in this paper with the algorithm in [11], which are in time-varying channels for comparison.

In this paper, we initialize values for frequency offset using the method in [12]. After having these values, Eq. (21) is used to get the initial estimates for channel coefficients. In addition, our proposed algorithms stop when difference between log-likelihood function of the two consecutive iterations is less than 0.001. In Fig.1 and



**Fig. 1:** Comparison of MSE performances of channel of [11] and proposed algorithm



Fig. 2: Comparison of MSE performances of CFO of [11] and proposed algorithm

Fig.2, the MSE performance of channel and carrier frequency offset is illustrated, respectively.

Fig.1 shows the comparison of the MSE performance of channel of [11] and the algorithm in this paper. Fig.2 shows the comparison of the MSE performance of carrier frequency offsets (CFO) of [11] and the algorithm in this paper. We can see that the MSE performance of the algorithm in [11] is relatively poor. What's worse, the performance is still poor at high-SNR region. It indicates that the algorithm in [11] is inappropriate in high-speed environments. We can also see from the two figures that the MSE performance of the proposed algorithm is very satisfying, especially in high-SNR region. Obviously, the performance of the proposed algorithm is better than the algorithm in [11] in high-speed environment. Thus we can conclude that the proposed algorithm can significantly compensate for the performance loss caused

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by time-variability and achieve better performance for time-varying distributed MIMO systems.

# **5** Conclusion

In this paper, two algorithms that jointly estimate frequency offsets and channel for distributed MIMO systems in time-varying channels have been presented. The two algorithms are based on ECM and SAGE algorithm, respectively. We promote the multi-parameter estimation based on ECM and SAGE algorithms in distributed MIMO flat-fading channels to the multi-parameter estimation using EM algorithm in distributed MIMO time-varying channels. Simulations indicates that the improvement of the algorithms can significantly compensate for the performance loss caused by time-variability and achieve better performance in time-varying distributed MIMO systems.

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