Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/080646

Discrete Adaptive Sliding Mode Control via Wavelet Network for a Class of Nonlinear Systems

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Received: 16 Nov. 2013, Revised: 14 Feb. 2014, Accepted: 15 Feb. 2014
Published online: 1 Nov. 2014

Abstract: Unmodelled dynamics and perturbations are always immeasurable. In this paper, an adaptive sliding mode control (ASMC) based on wavelet network (WN) for a class of non-affine multi-variable nonlinear discrete systems is presented in order to compensate them. Wavelet network which parameters are tuned on-line is adopted to realize the equivalent control, and hitting controls are added in order to satisfy reaching conditions. By combining the adaptive WN with SMC strategy, the constructed control law has many advantages such as robustness, adaptive characters, and the precise mathematic models of controlled plants are not required. Finally, experiment on an inverted pendulum control system based on the proposed control design method is given to verify its effectiveness and performance.

Keywords: Discrete, nonlinear system, sliding mode control, adaptive, wavelet network

1 Introduction

In practical control systems, highly unknown uncertainties, disturbances and nonlinearities always exist. Many efforts on this problem have been made by researchers of robust control, adaptive control and intelligent control etc. In recent years, wavelet network (WN) is used as a powerful tool for signal and data processing, time-series analysis and the approximation of arbitrary unknown functions such as literatures [1],[2],[3],[4] and [5]. Using WN for function approximation and identification of nonlinear systems has been studied by literatures[1],[2],[3] etc.

Adaptive neural network control has been widely investigated by many researchers such as literatures [6],[7],[8],[9] and [10]. The parameters of neural network (NN) are tuned on-line to approximate the unknown nonlinear dynamic. However the precision depends on the structure selection which is a difficult problem at present. Inspired by the theory of adaptive NN, adaptive wavelet network methods are reported dealing with on-line application in the control problem of dynamic nonlinear systems, it refers to literatures [11]-[18]. WN can be regarded as a class of NN, but it has its special characteristics such as the linearity in parameter space and the orthonormality. These make WN is suitable for on-line estimating, and there is not the problem of structure selection in adaptive wavelets networks. Therefore, successful application of adaptive WN to nonlinear systems is researched[17]. Based on the conception of multi-resolution approximation (MRA), WN is a three-layer network consisting of orthonormal father wavelets and mother wavelets. Because of the orthonormal property, it is possible to regulate the network structure and parameters on-line. Moreover, the multi-resolution approximation ensures that the approximation precision can be improved quickly as resolution increases. Although the precision can be improved arbitrarily, there exist many perturbation and disturbance that impact on the system performance such as stability, steady-state error and so on.

Sliding mode control (SMC) theory has been proved to be an effective way to control nonlinear dynamic system with strong robustness[19][20]. If we combine SMC theory into the adaptive wavelet network, the designed controller will possess many advantages. The wavelets neural network control (WNNC) based on SMC control theory and adaptive theory for the linear motor and induction motor drive has been studied by [15][18]. Nevertheless, their research works aim at special plants.

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Furthermore, the adaptive wavelets sliding mode control (AWSMC) for multi-variable affine nonlinear system has been studied by [14].

The purpose of this paper is to combine the advantages of SMC and the adaptive wavelet network to develop a control strategy with robustness and self-tuning property for non-affine multi-variable discrete nonlinear systems. Firstly, appropriate sliding surfaces are selected; secondly, the parameters of the wavelet network are tuned on-line to approximate the equivalent control; thirdly, hitting controls are added to ensure that the reaching condition can be satisfied. The final AWSMC controller comprises three parts: the equivalent control approximator, hitting control and the adaptive machine. The closed-loop system is proved to be asymptotically stable globally.

This paper is organized as follows. Problem formulation is given in section 2. The fundamentals of MRA and the function approximation are stated in section 3. The adaptive wavelet network and the hitting control designs, stability and robustness analysis are in section 4, additionally the network structure of the AWSMC. In section 5, the presented AWSMC is applied to an inverted pendulum system to confirm its validity and performance. Experiment results are presented. In the end conclusions are in section 6.

The signs of mathematics in this paper are general regular as follows.

R represents real number space, and so, R^n and $R^{n\times m}$ denote n vector space and $n\times m$ vector space, respectively. Z expresses the whole round number, $|\cdot|$ indicates absolute value and $\|\cdot\|$ denotes Euclidean norm. If matrix $A\in R^{n\times m}$ then, $\|A\|$ denotes its induced norm and $\lambda_{min}(\cdot)/\lambda_{max}(\cdot)$ defines its minimum or maximum characteristic value. L_∞ denotes Lebesgue integrable function space, namely for all functions $f\in R$ which satisfy $\int_{-\infty}^{\infty} f(t)dt<\infty$ form L_∞. All such functions $f:R^n\to R$ that fulfill $\int_{-\infty}^{\infty}\|f(t)\|^2dt<\infty$ form the space $L^2(\mathbf{R})$.

2 Problem Formulation

Consider a class of multi-variable non-affine nonlinear system with the form of

$$y_k^{(r)} = f(x_k,u_k), \tag{1}$$

where $y_k=[y_{1,k},y_{2,k},\cdots,y_{m,k}]^T$ denotes the output vector and $y_k^{(r)}=[y_{1,k}^{(r_1)},y_{2,k}^{(r_2)},\cdots,y_{m,k}^{(r_m)}]^T$ denotes its derivative for $i=1,2,m$. $r=[r_1,r_2,\cdots,r_m]^T$ is defined as the system relative degree with $\sum_{i=1}^m r_i = n$ and $u_k=[u_{1,k},u_{2,k},\cdots,u_{m,k}]^T$ denotes the control input vector. Following $x_k=[x_{1,k},x_{2,k},\cdots,x_{n,k}]^T=[\mathbf{y}_{1,k}^T,\mathbf{y}_{2,k}^T,\cdots,\mathbf{y}_{m,k}^T]^T$ constitutes the system state vector with depicted as $\mathbf{y}_{i,k} = [y_{i,k},y_{i,k+1},\cdots,y_{i+r_i-1}]^T$, $i=1,2,\cdots,m$. Additionally,

$$f(x_k,u_k)=[f_1(x_k,u_{1,k}),f_2(x_k,u_{2,k}),\cdots,f_m(x_k,u_{m,k})]$$

is defined as the nonlinear dynamic vector, which components $f_i(x_k,u_{i,k})\in L^2(\mathbf{R})$, $i=1,2,\cdots,m$ are all unknown. And about all, the subscript k denotes the discrete time instant.

If $\bar{y}_k=[\bar{\mathbf{y}}_{1,k},\bar{\mathbf{y}}_{2,k},\cdots,\bar{\mathbf{y}}_{m,k}]^T$ represents the known trajectory and comprises vector

$$\bar{\mathbf{y}}_{i,k}=[\bar{y}_{i,k},\bar{y}_{i,k+1},\cdots,\bar{y}_{i,k+r_i-1}]^T(i=1,2,\cdots,m),$$

the problem is to design a controller for the system (1) such that the tracking error

$$\mathbf{e}_k=[\mathbf{e}_{1,k}^T,\mathbf{e}_{2,k}^T,\cdots,\mathbf{e}_{m,k}^T]^T, \tag{2}$$

where $\mathbf{e}_{i,k}=\mathbf{y}_{i,k}-\bar{\mathbf{y}}_{i,k}=[e_{i,k},e_{i,k+1},\cdots,e_{i,k+r_i-1}]^T$ with $e_{i,k}=y_{i,k}-\bar{y}_{i,k}$, $i=1,2,\cdots,m$, will converge to the origin asymptotically.

Define the sliding manifold as follows,

$$S_k=[S_{1,k},S_{2,k},\cdots,S_{m,k}]^T, \tag{3}$$

where $S_{i,k}=C_i\mathbf{e}_i$, $i=1,2,\cdots,m$ and $C_i=[c_{i,1},c_{i,2},\cdots,c_{i,r_i-1},1]^T$ which satisfy Hurwitz polynomial

$$h_i(e_{i,k})=e_{i,k+r_i-1}+c_{i,r_i-1}e_{i,k+r_i-2}+\cdots+c_{i,1}e_{i,k}. \tag{4}$$

Then the main problem is to look for a SMC controller

$$u_{i,k}=u_{eq,i}+u_{v,i},\ i=1,2,\cdots,m, \tag{5}$$

where $u_{eq,i}$ is the equivalent control, $u_{v,i}$ is the hitting control such that the manifold S_k can be reached.

Because the function $f_i(x_k,u_{i,k})$ is unknown, the control input u_k cannot be designed directly. In the following section, an adaptive wavelet network will be adopted to realize a SMC controller. The following are necessary to underlie the reminder of this paper.

Assumption 1. The function $f_i(x_k,u_{i,k})\in L^2(\mathbf{R})$ and

$$f_{i,u_i}\overset{def}{=}\frac{\partial f_i(x_k,u_{i,k})}{\partial u_{i,k}}\neq 0,\ \forall\, x_k\in\Omega_{x_k},$$

where Ω_{x_k} is the compact set of the state x_k.

For all $x_k\in\Omega_{x_k}$ the smooth function satisfies $f_{i,u_i}>0$ or $f_{i,u_i}<0$. Without loss of generality, it is assumed that $f_{i,u_i}>0$.

Assumption 2. There exist positive constants $\zeta_{i,1}$, $\zeta_{i,2}$ and $\zeta_{i,3}$ such that $\forall\, x\in\Omega_x$, $\zeta_{i,1}\le f_{i,u_i}\le\zeta_{i,2}$ and $0\le|\Delta f_{i,u_i}|\le\zeta_{i,3}$, $i=1,2,\cdots,m$.

According to the Implicit Function Theorem, the following lemma is introduced,

Lemma 1. Considering the equation(1), there exist a subset $\Omega_{x,0}\in\Omega_{x_k}$ and a unique local solution $u_{i,k}=u_i(x_k,v_i)$ such that $f_i(x_k,u_{i,k})+v_i=0$ for $\forall\ x(0)\in\Omega_{x,0}$ holds where $v_i=v_i(x)$ is an arbitrary smooth function of the system state variable x_k for $i=1,2,\cdots,m$.

3 Preliminaries on Wavelet Network

Wavelet network is a type of building block for function approximation with the conception of multi-resolution analysis (MRA). Many statements about MRA can be found in [21]. For simplicity, we only describe its basic structure and frame theories here.

A successive close subspace serial $\{V_j\} \in L^2(\mathbf{R})$, $j \in \mathbf{Z}$ composes a MRA in space $L^2(\mathbf{R})$, in which $\varphi_{j,k}(x) = 2^{\frac{j}{2}}\varphi(2^j x - k)$ with $\varphi \in V_j$. $\varphi_{j,k}$ is an orthonormal basis of V_j namely scalling function. For $\forall\, j \in \mathbf{Z}$, W_j is defined to be the orthonormal complement of V_j in V_{j+1} which satisfies

$$V_{j+1} = V_j \oplus W_j, W_j \perp W_i,\ i \neq j, \forall\, i \in \mathbf{Z}.$$

Therefore $f_j(x) = \sum_{k\in\mathbf{Z}} < f(x), \varphi_{j,k} > \varphi_{j,k}$ is the approximation of $f(x)$ at the resolution $\frac{j}{2}$, or we call at the resolution j shortly. Here $< \cdot >$ is defined as the inner product in $L^2(\mathbf{R})$. In practical terms $f_j(x)$ is a projection of $f(x)$ in the space V_j. Consequently in the space V_j, an arbitrary function $f(x)$ can be depicted as

$$f(x) = \sum_{k\in\mathbf{Z}} < f(x), \varphi_{j,k} > \varphi_{j,k} + e_{f,j},$$

where $e_{f,j}$ is the approximation error at the resolution j. And with $j \to \infty$, $lim_{j\to\infty}|e_{f,j}| = 0$. At the resolution $j+1$, the approximation of $f(x)$ has the projection in the space W_j which is called detail.

It is clear that $|e_{f,j}| > |e_{f,j+1}| > \cdots$ and more details will be added to the approximation of function $f(x)$ as $j \to \infty$. So for a given resolution J, the projection of $f(x)$ in the space V_J is called the coarse approximation. With the increasing of the resolution j, the detail approximations are cumulating. Naturally, there exists a mother wavelet $\{\psi_{j,k}\}_{k\in\mathbf{Z}}$, which are canonical bases of the space W_j. Like the scaling function, the wavelet function $\psi_{j,k}$ also satisfies

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k),\ j,k \in \mathbf{Z}.$$

Then $\forall\, f(x)$ in $L^2(\mathbf{R})$, it can be described as

$$f(x) = \sum_{k\in\mathbf{Z}} < f(x), \varphi_{J,k} > \varphi_{J,k} + \sum_{j\geq J, k\in\mathbf{Z}} < f(x), \psi_{j,k} > \psi_{j,k},$$

where J is the lowest resolution. Define $\upsilon_{J,k} =< f(x), \varphi_{J,k} >$ and $\gamma_{j,k} =< f(x), \psi_{j,k} >$ as the coefficients of the basis $\{\varphi_{J,k}\}_{k\in\mathbf{Z}}$ and $\{\psi_{j,k}\}_{j\geq J,k\in\mathbf{Z}}$ in the corresponding space V_J and $W_J, W_{J+1}, \cdots$, the approximation of $f(x)$ becomes

$$f(x) = \sum_{k\in\mathbf{Z}} \upsilon_{J,k}\varphi_{J,k}(x) + \sum_{j\geq J,k\in\mathbf{Z}} \gamma_{j,k}\psi_{j,k}(x). \tag{6}$$

The equation (6) is the approximation of scaling function series and wavelet series for $f(x)$ in $L^2(\mathbf{R})$. In actual application, function approximation is carried out in space with finite dimension. So the equation (6) can not be used. Fortunately, the following theorem is introduced by [21].

Theorem 1. For arbitrary scalar $\varepsilon > 0$, there exists finite integers M, N and the parameters $\gamma^*_{j,k}$ such that the following inequality holds $\forall f(x) \in L^2(\mathbf{R})$.

$$\left\| f(x) - \sum_{j=-M}^{M}\sum_{k=-N}^{N} \gamma^*_{j,k}\psi_{j,k} \right\| \leq \varepsilon \tag{7}$$

It indicates that there always exists an optimal approximation $f^*(x) = \sum_{j=-M}^{M}\sum_{k=-N}^{N}\gamma^*_{j,k}\psi_{j,k}$ in the form of wavelet series. Thereby the accuracy of the following approximation

$$f^*(x) = \sum_{k=-N}^{N} \upsilon^*_{J,k}\varphi_{J,k}(x) + \sum_{j=J}^{M}\sum_{k=-N}^{N}\gamma^*_{j,k}\psi_{j,k}(x)$$

is better than that of the equation (7) when $J \leq -M$, where $\upsilon^*_{J,k}$ are the corresponding coefficients of the scaling basis. There is a problem that the functions in control systems are multi-dimensional, i.e. the variable x is a vector. The scaling function and the wavelet must be expanded to multi-dimension as follows

$$\varphi(x) = \varphi(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n}\varphi(x_i),$$

$$\psi(x) = \psi(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n}\psi(x_i).$$

The approximation of $f(x)$ can then be given as

$$f(x) = \Upsilon^T \Phi_J + \sum_{j=J}^{M}\Gamma_j^T \Psi_j, \tag{8}$$

where

$$\Upsilon = [\upsilon_{J,-N}, \upsilon_{J,-N+1}, \cdots, \upsilon_N]^T, \tag{9}$$

$$\Gamma_j = [\gamma_{j,-N}, \gamma_{j,-N+1}, \cdots, \gamma_{j,N}]^T, \tag{10}$$

$$\Phi_J = [\varphi_{J,-N}(x), \varphi_{J,-N+1}(x), \cdots, \varphi_{J,N}(x)]^T, \tag{11}$$

$$\Psi_j = [\psi_{j,-N}(x), \psi_{j,-N+1}(x), \cdots, \psi_{j,N}(x)]^T. \tag{12}$$

In this section, it is confirmed that there exists the optimal approximation $f^*(x)$ of $f(x)$ in $L^2(\mathbf{R})$, though x is a multi-dimensional variable. The equation (8) will be used in the next sections.

4 SMC based on Adaptive Wavelet Networks

Now seek for the time derivative of the element $S_{i,k}$ defined by the equations (2)(3)(4) (in the following analysis only the element $S_{i,k}$ is considered),

$$\begin{aligned}\Delta S_{i,k} &= C_i\left(\mathbf{e}_{i,k+1}-\mathbf{e}_{i,k}\right)\\ &= \sum_{j=1}^{r_i-1} c_{i,j}e_{i,k+j} - \sum_{j=1}^{r_i-1} c_{i,j}e_{i,k+j-1}\\ &+ e_{i,k+r_i} - e_{i,k+r_i-1}\\ &= \sum_{j=1}^{r_i-1} c_{i,j}\left(e_{i,k+j}-e_{i,k+j-1}\right) - e_{i,k+r_i-1}\\ &- \bar{y}_{i,k+r_i} + f_i(x_k,u_{i,k}) + d_i(x_k,k).\end{aligned}$$

If we define

$$v_i = \sum_{j=1}^{r_i-1} c_{i,j}\left(e_{i,k+j}-e_{i,k+j-1}\right) - e_{i,k+r_i-1} - \bar{y}_{i,k+r_i},$$

then

$$\Delta S_{i,k} = v_i + f_i(x_k,u_{i,k}) + d_i(x_k,k). \tag{13}$$

The objective is to design a sliding mode controller $u_{i,k}$ (the equation(5)) such that $S_{i,k}=0$. The SMC controller composes two parts like the equation(5): the equivalent control $u_{eq,i}$ and the hitting control $u_{v,i}$. Here The approach of function approximation in section 3 which is based on wavelet network will be adopted to approximate the equivalent control in SMC controller. Well then the equivalent control $u_{eq,i}$ would be designed firstly.

4.1 Equivalent Control Design

The system dynamic function $f_i(x_k,u_{i,k})$ satisfies the Assumption 1-2. According to Lemma 1 in section 2, when the external disturbance $d_i(x_k,k)$ is not considered, there exists a unique equivalent control $u_{eq,i}(x_k,v_i)$ that makes $S_{i,k}=0$. Namely the following equation holds by the equation(13),

$$\Delta S_{i,k} = v_i + f_i(x_k,u_{i,k}) = 0. \tag{14}$$

Because the function $f_i(x_k,u_{i,k}) \in L^2(\mathbf{R})$, the equivalent control $u_{eq,i}(x_k,v_i)$ is in $L^2(\mathbf{R})$ space as well. According to Theorem 1 and the equation (8) in section 3, there exists the optimal approximation of $u_{eq,i}(x_k,v_i)$ as the following,

$$u^*_{eq,i}(x_k,v_i) = \Upsilon_i^{*T}\Phi_J^i + \sum_{j=J}^{M}\Gamma_{i,j}^{*T}\Psi_j^i,$$

where Υ_i^* and $\Gamma_{i,j}^*$ are the optimal parameters of wavelet network for $u_{eq,i}(x_k,v_i)$ defined as the equations(9),(10).

The scaling function basis $\Phi_J^i = \Phi_J(x,v_i)$ and the wavelet function basis $\Psi_j^i = \Psi_j(x,v_i)$ correspondingly satisfy

$$\Psi_j(x_k,v_i) = \left[\psi_{j,-N}(x_k,v_i),\cdots,\psi_{j,N}(x_k,v_i)\right]^T,$$

$$\Phi_J(x_k,v_i) = \left[\varphi_{J,-N}(x_k,v_i),\cdots,\varphi_{J,N}(x_k,v_i)\right]^T,$$

$$\varphi(x_k,v_i) = \varphi(x_{1,k},x_{2,k},\cdots,x_{n,k},v_i) = \prod_{l=1}^{n}\varphi(x_{l,k})\varphi(v_i),$$

$$\psi(x_k,v_i) = \psi(x_{1,k},x_{2,k},\cdots,x_{n,k},v_i) = \prod_{l=1}^{n}\psi(x_{l,k})\psi(v_i).$$

An adaptive wavelet network(AWN) is used to approximate the equivalent control $u_{eq,i}(x_k,v_i)$ for $i=1,2,\cdots,m$. Denote the approximation value as $\bar{u}_{eq,i}$, according to the equation(8), it can be depicted as

$$\bar{u}_{eq,i}(x_k,v_i) = \Upsilon_{i,k}{}^T\Phi_J^i + \sum_{j=J}^{M}\Gamma_{i,j}{}^T\Psi_j^i, \tag{15}$$

where Υ_i and $\Gamma_{i,j}$ are coefficients of the wavelet basis which need self-tuning on-line by the following adaptive rules,

$$\Delta\Upsilon_{i,k} = \alpha_i\Phi_J^i\left[\Delta S_{i,k}\right], \Delta\Gamma_{i,j,k} = \beta_{i,j}\Psi_j^i\left[\Delta S_{i,k}\right], \tag{16}$$

where $\alpha_i,\beta_{i,j}$ are positive scalars. Thus $\bar{u}_{eq,i}$ can approximate to the optimal approximation $u^*_{eq,i}$ accurately by the online tuning. However there exists the approximation error $\varepsilon = u^*_{eq,i} - u_{eq,i}$. Thus the equation (14) can be written as

$$\begin{aligned}\Delta S_{i,k} &= v_i + f_i(x_k, \underset{eq,i}{u}(x_k,v_i))\\ &= f_i(x_k, \underset{eq,i}{u}) - f_i(x_k,u^*_{eq,i}) + f_i(x_k,u^*_{eq,i}) - f_i(x_k,u_{eq,i})\\ &= f_{i,u_{i,k}}(\bar{u}_{eq,i} - u^*_{eq,i}) + \varepsilon f_{i,u_{i,k}}.\end{aligned} \tag{17}$$

The error impels the sliding mode $S_{i,k}$ to tend $\Delta S_{i,k} \neq 0$. A hitting control $u_{v,i}$ is required to be added such that the sliding mode satisfies the reaching condition. The following assumption is necessary and with practical significance.

Assumption 3. The error between approximation $\bar{u}_{eq,i}$ and the optimal approximation $u^*_{eq,i}$is bounded, i.e. $\left|\bar{u}_{eq,i} - u^*_{eq,i}\right| \leq \zeta_{i,4}$.

4.2 Hitting Control Design

Obviously, the equivalent control can not insure the stability of closed-loop system. According to the basic theory of SMC, the hitting control $u_{v,i}$ can be designed as

$$u_{v,i} = -k_iS_{i,k} - \eta_i\mathbf{sgn}S_{i,k}, \tag{18}$$

where the designed parameters k_i and η_i correspondingly satisfy

$$0 \leq k_i \leq \frac{2}{\zeta_{i,2}(1+\xi_i)}, \eta_i \geq \zeta_{i,4} + \varepsilon \tag{19}$$

with

$$\xi_i = \alpha_i \left[\Phi_J^i\right]^T \Phi_J^i + \sum_{j=J}^{M} \beta_{i,j} \left[\Psi_j^i\right]^T \Psi_j^i$$

that satisfies

$$\xi_i \le \max_{i,j} \left\{\alpha_i, \beta_{i,j}\right\}$$

according to the properties of the wavelet basis functions.

Well then the whole SMC control law can be obtained by

$$u_{i,k} = \bar{u}_{eq,i} + u_{v,i}. \tag{20}$$

Consequently the forward difference of the embranchment $S_{i,k}$ can be written as

$$\begin{aligned}\Delta S_{i,k} &= f_{i,u_{i,k}}(u_{i,k} - u_{eq,i}^*) + \varepsilon f_{i,u_{i,k}} \\ &= f_{i,u_{i,k}} u_{v,i} + f_{i,u_{i,k}}(\bar{u}_{eq,i} - u_{eq,i}^*) + \varepsilon f_{i,u_{i,k}}.\end{aligned} \tag{21}$$

4.3 Stability Analysis of Closed-Loop System

The obtained control law (20) is designed such that the embranchment $S_{i,k}$ could reach the origin $S_{i,k} = 0$ for $i = 1,2,\cdots,m$. The main results are presented by the following theorem.

Theorem 2. Under the assumption 1-3, the sliding mode (3) for multiple variable nonlinear system (1) can be reached the switching band

$$\mathbf{B}^i = \left\{x_k \left| \left|S_{i,k}\right| \le \varphi_i \right.\right\}, i = 1,2,\cdots,m$$

by the adaptive wavelet sliding mode control (AWSMC) (15),(18),(19),(20) with the adaptive law (16), where

$$\varphi_i = \frac{\left(\zeta_{i,2}\left(1+\xi_i\right)+1\right)\eta_i + \varepsilon}{1 - 0.5\zeta_{i,2}\left(1+\xi_i\right)k_i}. \tag{22}$$

Proof. Choose a Lyapunov function about $S_{i,k}$ as

$$V_{i,k} = S_{i,k}^2 + \frac{1}{\alpha_i}\tilde{\Upsilon}_{i,k}^T\tilde{\Upsilon}_{i,k} + \sum_{j=J}^{M}\frac{1}{\beta_{i,j}}\tilde{\Gamma}_{i,j,k}^T\tilde{\Gamma}_{i,j,k},$$

where $\tilde{\Upsilon}_i = \Upsilon_i - \Upsilon_i^*$ and $\tilde{\Gamma}_{i,j} = \Gamma_{i,j} - \Gamma_{i,j}^*$ are the parameter estimation error. Apparently, $V_{i,k} > 0$ with $V_{i,k} \to \infty$ as $S_{i,k} \to \infty$. Now seek the forward difference of the Lyapunov function$V_{i,k}$,

$$\begin{aligned}\Delta V_{i,k} &= \Delta S_{i,k}\left[\Delta S_{i,k} + 2S_{i,k}\right] + \frac{1}{\alpha_i}\left[\Delta\Upsilon_{i,k}\right]^T\left[\Delta\Upsilon_{i,k} + 2\tilde{\Upsilon}_{i,k}\right] \\ &+ \sum_{j=J}^{M}\frac{1}{\beta_{i,j}}\left[\Delta\Gamma_{i,j,k}\right]^T\left[\Delta\Gamma_{i,j,k} + 2\tilde{\Gamma}_{i,j,k}\right] \\ &= \left[\Delta S_{i,k}\right]^2 + 2\Delta S_{i,k}S_{i,k} + \frac{1}{\alpha_i}\left[\Delta\Upsilon_{i,k}\right]^T\left[\Delta\Upsilon_{i,k} + 2\tilde{\Upsilon}_{i,k}\right] \\ &+ \sum_{j=J}^{M}\frac{1}{\beta_{i,j}}\left[\Delta\Gamma_{i,j,k}\right]^T\left[\Delta\Gamma_{i,j,k} + 2\tilde{\Gamma}_{i,j,k}\right].\end{aligned} \tag{23}$$

Substitute the adaptive law (16) into the above, we have

$$\begin{aligned}\Delta V_{i,k} &= \left[\Delta S_{i,k}\right]^2 + 2\Delta S_{i,k}S_{i,k} + 2\left(\bar{u}_{eq,i} - u_{eq,i}^*\right)\Delta S_{i,k} \\ &+ \xi_i\left[\Delta S_{i,k}\right]^2.\end{aligned} \tag{24}$$

Then from the equation (23) we have

$$\begin{aligned}\Delta V_{i,k} &= \left(1+\xi_i\right)\left[\Delta S_{i,k}\right]^2 + 2\Delta S_{i,k}S_{i,k} \\ &+ 2\left(\bar{u}_{eq,i} - u_{eq,i}^*\right)\Delta S_{i,k} \\ &= \Delta S_{i,k}\left\{\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k}\right. \\ &+ \left. 2S_{i,k} - 2\left(u_v + \varepsilon\right)\right\}.\end{aligned} \tag{25}$$

Consequently, two situations are considered:

(1) If $S_{i,k} > \varphi_i$, then

$$-f_{i,u_{i,k}}\left(k_i\varphi_i + 2\eta_i\right) \le \Delta S_{i,k} \le -f_{i,u_{i,k}}k_i\varphi_i$$

holds according to (19),(21),(22) and assumption 3. Then further we have

$$\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k} + 2S_{i,k} - 2\left(u_v + \varepsilon\right) \ge$$

$$\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k} + 2\left(1 + k_i\right)\varphi_i + 2\eta_i - 2\varepsilon,$$

namely

$$\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k} + 2S_{i,k} - 2\left(u_v + \varepsilon\right) \ge 0.$$

Therefore $\Delta V_{i,k} \le 0$ holds.

(2) If $S_{i,k} < -\varphi_i$, then

$$f_{i,u_{i,k}}\left(k_i\varphi_i + 2\eta_i\right) \ge \Delta S_{i,k} \ge f_{i,u_{i,k}}k_i\varphi_i$$

holds according to (19),(21),(22) and assumption 3. Then further we have

$$\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k} + 2S_{i,k} - 2\left(u_v + \varepsilon\right) \le$$

$$\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k} - 2\left(1 + k_i\right)\varphi_i - 2\eta_i - 2\varepsilon,$$

namely

$$\left(1 + 2f_{i,u_{i,k}}^{-1} + \xi_i\right)\Delta S_{i,k} + 2S_{i,k} - 2\left(u_v + \varepsilon\right) \le 0.$$

Therefore $\Delta V_{i,k} \le 0$ also holds.

From the above $\Delta V_{i,k} \le 0$ holds when $\left|S_{i,k}\right| \ge \varphi_i$. Obviously the sliding mode switching surface $\mathbf{B}^i = \left\{x_k \left| \left|S_{i,k}\right| \le \varphi_i \right.\right\}, i = 1,2,\cdots,m$ can be reached and asymptotically stable for all $x(0) \in \Omega_{x,0}$. □

4.4 Chattering Free

However, there is an important problem for the proposed control. It is well known that SMC has a notorious shortcomingchattering. Luckily, it has been studied by many researchers [22]-[25] and all of their results can be used directly for the presented design method in this paper. For example simple modification can be taken for the hitting control[22],

$$u_{v,i} = -k_i S_i - \eta_i sat(S_i), \tag{26}$$

where $sat(x)$ is saturation function defined as

$$sat(x) = \begin{cases} 1 & x > \Delta \\ \frac{1}{\Delta} & |x| \le \Delta \\ -1 & x < -\Delta \end{cases} \tag{27}$$

with $\Delta > 0$ is a selected scalar parameter.

4.5 Controller Structure

In the adaptive wavelet sliding mode control (AWSMC) proposed in section 4, the wavelet basis and the scaling function basis compose a wavelet network (WN) which has a three-layer network structure like neural network (NN)[15][16][18].

An arbitrary function in $L^2(\mathbf{R})$ can be decomposed as the form of the wavelet series or the sum of the coarse series and the detail series, but all these series approximations are infinite. In practice, it is impossible to use the entire infinite basis, and the equation (15) gives a potential approximation. In the design of general NN, the approximation error mainly depends on the structure selection. Although it has been proven that NN is able to approximate any nonlinear function defined on a compact set for a pre-specified accuracy, how to choose the NN structure is not an easy work. Moreover, any structure change of NN will affect whole network and the approximation accuracy. However, wavelet network is different from NN. Firstly it composes infinite dimensions orthonormal basis of the subspace V_j or W_j in $L^2(\mathbf{R})$ space. Therefore every branch of the network is orthonormal with each other. Namely the change of any part of the structure does not affect other parts. Secondly, the three-layer structure of WN is fixed because WN is based on the MRA in $L^2(\mathbf{R})$. In result the approximation accuracy mainly depends on the sufficiency of the bases in the subspace V_j or W_j and the selection of the resolution j. Thirdly, it has feed-forward properties and both the structure and the parameters can be tuned on-line. These are the most important advantages of WN compared with general NN. In this way it doesnt need training and can be directly used to the on-line control by the adaptive control theory.

The proposed controller is composed with three parts: the equivalent control approximator, the hitting control and the adaptive machine. The predigestion map of the AWSMC structure is shown in Fig.1.

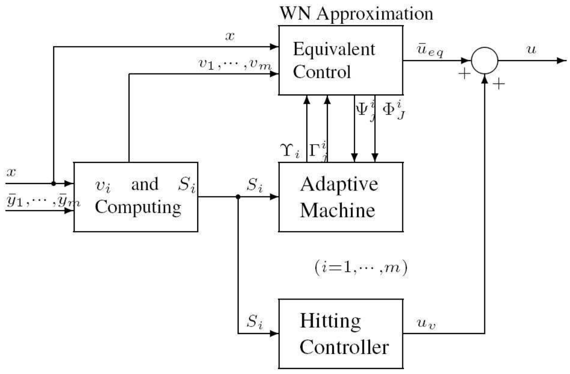

Fig.1 The structure of the AWSMC controller.

5 Simulations

In this section, the proposed controller design approach in this paper was applied to a single inverted pendulum system. It is a typical nonlinear system with high uncertainties.

The proposed AWSMC design does not need the accurate mathematic model of the inverted pendulum, that was regarded as a rigid body and the friction for the dolly was not taken into account. Consequently, its rough mathematic model is

$$x_{1,k+1} = x_{2,k},\ x_{2,k+1} = f(x_k, u_k), \tag{28}$$

where $x_k = [x_{1,k}, x_{2,k}]^T = [\theta_k, \theta_{k+1}]^T$ is the state variable(θ_k is the angle excursion of the pendulum), u_k is the control volts of the motor in the dolly. The boundaries of unmodelled dynamics and its forward difference are estimated as follow, $20 \le f_{u_{i,k}} \le 50$, $\Delta f_{u_{i,k}} \le 200$.

In case of the tracked system was $\bar{y}_k = 0$, the variable $v_k = cx_{2,k}$. Afterwards the sliding mode $S_k = [c, 1]x_k$ with $c = 0.2$ and the parameters of the adaptive law (16) were chosen as $\alpha_i = 8$ and $\beta_{i,j} = 4$.

In our simulation Haar wavelet was used for wavelet base function, namely

$$\psi(x) = \begin{cases} 1, & 0 < x \le 0.5 \\ -1, & 0.5 < x \le 1 \\ 0, & \text{others} \end{cases} \tag{29}$$

Its corresponding scale base function is

$$\phi(x) = \begin{cases} 1, & 0 < x \le 1 \\ 0, & \text{others.} \end{cases} \tag{30}$$

Obviously it can be verified that Haar wavelet function and its shift and scale dropping are orthogonal. Furthermore, Haar wavelet functions are compact support functions.

To compare the performance of presented AWSMC, a LQR controller was designed for the system (28) based on linearization, in which the poles of the closed-loop system were set at $-35.45 \pm 34.65i$ with the corresponding feedback control signal $u = -\begin{bmatrix}173.21 & 60.51\end{bmatrix} x$.

The simulation results of the above two controllers are shown in Fig.2 and Fig.3.

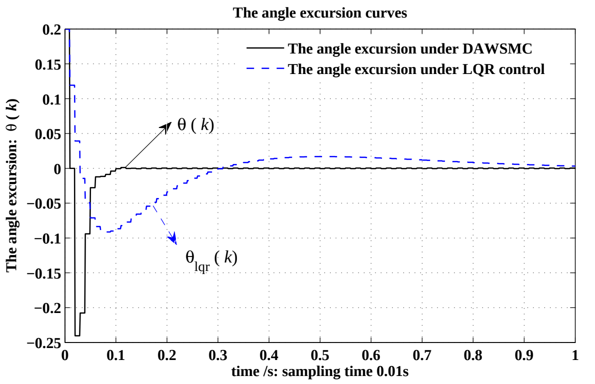

Fig.2 The angle excursions using AWSMC and LQR via linearization.

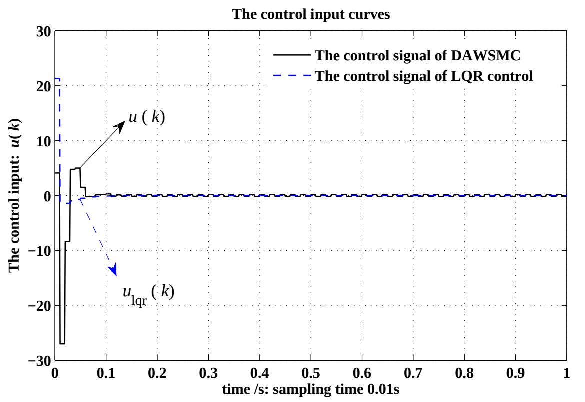

Fig.3 The control signals using AWSMC and LQR via linearization.

What is shown in Fig.2 and Fig.3 is that the AWSMC controller performance is as good as that of LQR controller. Each of the above two figures has compared the two curves of the two controllers correspondingly. Additionally, the sliding mode value curve is shown in Fig.4.

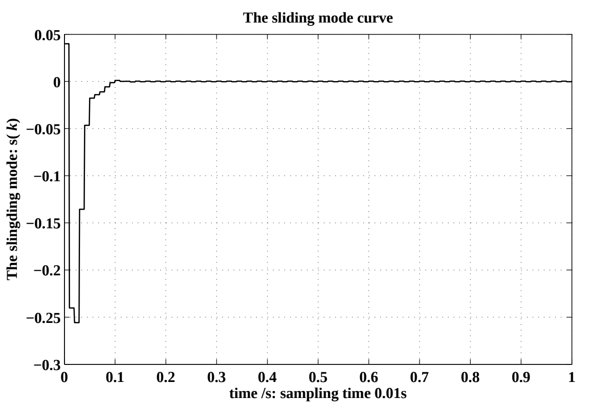

Fig.4 The sliding mode value curve when using AWSMC.

Next, when there was intense disturbance, the robustness of the AWSMC closed-loop system was examined as well. Fig.5 and Fig.6 show that the AWSMC controller is with robustness.

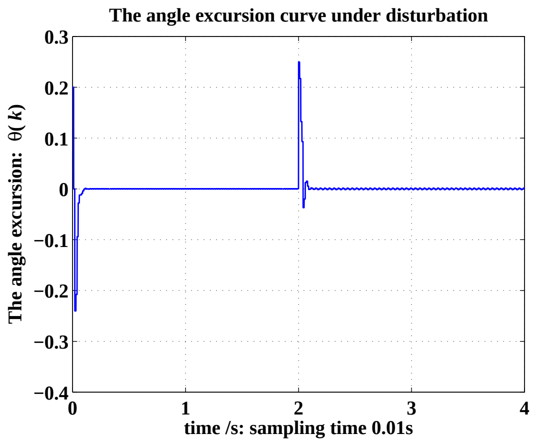

Fig.5 The angle excursion curve when the inverted pendulum was disturbed at time instant using AWSMC.

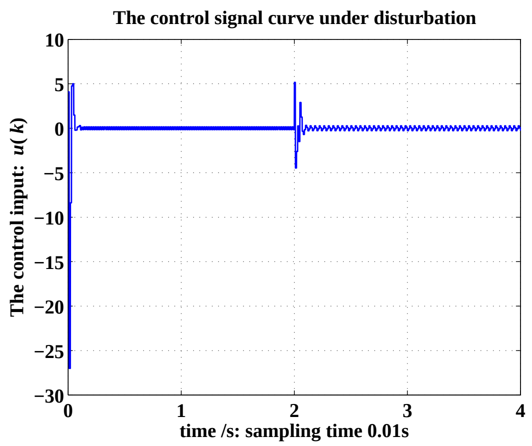

Fig.6 The control input signal curve when the inverted pendulum was disturbed at time instant using AWSMC.

6 Conclusions

An adaptive wavelet sliding mode control (AWSMC) design approach for a class of non-affine multi-variable discrete nonlinear systems is constructed. With the using of multi-resolution analysis (MRA) in wavelet theory, the equivalent control is approximated on-line by an adaptive wavelet network (WN) with appropriate adaptive laws. The reachability of the sliding mode can be guaranteed and proved to be asymptotically stable by Lyapunov stability theory. The controller performance possesses both the merits of SMC and the adaptive characters. The results of simulation on an inverted pendulum system validate its efficiency.

Acknowledgement

Project supported by National Natural Science Foundation of China (61304024).
This work is also supported by the Fundamental Research Funds for the Central Universities (3142013055), the Natural Science Foundation of Hebei Province (F2013508110) and the Science and Technology plan projects of Hebei Provincial Education Department (Z2012089).
The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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