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A Fast and Efficient Parameter Estimation Algorithm for Generalized Output Error Models

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Abstract: One kind of the colored noise interference systems is generalized output error model (OEARMA). This paper presents a two-stage recursive least squares algorithm for OEARMA. Aiming at the OEARMA, this paper puts forward a two-stage recursive least squares algorithm. The basic idea of the algorithm is to combinie the auxiliary model identification idea and the decomposition technique to decompose a system into two subsystems. Each subsystem contains a parameter vector. With auxiliary model-based recursive extended least squares theory, an unknown intermediate variable output instead of the auxiliary model identification model vector, instead of unmeasurable noise terms in the information vector with the estimated residuals, which can use recursive identification idea to estimated all the parameters of the system, the algorithm has a high computational efficiency. The example of simulation states the effectiveness of the proposed algorithm.

Keywords: Stochastic system, Least squares, Two-stage algorithm, Auxiliary model

1 Introduction

System identification of the colored noise has always been the domestic and foreign scholars are concerned about areas of research [1,2]. Stochastic systems with colored noise, the conventional least squares parameter estimation is biased [2]. Many scholars have done a lot of work, but also made a lot of effective methods for least squares algorithm identification system there is a deviation of colored noise, For example, Bias Compensation Least Squares algorithm (BCLS) [3], Recursive Generalized Least Squares (RGLS) algorithm [4], Recursive Generalized Extended Least Squares (RGELS) algorithm [4], two-stage recursive least squares parameter estimation algorithm [5], and in 1991 the thinking of the auxiliary model identification proposed by Ding Feng [6], the hierarchical identification principle [7], multi-innovation identification theory [8] and parameter estimation error bounds theory [9,10,11,12] and so on.

These methods can not only give the system model parameter estimation, and the latter two can produce noise model parameter estimation.

However, theoretical analysis shows that the unbiased estimate of bias compensation least squares algorithm for OEARMA model is difficult to do, and asked to enter is smooth (Stationary), ergodic (Ergodic). Ding Feng, improved bias compensation least squares algorithm to overcome these shortcomings [13], [14] using the filter to filter the input data, is bound to increase the amount of calculation. RGLS algorithm [15] in the process of the output signal to noise ratio is relatively large or the model parameters for a long time, this white processing of the data reliability will drop. May appear multiple local convergence point recognition accuracy is also low, so that the final identification result is biased. RGELS algorithm convergence as well as the convergence of the theory under what conditions proved challenging subject gives only an approximate analysis of the literature. Feng Ding et al. [16] proposed an iterative identification method identify CARAR model to obtain a satisfactory

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accuracy, but because of the complexity of practical problems, not online identification, model selection is more difficult to whiten at the same time the algorithm is also more complex, under what conditions convergence is also not a good solution. However, theoretical analysis shows that the RELS convergence of the algorithm requirements for noise model is strictly positive real transfer function [17,18,19].

The recursive least squares algorithm to solve the ARX model identification problem [20], the recursive extended least squares algorithm to solve the problem of identification of ARMAX model [20], the auxiliary model identification method [21,22,23,24,25] and bias compensation method to solve the identification problem of the output error model. Auxiliary variable least squares algorithm can be used to identify the system, but can not be given parameter estimation of the noise model. For output error model, in addition to the above-mentioned method, using the rational fraction equivalence method, and further using of relevant technology, proposed parameter estimation algorithm according to the finite impulse response model order incremental. Using rational fraction equivalence method, studied multi-input single-output system identification problem; using rational fraction equivalent ways to simplify the colored noise stochastic system, use the approximate simplified model can be extended least squares algorithm to estimate the parameters, and then determine the parameters of the original system.

The above-mentioned method only solves the special OEARMA model identification when a polynomial or two polynomials is 1. For the general form of stochastic systems, this paper proposes a class of generalized output error model two-stage recursive extended least squares parameter estimation algorithm. The basic idea of this algorithm is a combination of the auxiliary model identification of ideas and the decomposition technique, the system is decomposed into two subsystems, each subsystem contains a parameter vector.

With auxiliary model and the recursive extended least squares theory, using the auxiliary model's output instead of unknown intermediate variable of the identification model vector, and using the estimated residuals instead of unpredictable noise of the information vector, thus we can use recursive identification ideas estimated all the parameters of the system.

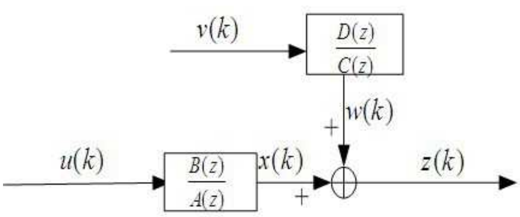

Fig. 1 The Random system block diagram

2 System description and identification model

Defining that "A:=X" or "X:=A" signifies "A is equivalent to X"; I signifies a unit matrix of appropriate sizes ($n\times n$); the superscript T signifies the matrix/vector transpose; I_n signifies an n-dimensional column vector whose elements are 1.

Consider the output error system, described in Fig. 1,

$$z(k)=\frac{B(z)}{A(z)}u(k)+\frac{D(z)}{C(z)}v(k) \tag{1}$$

Among them, $\{u(k)\}$ is a system input at time K , $\{z(k)\}$ is a system output at time K, $\{v(k)\}$ is an uncorrelated random white noise with zero mean and its variance is σ^2, $A(z)$, $B(z)$, $C(z)$ and $D(z)$ are polynomials in the unit backward shift functor $z^{-1}[i.e., z^{-1}y(k)=y(k-1)]$, moreover,

$$\begin{aligned}
A(z) &:= 1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n_a}z^{-n_a}\\
B(z) &:= b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_b}z^{-n_b}\\
C(z) &:= 1+c_1z^{-1}+c_2z^{-2}+\cdots+c_{n_c}z^{-n_c}\\
D(z) &:= 1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_d}z^{-n_d}
\end{aligned}$$

We may assume that $k\le 0$, $u(k)=0$, $z(k)=0$, $v(k)=0$, and the order known. The goal of this article is based on the separation of two-stage identification algorithm, the original recognition system into two sub-problems of smaller order. Define parameter vectors,

$$\begin{aligned}
\theta &:= \begin{bmatrix}\theta_s\\ \theta_n\end{bmatrix}\in R^n, n=n_a+n_b+n_c+n_d\\
\theta_s &:= [a_1,a_2,\cdots,a_{n_a},b_1,b_2,\cdots,b_{n_b}]^T\in R^{n_a+n_b}\\
\theta_n &:= [c_1,c_2,\cdots,c_{n_c},d_1,d_2,\cdots,d_{n_d}]^T\in R^{n_c+n_d}
\end{aligned}$$

Define the information vectors,

$$\begin{aligned}
\varphi(k) &:= \begin{bmatrix}\varphi_s(k)\\ \varphi_n(k)\end{bmatrix}\in R^n, n=n_a+n_b+n_c+n_d\\
\varphi_s(k) &:= [-x(k-1),-x(k-2),\cdots,-x(k-n_a),\\
&\quad u(k-1),u(k-2),\cdots,u(k-n_b)]\in R^{(n_a+n_b)}\\
\varphi_n(k) &:= [-w(k-1),-w(k-2),\cdots,-w(k-n_c),\\
&\quad v(k-1),v(k-2),\cdots,v(k-n_d)]\in R^{(n_c+n_d)}
\end{aligned}$$

Define the intermediate variables $x(k)$, $w(k)$, as follows,

$$x(k):=\frac{B(z)}{A(z)}u(k) \tag{2}$$

or

$$\begin{aligned}x(k) &= [1-A(z)]x(k)+B(z)u(k)\\ &= (-a_1z^{-1}-a_2z^{-2}-\cdots-a_{n_a}z^{-n_a})x(k)\\ &\quad +(b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_b}z^{-n_b})u(k)\\ &= -a_1x(k-1)-a_2x(k-2)-\cdots-a_{n_a}x(k-n_a)\\ &\quad +b_1u(k-1)+b_2u(k-2)+\cdots+b_{n_b}u(k-n_b)\\ &= \varphi_s^T(k)\theta_s\end{aligned}$$

$$w(k) := \frac{D(z)}{C(z)}v(k) \tag{3}$$

or

$$\begin{aligned}w(k) &= [1-C(z)]w(k)+D(z)v(k)\\ &= (-c_1z^{-1}-c_2z^{-2}-\cdots-c_{n_c}z^{-n_c})w(k)\\ &\quad +(1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_d}z^{-n_d})v(k)\\ &= -c_1w(k-1)-c_2w(k-2)-\cdots-c_{n_c}w(k-n_c)\\ &\quad +v(k)+d_1v(k-1)+d_2v(k-2)+\cdots+d_{n_d}v(k-n_d)\\ &= \varphi_n^T(k)\theta_n+v(k)\end{aligned}$$

Using (2) and (3), (1) can be expressed as

$$\begin{aligned}z(k) &= x(k)+w(k)\\ &= \varphi_s^T(k)\theta_s+\varphi_n^T(k)\theta_n+v(k)\\ &= \varphi^T(k)\theta+v(k) \qquad (4)\end{aligned}$$

3 Two-stage recursive least squares algorithm

The basic idea of the algorithm is to transform the system into two subsystems, the parameter vector and the information vector were also transformed into two sub-parameter vectors and two sub-information vectors. Then the auxiliary model identification idea is used to identify the parameters of each subsystem. Define two intermediate variables,

$$z_1(k) := z(k)-\varphi_n^T(k)\theta_n \tag{5}$$
$$z_2(k) := z(k)-\varphi_s^T(k)\theta_s \tag{6}$$

The system in (4) can be transformed into the two virtual identification subsystems, as follows,

$$z_1(k) = \varphi_s^T(k)\theta_s+v(k)$$
$$z_2(k) = \varphi_n^T(k)\theta_n+v(k)$$

These two subsystems contain the parameter vectors θ_s and θ_n, separately. Define two criterion functions,as follows,

$$J_1(\theta_s) := \sum_{j=1}^{k}[z_1(k)-\varphi_s^T(k)\theta_s]^2$$
$$J_2(\theta_n) := \sum_{j=1}^{k}[z_2(k)-\varphi_n^T(k)\theta_n]^2$$

Make the partial derivatives of $J_1(\theta_s)$ and $J_2(\theta_n)$ for θ_s and θ_n be zero, separately,

$$\frac{\partial J_1(\theta_s)}{\partial\theta_s} = -2\varphi_s(j)\sum_{j=1}^{k}[z_1(j)-\varphi_s^T(j)\theta_s]=0$$
$$\frac{\partial J_2(\theta_n)}{\partial\theta_n} = -2\varphi_n(j)\sum_{j=1}^{k}[z_2(j)-\varphi_n^T(j)\theta_n]=0$$

Make $\hat\theta(k):=\begin{bmatrix}\hat\theta_{s(k)}\\ \hat\theta_{n(k)}\end{bmatrix}\in R^n$ be the estimate of $\theta:=\begin{bmatrix}\theta_s\\ \theta_n\end{bmatrix}\in R^n$ at time k. Then minimizing the criterion functions, so we can get the recursive least squares (RLS) algorithm,

$$\hat\theta_s(k) = \hat\theta_s(k-1)+K_s(k)[z_1(k)-\varphi_s^T(k)\hat\theta_s(k-1)] \tag{7}$$
$$K_s(k) = P_s(k-1)\varphi_s(k)[1+\varphi_s^T(k)P_s(k-1)\varphi_s(k)]^{-1} \tag{8}$$
$$P_s(k) = [I-K_s(k)\varphi_s^T(k)]P_s(k-1), P_s(0)=P_0I \tag{9}$$
$$\hat\theta_n(k) = \hat\theta_n(k-1)+K_n(k)[z_2(k)-\varphi_n^T(k)\hat\theta_n(k-1)] \tag{10}$$
$$K_n(k) = P_n(k-1)\varphi_n(k)[1+\varphi_n^T(k)P_n(k-1)\varphi_n(k)]^{-1} \tag{11}$$
$$P_n(k) = [I-K_n(k)\varphi_n^T(k)]P_n(k-1), P_n(0)=P_0I \tag{12}$$

Substituting (5) and (6) into (7) and (10), separately, then

$$\begin{aligned}\hat\theta_s(k) &= \hat\theta_s(k-1)+K_s(k)[z(k)-\varphi_n^T(k)\theta_n\\ &\quad -\varphi_s^T(k)\hat\theta_s(k-1)] \qquad (13)\\ \hat\theta_n(k) &= \hat\theta_n(k-1)+K_n(k)[z(k)-\varphi_s^T(k)\theta_s\\ &\quad -\varphi_n^T(k)\hat\theta_n(k-1)] \qquad (14)\end{aligned}$$

Using the estimates $\hat\theta_n(k-1)$ and $\hat\theta_s(k-1)$ to replace the unknown parameter vectors at the right-hand sides of (13) and (14), separately, then we can get

$$\begin{aligned}\hat\theta_s(k) &= \hat\theta_s(k-1)+K_s(k)[z(k)-\varphi_n^T(k)\hat\theta_n(k-1)\\ &\quad -\varphi_s^T(k)\hat\theta_s(k-1)] \qquad (15)\\ \hat\theta_n(k) &= \hat\theta_n(k-1)+K_n(k)[z(k)-\varphi_s^T(k)\hat\theta_s(k-1)\\ &\quad -\varphi_n^T(k)\hat\theta_n(k-1)] \qquad (16)\end{aligned}$$

Using the estimates $\varphi_s(k)$ and $\varphi_n(k)$ to replace the unknown information vectors at the right-hand sides of (8), (11), (15) and (16). Finally, we can get

$$\begin{aligned}\hat\varphi_s(k) &:= [-\hat x(k-1),-\hat x(k-2),\cdots,-\hat x(k-n_a),\\ &\quad \hat u(k-1),\hat u(k-2),\cdots,\hat u(k-n_b)]\in R^{n_a+n_b} \qquad (17)\\ \hat\varphi_n(k) &:= [-\hat w(k-1),-\hat w(k-2),\cdots,-\hat w(k-n_c),\\ &\quad \hat v(k-1),\hat v(k-2),\cdots,\hat v(k-n_d)]\in R^{n_c+n_d} \qquad (18)\end{aligned}$$

Define,

$$\hat\varphi(k) := \begin{bmatrix}\hat\varphi_s(k)\\ \hat\varphi_n(k)\end{bmatrix}\in R^n$$

Replacing $\varphi_s(k)$, $\varphi_n(k)$, $\theta_s(k)$, $\theta_n(k)$ in (2), (3) and (4) with $\hat{\varphi}_s(k)$, $\hat{\varphi}_n(k)$, $\hat{\theta}_s(k)$, $\hat{\theta}_n(k)$, giving,

$$\begin{aligned}\hat{x}(k) &= \hat{\varphi}_s^T(k)\hat{\theta}_s,\\ \hat{w}(k) &= z(k) - \hat{\varphi}_s^T(k)\hat{\theta}_s,\\ \hat{v}(k) &= z(k) - \hat{\varphi}^T(k)\hat{\theta} \end{aligned} \tag{19}$$

We can obtain the two-stage recursive least squares identification algorithm (TS-RLS) for estimating the parameter vectors θ_n and θ_s of the OE models, as follows,

$$\hat{\theta}_s(k) = \hat{\theta}_s(k-1) + K_s(k)[z(k) - \hat{\varphi}_n^T(k)\hat{\theta}_n(k-1) - \hat{\varphi}_s^T(k)\hat{\theta}_s(k-1)] \tag{20}$$

$$K_s(k) = P_s(k-1)\hat{\varphi}_s(k)[1+\hat{\varphi}_s^T(k)P_s(k-1)\hat{\varphi}_s(k)]^{-1} \tag{21}$$

$$P_s(k) = [I - K_s(k)\hat{\varphi}_s^T(k)]P_s(k-1), P_s(0) = P_0 I \tag{22}$$

$$\hat{\varphi}_s(k) = [-\hat{x}(k-1), -\hat{x}(k-2), \cdots, -\hat{x}(k-n_a), u(k-1), u(k-2), \cdots, u(k-n_b)] \tag{23}$$

$$\hat{\theta}_n(k) = \hat{\theta}_n(k-1) + K_n(k)[z(k) - \hat{\varphi}_s^T(k)\hat{\theta}_s(k) - \hat{\varphi}_n^T(k)\hat{\theta}_n(k)] \tag{24}$$

$$K_n(k) = P_n(k-1)\hat{\varphi}_n(k)[1+\hat{\varphi}_n^T(k)P_n(k-1)\hat{\varphi}_n(k)]^{-1} \tag{25}$$

$$P_n(k) = [I - K_n(k)\hat{\varphi}_n^T(k)]P_n(k-1), P_n(0) = P_0 I \tag{26}$$

$$\hat{\varphi}_n(k) = [-\hat{w}(k-1), -\hat{w}(k-2), \cdots, -\hat{w}(k-n_c), \hat{v}(k-1), \hat{v}(k-2), \cdots, \hat{v}(k-n_d)] \tag{27}$$

$$\hat{x}(k) = \hat{\varphi}_(s)^T(k)\hat{\theta}_s, \hat{w}(k) = z(k) - \hat{\varphi}_s^T(k)\hat{\theta}_s, \quad \hat{v}(k) = z(k) - \hat{\varphi}^T(k)\hat{\theta} \tag{28}$$

$K_s(k)$ and $K_n(k)$ are two gain vectors, and $P_s(k)$ and $P_n(k)$ are two covariance matrices.

The steps of computing $\hat{\theta}_s(k), \hat{\theta}_n(k)$ in the RLS algorithm in (20)–(28) are listed in the following:

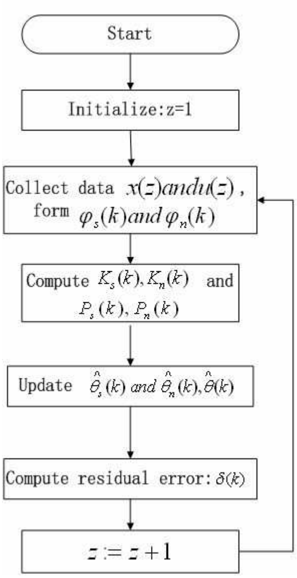

Fig. 2 The flowchart of computing the OEARMA parameter estimation $\hat{\theta}_s(k), \hat{\theta}_n(k)$

4 The recursive extended least squares algorithm-RELS

To compare with the proposed algorithm, the auxiliary model based recursive least square algorithm is introduced in this section. Recursive Extended Least Squares algorithm is an identification method, which is used to deal with colored noise of the CARMA model by increasing the dimension of the parameter vector and dope-vector. That is to say, noise regression item is added in the information vector, while noise model parameters are mixed in the parameter vector.

$$\theta := [a_1, a_2, \cdots, a_{n_a}, b_1, b_2, \cdots, b_{n_b}, c_1, c_2, \cdots, c_{n_c}, d_1, d_2, \cdots, d_{n_d}] \in R^{n_a+n_b+n_c+n_d}$$

$$\varphi(k) := [-z(k-1), -z(k-2), \cdots, -z(k-n_a), u(k-1), u(k-2), \cdots, u(k-n_b), -w(k-1), -w(k-2), \cdots, -w(k-n_c), v(k-1), v(k-2), \cdots, v(k-n_d)]$$

$$J(\theta) := \sum_{j=1}^{k} [z(k) - \varphi^T(k)\theta]^2$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\hat{\varphi}(k)[z(k) - \hat{\varphi}^T(k)\hat{\theta}(k-1)]$$

$$P(k) = P(k-1) - \frac{P(k-1)\hat{\varphi}(k)\hat{\varphi}^T(k)P(k-1)}{1+\hat{\varphi}^T(k)P(k-1)\hat{\varphi}(k)}, P(0) = p_0 I$$

$$\hat{v}(k) = z(k) - \hat{\varphi}^T(k)\hat{\theta}(k-1)$$

$$\hat{\varphi}(k) = [-z(k-1), -z(k-2), \cdots, -z(k-n_a), u(k-1), u(k-2), \cdots, u(k-n_b), -\hat{w}(k-1), -\hat{w}(k-2), \cdots, -\hat{w}(k-n_c), \hat{v}(k-1), \hat{v}(k-2), \cdots, \hat{v}(k-n_d)]$$

$$\hat{\theta} = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_{n_a}, \hat{b}_1, \hat{b}_2, \cdots, \hat{b}_{n_b}, \hat{c}_1, \hat{c}_2, \cdots, \hat{c}_{n_c}, \hat{d}_1, \hat{d}_2, \cdots, \hat{d}_{n_d}]$$

5 Example

Consider the following OEARMA system,

$$Z(k) = \frac{B(k)}{A(k)}u(k) + \frac{D(k)}{C(k)}v(k)$$

$$w(k) = \frac{D(k)}{C(k)}v(k)$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 + 1.60z^{-1} + 0.8z^{-2}$$

$$B(z) = b_1 z^{-1} + b_2 z^{-2} = 0.412z^{-1} + 0.309z^{-2}$$

$$C(z) = 1 + c_1 z^{-1} = 1 + 0.8z^{-1}$$

$$D(z) = 1 + d_1 z^{-1} = 1 - 0.64z^{-1}$$

$$\theta^T = [1.60, 0.8, 0.412, 0.309, 0.8, -0.64]$$

In simulation, the system input $\{u(k)\}$ is an uncorrelated random signal sequence with zero mean and unit variance, and $\{v(k)\}$ as a white noise sequence with zero mean and its variance is σ^2. Using the RLS algorithm to estimate the parameters of this system, the parameter estimate and estimation errors $\delta := \frac{\|\hat{\theta}-\theta\|}{\|\theta\|}$ versus k are are presented in Tables 1 and 2. The parameter estimation errors of the RLS algorithm with different variances are presented in Fig. 3. The parameter estimation errors for the two algorithms are presented in Fig. 4. From these Tables and Figs, we can find that:

• With the gradual increasing of noise variance decreases, the precision of the parameter estimates gradually improved, referring to Fig.3 to control;

• With the increase of the length of data, the parameter estimation error is smaller and smaller, and in Fig.4, the RLS algorithm can visually see superior RELS algorithm;

• The reference to Table 3 that the amount of calculation of the RLS algorithm respect RELS algorithm to be greatly simplified, wherein $n = n_a + n_b + n_c + n_d$.

Table 1 RLS algorithm parameter estimates and residuals

k	a_1	a_2	b_1	b_2	c_1	d_1	$\delta(\%)$
100	1.20260	0.50691	0.44384	0.10396	0.79078	-0.61422	25.24060
200	1.38256	0.62701	0.43945	0.20664	0.80789	-0.73193	14.65398
500	1.50781	0.73472	0.41297	0.25746	0.82262	-0.63651	5.94194
1000	1.56554	0.77144	0.42204	0.29850	0.79953	-0.61600	2.48625
2000	1.56720	0.77511	0.40969	0.30610	0.79876	-0.63711	1.95130
3000	1.57315	0.77594	0.41251	0.30417	0.79270	-0.67137	2.28639
4000	1.61920	0.81367	0.40720	0.32474	0.79790	-0.66544	1.80940
5000	1.60369	0.79932	0.40738	0.31820	0.79974	-0.66888	1.45374
6000	1.61022	0.80409	0.40868	0.32003	0.80252	-0.65743	1.11775
7000	1.60706	0.80172	0.41220	0.31841	0.80221	-0.66047	1.11907
True values	1.60000	0.80000	0.41200	0.30900	0.80000	-0.64000	0.00000

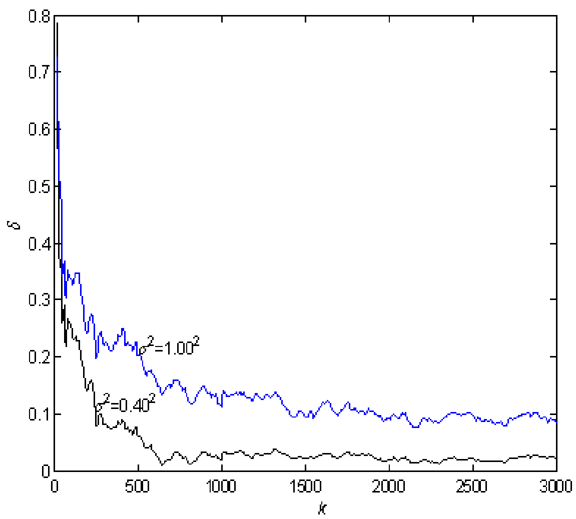

Fig. 3 The RLS parameter estimation errors δ versus $k(\sigma^2 = 0.40^2$ and $\sigma^2 = 1.00^2)$

Table 2 RLS algorithm parameter estimates and residuals

k	a_1	a_2	b_1	b_2	c_1	d_1	$\delta(\%)$
100	1.64648	0.77137	0.42953	0.30980	0.54707	-0.60255	12.33223
200	1.63711	0.78087	0.41288	0.32958	0.68643	-0.68990	6.23565
500	1.59082	0.74141	0.40771	0.28768	0.77035	-0.65435	3.35251
1000	1.58187	0.73913	0.42097	0.30178	0.76132	-0.65371	3.60007
2000	1.56228	0.72573	0.40866	0.30378	0.78236	-0.68412	4.52285
3000	1.55633	0.72314	0.41106	0.29813	0.78266	-0.72008	5.69570
4000	1.54884	0.72009	0.40550	0.29869	0.80517	-0.70123	5.35111
5000	1.54388	0.71793	0.40621	0.29756	0.81598	-0.70100	5.57394
6000	1.54133	0.71753	0.40815	0.29671	0.82358	-0.68668	5.39588
7000	1.53665	0.71462	0.41102	0.29458	0.82989	-0.68782	5.70455
True values	1.60000	0.80000	0.41200	0.30900	0.80000	-0.64000	0.00000

Table 3 Comparison of the computational efficiency of the RLS and RELS algorithms

Computational efficiency \ Algorithms	RLS	RELS
Number of multiplication	$2(n_a+n_b)^2+2(n_c+n_d)^2+4n$	$2n^2+4n$
Number of additions	$2(n_a+n_b)^2+2(n_c+n_d)^2+2n$	$2n^2+2n$
The total number of calculation	$4(n_a+n_b)^2+4(n_c+n_d)^2+6n$	$4n^2+6n$

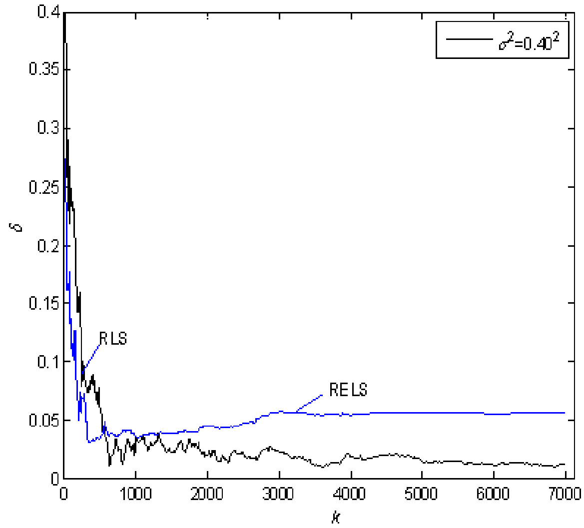

Fig. 4 The parameter estimation errors versus t for the two algorithms $(\sigma^2 = 0.40^2)$

6 Conclusion

A class of generalized output error model of two-stage recursive least squares parameter estimation algorithm has been derived in the paper. with the help of the auxiliary model identification idea and decomposition techniques ,the system is converted to the two-step process in the proposed algorithm. Use the measurable information of the system to build a auxiliary model, of which inputs can replace the unmeasurable variates. By choosing the parameters of the auxiliary model, the inputs of the auxiliary model can approach to these unmeasurable variates, thus obtaining the concordant estimation of the system variates. The derivation of the algorithm is simple, less computation, high accuracy. Theoretical analysis and simulation results verify these conclusions.

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References

[1] Liu SXHuang M, An auxiliary models Identification method of Multi-variable Systems with colored noise,Computer Measurement and Control, **17**, 145-147 (2009).

[2] Cui GM, Guan YH, Zhang Y, The identification research of one Kind of colored noise interference system, Science Technology and Engineering, **10**, 4358-4362 (2010).

[3] Feng CB, Zheng WX, Liu B, The error compensation least squares of System parameter estimation, Control and Decision, 2-8 (1986).

[4] Ding F. The parameters identification of Box-Jenkins model based Recursive Extended Least Squares algorithm, Control and Decision, **5**, 53-561 (1990).

[5] Duan HH, Jia J, Ding RF. Two-stage recursive least squares parameter estimation algorithm for output error models, Mathematical and Computer Modelling, **55**, 1151-1159 (2012).

[6] Ding F. system identification (4)Auxiliary model identification ideas and methods, Nanjing Information engineering journal, **3**, 289-318 (2011).

[7] Ding F. system identification (7): Hierarchical recognition principle and method, Nanjing Information engineering journal, **4**, 97-124 (2012).

[8] Ding F. system identification (6): multi-innovation Identification theory and method, Nanjing Information engineering journal, **4**, 1-28 (2012).

[9] Ding F, Ding T, The Least Squares's astringency of stochastic system in the condition of weakening Motivation, Hubei Journal of engineering, **16**, 5-7 (2001).

[10] Ding T, Ding F, The upper bound and convergence rate of the least Squares Parameter estimation error, Basic Automation, **8**, 31-33 (2001).

[11] Ding F, Yang Jb, The least Squares Convergence analysis about Martingale super convergence theorem and forgetting factor, Control Theory and Applications, **16**, 569-572 (1999).

[12] Yu L, Zhang JB, Liao YW, Ding J, Parameter estimation error bounds for Hammerstein finite impulsive response models, Applied Mathematics and Computation, **202**, 472-480 (2008).

[13] Ding J, Ding F, Bias compensation based parameter estimation for output error moving average systems, International Journal of Adaptive Control and Signal Processing, **25**, 1100-1111 (2011).

[14] Xie L, Yang HZ, Ding F, Recursive least squares parameter estimation for non-uniformly sampled systems based on the data filtering, Mathematical and Computer Modelling, **54**, 315-324 (2011).

[15] Li M, Tang Q and Zhou Z, Recent Computational Explorations for Nano structured Hydrogen Storage Materials,J. Comput. Theor. Nanosci., **8**, 2398-2405 (2011)

[16] Wang JH, Ding F, The off-line least Squares Iterative identification method about CARMA model ,Science Technology and Engineering, **12**, (2007).

[17] Ding F, Chen T, Author's Reply to Comments on Identification of Hammerstein Nonlinear ARMAX Systems, Automatica, **43**, 1497 (2007).

[18] Ding F, Chen T. Identification of Hammerstein nonlinear ARMAX systems, Automatica, **41**, 1479-1489 (2005).

[19] Ding F, Xie XM. The Recursive Extended Least Squares algorithm Convergence analysis of multivariable system, Control and Decision, **7**, 443-447 (1992).

[20] Xie XM, Ding F. Self-adaptive control system. Beijing: Tsinghua University Press, 68-75 (2002).

[21] Ding F, Chen T. Combined parameter and output estimation of dual- rate systems using an auxiliary model, Automatica, **40**, 1739-1748 (2004).

[22] Gao XF, Jiang D, Zhao YL, Shigeru Nagase, Zhang SB, and Chen ZFTheoretical Insights into the Structures of Graphene Oxide and Its Chemical Conversions Between GrapheneJ. Comput. Theor. Nanosci., **8**, 2406-2422 (2011).

[23] Zheng WX. Least-squares identification of a class of multivarable systems with correlated disturbances. Journal of the Franklin Institute, **336**, 1309-1324 (1999).

[24] Zheng WX. Abias correction method for identification of linear dynamic errors-in-variables models. IEEE Transactions on Automatic Control, **47**, 1142-1147 (2002).

[25] Zheng WX. Parameter estimation of stochastic linear systems with noisy input. International Journal of Systems Science, **35**, 185-190 (2004).

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