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A Domian Decomposition Sumudu Transform Method for Solving Fractional Nonlinear Equations

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Abstract: In this paper, we use a new technique called a domain decomposition Sumudu transform method (ADSTM) to solve generalized nonlinear fractional Fokker-Planck equation. We show that the new technique finds solution without any discretization or restrictive assumptions and avoids the round-off errors. The new technique shows that the approach is very efficient, simple and can be applied to other nonlinear problems.

Keywords: Sumudu transform, Adomian decomposition method, Adomian decomposition Sumudu transform method, Linear and nonlinear fractional Fokker-Planck equations.

1 Introduction

Most of phenomena in nature are described by nonlinear differential equations. Different analytical methods have been applied to find a solution to them. For example, Adomian has presented and developed a so-called decomposition method for solving differential equations [1]. The Fokker-Planck equation was first introduced by Fokker and Planck to describe the Brownian motion of particles [2]. This equation has been used in different fields in natural sciences such as quantum optics, solid state physics, chemical physics, theoretical biology and circuit theory. Fokker-Planck equations describe the erratic motions of small particles that are immersed in fluids, fluctuations of the intensity of laser light, velocity distributions of fluid particles in turbulent flows and the stochastic behavior of exchange rates [3]. It is a more general form of linear one which has also been applied in vast areas such as plasma physics, surface physics, astrophysics, the physics of polymer fluids and particle beams, nonlinear hydrodynamics, theory of electronic-circuitry and laser arrays, engineering, biophysics, population dynamics, human movement sciences, neurophysics, psychology and marketing [4,5,6,7,8] in this paper, we introduce a new approximate method, namely a domain decomposition Sumudu transform method (ADSTM) for solving the nonlinear differential equations.

2 Definition and Preliminaries

In this section, we give some definitions and lemmas which are used in this paper.
Definition 2.1. In early 90's, Watugala [9] introduced a new integral transform, named the Sumudu transform which applied to the solution of ordinary differential equation in control engineering problems. The Sumudu transform is defined over the set of functions:

$$A=\left\{f(t):\exists M,\tau_1,\tau_2>0,\mid f(t)\mid<Me^{\mid t\mid/\tau_j}, if\, t\in(-1)^j\times[0,\infty)\right\}.$$

By the following formula:

$$G(u)=S[f(t);u]=\int_0^\infty f(ut)e^{-t}dt, u\in(-\tau_{1,},\tau_2).$$

Definition 2.2. Sumudu transform of the Caputo fractional derivative is defined as follows [3]:

$$S[D_t^\alpha f(t)]=u^{-\alpha}S[f(t)]-\sum_{k=0}^{m-1}u^{-\alpha+k}f^{(k)}(0+),(m-1<\alpha\leq m).$$

Definition 2.3. The Mittage-Leffler function $E_\alpha(z)$with $\alpha>0$ is defined by the following series representation,

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valid in the whole complex plane [10] **:**

$$E_\alpha(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}.$$

Definition 2.4. Some properties of the Sumudu transform and its derivatives found in [11,12]

3 A domain Decomposition Method

The adomian decomposition method was introduced and developed by George Adomian [1]. A considerable amount of research work has been invested recently in applying this method to a wide class of linear and nonlinear ordinary differential equations, partial differential equations and integral equations. The Adomian decomposition method consists of decomposing the unknown function $u(x,y)$ of any equation into a sum of an infinite number of components defined by the decomposition series:

$$u(x,y)=\sum_{n=0}^{\infty}u_n(x,y). \tag{1}$$

where the components $u_n(x,y), n\geq 0$ are to be determined in a recursive manner. The decomposition method concerns itself with finding the components $u_0,u_1,u_2,..$ individually. As will be seen through the text, the determination of these components can be achieved in an easy way through a recursive relation that usually involve simple integrals. To give a clear overview of Adomian decomposition method, we first consider the linear differential equation written in an operator form by:

$$Lu+Ru=g. \tag{2}$$

where $L$is, mostly, the lower order derivative which is assumed to be invertible, R is other linear differential operator, and$g$ is a source term. We next apply the inverse operator L^{-1} to both sides of equation (2) and using the given condition to obtain:

$$u=f-L^{-1}(Ru) \tag{3}$$

where the function $f$represents the terms arising from integrating the source term $g$and from using the given conditions that are assumed to be prescribed. As indicated before, Adomian method defines the solution $u$by an infinite series of components given by:

$$u=\sum_{n=0}^{\infty}u_n \tag{4}$$

where the component $u_0,u_1,u_2,..$ are usually recurrently determined. Substituting (4) into both sides of (3) leads to:

$$\sum_{n=0}^{\infty}u_n=f-L^{-1}(R(\sum_{n=0}^{\infty}u_n)) \tag{5}$$

For simplicity, Equation (5) can be rewritten as:

$$u_0+u_1+u_2+.........=f-L^{-1}(R(u_0+u_1+u_2+....)) \tag{6}$$

To construct the recursive relation needed for the determination of the components $u_0,u_1,u_2,....$ It is important to note that Adomian method suggests that the zeroth component f is usually defined by the function u_0 described above, i.e. by all terms, that are not included under the inverse operator L^{-1}, which arise from the initial data and from integrating the inhomogeneous term. Accordingly, the formal recursive relation is defined by:

$$\begin{aligned}&u_{k+1}=-L^{-1}(R(u_k)),k\geq 0,\\&u_0=f\end{aligned} \tag{7}$$

or equivalently

$$\begin{aligned}&u_0=f\\&u_1=-L^{-1}(R(u_0))\\&u_2=-L^{-1}(R(u_1))\end{aligned} \tag{8}$$

It is clearly seen that the relation (8) reduced the differential equation under consideration into an elegant determination of computable components. Having determined these components, we then substitute it into (4) to obtain the solution in a series form.

4 Fokker-Planck equation

The general form of Fractional Fokker-Plank equation is :

$$\frac{\partial^\alpha U}{\partial t^\alpha}=[-\frac{\partial}{\partial x}A(x)+\frac{\partial^2}{\partial x^2}B(x)]U \tag{9}$$

With initial condition :

$$u(x,0)=f(x),x\in R$$

where $u(x,y)$is an unknown function, $A(x)$ and $B(x)$are called diffusion and drift coefficients such that. The diffusion and drift coefficients in equation (9) can be functions of x and t as well as:

$$\frac{\partial^\alpha U}{\partial t^\alpha}=[-\frac{\partial}{\partial x}A(x,t)+\frac{\partial^2}{\partial x^2}B(x,t)]U.$$

Equation (9) is also well known as a forward Kolmogorov equation. There exists another type of this equation is called a backward one as [2]:

$$\frac{\partial^\alpha U}{\partial t^\alpha}=[-A(x,t)\frac{\partial}{\partial x}+B(x,t)\frac{\partial^2}{\partial x^2}]U.$$

A generalization of equation (9) to N-variables of $x_1,x_2,x_3,...,x_N$, yields to

$$\frac{\partial^\alpha U}{\partial t^\alpha}=[-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}A_i(x)+\sum_{i,j=1}^{N}\frac{\partial^2}{\partial x_i\partial x_j}B_{I,J}(x)]U.$$

with the initial condition:

$$U(x,0) = f(x), x = (x_1, x_2, ..., x_N) \in R^N.$$

The nonlinear form of the Fokker-Planck equation can be expressed in the following form:

$$\frac{\partial^\alpha U}{\partial t^\alpha} = [-\frac{\partial}{\partial x}A(x,t,U) + \frac{\partial^2}{\partial x^2}B(x,t,U)]U \quad (10)$$

A generalization of equation(10) to N-variables of $x_1, x_2, x_3, ..., x_N$, yields to

$$\frac{\partial^\alpha U}{\partial t^\alpha} = [-\sum_{i-1}^{N}\frac{\partial}{\partial x_i}A_i(x,t,U) + \sum_{i,j=1}^{N}\frac{\partial^2}{\partial x_i \partial x_j}B_{i,j}(x,t.U)]U.$$

5 A domain Decomposition Sumudu Transform Method (ADSTM)

We consider the general inhomogeneous nonlinear equation with initial conditions given below [2]:

$$LU + RU + NU = h(x,t)$$

where $L$is the lowest order derivative which is assumed to be easily invertible, R is a linear differential operator of order less than L, NU represents the nonlinear terms and $h(x,t)$ is the source term. First we explain the main idea of SDM: the method consists of applying Sumudu transform :

$$S[LU] + S[RU] + S[NU] = S[h(x,t)]$$

Using the differential property of Sumudu transform and initial conditions we get:

$$S[h(x,t)] = \frac{1}{u^n}S[U(x,t)] - \frac{1}{u^n}U(x,0) - \frac{1}{u^{n-1}}U^{'}(x,0) - ... - \frac{U^{(n-1)}(x,0)}{u} + S[RU] + S[NU].$$

By arrangement we have:

$$\begin{aligned} S[U(x,t)] &= U(x,0) + uU^{'}(x,0) + ... \\ &+ u^{n-1}U^{n-1}(x,0) - u^n S[RU] \\ &- u^n S[NU] + u^n S[h(x,t)]. \end{aligned} \quad (11)$$

The second step in Sumudu decomposition method is that we represent solution as an infinite series:

$$U(x,t) = \sum_{i=0}^{\infty} U_i(x,t) \quad (12)$$

and the nonlinear term can be decomposed as:

$$NU(x,t) = \sum_{i=0}^{\infty} A_i \quad (13)$$

where A_i are Adomian polynomial of $U_0, U_1, U_2, ..., U_n$ and it can be calculated by formula:

$$A_i = \frac{1}{i!}\frac{d^i}{d\lambda^i}[N(\sum_{i=0}^{\infty}\lambda^i U_i)]_{\lambda=0}, i = 0,1,2,3,...$$

Substitution of (13) and (12) into (11) yields:

$$\begin{aligned} S[\sum_{i=0}^{\infty}U_i(x,t)] &= U(x,0) + uU^{'}(x,0) + ... \\ &+ u^{n-1}U^{(n-1)}(x,0) - u^n S[RU(x,t)] \\ &- u^n S[\sum_{i=0}^{\infty}A_i] + u^n S[h(x,t)]. \end{aligned} \quad (14)$$

On comparing both sides of (14) and using standard ADM we have :

$$\begin{aligned} S[U_0(x,t)] &= U(x,0) + uU^{'}(x,0) + ... + u^{n-1}U^{(n-1)}(x,0) \\ &+ u^n S[h(x,t)] = Y(x,u). \end{aligned} \quad (15)$$

Then it follow that:

$$S[U_1(x,t)] = -u^n S[RU_0(x,t)] - u^n S[A_0],$$
$$S[U_2(x,t)] = -u^n S[RU_1(x,t)] - u^n S[A_1].$$

In more general, we have:

$$S[U_{i+1}(x,t)] = -u^n S[RU_i(x,t)] - u^n S[A_i], i \geq 0 \quad (16)$$

On applying the inverse Sumudu transform to (15) and (16), we get:

$$U_0(x,t) = K(x,t),$$
$$U_{i+1}(x,t) = -S^{-1}[u^n S[RU_i(x,t)] + u^n S[A_1]], i \geq 0.$$

where$K(x,t)$represents the term that is arising from source term and prescribed initial conditions. On using the inverse Sumudu transform to $h(x,t)$ and using the given condition we get:

$$\Psi = \Phi + S^{-1}[h(x,t)]$$

where the functions Ψ, obtained from a term by using the initial condition is given by

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2 + ... + \Psi_n.$$

The terms $\Psi_0, \Psi_1, \Psi_2, ..., \Psi_n$ appears while applying the inverse Sumudu transform on the source term $h(x,t)$and using the given conditions. We define:

$$U_0 = \Psi_k + ... + \Psi_{k+r}$$

where $k = 0,1,2,3,...,n$, $r = 0,1,2,...,n-k$. Then we verify that U_0 satisfies the original equation.

6 Application

In this section, we use the ADSTM to solve linear and nonlinear Fokker–Plank equations.

6.1 Application 1

Consider the following linear Fokker-Planck equation:

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}}=\frac{\partial U}{\partial x}+\frac{\partial^{2}U}{\partial x^{2}},0\prec\alpha\leq1 \tag{17}$$

With the initial condition:

$$U(x,0)=x$$

We can write equation (17) in the operator form as the following :

$$LU=L_x(U)+L_{xx}(U) \tag{18}$$

Where $l=\frac{\partial}{\partial t^{\alpha}}^{\alpha},L_x=\frac{\partial}{\partial x},L_{xx}=\frac{\partial^2}{\partial x^2}$. By taking Sumudu transform for (18) and using initial condition we obtain:

$$S[U]=x+u^{\alpha}S[L_xU+L_{xx}U]$$

By applying the inverse Sumudu transform from the above equation, we get:

$$U(x,t)=x+S^{-1}[u^{\alpha}S[L_xU+L_{xx}U]] \tag{19}$$

which assumes a series solution of the function $U(x,t)$and is given by:

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x,t) \tag{20}$$

Using (19), (20) we get:

$$\begin{aligned}\sum_{n=0}^{\infty}U_n(x,t)&=x+S^{-1}[u^{\alpha}S[\partial_x(\sum_{n=0}^{\infty}U_n(x,t))\\&+\partial_{xx}(\sum_{n=0}^{\infty}U_n(x,t))]]\end{aligned}$$

From above we get:

$$U_0(x,t)=x,U_{k+1}=S^{-1}[u^{\alpha}S[U_{kx}+U_{kxx}]],$$

$$U_1=S^{-1}[u^{\alpha}]=\frac{t^{\alpha}}{\Gamma(\alpha+1)},U_2=0,U_3=0.$$

Hence,

$$\begin{aligned}U(x,t)&=\sum_{n=0}^{\infty}U_n(x,t)=U_0+U_1+U_2+U_3+\ldots\\&=x+\frac{t^{\alpha}}{\Gamma(\alpha+1)}\end{aligned}$$

See Figures (1,2). At $\alpha\to1$we have $U(x,t)=x+t$. Which is the exact solution of the standard form obtained by ADM [13], VIM [14] and HPM [15].

6.2 Application 2

Consider the following linear Fokker-Plank equation:

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}}=-\frac{\partial}{\partial x}A(x,t)U+\frac{\partial^2}{\partial x^2}B(x,t)U, \tag{21}$$

$$\begin{aligned}A(x,t)&=e^t\coth x\cosh x+e^t\sinh x-\coth x,\\B(x,t)&=e^t\cosh x\end{aligned} \tag{22}$$

With the initial condition:

$$U(x,0)=\sinh x,\ x\in R$$

According to the ADSTM, by taking Sumudu transform for (21)and using (22) we obtain:

$$S[U]=\sinh x+u^{\alpha}S[-L_x(AU)+L_{xx}(BU)]. \tag{23}$$

By applying the inverse Sumudu transform for (23) we obtain:

$$U(x,t)=\sinh x+S^{-1}[u^{\alpha}S[L_{xx}(BU)-L_x(AU)]]. \tag{24}$$

According to ADM, we assume a series solution of the function $U(x,t)$and is given by:

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x.t). \tag{25}$$

Using(24)and (25) we get :

$$\begin{aligned}\sum_{n=0}^{\infty}U_n(x,t)=\sinh x+S^{-1}[u^{\alpha}S[\partial_{xx}(B(\sum_{n=0}^{\infty}U_n(x,t)))\\-\partial_x(A(\sum_{n=0}^{\infty}U_n(x,t)))]].\end{aligned}$$

From above we get:

$$\begin{aligned}U_0(x,t)&=\sinh x,\\U_{k+1}(x,t)&=S^{-1}[u^{\alpha}S[\partial_{xx}(B(\sum_{n=0}^{\infty}U_n(x,t)))\\&-\partial_x(A(\sum_{n=0}^{\infty}U_n(x,t)))]],\end{aligned}$$

$$U_1(x,t)=\frac{t^{\alpha}}{\Gamma(\alpha+1)}\sinh x,U_2=\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\sinh x,\ldots$$

hence,

$$U(x,t)=\sinh xE_{\alpha}(t^{\alpha}).$$

See Figures (3,4). At $\alpha\to1$ we have $U(x,t)=e^t\sinh x$, which is the exact solution of the standard form obtained by ADM [13], VIM [14] and HPM [15].

6.3 Application 3

Consider the Backward Kolmogorov equation:

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}}=[-A(x,t)\frac{\partial U}{\partial x}+B(x,t)\frac{\partial^{2}U}{\partial x^{2}}], \tag{26}$$

$$A(x,t)=-(x+1),B(x,t)=x^{2}e^{t} \tag{27}$$

With the initial condition:

$$U(x,t)=x+1,x\in R$$

First we take Sumudu transform to (26) and using(27) we obtain:

$$S[U]=(x+1)+u^{\alpha}S[(x^{2}e^{t})L_{xx}U+(x+1)L_{x}U].$$

Applying the inverse Sumudu transform, we have

$$U(x,t)=(x+1)+S^{-1}[u^{\alpha}S[(x^{2}e^{t})L_{xx}U+(x+1)L_{x}U]]. \tag{28}$$

According to ADM, we assume a series solution of the function $U(x,t)$and is given by:

$$U(x,t)=\sum_{n=0}^{\infty}U_{n}(x.t), \tag{29}$$

Using (28) and (29) we get:

$$\sum_{n=0}^{\infty}U_{n}(x,t)=(x+1)+S^{-1}[u^{\alpha}S[(x^{2}e^{t})\partial_{xx}(\sum_{n=0}^{\infty}U_{n}(x,t))$$
$$+(x+1)\partial_{x}(\sum_{n=0}^{\infty}U_{n}(x,t))]]$$

The formal recursive relation is defined by:

$$U_{0}(x,t)=(x+1),$$
$$U_{k+1}(x,t)=S^{-1}[u^{\alpha}S[(x^{2}e^{t})\partial_{xx}(\sum_{n=0}^{\infty}U_{k}(x,t))$$
$$+(x+1)\partial_{x}(\sum_{n=0}^{\infty}U_{k}(x,t))]$$

Then

$$U_{1}=\frac{t^{\alpha}}{\Gamma(\alpha+1)}(x+1),\ U_{2}=\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}(x+1),$$
$$U_{3}=\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}(x+1),$$
$$U(x,t)=(x+1)\sum_{n=0}^{\infty}\frac{(t^{\alpha})^{k}}{\Gamma(\alpha k+1)}$$

Hence,

$$U(x,t)=(x+1)E_{\alpha}(t^{\alpha}).$$

Where $E_{\alpha}(t^{\alpha})=\sum_{k=0}^{\infty}\frac{(t^{\alpha})^{k}}{\Gamma(\alpha k+1)}$is the famous Mittag–Leffler function. See Figures (5,6) At $\alpha\to 1$, we have $U(x,t)=(x+1)e^{t}$. Which is the exact solution of the standard form obtained by ADM [13], VIM [14] and HPM [15].

6.4 Application 4

Consider the following nonlinear Fokker–Planck equation:

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}}=[-\frac{\partial}{\partial x}A(x,t,U)+\frac{\partial}{\partial x^{2}}^{2}B(x,t)]U \tag{30}$$

$$A(x,t)=\frac{4}{x}U-\frac{x}{3},\quad B(x,t,U)=U.$$

Then equation (30) becomes :

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}}=\frac{\partial}{\partial x}(\frac{xU}{3}-\frac{4}{x}U^{2})+\frac{\partial^{2}}{\partial x^{2}}(U^{2}), \tag{31}$$

Subject to initial condition

$$U_{0}(x,t)=x^{2},x\in R \tag{32}$$

According to ADSTM by applying Sumudu transform of both sides of equation (31) and using (32) we get:

$$S[U]=x^{2}+u^{\alpha}S[\partial_{x}(\frac{xU}{3}-\frac{4}{x}U^{2})+\partial_{xx}(U^{2})]$$

The second step in Sumudu decomposition method is that we represent solution as an infinite series:

$$U(x,t)=\sum_{n=0}^{\infty}U_{n}(x,t)$$

hence,

$$S[\sum_{n=0}^{\infty}U_{n}(x,t)]=x^{2}+u^{\alpha}S[\partial_{x}(\frac{x}{3}(\sum_{n=0}^{\infty}U_{n}(x,t)))$$
$$-\partial_{x}(\frac{4}{x}(\sum_{n=0}^{\infty}A_{i})+\partial_{xx}(\sum_{n=0}^{\infty}B_{i})].$$

Then recursive relations are:

$$U_{0}(x,t)=x^{2},$$
$$U_{k+1}=S^{-1}[u^{\alpha}S[\partial_{x}(\frac{x}{3}\sum_{k=0}^{\infty}U_{k}(x,t))$$
$$-\partial_{x}(\frac{4}{x}\sum_{k=0}^{\infty}A_{k})+\partial_{xx}(\sum_{k=0}^{\infty}B_{k})]$$

And nonlinear terms can be decomposed as:

$$N_{1}U(x,t)=\sum_{k=0}^{\infty}A_{k},N_{2}U(x,t)=\sum_{k=0}^{\infty}B_{k},U_{0},U_{1},....,U_{k}$$

are A domain polynomials of [10] , and they can calculated by formula A_k, B_k:

$$A_{i}(B_{i})=\frac{1}{i!}\frac{d^{i}}{d\lambda^{i}}[N\sum_{i=0}^{\infty}\lambda^{i}U_{i}]_{\lambda=0}^{2},i=0,1,2,3,...$$
$$A_{0}(B_{0})=U_{0},\ A_{1}(B_{1})=2U_{0}U_{1},...$$

Then we get:

$$U_{1}=x^{2}\frac{t^{\alpha}}{\Gamma(\alpha+1)},\ U_{2}=x^{2}\frac{t^{\alpha}}{\Gamma(\alpha+1)},...$$

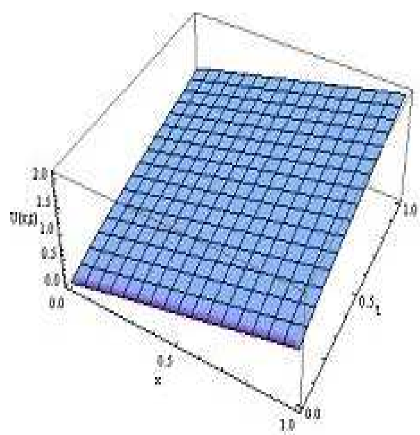
Hence

$$U(x,t) = x^2 E_\alpha(t^\alpha).$$

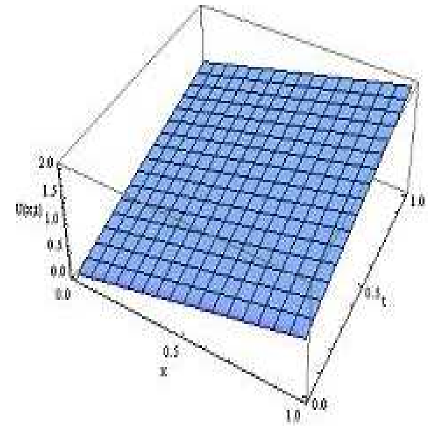
See Figures (7,8). At $\alpha \to 1$ we have $U(x,t) = x^2 e^t$. Which is the exact solution of the standard form obtained by ADM [13], VIM [14] and HPM [15].

7 Conclusion

The present paper develops the Adomian decomposition Sumudu transform method (ADSTM) for solving fractional nonlinear problems. Figures (1-8) show the numerical results of the probability density function$U(x,t)$for different time-fractional Brownian motions α. We also show that, whenα=1, the approximate solution obtained by the present method is the exact solution of the standard form. It is evident that the proposed technique has shown to computationally efficient in these examples that are important to researchers in the field of applied sciences. It is worth mentioning that the method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result amounts to an improvement of the performance of the approach. In conclusion, the (ADSTM) may be considered as a nice refinement in existing numerical techniques and may cover the wide applications.



(a)



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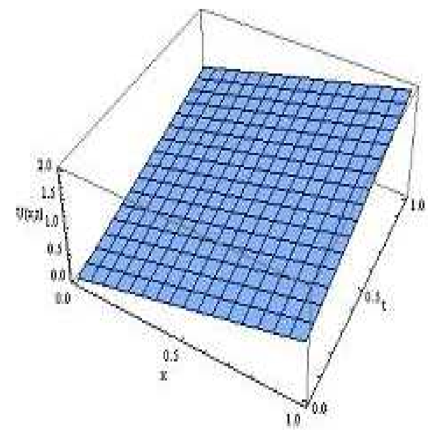
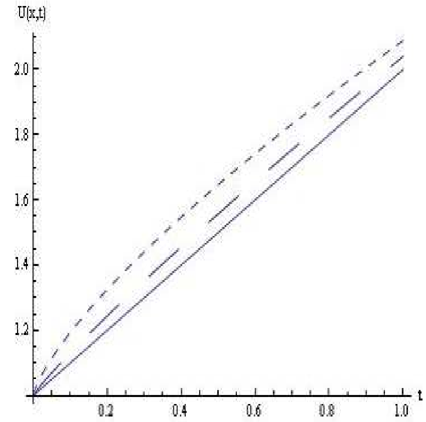


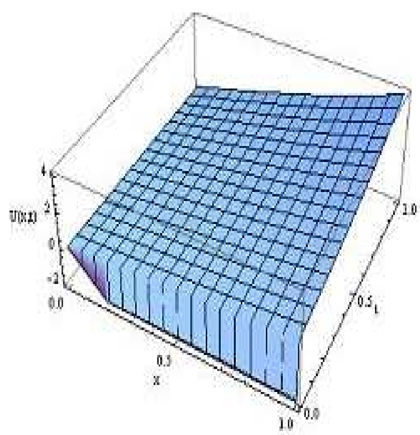
(c)

Fig. 1: The behavior of the $U(x,t)$, x and t are obtained when (a) α = 0.75, (b) α = 0.90, (c) α=1 which is the exact solution.

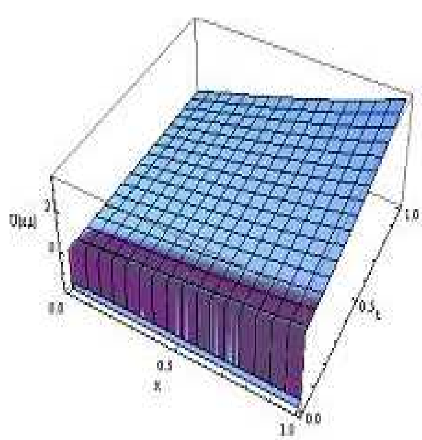


(d)

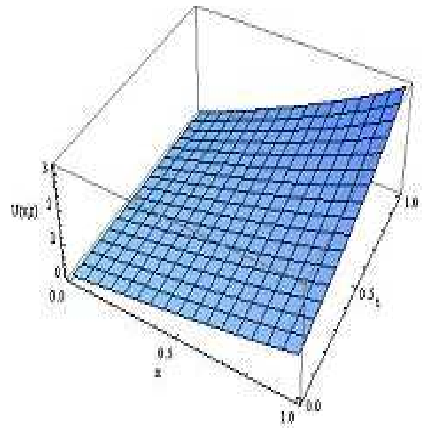
Fig. 2: Plots of $U(x,t)$ versus t at x = 1 for different values of α, $\{$(- - -) α = 0.75, (— —) α = 0.90, (——) α = 1. $\}$



(a)

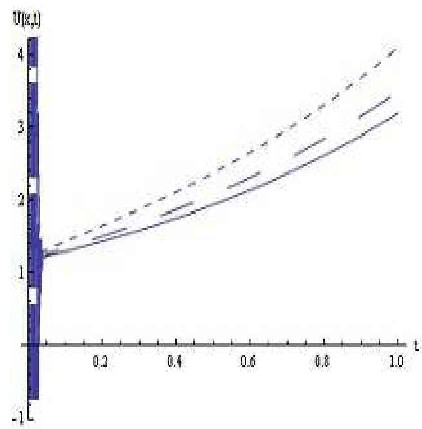


(b)



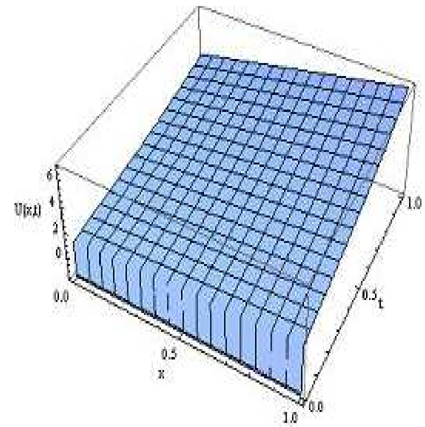
(c)

Fig. 3: The behavior of the $U(x,t)$, x and t are obtained when (a) α = 0.75, (b) α = 0.90, (c) α=1 which is the exact solution.

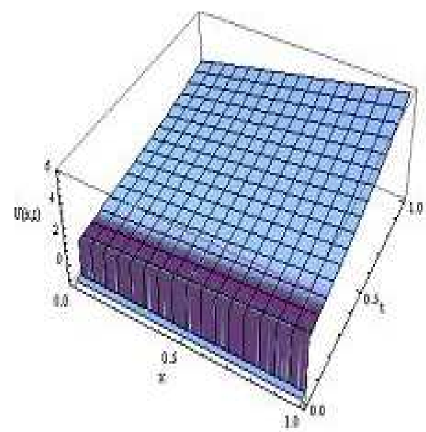


(d)

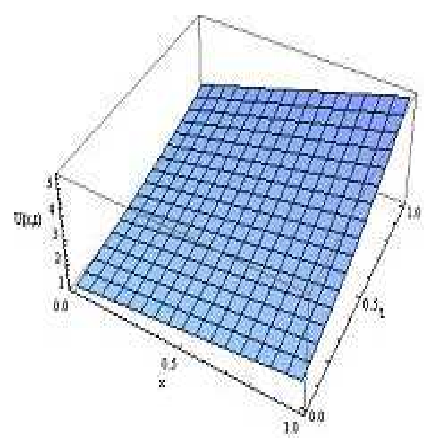
Fig. 4: Plots of $U(x,t)$ versus t at x = 1 for different values of α, $\{$(- - -) α = 0.75, (— —) α = 0.90, (——) α = 1. $\}$



(a)

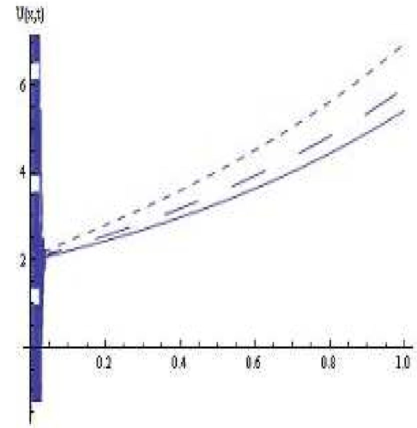


(b)



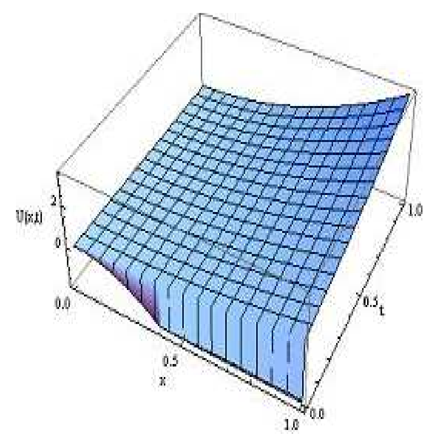
(c)

Fig. 5: The behavior of the $U(x,t)$, x and t are obtained when (a) α = 0.75, (b) α = 0.90, (c) α =1which is the exact solution.

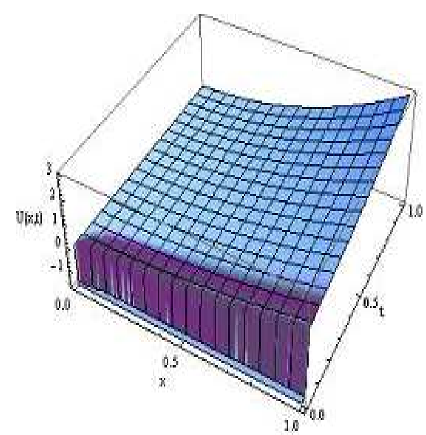


(d)

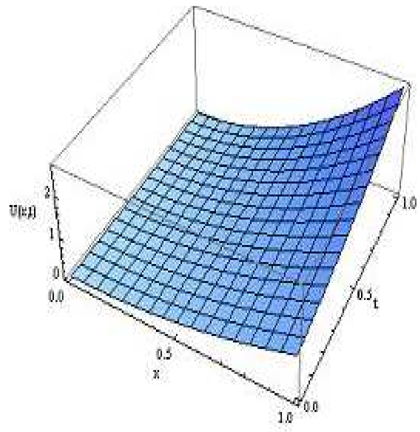
Fig. 6: Plots of $U(x, t)$ versus t at x = 1 for different values of α, {(- - -) α = 0.75, (— —) α = 0.90, (——) α = 1. }



(a)

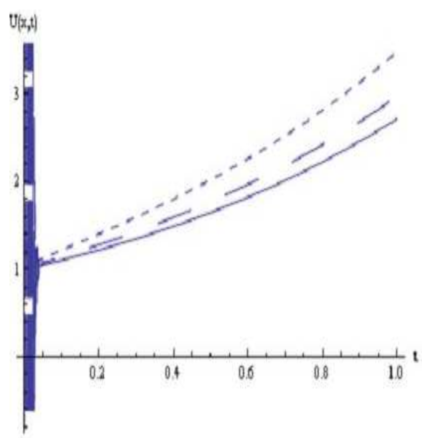


(b)



(c)

Fig. 7: The behavior of the $U(x,t)$, x and t are obtained when (a) α = 0.75, (b) α = 0.90, (c) α =1which is the exact solution.



(d)

Fig. 8: Plots of $U(x, t)$ versus t at x = 1 for different values of α, {(- - -) α = 0.75, (— —) α = 0.90, (——) α = 1. }

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