

Optimal Time and Random Inspection Policies for Computer Systems

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Received: 28 May. 2013, Revised: 24 Sep. 2013, Accepted: 25 Sep. 2013

Published online: 1 Apr. 2014

Abstract: Faults in computer systems sometimes occur intermittently. This paper applies a standard inspection policy with imperfect inspection to a computer system: The system is checked at periodic times and its failure is detected at the next checking time with a certain probability. The expected cost until failure detection is obtained, and when the failure time is exponential, an optimal inspection time to minimize it is derived. Next, when the system executes computer processes, it is checked at random processing times and its failure is detected at the next checking time with a certain probability. The expected cost until failure detection is obtained, and when random processing times are exponential, an optimal inspection time to minimize it is derived. This paper compares optimal times for two inspection policies and shows that if the random inspection cost is the half of the periodic one, then two expected costs are almost the same. Finally, we consider the random inspection policy in which the system is checked at the N th interval of random times and derive an optimal number N^* which minimizes the total expected cost.

Keywords: Periodic inspection, random inspection, imperfect inspection, checking time, expected cost

1 Introduction

It has been well-known that faults in computer systems sometimes occur intermittently [1], [2] and [3]: Faults are hidden and become permanent failure when the duration of hidden faults exceeds a threshold level [4] and [5]. To prevent such faults, some inspection policies for computer systems were considered [1], [6] and [7], data transmission strategies for communication systems were considered [8], and some properties for security measures of software in computer systems were observed [9]. The reliability models and a variety of maintenance models play an important role in manufacturing or computer systems. Some applications of reliability models in computer systems, such as communications, backup policies, checkpoint intervals, were summarized [10]. The latest work proposed new reliability and fault-tolerant methods which were applied in manufacturing modules [11], system reliability allocation based on Bayesian network [12], supporting real-time data services [13], and software reliability modeling based on gene expression [14].

Most systems in offices and industries successively execute computer processes. For such systems, it would be impossible to maintain them in a strictly periodic fashion, because sudden suspension of the working processes would cause serious production losses. Optimal periodic and random inspection policies were summarized in [15]. So that we consider in this paper that the system executes a job with random processing times [16] and apply the inspection policy with imperfect inspection [1] to the system.

First, we apply a standard inspection policy with imperfect inspection to a computer system in this paper: The system has to be operated for an infinite time span and fails. To detect the failure, the system is checked at periodic times. System failure is detected at the next checking time with a certain probability, and undetected failure is detected at the next checking time with the same probability. Such procedures are continued until the failure is detected. The expected cost until failure detection is obtained, and when the failure time is exponential, an optimal inspection time which minimizes it is derived analytically and computed numerically.

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Second, it is assumed that the system is checked at random processing times. System failure is detected at the next checking time with a certain probability. By the methods similar to the periodic inspection policy, the expected cost until failure detection is obtained, and when each processing time is exponential, an optimal random time which minimizes it is derived.

Third, we compare optimal times for periodic and random inspection policies when both failure and processing times are exponential. It is shown that the periodic inspection is better than the random one; however, if the random inspection cost is the half of the periodic one, then two expected costs are almost the same.

Furthermore, we consider the random inspection policy in which the system is checked at the N th interval of random times and obtain the total expected cost, using the result of random inspection. An optimal N^* which minimizes the expected cost is derived analytically when the failure time is exponential.

2 Time Inspection

Consider a standard inspection policy [3] with imperfect inspection: A system should operate for an infinite time span and is checked at periodic times kT ($k = 1, 2, \dots$). System failure is detected at the next checking time with probability q ($0 < q \leq 1$) and is replaced immediately, and is not done with probability $p \equiv 1 - q$. The undetected failure is detected at the next checking time with the same probability q . Such procedures are continued until the failure is detected.

It is assumed that the system has a failure distribution $F(t)$ with finite mean $1/\lambda$, irrespective of any inspection. All times for checks and replacement are negligible. Let c_T be the cost of one check and c_D be the loss cost per unit of time for the time elapsed between a failure and its detection at some checking time. Then, the probability that the system fails between the j th and $(j+1)$ th ($j = 0, 1, 2, \dots$) checking times, and its failure is detected after the $(k+1)$ th ($k = 0, 1, 2, \dots$) check, i.e., at the $(j+k+1)$ th checking time, is

$$[F((j+1)T) - F(jT)]p^kq.$$

Clearly,

$$\sum_{j=0}^{\infty} [F((j+1)T) - F(jT)] \sum_{k=0}^{\infty} p^k q = 1.$$

Then, the expected number of checks until replacement is

$$\begin{aligned} & \sum_{j=0}^{\infty} j[F((j+1)T) - F(jT)] + \sum_{k=0}^{\infty} (k+1)p^k q \\ &= \sum_{j=1}^{\infty} \bar{F}(jT) + \frac{1}{q}, \end{aligned} \quad (1)$$

and the mean time from failure to its detection is

$$\begin{aligned} & \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} p^k q \int_{jT}^{(j+1)T} [(j+k+1)T - t] dF(t) \\ &= T \left[\sum_{j=1}^{\infty} \bar{F}(jT) + \frac{1}{q} \right] - \frac{1}{\lambda}. \end{aligned} \quad (2)$$

Therefore, the total expected cost until replacement is, from (1) and (2),

$$C_P(T) = (c_T + c_D T) \left[\sum_{j=1}^{\infty} \bar{F}(jT) + \frac{1}{q} \right] - \frac{c_D}{\lambda}. \quad (3)$$

In particular, when $F(t) = 1 - e^{-\lambda t}$ ($0 < 1/\lambda < \infty$),

$$C_P(T) = (c_T + c_D T) \left(\frac{1}{1 - e^{-\lambda T}} + \frac{p}{q} \right) - \frac{c_D}{\lambda}. \quad (4)$$

Differentiating $C_P(T)$ with respect to T and setting it equal to zero,

$$\frac{p}{q} (1 - e^{-\lambda T})^2 e^{\lambda T} + e^{\lambda T} - 1 - \lambda T = \frac{\lambda c_T}{c_D}, \quad (5)$$

whose left-hand side increases from 0 to ∞ . Thus, there exists an optimal T^* ($0 < T^* < \infty$) which satisfies (5), and the resulting cost rate is

$$\frac{C_P(T^*)}{c_D/\lambda} = (e^{\lambda T^*} - 1) \left[\frac{p^2}{q^2} (1 - e^{-\lambda T^*}) + \frac{p+1}{q} \right]. \quad (6)$$

Clearly, T^* increases strictly with q from 0 to a solution of the equation

$$e^{\lambda T} - 1 - \lambda T = \frac{\lambda T}{c_D}.$$

3 Random Inspection

Consider a random inspection policy [3] with imperfect inspection: Suppose that the system is checked at successive times S_j ($j = 1, 2, \dots$), where $S_0 \equiv 0$ and $Y_j \equiv S_j - S_{j-1}$ ($j = 1, 2, \dots$) are independently and identically distributed random variables, and also, independent of its failure time. It is assumed that each Y_j has an identical distribution $G(t)$ with finite mean $1/\mu$. The system is checked at successive times S_j and its cost for one check is c_R . The other assumptions are the same as those in Section 2.

The probability that the system fails between the j th and $(j+1)$ th ($j = 0, 1, 2, \dots$) checking times and its failure is detected after the $(k+1)$ th check is

$$\int_0^{\infty} dG^{(j)}(t_1) \int_{t_1}^{\infty} [F(t_2) - F(t_1)] dG(t_2 - t_1) p^k q. \quad (7)$$

Clearly,

$$\begin{aligned} & \sum_{k=0}^{\infty} p^k q \sum_{j=0}^{\infty} \int_0^{\infty} dG^{(j)}(t_1) \\ & \times \int_{t_1}^{\infty} [F(t_2) - F(t_1)] dG(t_2 - t_1) \\ & = \sum_{j=0}^{\infty} \int_0^{\infty} dG^{(j)}(t_1) \int_0^{\infty} [\bar{F}(t_1) - \bar{F}(t_1 + t_2)] dG(t_2) \\ & = \sum_{j=0}^{\infty} \int_0^{\infty} \bar{F}(t) dG^{(j)}(t) - \sum_{j=1}^{\infty} \int_0^{\infty} \bar{F}(t) dG(t) = 1. \end{aligned}$$

Then, the expected number of checks until replacement is

$$\begin{aligned} & \sum_{j=0}^{\infty} j \int_0^{\infty} dG^{(j)}(t_1) \int_{t_1}^{\infty} [F(t_2) - F(t_1)] dG(t_2 - t_1) \\ & + \sum_{k=0}^{\infty} (k+1) p^k q = \int_0^{\infty} M(t) dF(t) + \frac{1}{q}, \end{aligned} \quad (8)$$

where $M(t) \equiv \sum_{j=1}^{\infty} G^{(j)}(t)$ which is the expected number of checks in $[0, t]$ and is called a renewal function in stochastic processes [17]. The mean time from failure to its detection is

$$\begin{aligned} & \sum_{j=0}^{\infty} \int_0^{\infty} dG^{(j)}(t_1) \int_{t_1}^{\infty} dG(t_2 - t_1) \int_{t_1}^{t_2} dF(t) \\ & \times \sum_{k=0}^{\infty} p^k q \int_{t_2}^{\infty} (t_3 - t) dG^{(k)}(t_3 - t_2) \\ & = \sum_{j=0}^{\infty} \int_0^{\infty} dG^{(j)}(t_1) \int_{t_1}^{\infty} dG(t_2 - t_1) \\ & \times \int_{t_1}^{t_2} \left(\frac{p}{q\mu} + t_2 - t \right) dF(t) \\ & = \frac{p}{q\mu} + \sum_{j=0}^{\infty} \int_0^{\infty} dG^{(j)}(t_1) \int_0^{\infty} dG(t_2) \\ & \times \int_{t_1}^{t_1+t_2} [\bar{F}(t_1) - \bar{F}(t)] dt \\ & = \sum_{j=0}^{\infty} \int_0^{\infty} dG^{(j)}(t_1) \int_0^{\infty} \bar{G}(t) [\bar{F}(t_1) - \bar{F}(t + t_1)] dt \\ & + \frac{p}{q\mu} = \frac{1}{q\mu} + \frac{1}{\mu} \int_0^{\infty} M(t) dF(t) - \frac{1}{\lambda}. \end{aligned} \quad (9)$$

Therefore, the total expected cost until replacement is, from (8) and (9),

$$C_R(G) = \left(c_R + \frac{c_D}{\mu} \right) \left[\int_0^{\infty} M(t) dF(t) + \frac{1}{q} \right] - \frac{c_D}{\lambda}.$$

In particular, when $G(t) = 1 - e^{-\mu t}$ ($0 < 1/\mu < \infty$), i.e., $M(t) = \mu t$, the expected cost is a function of μ which is given by

$$C_R(\mu) = \left(c_R + \frac{c_D}{\mu} \right) \left(\frac{\mu}{\lambda} + \frac{1}{q} \right) - \frac{c_D}{\lambda}. \quad (10)$$

Differentiating $C_R(\mu)$ with respect to μ and setting it equal to zero,

$$\left(\frac{\lambda}{\mu} \right)^2 = \frac{q\lambda c_R}{c_D}, \quad (11)$$

and the resulting cost is

$$\frac{C_R(\mu^*)}{c_D/\lambda} = \frac{\lambda}{q\mu} \left(\frac{\lambda}{q\mu} + 2 \right). \quad (12)$$

Table 1. Optimal T^* , $1/\mu^*$, and $C_P(T^*)/c_D$, $C_R(\mu^*)/c_D$ when $q = 0.9$ and $\lambda = 1$.

c_T/c_D	T^*	$C_P(T^*)/c_D$	$1/\mu^*$	$C_R(\mu^*)/c_D$
0.001	0.0403	0.0502	0.0300	0.0678
0.002	0.0566	0.0714	0.0424	0.0965
0.005	0.0893	0.1143	0.0671	0.1546
0.010	0.1256	0.1639	0.0949	0.2219
0.020	0.1764	0.2363	0.1342	0.3204
0.050	0.2746	0.3879	0.2121	0.5270
0.100	0.3824	0.5716	0.3000	0.7778
0.200	0.5282	0.8556	0.4243	1.1650
0.500	0.7994	1.5052	0.6708	2.0463
1.000	1.0757	2.3807	0.9487	3.2193

Suppose that $F(t) = 1 - e^{-\lambda t}$, $G(t) = 1 - e^{-\mu t}$, $c_T = c_R$, and $q = 0.9$ and $\lambda = 1$. Then, Table 1 presents optimal T^* , $1/\mu^*$ and their resulting costs for c_T/c_D . This indicates as estimated previously that $T^* > 1/\mu^*$ and $C_P(T^*) < C_R(\mu^*)$, i.e., the periodic inspection time is larger than the random one, and hence, when $c_T = c_R$, the periodic policy is better than the random one.

It has been assumed in Table 1 that two checking costs c_T and c_R are the same. In general, cost c_R would be lower than c_T because the system is checked at random times. Such random inspections may not break off the random procedures in computers. We compute $\hat{c}_R$ when the expected costs of two inspection policies are the same. From (6) and (12), we compute $\hat{\mu}$ which satisfies

$$\begin{aligned} & (e^{\lambda T^*} - 1) \left[\frac{p^2}{q} (1 - e^{-\lambda T^*}) + p + 1 \right] \\ & = \frac{1}{q} \left(\frac{\lambda}{\hat{\mu}} \right)^2 + \frac{2\lambda}{\hat{\mu}}, \end{aligned}$$

and compute

$$\frac{\hat{c}_R}{c_D/\lambda} = \frac{1}{q} \left(\frac{\lambda}{\hat{\mu}} \right)^2.$$

Table 2 presents $1/\hat{\mu}$, $\hat{c}_R/c_D$ and $\hat{c}_R/c_T$ for c_T/c_D when $q = 0.9$ and $\lambda = 1$. This indicates that $\hat{c}_R$ is a little more than the half of c_T . In other words, when $c_R \approx c_T/2$, two expected costs are almost the same.

Table 2. Values of $1/\hat{\mu}$, $\hat{c}_R/c_D$ and $\hat{c}_R/c_T$ when $q = 0.9$ and $\lambda = 1$.

c_T/c_D	$1/\hat{\mu}$	$\hat{c}_R/c_D$	$\hat{c}_R/c_T$
0.001	0.0224	0.0005	0.5039
0.002	0.0318	0.0010	0.5054
0.005	0.0504	0.0025	0.5086
0.010	0.0715	0.0051	0.5118
0.020	0.1016	0.0103	0.5160
0.050	0.1621	0.0263	0.5253
0.100	0.2313	0.0535	0.5352
0.200	0.3312	0.1097	0.5485
0.500	0.5355	0.2868	0.5735
1.000	0.7738	0.5987	0.5987

4 Nth Checking Time

Suppose that the system is checked at times S_{jN} ($j = 1, 2, \dots, N = 1, 2, \dots$), i.e., at times $S_{1N}, S_{2N}, \dots$ [11]. When $N = 1$, the system is checked at every S_j in Section 3. Then, from $C_R(G)$, replacing $G(t)$ with $G^{(N)}(t)$, $1/\mu$ with N/μ , and $M(t)$ with $M^{(N)}(t) = \sum_{j=1}^{\infty} G^{(jN)}(t)$ ($N = 1, 2, \dots$), the total expected cost until replacement is

$$C_R(N) = \left(c_R + \frac{Nc_D}{\mu} \right) \left[\int_0^{\infty} M^{(N)}(t) dF(t) + \frac{1}{q} \right] - \frac{c_D}{\lambda} \quad (N = 1, 2, \dots). \quad (13)$$

In particular, when $F(t) = 1 - e^{-\lambda t}$,

$$\int_0^{\infty} e^{-\lambda t} dM^{(N)}(t) = \frac{[G^*(\lambda)]^N}{1 - [G^*(\lambda)]^N},$$

where $G^*(\lambda) \equiv \int_0^{\infty} e^{-\lambda t} dG(t)$, and the expected cost in (13) is

$$C_R(N) = \left(c_R + \frac{Nc_D}{\mu} \right) \left(\frac{1}{1 - A^N} + \frac{p}{q} \right) - \frac{c_D}{\lambda}. \quad (14)$$

where $A = G^*(\lambda) < 1$. From the inequality $C_R(N+1) - C_R(N) \geq 0$,

$$\frac{1 - A^N}{(1 - A)A^N} \left[1 + \frac{p}{q}(1 - A^{N+1}) \right] - N \geq \frac{c_R}{c_D/\mu}, \quad (15)$$

which increases strictly with N to ∞ . Thus, there exists a unique minimum N^* ($1 \leq N^* < \infty$) which satisfies (15). Clearly, N^* increases strictly with q from 1 to a solution of the equation

$$\frac{1 - A^N}{(1 - A)A^N} - N \geq \frac{c_R}{c_D/\mu}.$$

Note that when $G(t) = 1 - e^{-\mu t}$, $A = \mu/(\lambda + \mu)$. In addition, when $p = 0$, (15) agrees with the result of [11].

Table 3 presents the optimal N^* and the resulting cost $C_R(N^*)/c_D$ for $1/\mu$ and c_R/c_D when $q = 0.9$ and $1/\lambda = 1$. This indicates that optimal N^* decreases with $1/\mu$ and increases with c_R/c_D , and N^*/μ are almost the same for

Table 3. Optimal N^* and $C_R(N^*)/c_D$ when $q = 0.9$ and $\lambda = 1$.

$1/\mu$	$c_R/c_D = 0.5$		$c_R/c_D = 1.0$	
	N^*	$C_R(N^*)/c_D$	N^*	$C_R(N^*)/c_D$
0.01	80	1.5129	108	2.3894
0.02	40	1.5206	54	2.3981
0.05	16	1.5435	22	2.4241
0.10	8	1.5812	11	2.4666
0.20	4	1.6553	6	2.5522
0.50	2	1.8667	2	2.8222
1.00	1	2.1667	1	3.2222

small $1/\mu$. Compared Table 3 with Table 1, if $c_T = c_R$, when $c_R/c_D = 0.5$, $1/\mu^* < N^*/\mu$ and $C_R(N^*) > C_R(\mu^*)$ as $1/\mu$ is big enough, other wise, $C_R(N^*) < C_R(\mu^*)$; if $c_R \approx c_T/2$, when $c_R/c_D = 0.5$ and $c_T/c_D = 1.0$, $1/\mu^* > N^*/\mu$ and $C_R(N^*) < C_R(\mu^*)$.

5 Conclusion

We have applied a standard inspection policy with imperfect inspection to a computer system. The expected costs of the periodic and random inspection policies are obtained and the optimal inspection times which minimize them are derived analytically, when the failure and random times are exponential. It is shown numerically that when the costs for periodic and random inspection are the same, the periodic policy is better than the random one. However, it is of interest that if the cost for random inspection is the half of the periodic one, two expected costs are almost the same.

Furthermore, we have considered the inspection policy in which the system is checked at the N th interval and derived analytically optimal N^* when the failure time is exponential. It shows from numerical examples that if cost for random inspection is the half of the periodic one, policy in Section 4 is better than Section 3, and if the two costs are the same, then both two cases occur.

Acknowledgement

This work is partially supported by National Science Council of Taiwan NSC 100-2628-E-0330-002; National Natural Science Foundation of China 71171110; Natural Science Foundation of Jiangsu Province of China BK2010555; Grant-in-Aid for Scientific Research (C) of Japan Society for the Promotion of Science under Grant No. 22500897 and No. 24530371.

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