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Robust Blind Algorithm based on Oblique Projection and Worst-Case Optimization

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Abstract: When adaptive arrays are applied to practical systems, the performance of the convention constant modulus algorithm degrades severely in the presence of array steering vector errors. The similar situation of performance degradation can occur even when the array steering vector is known exactly, but the training sample size is small. In this paper, we propose a novel doubly constrained robust constant modulus algorithm based on the worst case performance optimization and oblique projection technique. The proposed algorithm uses explicit modeling of uncertainties in the desired signal array response and in data snapshots, which provides sufficiently robustness to uncertainty in source DOA, and makes the mean output array SINR consistently close to the optimal one. The array weight vector is derived iteratively by the Lagrange multiplier approach and descent gradient technique, in which the factors can be precisely obtained at each step. A theoretical analysis for our proposed algorithm in terms of the optimal step size, convergence and array output SINR performance is presented in this paper. As compared with the linearly constrained constant modulus algorithm, our proposed robust constant modulus algorithm resolves the interference capture problem, has faster convergence speed, and enhances the array output performance under practical situations. Computer simulation results are presented to show the superiority of our proposed algorithm on output SINR enhancement and signal sampling resolution.

Keywords: Robust adaptive beamformer, array steering vector errors, oblique projection technique, worst-case optimization.

1 Introduction

The constant modulus algorithm (CMA) is known to enjoy widespread popularity as methods for blind source separation, equalization of communication signals [1]-[5] and blind beamforming. CMA is a preferred choice for blind algorithm because of its robustness and its ease of implementation. In practice, circumstances like local scattering imperfectly calibrated arrays and imprecisely known wave field propagation conditions can lead to performance degradation of the conventional algorithms. Therefore, robust adaptive beamforming has emerged as a necessary constituent of most systems using an array of sensors. To combat DOA uncertainty, linearly constrained minimum variance beamformer is proposed [6], but the method is conservative and thus suitable only for small DOA errors. Robust adaptive beamformer based on Bayesian method [7] is able to estimate signal when the DOA is uncertain or completely unknown. In order to solve the robustness problem against DOA error, robust beamforming algorithms based on additional linear constraints [8,9] can broaden the main beam of adaptive array. These algorithms have good performance if there are no other array imperfections. The most widely used method, due to its simplicity and effectiveness, is diagonal loading [10]-[13]. The approach addes a scaled identity matrix to the covariance matrix prior to inversion. Diagonal loading can either be viewed as a method to equalize the least significant eigenvalues of the sample covariance matrix, or to constrain the white noise array gain. In recent years, novel robust approaches have been proposed [14]-[21]. Many robust adaptive beamforming methods belong to the family of the diagonal loading method. Robust Capon beamformer [14]-[15] has been designed by assuming that the array steering vector belongs to an ellipsoidal uncertainty set. It is dependent on the choice of a user parameter related to the size of the uncertainty set. This guarantees a desired array response from a specific direction, whose steering vector is expected to lie in the ellipsoid. However, it is difficult to choose this user parameter. With the generalization of sphere uncertainty set to ellipsoid, the second order cone programming (SOCP) in [16,17] is avoided by the

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proposed algorithm [18]. The approach is developed for the most general case of an arbitrary dimension of the desired signal subspace and is applicable to both the rank-one and higher rank desired signal models in [19], which can efficiently calculate the corresponding diagonal loading level. Due to its high computational load, there is room for simplification and the development of low-complexity algorithm [20], which belong to the category of the above algorithms. The algorithm uses a modified conjugate gradient algorithm performing only one iteration per snapshot. In multicall coordinated beamforming, an efficient approximation method solves the nonconvex centralized problem, using semidefinite relaxation, an approximation technique based on convex optimization. This paper extends the worst-case robust beamforming design as well as its decentralized implantation method to a fully coordinated scenario [21].

In this paper, to combat the array steering vector errors and the small training sample size, we proposed robust constant modulus algorithm with double constraints, based on the worst case optimization and the oblique projection of the array steering vector. Unlike the existing robust adaptive beamforming based on the worst case optimization via a second-order cone program, our algorithm employs descent gradient technique, which has the low complexity cost. The array weight vector is updated iteratively by minimizing the objective function subject to double constraints on the array response. The proposed algorithm is parameter-free and can be implemented simply as an iterative process. Some performances of our proposed algorithm are analyzed. In contrast to the traditional constant modulus algorithm, the results show that our algorithm can have a fast convergence speed, yield better array output performance, and provide significant robustness to the array steering vector errors. Simulations display that the performance of our algorithm are better than the traditional constant modulus algorithm.

2 Background

2.1 Signal Model

We consider an array composed of M sensors with inter-element distance d. The desired signal impinges on the antenna array from a certain direction θ_0, together with directional interferences from other directions $\{\theta_1,\cdots,\theta_{k-1}\}$. The interferences are assumed to be uncorrelated with the desired signal. The received array vector is expressed as

$$\begin{aligned}\mathbf{X}(n) &= \mathbf{a}(\theta_0)s(n)+\mathbf{i}(n)+\mathbf{c}(n)\\ &= \mathbf{A}\mathbf{S}(n)+\mathbf{c}(n)\end{aligned} \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_0),\mathbf{a}(\theta_1),\ldots,\mathbf{a}(\theta_{D-1})]$ is the array manifold matrix, $\mathbf{a}(\theta_0)$ is the desired signal steering vector, $\mathbf{S}(n)$ is the $k\times 1$ vector of transmitting signals, $\mathbf{i}(n)$ is the $k\times 1$ vector of interference signals, and $\mathbf{c}(n)$ is additive white Gaussian noise. The interference signals and noise are considered to be statistically independent. The complex output may be written as

$$y(n) = \mathbf{W}^H\mathbf{X}(n) \tag{2}$$

where $\mathbf{W} = [W_1,W_2,\ldots,W_M]^T$ is the complex vector of beamformer weights.

The Signal to Interference Noise Ratio(SINR) performance is given by

$$\mathrm{SINR}_{opt} = \frac{\mathbf{W}^H\mathbf{R_s}\mathbf{W}}{\mathbf{W}^H\mathbf{R_{i+c}}\mathbf{W}} \tag{3}$$

where $\mathbf{R_s}$ is the signal covariance matrix

$$\mathbf{R_s} = \mathrm{E}\{\mathbf{s}(n)\mathbf{s}^H(n)\} \tag{4}$$

and $\mathbf{R_{i+c}}$ is the interference plus noise covariance matrix

$$\mathbf{R_{i+c}} = \mathrm{E}\{(\mathbf{i}(n)+\mathbf{n}(n))(\mathbf{i}(n)+\mathbf{n}(n))^H\} \tag{5}$$

where E is the statistical expectation.

2.2 Linearly Constrained Constant Modulus Algorithm

In the conventional constant modulus algorithm, a linear receiver is chosen comprising the array weight vector $\mathbf{W}$ [22]. The cost function of the constant modulus algorithm is of the form

$$J(n) = E[(|y(n)|^l-\delta_k^l)^m] \tag{6}$$

where δ_k is the desired signal amplitude at the array output. The constant modulus algorithm requires no knowledge about the signal except that the signal waveform has a constant envelop.

By minimizing $J(n)$ with respect to $\mathbf{W}$, the following descent gradient adaptive algorithm can be written as

$$\mathbf{W}(n+1) = \mathbf{W}(n)-2\mu\zeta(n)\mathbf{X}(n+1) \tag{7}$$

where μ is the step size factor, and the parameter $\zeta(k)$ is written as

$$\zeta(n) = (|y(n)|^2-1)y(n) \tag{8}$$

However, in practical applications, the constant modulus algorithm can converge to the transmitted signal which has stronger power according to the feature of constant modulus. This leads to interference capture problem. To solve the above problem, the linearly constrained constant modulus algorithm was proposed [23]-[24].

The cost function of the linearly constrained CMA is the following form

$$\min_{\mathbf{W}} E[(|y(n-1)|^2-|y(n)|^2)^2] \quad \text{s.t.} \quad \mathbf{W}^H(n)\mathbf{a}(\theta_0) = 1 \tag{9}$$

The optimization approach used to obtain the array weight vector will use Lagrange multiplier algorithm, thus we have

$$\mathbf{W}(n+1)=\mathbf{B}[\mathbf{W}(n)-\mu\mathbf{d}(\mathbf{W}(n))]+\mathbf{a}(\theta_0)[\mathbf{a}^H(\theta_0)\mathbf{a}(\theta_0)]^{-1} \tag{10}$$

where $\mathbf{B}=\mathbf{I}-\mathbf{a}(\theta_0)[\mathbf{a}^H(\theta_0)\mathbf{a}(\theta_0)]^{-1}\mathbf{a}^H(\theta_0)$ is a projection matrix, and $\mathbf{d}(\mathbf{W}(n))$ is an estimation of the gradient of the objective function (9).

Note that the linearly constrained constant modulus algorithm requires direction of arrival (DOA) of the desired signal. The performance of linearly constrained constant modulus algorithm may degrade significantly in the presence of array steering vector errors because there are many array imperfections, such as the sensor location errors, unknown deformation of the antenna, steering direction errors, and wavefront distortion. The worst case is that the errors cause the linear programming problem to have no feasible solution. It is difficult that many array imperfections are formulated.

3 Robust Constant Modulus Algorithm under Double Constraints

To solve the interference capture problem, robust constant modulus algorithm under double constraints is proposed. In this algorithm, we use the oblique projection technique and the worst-case performance optimization approach to improve robustness to the signal vector errors and the small sample size. The minimization problem of robust constant modulus algorithm is expressed as

$$\begin{aligned}&\min_{\mathbf{W}} E[(|y(n-1)|^2-|y(n)|^2)^2]\\ &\text{s.t.}\quad \mathbf{W}^H(\mathbf{R}_s+\mathbf{v})\mathbf{W}\geq 1\\ &\text{for all}\|\mathbf{v}\|\leq r\end{aligned} \tag{11}$$

From (11), we note that the optimization problem is based on the worst-case optimization. The objective function is optimized so that the distortionless response to the desired signal can be maintained. With this new optimization method, the performance of our robust beamformer is guaranteed.

In this paper, our proposed method has the advantage of the constraint on array response vector to provide robustness to the small training sample size. We define the error $\mathbf{v}$ as the difference between the actual and presumed sequences. The constrained condition in (11) is written as simply

$$\min_{\mathbf{v}} \mathbf{W}^H(\mathbf{R}_s+\mathbf{v})\mathbf{W}=1 \qquad \text{s.t.}\quad \|\mathbf{v}\|\leq\beta \tag{12}$$

To solve the minimization problem, the error $\mathbf{v}$ is expressed as [18]

$$\mathbf{v}=-\frac{\beta}{\mathbf{W}^H\mathbf{W}}\mathbf{W}\mathbf{W}^H \tag{13}$$

Such formulation (11) can be equivalently converted to

$$\min_{\mathbf{W}}[(|y(n-1)|^2-|y(n)|^2)^2] \quad \text{s.t.}\mathbf{W}^H(\mathbf{R}_s-\beta\mathbf{I})\mathbf{W}=1 \tag{14}$$

The weight vector is derived by the Lagrange multiplier method. So, we can give the Lagrange function $H(\mathbf{W},r)$

$$H(\mathbf{W},r)=(|y(n-1)|^2-|y(n)|^2)^2+r(\mathbf{W}^H(n)\mathbf{P}\mathbf{W}(n)-1) \tag{15}$$

where r is the Lagrange multiplier and $\mathbf{P}=\mathbf{R}_s-\beta\mathbf{I}$. Applying the descent gradient approach, the gradient vector of (15) is

$$f(\mathbf{W},r)=-\varepsilon^*(n)\mathbf{X}(n)+r\mathbf{P}\mathbf{W}(n) \tag{16}$$

where

$$\varepsilon(n)=(|y(n-1)|^2-|y(n)|^2)y(k) \tag{17}$$

Applying (16) and (17) to robust constant modulus algorithm, the array weight vector is obtained iteratively

$$\begin{aligned}\mathbf{W}(n+1)&=\mathbf{W}(n)-\mu_c f(\mathbf{W},r)\\ &=\mathbf{W}(n)-\mu_c(r\mathbf{P}\mathbf{W}(n)-\varepsilon^*(n)\mathbf{X}(n))\end{aligned} \tag{18}$$

Inserting (18) into the quadratic constraint in (11), we can get

$$\begin{aligned}\mathbf{F}^H(n)\mathbf{P}\mathbf{F}(n)\ &-2\mu_c r\mathrm{Re}[\mathbf{F}^H(n)\mathbf{P}\mathbf{G}(n)]\\ &+\mu_c^2r^2\mathbf{G}^H(n)\mathbf{P}\mathbf{G}(n)=1\end{aligned} \tag{19}$$

where

$$\begin{aligned}\mathbf{F}(n)&=\mathbf{W}(n)+\mu_c\varepsilon^*(n)\mathbf{X}(n)\\ \mathbf{G}(n)&=\mathbf{P}\mathbf{W}(n)\end{aligned} \tag{20}$$

The Lagrange factor can be given by

$$r(n)=-\frac{\mathrm{Re}[\chi(n)]-\mathrm{Re}[\mathbf{F}^H(n)\mathbf{P}\mathbf{G}(n)]}{\mu_c\mathbf{G}^H(n)\mathbf{P}\mathbf{G}(n)} \tag{21}$$

where

$$\begin{aligned}\chi^*(n)\chi(n)=\mathbf{G}^H(n)\mathbf{P}\mathbf{G}(n)\ &-\mathbf{G}^H(n)\mathbf{P}\mathbf{G}(n)\mathbf{F}^H(n)\mathbf{P}\mathbf{F}(n)\\ &+(\mathrm{Re}[\mathbf{F}^H(n)\mathbf{P}\mathbf{G}(n)])^2\end{aligned} \tag{22}$$

The another constraint on the array steering vector via the oblique projection technique [25] is adjoined to the minimization function, thus our proposed algorithm provides better robustness to the signal steering vector errors. Therefore, the optimization function can be reformulated as

$$\begin{aligned}&\min_{\mathbf{W}} E[(|y(n-1)|^2-|y(n)|^2)]\\ &\text{s.t.}\quad \mathbf{W}^H(n)\bar{\mathbf{a}}_r=1,\quad \mathbf{W}^H(\mathbf{R}_s-\beta\mathbf{I})\mathbf{W}=1\end{aligned} \tag{23}$$

where $\bar{\mathbf{a}}_r$ is the steering vector of oblique projection

$$\bar{\mathbf{a}}_r=Z_{\mathbf{a}(\theta_i)\mathbf{A}_i}\mathbf{a}(\theta_0) \tag{24}$$

where $Z_{\mathbf{a}(\theta_i)\mathbf{A}_i}=\mathbf{a}(\theta_i)(\mathbf{a}^H(\theta_i)\mathbf{R}_P^+\mathbf{a}(\theta_i))\mathbf{a}(\theta_i)\mathbf{R}_P^+$, here $\mathbf{R}_P^+=[\mathbf{ASA}^H]^+$ is the pseudo-inverse matrix, $\mathbf{A}_i=[\mathbf{a}(\theta_1),\ldots,\mathbf{a}(\theta_{k-1}]$.

First form the following Lagrange function

$$L(\mathbf{W},\alpha,v)=[(|y(n-1)|^2 \;\; -|y(n)|^2)^2]+2\alpha(\mathbf{W}^H(n)\bar{\mathbf{a}}_r-1) +v(\mathbf{W}^H(k)\mathbf{P}\mathbf{W}(k)-1) \quad (25)$$

where α, v are the Lagrange multipliers. The array weight vector is updated iteratively by computing the gradient of (25)

$$\mathbf{W}(n+1)=\mathbf{W}(n)-\hat{\mu}_r[\alpha\bar{\mathbf{a}}_r+v\mathbf{P}\mathbf{W}(n)-\varepsilon^*(n)\mathbf{X}(n)] \quad (26)$$

Let the gradient of (25) be equal to zero, we can obtain the optimal array weight vector

$$\mathbf{W}_z=\frac{1}{v}\mathbf{P}^{-1}[\varepsilon^*(n)\mathbf{X}(n)-\alpha\bar{\mathbf{a}}_r] \quad (27)$$

Substituting (26) into the oblique projection constraint, the Lagrange factor $\alpha(n)$ is given by

$$\alpha(n)=\frac{1}{\hat{\mu}_r}[\bar{\mathbf{a}}_r^H\bar{\mathbf{a}}_r]^{-1} \;\; [\bar{\mathbf{a}}_r^H\mathbf{W}(n)+\hat{\mu}_r\bar{\mathbf{a}}_r^H\varepsilon^*(n)\mathbf{X}(n)] -\frac{1}{\hat{\mu}_r}[\bar{\mathbf{a}}_r^H\bar{\mathbf{a}}_r]^{-1}[\hat{\mu}_r v\bar{\mathbf{a}}_r^H\mathbf{P}\mathbf{W}(n)+1] \quad (28)$$

Inserting the factor $\alpha(n)$ into (26), we have the formulation of the array weights

$$\mathbf{W}(n+1)=\mathbf{J}\mathbf{W}(n)+\hat{\mu}_r\varepsilon^*(n)\mathbf{J}\mathbf{X}(n)- \;\; \hat{\mu}_r v\mathbf{J}\mathbf{P}\mathbf{W}(n) +[\bar{\mathbf{a}}_r^H\bar{\mathbf{a}}_r]^{-1}\bar{\mathbf{a}}_r \quad (29)$$

where $\mathbf{J}=\mathbf{I}-\bar{\mathbf{a}}_r[\bar{\mathbf{a}}_r^H\bar{\mathbf{a}}_r]^{-1}\bar{\mathbf{a}}_r$ is the oblique projection matrix.

Substituting (29) into the quadratic constraint in (23), we can get the following equation

$$(\mathbf{Q}(n)-v\hat{\mu}_r\mathbf{J}\mathbf{P}\mathbf{W}(n))^H\mathbf{P}(\mathbf{Q}(n)-v\hat{\mu}_r\mathbf{J}\mathbf{P}\mathbf{W}(n))=1 \quad (30)$$

where

$$\mathbf{Q}(n)=\mathbf{J}\mathbf{W}(n)+\hat{\mu}_r\varepsilon^*(n)\mathbf{J}\mathbf{X}(n)+\frac{\bar{\mathbf{a}}_r}{\bar{\mathbf{a}}_r^H\bar{\mathbf{a}}_r} \quad (31)$$

Solve the above equation (30) to derive the Lagrange factor

$$v(n)=-\frac{\mathrm{Re}[\varphi(n)]-\mathrm{Re}[\mathbf{Q}^H(n)\mathbf{P}\mathbf{J}\mathbf{P}\mathbf{W}(n)]}{\hat{\mu}_r\mathbf{W}^H(n)\mathbf{P}^H\mathbf{J}^H\mathbf{P}\mathbf{J}\mathbf{P}\mathbf{W}(n)} \quad (32)$$

where

$$\varphi^*(n)\varphi(n)= \;\; (\mathrm{Re}[\mathbf{Q}^H(n)\mathbf{P}\mathbf{J}\mathbf{P}\mathbf{W}(n)])^2 +\mathbf{W}^H(n)\mathbf{P}^H\mathbf{J}^H\mathbf{P}\mathbf{J}\mathbf{P}\mathbf{W}(n) -\mathbf{W}^H(n)\mathbf{P}^H\mathbf{J}^H\mathbf{P}\mathbf{J}\mathbf{P}\mathbf{W}(n)\mathbf{Q}^H(n)\mathbf{P}\mathbf{Q}(n) \quad (33)$$

4 Performance Analysis

4.1 The Optimal Step Size $\hat{\mu}_r$

The array weights (26) can be rewritten as

$$\mathbf{W}(n+1)=-[\hat{\mu}_r\mathbf{D}-\mathbf{I}]\mathbf{W}(n)+\hat{\mu}_r(\varepsilon^*(n)\mathbf{X}(n)-\alpha\bar{\mathbf{a}}_r) \quad (34)$$

where $\mathbf{D}=\gamma\mathbf{P}$.

Define $\mathbf{D}$ as

$$\mathbf{D}=\mathbf{U}_g\mathbf{V}_g\mathbf{U}_g^H \quad (35)$$

where the diagonal elements of diagonal matrix $\mathbf{V}_g$, $\sigma_1\geq\sigma_2\geq\ldots\geq\sigma_M$, are the corresponding eigenvalues, and the columns of $\mathbf{U}_g$ contain the eigenvectors of $\mathbf{D}$.

We can multiply $\mathbf{U}_g^H$ in the both sides of equation (34) to obtain

$$\mathbf{U}_g^H\mathbf{W}(n+1)=-[\hat{\mu}_r\mathbf{V}_g-\mathbf{I}]\mathbf{U}_g^H\mathbf{W}(n) \;\; -\hat{\mu}_r\mathbf{U}_g^H(\varepsilon^*(n)\mathbf{X}(n) -\bar{\mathbf{a}}_r\alpha) \quad (36)$$

From equation (36), note that if the proposed method converges, the parameter is required to satisfy

$$|\hat{\mu}_r\sigma_i-1|<1 \quad (37)$$

Solving (37), the range of step size $\hat{\mu}_r$ is expressed as

$$0<\hat{\mu}_r<\frac{2}{\sigma_{\max}} \quad (38)$$

where $\sigma_{\max}$ is the maximum eigenvalue

$$\sigma_{\max}<\sum_{i=1}^{M}\sigma_i=\mathrm{tr}[v\mathbf{P}] \quad (39)$$

It is known that the step size $\hat{\mu}_r$ is very important in descent gradient approach. If $\hat{\mu}_r$ is small, the approach may converge slowly but have little error. If $\hat{\mu}_r$ is large, the approach may converge quickly but the misadjustment error is large. If $\hat{\mu}_r$ is too large, the approach diverges.

So, we can choose the variable step size $\hat{\mu}_r$ in our algorithm to assure convergence performance

$$\hat{\mu}_r=\frac{5}{\mathrm{tr}[v\mathbf{P}]} \quad (40)$$

4.2 The Convergence of Our Algorithm

To present the convergence performance, we can take the statistical expectation in (26)

$$E[\mathbf{W}(n+1)]=-(\hat{\mu}_r v\mathbf{P}-\mathbf{I})E[\mathbf{W}(n)] \;\; +\hat{\mu}_r E[\varepsilon^*(n)\mathbf{X}(n)] -\hat{\mu}_r\alpha\bar{\mathbf{a}}_r \quad (41)$$

Let

$$\begin{aligned}\mathbf{T}(n+1)=&\;E[\mathbf{W}(n+1)]-\mathbf{W}_z\\ =&\;(\mathbf{I}-\hat{\mu}_r v\mathbf{P})(E[\mathbf{W}(n)]-\mathbf{W}_z)-\hat{\mu}_r\alpha\bar{\mathbf{a}}_r-\mathbf{W}_z\\ &+\hat{\mu}_r E[\varepsilon^*(n)\mathbf{X}(n)]+(\mathbf{I}-\hat{\mu}_r v\mathbf{P})\mathbf{W}_z \end{aligned} \quad (42)$$

Using (27), the following equation is obtained

$$(\mathbf{I}-\hat{\mu}_r v\mathbf{P})\mathbf{W}_z=\mathbf{W}_z-\hat{\mu}_r[\varepsilon^*(n)\mathbf{X}(n)]+\hat{\mu}_r\alpha\bar{\mathbf{a}}_r \quad (43)$$

The equation (42) can be reformulated as

$$\begin{aligned}\mathbf{T}(n+1)=&\;(\mathbf{I}-v\hat{\mu}_r\mathbf{P})\mathbf{T}(n)\\ =&\;(\mathbf{I}-v\hat{\mu}_r\mathbf{P})^{n+1}\mathbf{T}(0)\end{aligned} \quad (44)$$

The nonzero eigenvalue ψ_i satisfies the following condition

$$\psi_{\min} \leq \psi_i \leq \psi_{\max} \tag{45}$$

$\hat{\mu}_r$ is chosen as

$$0 \leq \hat{\mu}_r \leq 1/\psi_{\max} \tag{46}$$

Applying (46), we can obtain

$$\begin{aligned}(1-\hat{\mu}_r\psi_{\max})^{n+1}\|\mathbf{T}(0)\| &\leq \|\mathbf{T}(n+1)\| \\ &\leq (1-\hat{\mu}_r\psi_{\min})^{n+1}\|\mathbf{T}(0)\|\end{aligned} \tag{47}$$

If the initial difference vector length is finite, we have

$$\lim_{n\to\infty}\frac{1}{n}[\mathbf{W}(n)] = \mathbf{W}_z \tag{48}$$

4.3 Array Output SINR of Our Algorithm

The array output SINR_d is given by

$$\mathrm{SINR}_d = \frac{\mathbf{W}_z^H(\mathbf{a}_p\mathbf{a}_p^H)\mathbf{W}_z}{\mathbf{W}_z^H(\mathbf{A}_i\mathbf{A}_i^H+\sigma_n^2\mathbf{I}-\mathbf{a}_p\mathbf{a}_p^H)\mathbf{W}_z} \tag{49}$$

where $\mathbf{a}_p$ is the presumed steering vector, and σ_n^2 is the variance of noise.

In steering vector errors, $\mathbf{a}_c \neq \mathbf{a}_p$, so $|\mathbf{a}_c^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_p| < |\mathbf{a}_c^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_c|$.

The array output SINR is expressed as

$$\begin{aligned}\mathrm{SINR}_d &= \sigma_s^2\mathbf{a}_c^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_c\frac{|\mathbf{a}_p^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_c|^2}{(\mathbf{a}_c^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_c)(\mathbf{a}_p^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_p)} \\ &= \mathrm{SINR}_0\cos^2(\mathbf{a}_p,\mathbf{a}_c;\mathbf{R}_{i+n}^{-1})\end{aligned} \tag{50}$$

where $\mathrm{SINR}_0 = \sigma_s^2\mathbf{a}_c^H\mathbf{R}_{i+c}^{-1}\mathbf{a}_c$, $0 \leq cos^2(\mathbf{a}_p,\mathbf{a}_c;\mathbf{R}_{i+c}^{-1}) \leq 1$.

The linearly constrained constant modulus algorithm is very sensitive to the array steering vector errors. Thereby, it tends to treat the desired signal as interference signal and yields nulling on the desired signal. So, we can obtain the array output SINR

$$\mathrm{SINR}_{con} = 0 \tag{51}$$

This causes the signal cancellation problem. The double constraints of robust constant modulus algorithm are expressed as

$$\mathbf{W}^H\overline{\mathbf{a}}_r = 1, \quad \mathbf{W}^H\mathbf{P}\mathbf{W} = 1 \tag{52}$$

Using (52), the term can be approximated as

$$cos^2(\mathbf{a}_p,\mathbf{a}_c;\mathbf{R}_{i+c}^{-1}) \approx 1 \tag{53}$$

Using the double constraints, the following inequation is derived

$$\mathrm{SINR}_{rob} > \mathrm{SINR}_{con} \tag{54}$$

where SINR_{rob} is the output SINR of the our proposed method, and SINR_{con} is the output SINR of linearly constrained constant modulus algorithm.

From the performance analysis, we can see that our proposed algorithm can improve the beamformer performance in the presence of array steering vector errors.

5 Numerical Examples

We evaluate the proposed algorithm using Monte-Carlo simulations considering $N = 100$ training samples from a ULA with $M = 10$ omni-directional sensors with half wavelength spacing. In the examples, we assumed that one desired signal impinges on the antenna array from the direction of arrival 5°. The directions of arrival (DOA) of the two interference signals are $-50°$ and $50°$. We evaluate the performances in array beampattern and array output SINR. The factor $\beta = 9$ is chosen in our robust constant modulus algorithm. The noise is spatially and temporally white and it has a complex Gaussian zero mean distribution.

1) Comparison of the beampattern performance

The signal-to-noise ratio is 10dB The simulated array beampatterns are shown in Fig. 1, where the vertical long lines represent the direction of the desired signal. It can be seen that the main beams are all at the desired signal, whereas nulls both appear in the directions of the interference signals. Fig. 2 shows the array beampatterns of the two algorithms in a mismatch situation. In Fig. 2, the vertical long lines represent the direction of the actual signal. We note that the linearly constrained constant modulus algorithm is sensitive to the array steering vector error and suppress the desired signal. This yields the signal cancellation problem. Our proposed algorithm can provide better robustness and have signal sampling resolution in contrast to the linearly constrained constant modulus algorithm.

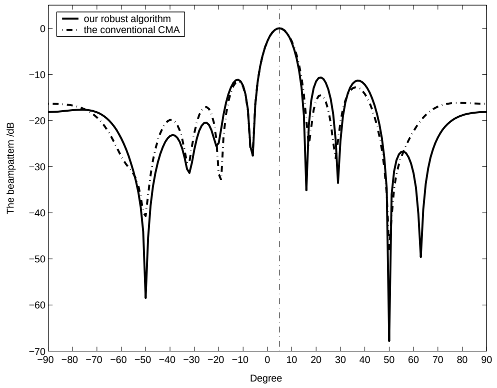

Fig. 1: Comparison of array output beampattern in no mismatch

2) Comparison of array output SINR performance versus N

Fig. 3 shows array output SINR performance versus N in no mismatch. The array output SINR in the array imperfections is shown in Fig. 4. According to Fig. 3, our proposed algorithm gives better SINR performance and maintains output performance close the optimum. From Fig. 4, it is obvious that array output SINR of the linearly constrained constant modulus algorithm decreases significantly in nonideal situations. In Fig. 4, our algorithm improves about 27dB over conventional

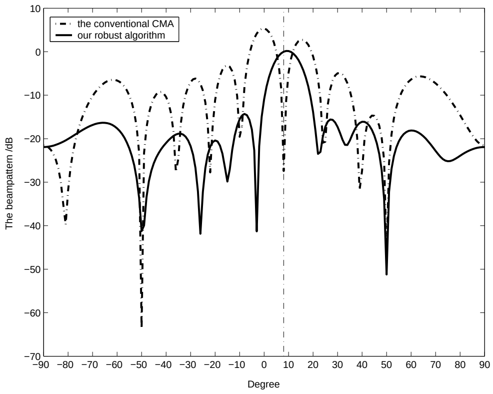

Fig. 2: Comparison of array output beampattern in a mismatch

constant modulus algorithm. The proposed algorithm achieves better output performance in the presence of array steering vector errors.

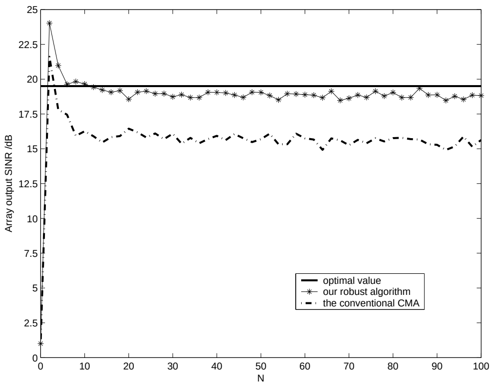

Fig. 3: Array output SINR versus N in no mismatch

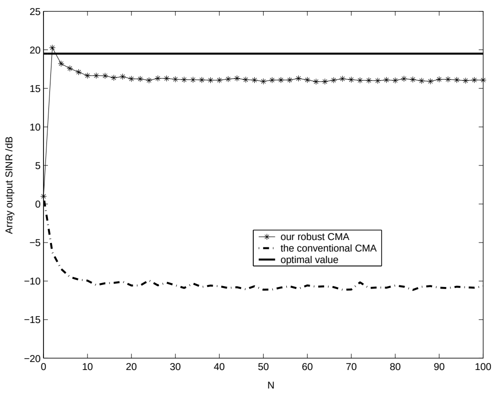

Fig. 4: Array output SINR versus N in a mismatch

6 Conclusions

By solving an optimization criterion and satisfying the double constraints, a new robust constant modulus algorithm based on the worst-case performance optimization and oblique projection technique is proposed to solve the problem of array steering vector errors. In this paper, we analyze the output performance of the proposed algorithm. The analysis is carried out assuming that random steering vector errors are present. We can derive a theoretical expression of the output SINR, analyze the convergence performance, and obtain the optimal step size. With one constraint imposed on the array steering vector and another constraint imposed on the data snapshots, our proposed robust beamformer provides significant robustness against array steering vector errors and the small training sample size. In this paper, we can update the weights via the Lagrange multiplier method and descent gradient technique. Our algorithm has simple on-line implementation, flexible beampattern control and significant SINR improvement, as compared with the linearly constrained constant modulus algorithm. Our simulations suggest that the proposed method is effective for practical applications.

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