Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/081L30

A Vehicle Routing Optimization Method with Constraints Condition based on Max-Min Ant Colony Algorithm

Wei-Cheng Xie[1,*], *Chang-Min Chen*[1], *Song-Song Fan*[2] *and Ling-Ling Li*[1]

[1] School of Electrical and information Engineering, Xihua University, Chengdu 610039, China
[2] School of communication and information engineering UESTC, Chengdu, China

Received: 6 May. 2013, Revised: 31 Aug. 2013, Accepted: 1 Sep. 2013
Published online: 1 Apr. 2014

Abstract: According to the command quantity of customers and limits of distribution vehicle, in order to obtain more profit, optimization study on vehicle routing and quantity is needed. Basic ant colony algorithm has slow convergence speed and is easy to falling into local optimum when used to solve this problem, while max-min ant colony algorithm can overcome these shortcomings and is the best algorithm to solve this problem at present. In this paper, vehicle routing optimization on constraints condition is simulated with max-min ant colony algorithm on MATLAB, and the direction of vehicle routing and the least number of vehicle can be obtained from the simulation results.

Keywords: MMAS, the optimization of VRP, constraint conditions, MATLAB

1 Introduction

With the rapid development of scientific technology and increase of economic globalization, the world economy is facing unprecedented opportunities and challenges, and modern logistics is an important part of modern economy. In 2008, China's total social logistics was 89.9 trillion Yuan, 4.2 times increase over 2000, an average increase of 23%; logistics industry realized an added value of 2.0 trillion Yuan, 1.9 times increase over 2000, an average increase of 14%. As can be seen from these data, the logistics industry makes great interests, but the overall level of China's logistics industry is low, such as: first, low operation efficiency of the whole social, China's total logistics costs to GDP ratio of about 1 times higher than developed countries; second, the less demand on socialization logistics and insufficient ability of professional supply [1,4].

Vehicle routing problem (VRP) is an important part of the logistics system research. In order to reduce the service providers' operating costs, logistics system will be done optimization on the vehicle's path. Selecting the appropriate transport routes can speed up response time to customer needs, improve service quality, and enhance customer satisfaction with the logistics system [4].

Vehicle routing problem is generally defined as: a series of delivery point and/or receiving point, selected the proper route with certain constraints orderly through them.

To save logistics costs, two aspects are available [5]: Improving efficiency, which is to reduce the costs of the vehicles from the economic perspective; the reasonable arrangement of vehicle distribution to improve the professional logistics supply. At present, Max-min ant colony algorithm is the better algorithm to solve the vehicle path 1-4. On existing conditions with two constraints:

Firstly, the vehicle maximum driving distances.

Secondly, the vehicle's maximum loading, can fundamentally reduce the cost of the vehicle to gain greater benefits.

2 The Description of VRP Problem with Constraints and the Model

A. The Description of VRP Problem with Constraints

VRP (vehicle routing problem), the vehicle routing problem can been described as follows1: There are a number of customer points, and distance and demand for

* Corresponding author e-mail: scxweicheng@aliyun.com

goods between two customers are given. Each vehicle starts from the distribution station, and come back when reaching the limitations [6,8]. Vehicle's maximum load capacity and driving range are given.

B. The Model of VRP Problem with Constraints

Record the maximum load per vehicle for QV. The maximum driving range for Lengthmax,the demand of customer i for gi ($i \in C$). C is a collection of customers. Set up:

$$X_{ijk} = \begin{cases} 1, \text{ If the vehicle k along the path(i,j),} \\ 0, \text{ else.} \end{cases}$$

Then we obtain the mathematical model [2]:

$$minZ = \left\{ \sum_i \sum_j \sum_k X_{ijk}, \sum_j \sum_k X_{0jk} \right\}$$

$$s.t \begin{cases} \sum_i g_i y_{ki} \leq QV, \forall k & (a) \\ \sum_i \sum_j D_{ij} y_{kij} \leq lengthmax, \forall k & (b) \\ \sum_k y_{ki} = 1, i \in C, & (c) \\ \sum_i x_{ijk} = y_{kj}, j \in C, \forall k & (d) \\ \sum_j x_{ijk} = y_{ki}, i \in C, \forall k & (e) \\ \sum_{i,j \in S*S} \sum X_{ijk} \leq |S| - 1, S \subseteq C & (f) \\ x_{ijk}, y_{ki} \in (0,1), i, j \in C, \forall k \end{cases}$$

Of which:
Constraints (a) for the vehicle load limits;
Constraint (b) for the vehicle journey restrictions;
Constraint (c) to ensure that every vehicle access to each customer only once;
Constraints (d) - (f) to ensure that form a viable circuit;
The targets are the least of vehicles' number and the minimum of the total road length.

3 Basic Ant Colony Algorithm

According to long-term study on bionics, the ant has no vision, but it can release a special kind of secretion pheromone on the path when looking for food, so that the later ants can quickly find food. They will randomly choose a path forward and release the pheromone when coming across the path not passed by. The longer path is going, the less amount of information is released 5-6. The ants coming later will choose the path of large amount is larger, this probability form a positive feedback mechanism Over time, the path chosen by most ants the greater the amount of information, while others gradually reduce the amount of information on the path, eventually the whole colony will find the optimal path.

Ant also able to adapt to environmental changes, when the colony there is an obstacle on the path of movement, the ants can quickly identify the optimal path. Result that, in the ant routing process, a single ant has the ability, though limited selection, but by the positive feedback of pheromone the ant colony behavior can have very high self-organization.

Ant path between the exchange of information, and ultimately self-catalyzed by the collective behavior of ant colony to find the optimal path5. Ants search path is shown as Figure 1.

Assuming the ants are starting from the nest, there are four food points, each ant needs to reach four food points and then back to the nest, which is equivalent to a car to four different customer sites and distribution of goods (with the traveling salesman problem (TSP) related) [9, 12]. Ants depart from the nest, there are four selection lines (1, 5, 6, 7), and each route is a two-way. In the figure the length of 6, 7 were clearly longer than the length of 1, 5, and the pheromone on the route of 6, 7 would less and less over time, so there will be more and more ants select the routes 1and 5. And so, eventually the ants will be based on the strength of the pheromone to find the optimal path.

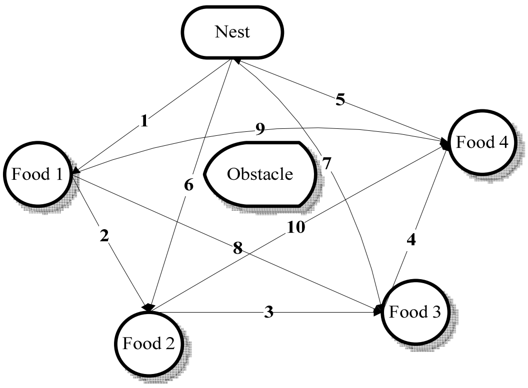

Fig. 1: The figure of ant search path.

4 Max-Min Ant Colony Algorithm

Because the shortcomings of the basic ant colony algorithm can not be ignored, MMAS can avoid premature stagnation phenomenon 6-8 [13,17]. At present, Max-min ant colony algorithm is the better ant colony algorithm to solve TSP problem (ACO) compared with other algorithms. MMAS limits possible residue pheromone limit of every path to $[\tau_{min}, \tau_{max}]$, and keep the optimum path after one cycle.

A. The Improvements over ACO

MMAS came directly from the ACO, mainly made the following improvements:

1) After each iterating, only the pheromone on the optimal path is updated;

2) In order to overcoming the ACO's shortcoming of being easy to fall into local optimum, MMAS limit the pheromone of each path to $[\tau_{min}, \tau_{max}]$. Pheromone will be forced to set to τ_{min} or τ_{max}, if it excesses the value. This can effectively avoid information of certain path far outweigh the other path, keep all the ants on the same path, make the algorithm falling into the local optimum and not spread the path;

3) Initially, the pheromone on each path is , and when ρ (the pheromone evaporation coefficient) take smaller value, MMAS has a better ability to find better solutions, and τ_{max} and τ_{min} are determined by following formula:

Pheromone has not been updated:

$$\tau_{max}(t) = \frac{1}{2(1-\rho) * L_{best}}$$

$$\tau_{min}(t) = \frac{\tau_{max}(t)}{20}$$

Which $\mathbf{L}_{best}$ is the corresponding path length of the global best solution, after pheromone updated, τ_{max} and τ_{min} are decided by the following formula, τ_{min} is the same as (1).

$$\tau_{max}(t) = \frac{1}{2(1-\rho) * L_{best}} + \frac{\sigma}{L_{best}}$$

Where σ is the number of "elite ant".

4) In the MMAS, all the pheromone re-initialization when the system is stalled.

B. Pheromone Update

In this paper, we use a DC motor as controlled object to analyze the system stability, the transfer function of the controlled plant is expressed as follows9-10: Pheromone update has two ways: local update and global update. In this paper, the global update is decided, the best ant (the top few "elite ant" in the path constructed) for the pheromone update, the update rules are as follows:

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \sum_{u=1}^{\sigma-1} \Delta\tau_{ij}^{u} + \sigma\Delta\tau_{ij}^{*}$$

Only when the trajectory (Di, Dj) is passing by elite ants μ, the pheromone on the path will increase $\Delta\tau_{ij}^{u}(\Delta\tau_{ij}^{u} = (\sigma-\mu)/L_u)$, which Lu causes the path length of ant μ . The pheromone on the entire best path are increased $\sigma\Delta\tau_{ij}^{*}$
$(\sigma\Delta\tau_{ij}^{*} = 1/L^{*})$, L* indicates the current best path length. Generally, σ 's value between 3 to 6 is more appropriate11.

C. The implementation of MMAS in a multi-constraint VRP

1) parameters initialization, set the initial amount of information on the path (Di, Dj) $\tau_{ij} = tau_{max}$, set up distribution points for the initial node, and put the initial node into the tabuk list, set the initial value for the parameter α,β,ρ,ρ; iteration number nc = 0, Ncmax is the maximum number of iterations;

2) Into the iterative process, nc = 0;

3) Let all not visited customers into the set J;

4) If L (Ci, Cj) + L (Cj, Cj +1) + ... + L (Cm-1, Cm) $\geqslant$ lengthmax or g (i, j) + g (j, j +1) + ... + g (k-1, k) $\geqslant$ QV, save the current path and re-open a path, which starts from the starting point that is the distribution point, repeat step (3);

5) Pheromone update;

6) If set J is empty, then jump to (8) step;

7) Iterations nc = nc +1, and nc $\leqslant$ Ncmax;

8) End and output the results.

5 MMAS's Application of Vehicle Distribution Logistics on Multi-Constraint

Suppose logistics distribution center distributed goods to 13 supermarkets on the outskirts. Every car has certain load and driving distance, and each path can not excess the load of vehicle. Each shop is served only by a car. The distance and demand size of each shop is on table 2.

Suppose that traffic control and congestion was not taken into account, maximum load of vehicle was 10 ton, maximum driving distance was 150km. MMAS algorithm is analyzed with MATLAB and results path are separately shown in figure 2 and sheet 1.

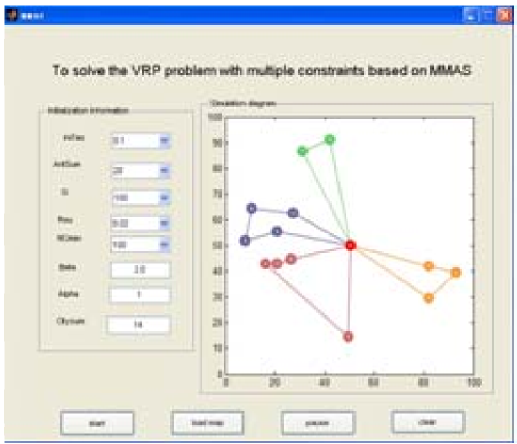

Fig. 2: MATLAB simulation diagram of the MMAS algorithm.

6 Conclusions

Vehicle routing optimization problem is a typical non-linear combination problem, and it occupies a large part in the modern logistics system's cost control. Max-Min ant algorithm is a good method to deal with

Table 2: The specific circumstances of each supermarket

Distri-bution	Distance between points														Demands
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	
0	0	48	47	40	33	46	27	46	38	46	41	29	33	35	0
1	48	0	80	55	80	93	72	85	83	93	16	76	78	12	2
2	47	80	0	86	46	57	55	13	61	45	81	36	59	70	1.5
3	40	55	86	0	55	62	42	83	48	70	39	59	45	47	2
4	33	80	46	55	0	15	13	37	15	15	72	11	14	68	2.8
5	46	93	57	62	15	0	22	46	13	14	84	24	17	81	2.5
6	27	72	55	42	13	22	0	47	11	28	62	20	6	60	2.2
7	46	85	13	83	37	46	47	0	52	33	84	27	50	74	3
8	38	83	61	48	15	13	11	52	0	25	73	25	5	71	2.2
9	46	93	45	70	15	14	28	33	25	0	86	18	26	81	2.5
10	41	16	81	39	72	84	62	84	73	86	0	70	68	14	1.6
11	29	76	36	59	11	24	20	27	25	18	70	0	23	63	2
12	33	78	59	45	14	17	6	50	5	26	68	23	0	66	3.2
13	35	12	70	47	68	81	60	74	71	81	14	63	66	0	4.8

Table 1: Vehicle routing optimization results

Vehicle i	Driving Directions
1	0–2–7–0
2	0–11–9–5–4–0
3	0–6–12–8–3–0
4	0–10–1–13–0

these problems from the angle of cost savings. Two constraints are added on this base in the paper, which optimize the path and have a great economic value in solving the vehicle routing optimization.

Acknowledgement

This work was supported by the project 10ZA098 of the Scientific Research Fund of Sichuan Provincial Education Department, the project Ycjj201145 of the graduate student innovation Fund of Xihua University and the Open Research Fund of the Key Laboratory of Signal and Information Processing, Xihua University (No. szjj2012-014).

References

[1] B. Yu, Z. Z. Yang, B. Z. Yao. An improved ant colony optimization for vehicle routing problem, European Journal of Operational Research, **196**, 171-176 (2009).

[2] S. H. Huang, P. C. Lin. A modified ant colony optimization algorithm for multi-item inventory routing problems with demand uncertainty, Transportation Research Part E: Logistics and Transportation Review, **46**, 598-611 (2010).

[3] L. Santos, J. Coutinho-Rodrigues, J. R. Current. An improved ant colony optimization based algorithm for the capacitated arc routing problem, Transportation Research Part B: Methodological, **44**, 246-266 (2010).

[4] B. Yu, Z. Z. Yang. An ant colony optimization model: The period vehicle routing problem with time windows, Transportation Research Part E: Logistics and Transportation Review, **47**, 166-181 (2011).

[5] Hai-bin Duan. Ant Colony Algorithms: Theory and Applications. Beijing: Science Press, 24-25 (2005).

[6] C. M. Chen, W. C. Xie, S. S. Fan, et al. The logistics vehicle routing optimization method and implementation based on ant colony algorithm, Journal of Computational Information Systems, **8**, 8439-8446 (2012).

[7] John E. Bell, Patrick R. McMullen. Ant colony optimization techniques for the vehicle routing problem, Advanced Engineering Informatics, **18**, 41-48 (2004).

[8] Q. Chen, B. Ning. Application of MMAS on vehicle routing problem with time window. Journal of Jiangsu University of Science and Technology (Natural Science Edition), 23, 263-268.

[9] M. Tripathi, G. Kuriger, H. D. Wan. An Ant Based Simulation Optimization for Vehicle Routing Problem with Stochastic Demands, Proceeding of the 2009 Winter Simulation Conference, 2476-2487 (2009).

[10] S. R. Balseiro, I. Loiseau, J. Ramonet. An Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows , Computers & Operations Research, **38**, 954-966 (2011).

[11] D. M. Zhao, L. Luo, K. Zhang. An improved ant colony optimization for the communication network routing problem, Mathematical and Computer Modelling, **52**, 1976-1981 (2010).

[12] C. Garcła-Martłnez, O. Cordn, F. Herrera. A taxonomy and an empirical analysis of multiple objective ant colony optimization algorithms for the bi-criteria TSP, European Journal of Operational Research, **180**, 116-148 (2007).

[13] Y. Yang, Mohamed S. Kamel. An aggregated clustering approach using multi-ant colonies algorithms, Pattern Recognition, **39**, 1278-1289 (2006).

[14] C. Rajendran, H. Ziegler. Ant-colony algorithms for permutation flow shop scheduling to minimize make span/total flow time of jobs, European Journal of Operational Research, **155**, 426-438 (2004).

[15] H. Wang, Z. Shi, S. Li. Multicast routing for delay variation bound using a modified ant colony algorithm, Journal of Network and Computer Applications, **32**, 258-272 (2009).
[16] X. Zhao. MAX-MIN Ant System and its Convergence, Computer Engineering and Applications, **8**, 70-72 (2006).
[17] J. G. Hu, H. N. Qi, F. Dong et al. Improved ant colony algorithm for path planning of tourist scenic area. Application Research of Computers, **5**, 1647-1650 (2011).
[18] Y. L. Zhu, Y. W. Chen, Han Kai and so on. Searching optimal rush repair path of power lines based on improved max-min ant colony algorithm. Application Research of Computers, **26**, 3436-3439 (2009).
[19] K. F. Zhang, D. Huang. Vehicle Routing Problem Research Based on an Improved Ant Colony Algorithm, Advances in Computer Technology and Applications, 259-264 (2007).
[20] C. M. Chen, W. C. Xie, S. S. Fan. The Research on VRP Based on Max-Min Ant Colony Algorithm. Advanced Materials Research, **218-219**, 1285-1288 (2011).
[21] A. Colorni , M. Dorigo , V. Maniezzo , et al. Distributed Optimization by Ant Colonies. Proceedings of the 1st European Conference on Artificial Life. France: Elsevier publishing, **2**, 134-142 (1991).
[22] I. C. Trelea. The particle swarm optimization algorithm: convergence analysis and parameter selection. Information Processing Letters, 85, 317-325 (2003).
[23] M. Jiang, Y. P. Luo, S. Y. Yang. Stochastic convergence analysis and parameter selection of the standard particle swarm optimization algorithm. Information Processing Letters, 102, 8-16 (2007).
[24] G. Radolph. Convergence analysis of canonical genetic algorithms [J]. IEEE Transactions on Neural Network, **5**, 96-101 (1994).
[25] C. Perey. Combinative optimization with use of guided evolutionary simulated annealing [J]. IEEE Transactions on Neural Network, **6**, 209-295 (1995).
[26] X. F. Qi. Palmieri theoretical analysis of evolutionary algorithms with an infinite population size in continuous space (Part I, II). IEEE Transactions on Neural Network, **5**, 102-129 (1994).

Wei-Cheng Xie, received the B.S. degree in applied physics (Electronic Information Engineering) from Xinan Normal University in June 1995, Chongqing, P. R. China,the M.S. degree in software engineering from Sichuan University in June 2005, Chengdu, P. R. China. Since July 1995, he has been with the School of Electrical and Information Engineering, Xihua University in July 1995, Chengdu, P. R. China, where he became a Professor in November 2011. His research interests include intelligent information processing, signal acquisition & processing and intelligent control.

Chang-Min Chen, born in 1987, received the B.S. degree in electronic information engineering from Leshan Normal University in June 2009, Leshan, Sichuan P. R. China, the M.S. candidate of Xihua University in June 2012, Chengdu, P. R. China. Her research interests include intelligent information processing and information system.

Song-Song Fan, born in 1987, received the B.S. degree in electronic information engineering from Leshan Normal University in June 2009, the M.S. candidate of University of Electronic Science and Technology of China (UESTC) in June 2012, Chengdu, P. R. China. His research interests

Ling-Ling Li, born in 1988, received the B.S. degree in the measurement and control technology and instrument from Xihua University in June 2010, Chengdu, Sichuan P. R. China, the M.S. degree in signal and information processing from Xihua University in June 2013, Chengdu,P. R. China. Her research interests include signal detection and information system.