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Attitude Dynamics and Control of Spacecraft using a Serial Manipulator

Yong-Lin Kuo[1,*] *and Tsung-Liang Wu*[2]

[1] Institute of Automation and Control, National Taiwan University of Science and Technology, Taipei City, 10607, Taiwan
[2] Mechanical and Systems Laboratory, Industrial Technology Research Institute,Hsinchu County, 31040 Taiwan

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Abstract: This paper studies the attitude control of spacecraft hinged by a serial manipulator under the effect of the earths gravitational field. The manipulator consists of multiple rigid body links. In literature, most of papers considered a one- or two-link manipulator. This paper proposes a general form of equations of motion of the spacecraft-manipulator system with any number of links. The equations of motion are nonlinear, and the linearized equations are also provided. Besides, a simple approach of the controller design is proposed, which is based on the combinations of the attitude dynamics and the desired system responses. The paper demonstrates three types of manipulators with one, two and three links. Several controllers for each manipulator are applied to demonstrate the feasibility of the proposed approach. The results show that the rotating angle of the spacecraft attitude can reach the steady state within 0.002 orbits based on the attitude dynamics and the desired system responses.

Keywords: Attitude control, serial manipulator, gravitational field

1 Introduction

Attitude control and stabilization is extremely significant in the operation of spacecraft because it constitutes a mandatory feature both for the survival of spacecraft and for the satisfactory achievement of space missions. There are a number of possible approaches to the control and stabilization of attitude dynamics developed during the past decades. Also, attitude control and stabilization of spacecraft has been an active research topic for quite sometime. Due to the nonlinearities of spacecraft dynamics models and the effects of coupling with the uncertainties both in parameters and disturbances, the relevant researches of attitude control and stabilization become more attractive and challenging. Numerous control design methods have been investigated to achieve control system performance and/or robustness. Recent works on spacecraft attitude control and stabilization include linear and nonlinear H^∞ control [1,2,3], fuzzy-neuro control [4,5], LQR/LRT [6,7,8], and adaptive control [9] among others.

A space manipulator implemented to spacecraft play an important role in space mission because of its capability to act in inaccessible environments for humans. Besides, it has characteristics such as light weight, less power requirement, ease of maneuverability and ease of transportability. Because of the light weight, spacecraft can be operated at high speed. In general, a space manipulator system composes of a base and a manipulator. The base usually refers to a spacecraft or a satellite, and the manipulator mounted on it. Since the base is free-floating or free-flying, it can be affected by the motion of the manipulator. Thus, this results in a set of coupling dynamic equations between the motions of the base and the manipulator. Also, it is necessary to incorporate the disturbance torques in space environment into the dynamic system. There are a numerous researchers devoted to the kinematics and dynamics of a free-floating or free-flying space manipulator system. Vafa et al. developed the virtual manipulator and proposed a planning technique, which employs small cyclical manipulator joint motions to modify an attitude of spacecraft [10,11]. Papadopoulos et al. studied the path-dependent dynamic singularities and showed that their inertial space location is a function of the dynamic properties of a system [12,13]. Umetani et al. presented the free-floating system generalized Jacobian, which reflects both momentum conservation laws and kinematic

* Corresponding author e-mail: kuo@mail.ntust.edu.tw

relations under the absence of external forces and torques [14]. Franch et al. used the flatness theory to plan trajectories for free-floating systems, which requires a selection of robot parameters so that the system is controllable and linearizable by prolongations [15]. Shui et al. studies the coordinated manipulator and spacecraft motion planning for free-floating space robots. The kinematics is analyzed based on momentum conservation law [16]. Nanos et al. studies the presence of the initial momentum, which renders the end-effector immune to angular momentum accumulation [17]. The relevant kinematics and dynamics are studied in 2D and 3D systems, and workspace subsets, where the end-effector can remain fixed, are identified. Furthermore, Ali et al. presented an overview paper addressing dynamics modeling, planning and control of free-flying robots in space [18].

This paper studies the attitude dynamics and control of a spacecraft-manipulator system on the free-floating mode. The spacecraft is hinged by a serial manipulator, and the attitude control of the spacecraft is achieved by applying external torques to each link of the serial manipulator. The modeling of the entire system is affected by the earths gravitational field. In the literatures, there are numerous papers studying the dynamics of the spacecraft-manipulator system. However, most of them consider that the manipulator has only one or two links. This paper presents a general form of equations of motion based on a spacecraft connected with a serial manipulator, which consists of any number of rigid body links. Also, one proposes a simple approach to design the controller, which is based on the attitude dynamics and the desired system responses. The performance criteria of system responses are specified first, and then the control torques can be determined by combining the attitude dynamics and the controller design criteria. The paper presents three manipulators with one, two and three links individually. One also proposes several kinds of controllers for each manipulator. This paper is organized as follows. Section 2 presents a general form of equations of motion based on a spacecraft hinged by a serial manipulator with any number of links. Section 3 presents the dynamics and control for a spacecraft controlled by a single-link manipulator. Two more complicated cases, two and three links, are demonstrated in Sections 4 and 5, respectively. Section 6 summarizes the results of the paper.

2 Formulation of equations of motion

This section presents the formulation of the equations of motion based on a spacecraft with a serial manipulator. One intends to utilize the motion of the serial manipulator to fulfill the attitude stabilization and control of spacecraft. The manipulator comprises of multiple rigid body links illustrated in Figure 1. Also, the motion of the mass center of the entire spacecraft-manipulator system follows the orbital trajectory, and the mass center may not be coincident with that of the spacecraft due to a large-size manipulator. Both the spacecraft and the manipulator are affected by the earths gravitational field.

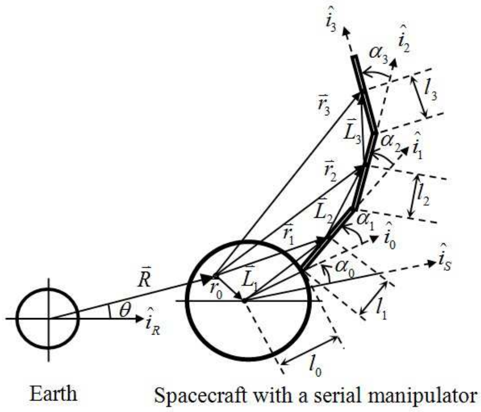

Fig. 1: A spacecraft-manipulator system

The spacecraft-manipulator system is illustrated in Figure 1, and several coordinate systems are defined. A reference axis is defined, and $\hat{i}_R$ represents its unit vector; $\hat{i}_S$ is a unit vector from the origin to the mass center of the spacecraft-manipulator system removed to the spacecraft origin. The spacecraft has a pitch angle α_0 , and a corresponding unit vector $\hat{i}_0$ is defined. Similarly, the ith link performs a rotating angle α_i, and $\hat{i}_i$ is the corresponding unit vector. Vector $\mathbf{R}$ is one from the origin to the mass center of the spacecraft-manipulator system, and $\mathbf{r}_i$ is the vector from the mass center of the system to that of each link. illustrates the spacecraft-manipulator system with only three links. If the number of links is greater than three, the related coordinate and nomenclature can be similarly defined.

One considers that the serial manipulator has n links. The kinetic energy and the potential energy of the spacecraft-manipulator system are derived as follows.

2.1 Kinetic Energy

The kinetic energy of the spacecraft-manipulator system is written as

$$T = \frac{1}{2}\sum_{k=0}^{n} m_k(\dot{\mathbf{R}}+\dot{\mathbf{r}}_k)^2 + \frac{1}{2}\sum_{k=0}^{n} I_{Zk}\omega_{Zk}^2, \tag{1}$$

where m_k and I_{Zk} are the mass and the moment of inertia, respectively; the subscript k represents the kth link ($k = 0$ refers to the spacecraft itself).

The summation of the product of the mass m_k and the relative vector $\mathbf{r}_k$ for the spacecraft and the links should be zero, which is written as

$$\sum_{k=0}^{n} m_k\mathbf{r}_k = 0, \tag{2}$$

or

$$(1-\sum_{k=1}^{n}\rho_k)\mathbf{r}_0+\sum_{k=1}^{n}\rho_k\mathbf{r}_k=0, \tag{3}$$

where ρ_k is the mass ratio of m_k to the total mass of the system.

Define the rotating angle β_k and total mass M respectively as

$$\beta_k=\theta+\sum_{i=0}^{k}\alpha_i, \tag{4}$$

$$M=\sum_{k=0}^{n}m_k, \tag{5}$$

Substitute Equations (3), (4) and (5) into (1), the kinetic energy can be simplified as

$$T=\frac{1}{2}M\dot{\mathbf{R}}^2+\frac{1}{2}\sum_{k=0}^{n}m_k\dot{\mathbf{r}}_k^2+\frac{1}{2}\sum_{k=0}^{n}I_{Zk}\dot{\beta}_k^2, \tag{6}$$

Define the relative vectors (see) as

$$\mathbf{L}_k=\mathbf{r}_k-\mathbf{r}_{k-1},\quad k=0,1,2,\cdots,n, \tag{7}$$

Solving Equations (3) and (7) leads to

$$\mathbf{r}_k=\sum_{i=1}^{n}a_{ki}\mathbf{L}_i,\quad k=0,1,2,\cdots,n, \tag{8}$$

where

$$a_{ki}=(-1)^n(\varepsilon_{ki}-\sum_{j=i}^{n}\rho_j),\quad k=0,1,2,\cdots,n, \tag{9}$$

$$\varepsilon_{ki}=\begin{cases}1, & \text{if}\quad k\geq i\\ 0, & \text{if otherwise}\end{cases} \tag{10}$$

The vector $\mathbf{L}_k$ can be alternatively expressed as (see Figure 1)

$$\mathbf{L}_k=l_{k-1}\hat{i}_{k-1}+l_k\hat{i}_k,\quad k=0,1,2,\cdots,n, \tag{11}$$

where l_k is a half length of the kth link, and l_0 is the distance between the mass center of spacecraft and the joint with the first link (see Figure 1). Hence, differentiating Equation (12) leads to

$$\dot{\mathbf{L}}_k=l_{k-1}\dot{\beta}_{k-1}\hat{j}_{k-1}+l_k\dot{\beta}_k\hat{j}_k,\quad k=0,1,2,\cdots,n, \tag{12}$$

where $\hat{j}_k$ is the unit vector corresponding perpendicular to $\hat{i}_k$ on the rotating plane.

Substitute Equations (7) and (12) into (5), one obtains the kinetic energy as the expression

$$T=\frac{1}{2}M\dot{\mathbf{R}}^2+\frac{1}{2}\sum_{k=0}^{n}I_{Zk}\dot{\beta}_k^2$$
$$+\frac{1}{2}\sum_{k=0}^{n}m_k\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}[a_{ki}a_{kj}(\sum_{l=i-1}^{n}\sum_{m=j-1}^{j}l_l l_m\dot{\beta}_l\dot{\beta}_m\cos\gamma_{lm})]\right\}, \tag{13}$$

where

$$\gamma_{lm}=\begin{cases}\sum_{p=\min(l,m)+1}^{\max(l,m)}\alpha_p, & l\neq m\\ 0, & l=m\end{cases} \tag{14}$$

2.2 Potential Energy

The potential energy of a small mass dm of the ith link in a spacecraft-manipulator system due to the earths gravity field is given as

$$U=-\sum_{k=0}^{n}\int_{m_i}\frac{\mu}{|\mathbf{R}+\mathbf{s}|}\mathrm{d}m \tag{15}$$

where μ is the gravitational parameter, and $\mathbf{s}$ is a position vector of a small mass dm from the mass center of the system. The distance $|\mathbf{s}|$ is assumed as much smaller than $|\mathbf{R}|$, and the products of inertia assumed as zeros with respect to the spacecraft body-fixed coordinate. By applying Binomial series expansion and carrying out the expansion till $O(1/|\mathbf{R}|^3)$, the potential energy is written as

$$U=-\frac{\mu M}{R}+\frac{\mu}{2R^3}\sum_{k=0}^{n}m_k\mathbf{r_k}^2-\frac{3\mu}{2R^5}\sum_{k=0}^{n}[m_k(\cdot\mathbf{r}_k)^2]$$
$$+\frac{\mu}{4R^3}\sum_{k=0}^{n}[(I_{Xk}+I_{Yk}+I_{Zk})-3(I_{Zk}+(I_{Yk}-I_{Xk})\cos(2\beta_k)] \tag{16}$$

where I_{Xk}, I_{Yk} and I_{Zk} are the moments of inertia of spacecraft; R is the magnitude of the vector $|\mathbf{R}|$.

Based on Figure 1, the vector R is written as

$$\mathbf{R}=R\hat{i_c} \tag{17}$$

Substitute Equations (7), (11) and (17) into (16), one obtains

$$U=-\frac{\mu M}{R}+\frac{\mu}{2R^3}\sum_{k=0}^{n}m_k\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ki}a_{kj}\sum_{l=i-1}^{i}\sum_{m=j-1}^{j}[l_l l_m(\cos\gamma_{lm}$$
$$-3\cos\phi_i\cos\phi_m)]+\frac{\mu}{4R^3}\sum_{k=0}^{n}[(I_{Xk}+I_{Yk}+I_{Zk})$$
$$-3(I_{Zk}+(I_{Yk}-I_{Xk})\cos(2\phi_k))] \tag{18}$$

where

$$\phi_k=\sum_{i=0}^{k}\alpha_i \tag{19}$$

2.3 Nonlinear Equations of Motion

Apply the Euler-Lagrange equation, the equation of motion for α_q $(q=0,1,2,\cdots,n)$ is written as

$$\frac{1}{2}\sum_{k=0}^{n}m_k\sum_{i=1,j=1}^{n,n}a_{ki}a_{kj}\sum_{l=i-1,m=j-1}^{i,j}l_l l_m[(\beta_{l,q}\ddot{\beta}_m+\beta_{m,q}\ddot{\beta}_l)\cos\gamma_{lm}$$
$$-(\beta_{l,q}\dot{\beta}_m+\beta_{m,q}\dot{\beta}_l)\dot{\gamma}_{lm}\sin\gamma_{lm}+\gamma_{lm,q}\dot{\beta}l\dot{\beta}_m\sin\gamma_{lm}]$$
$$+\sum_{k=0}^{n}\beta_{k,q}I_{Zk}\ddot{\beta}_k+\frac{3\mu}{2R^3}\sum_{k=0}^{n}\beta_{k,q}(I_{Yk}-I_{Xk})\sin(2\phi_k)$$

$$-\frac{\mu}{2R^3}\sum_{k=0}^{n} m_k \sum_{i=1,j=1}^{n,n} a_{ki}a_{kj} \sum_{l=i-1,m=j-1}^{i,j} l_l l_m(\gamma_{m,q}\sin\gamma_{lm}$$
$$-3\beta_{l,q}\sin\phi_i\cos\phi_m - 3\beta_{m,q}\cos\phi_l\sin\phi_m) = 0 \quad (20)$$

where

$$\beta_{k,q} = \frac{\partial\beta_k}{\partial\alpha_q} \quad (21)$$

and

$$\gamma_{m,q} = \frac{\partial\gamma_{lm}}{\partial\alpha_q} \quad (22)$$

Equation (20) can be alternatively expressed as (the first and fourth terms are rewritten)

$$\frac{1}{2}\sum_{i=1,j=1}^{n,n}\eta_{ij}\sum_{l=i-1,m=j-1}^{i,j} l_l l_m[(\beta_{l,q}\ddot{\beta}_l)\cos\gamma_{lm}$$
$$-(\beta_{l,q}\dot{\beta}_m + \beta_{m,q}\dot{\beta}_l)\dot{\gamma}_{lm}\sin\gamma_{lm} + \gamma_{m,q}\dot{\beta}_l\dot{\beta}_m\sin\gamma_{lm}]$$
$$+\sum_{k=0}^{n}\beta_{k,q}I_{Zk}\ddot{\beta}_k + \frac{3\mu}{2R^3}\sum_{k=0}^{n}\beta_{k,q}(I_{Yk}-I_{Xk})\sin(2\phi_k)$$
$$-\frac{\mu}{2R^3}\sum_{i=1,j=1}^{n,n}\eta_{ij}\sum_{l=i-1,m=j-1}^{i,j} l_l l_m(\gamma_{m,q}\sin\gamma_{lm}$$
$$-3\beta_{l,q}\sin\phi_i\cos\phi_m - 3\beta_{m,q}\cos\phi_i\sin\phi_m) = 0, \quad (23)$$

where

$$\eta_{ij} = \sum_{k=1}^{n} m_k a_{ki}a_{kj} \quad (24)$$

Equation (24) is the equation of motion of the space-manipulator system for variable α_k. There are four terms in the equation. The first two terms come from the kinetic energy, and the other two terms are from the potential energy. Note that it includes variables β_k, γ_{lm} and ϕ_k, which are functions of α_k. Since the set of equations are nonlinear, one can apply numerical methods to solve the $(n+1)$ equations with initial conditions, and the time responses of the variables α_k $(k=0,1,2,\cdots,n)$ can be obtained. Besides, the set of equations of motion can be expressed in a matrix form as

$$[A]\{\ddot{\alpha}\} + \{R\} = 0, \quad (25)$$

where $\{\alpha\}$ is a column vector in terms of variables α_q $(q=0,1,2,\cdots,n)$, and $[A]$ is a matrix associated with the system parameters, and $\{R\}$ is a column vector in terms of the system parameters and the derivatives of variables α_q.

In order to express α_q as a function of θ, one defines

$$\alpha_q' = \frac{\mathrm{d}\alpha_q}{\mathrm{d}\theta}, \quad q = 0,1,2,\cdots,n, \quad (26)$$

Thus, one can obtain

$$\dot{\alpha}_q = \dot{\theta}\alpha_q' \quad (27)$$

and

$$\ddot{\alpha}_q = \dot{\theta}^2\alpha_q'' + \ddot{\theta}\alpha_q' \quad (28)$$

Then, Equation (25) can be rewritten as

$$\dot{\theta}[A]\{\alpha''\} + \ddot{\theta}[A]\{\alpha'\} + \{R\} = 0 \quad (29)$$

Equations (25) and (29) are two sets of general form of the attitude dynamics of the spacecraft-manipulator system, where the manipulator composes of n rigid body links.

2.4 Linearized Equations of Motion

One considers that variable α_0 has a small variation $\delta\alpha_0$ at a reference angle α_R, which represents a desired orientation of spacecraft, and variables α_i $(i = 0,1,2,\cdots,n)$ has small variation $\delta\alpha_i$. Thus, quantities β_k, γ_{lm} and ϕ_k can be respectively written as

$$\beta_k = \theta + \alpha_R + \delta\phi_k, \quad (30)$$

$$\gamma_{lm} = \delta\gamma_{lm}, \quad (31)$$

$$\phi_k = \alpha_R + \delta\phi_k, \quad (32)$$

where

$$\delta\gamma_{lm} = \sum_{i=\min(l,m)+1}^{\max(l,m)}\delta\alpha_i, \quad (33)$$

$$\delta\phi_k = \sum_{i=0}^{k}\delta\alpha_i, \quad (34)$$

Substituting Equations (30), (31) and (32) into (23) leads to

$$\frac{1}{2}\sum_{i=1,j=1}^{n,n}\eta_{ij}\sum_{l=i-1,m=j-1}^{i,j} l_l l_m[[\beta_{l,q}(\ddot{\theta}+\delta\ddot{\phi}_m)+\beta_{m,q}(\ddot{\theta}+\delta\ddot{\phi}_l)]$$
$$+\gamma_{m,q}\dot{\theta}^2\delta\gamma_{lm}] + \sum_{k=0}^{n}\beta_{k,q}I_{Zk}(\ddot{\theta}+\delta\ddot{\phi}_k)$$
$$+\frac{3\mu}{2R^3}\sum_{k=0}^{n}\beta_{k,q}(I_{Yk}-I_{Xk})[\sin(2\alpha_R)$$
$$+2\cos(2\alpha_R)\delta\phi_k] - \frac{\mu}{2R^3}\sum_{i=1,j=1}^{n,n}\eta_{i,j}\sum_{l=i-1,m=j-1}^{i,j} l_l l_m[\gamma_{m,q}\delta\gamma_{lm}$$
$$-3\beta_{l,q}[(\cos^2\alpha_R)\delta\phi_l - (\sin^2\alpha_R)\delta\phi_m]$$
$$-3\beta_{m,q}[(\cos^2\alpha_R)\delta\phi_m - (\sin^2\alpha_R)\delta\phi_m]] = 0 \quad (35)$$

Equation (35) is a linearized equation of motion for variable α_q $(q=0,1,2,\cdots,n)$. Similar to Equations (25) and (29), (35) can be further rewritten as

$$[M]\{\ddot{\alpha}\} + [K]\{\alpha\} = 0 \quad (36)$$

and

$$\dot{\theta}^2[M]\{\alpha''\} + \ddot{\theta}[M]\{\alpha'\} + [K]\{\alpha\} = 0 \quad (37)$$

where $[M]$ and $[K]$ are matrices associated with the system parameters.

Table 1: Parameters of the spacecraft-manipulator system

Parameter	Mass	Half Length	Moment of Inertia w.r.t.		
			x-axis	y-axis	z-axis
Symbol	m_i	l_i	I_{Xi}	I_{Yi}	I_{Zi}
Unit	kg	m	$\mathrm{kg}\cdot\mathrm{m}^2$	$\mathrm{kg}\cdot\mathrm{m}^2$	$\mathrm{kg}\cdot\mathrm{m}^2$
Spacecraft	1	0.25	1	2	2
Rigid Link	0.1	1	0.1	0.3	0.3

Note: For the subscript of the symbols, $i=0$ refers to the spacecraft, and $i>0$ refers to the ith link of the manipulator.

2.5 Equations of Motion with Control Torques

The attitude control of the spacecraft is achieved by applying control torques on each link. Therefore, Equations (25), (29), (36), and (37) can be respectively rewritten as

$$[A]\{\ddot{\alpha}\}+\{R\}=\{u\}, \tag{38}$$

$$\dot{\theta}^2[A]\{\alpha''\}+\ddot{\theta}[A]\{\alpha'\}+\{R\}=\{u\}, \tag{39}$$

$$[M]\{\ddot{\alpha}\}+[K]\{\alpha\}=\{u\}, \tag{40}$$

$$\dot{\theta}^2[M]\{\alpha''\}+\ddot{\theta}[M]\{\alpha'\}+[K]\{\alpha\}=\{u\} \tag{41}$$

where $\{u\}$ is a column vector in terms of control torques.

A simple controller is proposed, which is based on a desired performance for the pitch angle α_0 of the spacecraft. One requires that the pitch angle α_0 should satisfy the equation as

$$\ddot{\alpha}_0+2\zeta_0\omega_0\dot{\alpha}_0+\omega_0^2\alpha_0=\omega_0^2\alpha_R \tag{42}$$

where ζ_0 and ω_0 are the damping ratio and natural frequency, which can be assigned by designers.

The first equation in Equations (25), (29), (36), or (37) should be transformed to (42) in order to obtain the control torques $\{u\}$.

3 Numerical Simulations

The spacecraft moves on a circle orbit, and the altitude is 500 km. The parameters of the spacecraft and the manipulator are listed in Table 1. The desired damping ratio and natural frequency of the time response of the pitch angle α_0 are 0.7 and 1 rad/s, respectively. The numerical simulation is based on that an initial value of the pitch angle is 10 degrees, and one hopes that it reduces to zero by adding a control torque on each link of the manipulator.

3.1 Attitude Control by One-Link Manipulator

This subsection presents the attitude control of the spacecraft by using a one-link manipulator. Based on the formulation in Section 2, the nonlinear equations of motion are expressed as Equation (25), where the components in matrix $[A]$ and vector $\{R\}$ are given as

$$A_{11}=\eta_{11}(l_0^2+l_1^2+2l_0l_1\cos\alpha_1)+I_{Z0}+I_{Z1} \tag{43}$$

$$A_{12}=A_{21}=\eta_{11}(l_1^2+l_0l_1\cos\alpha_1)+I_{Z1} \tag{44}$$

$$A_{22}=\eta_1 1l_1^2+I_{Z1} \tag{45}$$

$$\begin{aligned}R_1=&A_{11}\ddot{\theta}-\eta_1 1l_0l_1(2\dot{\theta}+2\dot{\alpha}_0+\dot{\alpha}_1)\dot{\alpha}_1\sin\alpha_1\\&+\frac{3\mu}{R^3}\eta_{11}[l_0\cos\alpha_0+l_1\cos(\alpha_0+\alpha_1)]\\&\times[l_0\sin\alpha_0+l_1\sin(\alpha_0+\alpha_1)]\\&+\frac{3\mu}{2R^3}[(I_{Y0}-I_{X0})\sin 2\alpha_0\\&+(I_{Y1}-I_{X1})\sin 2\alpha_1]\end{aligned} \tag{46}$$

$$\begin{aligned}R_2=&A_{11}\ddot{\theta}-\eta_{11}l_0l_1(\dot{\theta}+\dot{\alpha}_0)\dot{\alpha}_1\sin\alpha_1\\&+\eta_{11}l_0l_1(\dot{\theta}+\dot{\alpha}_0)(\dot{\theta}+\dot{\alpha}_0+\dot{\alpha}_1)\sin\alpha_1\\&-\frac{mu}{R^3}\eta_{11}l_0l_1\sin\alpha_1+\frac{3\mu}{R^3}\eta_{11}l_1\sin(\alpha_0+\alpha_1)\\&\times[l_0\cos\alpha_0+l_1\cos(\alpha_0+\alpha_1)]\\&+\frac{3\mu}{2R^3}(I_{Y1}-I_{X1})\sin 2\alpha_1\end{aligned} \tag{47}$$

where

$$\eta_{11}=\frac{m_0m_1}{m_0+m_1} \tag{48}$$

Also, based on the linear equations of motions as Equation (36), the components in matrices $[M]$ and vector $[K]$ are given as

$$M_{11}=\eta_{11}(l_0^2+l_1^2+2l_0l_1)+I_{Z0}+I_{Z1} \tag{49}$$

$$M_{12}=M_{21}=\eta_{11}(l_1^2+l_0l_1)+I_{Z1} \tag{50}$$

$$M_{22}=\eta_1 1l_1^2+I_{Z1} \tag{51}$$

$$\begin{aligned}K_{11}=&\frac{3\mu}{R^3}\eta_1 1(l_0+l_1)^2\cos 2\alpha_R\\&+\frac{3mu}{2R^3}(I_{Y0}+I_{Y1}-I_{X0}-I_{X1})\cos 2\alpha_R\end{aligned} \tag{52}$$

$$K_{12}=K21=\frac{3\mu}{R^3}\eta_{11}l_1(l_0+l_1)\cos 2\alpha_R$$

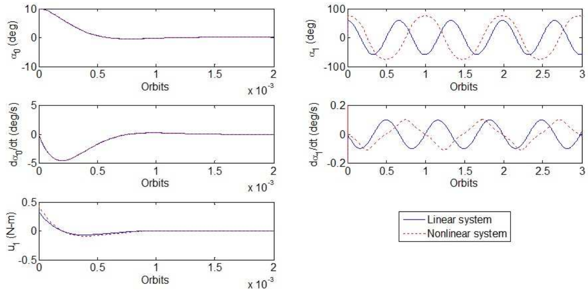

Fig. 2: Time responses of angles α_0, α_1 and control torque by a one-link manipulator

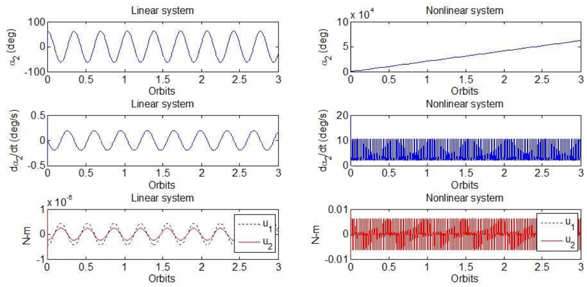

Fig. 3: Time responses of angle α_2 and control torques by a two-link manipulator based on $(\alpha_1)_{ss} = 0$

$$+\frac{3mu}{2R^3}(I_{Y1}-I_{X1})\cos 2\alpha_R \quad (53)$$

$$K_{22} = \eta_1 1 l_0 l_1(\dot{\theta}^2 - \frac{\mu}{R^3} + \frac{3\mu}{R^3}\eta_{11} l_1 (l_0 \cos^2 \alpha_R$$

$$+l_1 \cos 2\alpha_R + \frac{3\mu}{2R^3}(I_{Y1}-I_{X1})\cos 2\alpha_R \quad (54)$$

Based on Equation (42) and the nonlinear equations of motion shown in Equation (25), the control torque can be expressed as

$$u_1 = (2\zeta_0\omega_0\dot{\alpha}_0 + \omega_0^2\alpha_0 - \omega_0^2\alpha_R)\det([A])/A_{12}$$

$$+(R_2 - R_1 A_{21}/A12) \quad (55)$$

The controller shown in Equation (55) is a function of variables α_0, $\dot{\alpha}_0$, α_1, and $\dot{\alpha}_1$, and it performs a closed-loop state-feedback control system. A similar approach can be applied to linear systems.

Figure 2, which shows the rotating angles α_0 as well as α_1, their rotating rates, and the control torque. The results show that time response of the pitch angle α_0 meets the desired requirement, and it takes about 0.002 orbits to reach the steady state. Since there is no requirements about the motion of the angle α_1, its time response periodically vibrates at the steady state. Also, ones notice that there are large differences between the time responses of the angle α_1 of the linear and nonlinear models.

3.2 Attitude Control by Two-Link Manipulator

This subsection presents the attitude control by using a two-link manipulator. The equations of motion are complicated and not shown in this paper, but the equations can be obtained by directly applying the general form shown in Section 2. In this case, there are two control torques applied to each link. Based on the concept of the proposed controller design in Section 2.5, one needs to specify two controller design criteria. Same as the case presented in Section 3.1, the first criterion is the time response the pitch angle of spacecraft as Equation (42), which is expressed as $(\alpha_0)_{ss} = 0)$ and called the basic design criterion in this paper. Thus, the time response of the pitch angle is the same as that in Figure 2. For the second design criterion, it can be arbitrarily specified. Since there are numerous options for the second criterion, this subsection demonstrates two controllers based on the second design criterion.

The first controller requires zero rotating angle of the first link at the steady state $((\alpha_1)_{ss} = 0$. Thus, the rotating angle of the first link is zero at steady state, and the second link can rotate freely. Figure 3 shows the time responses of α_2, $\dot{\alpha}_2$ and the control torques. The results show that the α_2 response of the linear system vibrates with ± 62 degrees at steady state, but the α_2 response of the nonlinear system has a positive-direction and quasi-constant-speed rotation. Examining the response of the nonlinear system, it oscillates between 2.07 and 10.74 deg/s. For the control torques, the initial values u_1 and u_2 are 0.32 and 0.34 N-m respectively for the linear and the nonlinear systems, and they reach the steady state after around 0.002 orbits. Examining the steady state, u_1 and u_2 have small oscillations, which are within $\pm 4.31\times 10^{-6}$ and $\pm 2.38\times 10^{-6}$ N-m for the linear system, respectively. For the nonlinear system, they oscillate within $\pm 1.79\times 10^{-3}$ and $\pm 6.20\times 10^{-3}$ N-m, respectively.

The second controller requires zero rotating angle of the second link at the steady state $((\alpha_2)_{ss} = 0)$. Thus, the time response of the rotating angle of the second link is zero at steady state, but the first link can rotate freely, which implies that the entire manipulator performs the rotating angle α_1, and there is no relative motion between the two links at steady state. Figure 4 shows the time responses of α_1, $\dot{\alpha}_1$ and the control torques. The results show that the α_1 response vibrates within ± 22 degrees, but the vibrating frequency of the nonlinear systems is greater than that of the linear system. Examining the control torques, the initial values of u_1 and u_2 are 0.3267 and 0.1216 N-m respectively for both the linear and the nonlinear systems. At steady state, u_1 oscillates within $\pm 7.13\times 10^{-7}$ and $\pm 1.06\times 10^{-7}$ N-m respectively for the

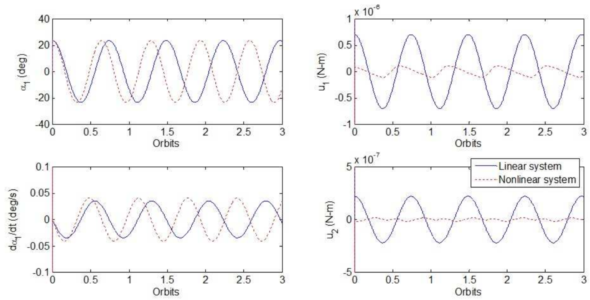

Fig. 4: Time responses of angles α_1 and control torques by a two-link manipulator based on $(\alpha_2)_{ss} = 0$

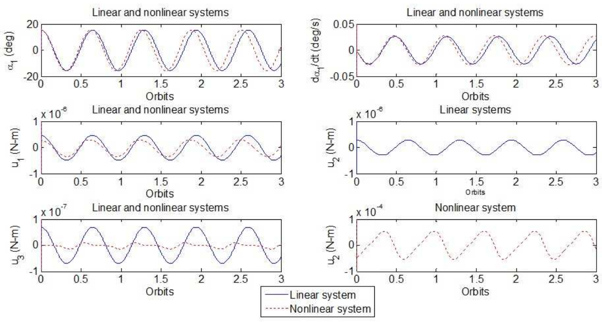

Fig. 5: Time responses of angle α_1 and control torques by a three-link manipulator based on $(\alpha_2)_{ss} = (\alpha_3)_{ss} = 0$

linear and the nonlinear systems, and u_2 are $\pm 2.21 \times 10^{-7}$ and $\pm 1.70 \times 10^{-7}$ for both systems.

3.3 Attitude Control by Three-Link Manipulator

Similar to the case presented in Section 3.2, this section presents the attitude control by using a three-link manipulator. The equations of motion are more complicated and not shown in this paper, but the equations can be obtained by directly applying the general form shown in Section 2. In this case, there are three control torques applied to each link. Thus, one can specify two additional controller design criteria besides for the basic one $(\alpha_0)_{ss} = 0$. Since there are numerous options for the two criteria, this subsection demonstrates three controllers.

The first controller requires zero rotating angles of the second and the third links at the steady state $((\alpha_2)_{ss} = (\alpha_3)_{ss} = 0)$. Thus, the first link can rotate freely. This implies that there are no relative motions between any two of the three links at steady state, and the vibration angle of the entire manipulator is α_1. Figure 5 shows the time responses of α_1, $\dot{\alpha}_1$, and the control torques for the linear and the nonlinear systems. The result shows that both of the vibration amplitudes of the angle α_1 for the two systems are very close, which are 15.54 and 15.63 degrees, respectively. However, their vibration frequencies are slightly different. Examining the three control torques, u_1 initiates at 0.3349 and 0.3413 N-m respectively for the linear and the nonlinear systems, u_2 are 0.1688 and 0.1723 N-m for both systems, and u_3 are 0.0673 and 0.0685 N-m. All of them take around 0.002 orbits to reach the steady state. At steady state, u_1, u_2 and u_3 of the linear system oscillate within $\pm 4.817 \times 10^{-7}$, $\pm 2.882 \times 10^{-7}$ and $\pm 6.982 \times 10^{-7}$, respectively, and they are $\pm 2.986 \times 10^{-7}$, $\pm 5.412 \times 10^{-5}$ and $\pm 8.735 \times 10^{-9}$ for the nonlinear system.

This controller requires zero rotating angles of the first link and zero sum of the rotating angle of the second and the third links at the steady state $((\alpha_2 + \alpha_3)_{ss} = 0)$, which implies that the directions of $\hat{i}_0$, $\hat{i}_1$ and $(\hat{i}_2 + \hat{i}_3)$ should be parallel to each other at the steady state. Figure 6 shows the time responses of α_2, $\dot{\alpha}_2$, and the control torques for the linear and the nonlinear systems. Since the angle α_3 is equal to the negative value of the angle α_2, the time response of α_3 is not shown in the figure. The results show that angle α_2 oscillates within ± 42.42 and ± 56.12 degrees for the linear and the nonlinear systems, respectively. Examining the control torques, u_1 initiates at 0.3392 and 0.3456 N-m respectively for the linear and the nonlinear systems, u_2 are 0.2866 and 0.2907 N-m for both systems, and u_3 are 1.967×10^{-4} and 6.220×10^{-4} N-m. All of them take around 0.002 orbits to reach the steady state. At steady state, u_1 oscillates within $\pm 6.027 \times 10^{-7}$ and $\pm 4.003 \times 10^{-7}$ N-m for both systems, u_2 oscillates within $\pm 1.841 \times 10^{-6}$ and $\pm 1.718 \times 10^{-6}$ N-m for both systems, and u_3 oscillates within $\pm 3.541 \times 10^{-7}$ and $\pm 4.672 \times 10^{-7}$ N-m for both systems.

This controller requires zero rotating angles of the third link and zero sum of the rotating angle of the first and the second links at the steady state $((\alpha_1 + \alpha_2)_{ss} = 0)$. This implies that the directions of $\hat{i}_0$ and $\hat{i}_1$ should be parallel to each other at the steady state, and there is no relative motion between the second and the third links at steady state. Figure 7 shows the time responses of α_1, $\dot{\alpha}_1$, and the control torques for the linear and the nonlinear systems. The results show that the time responses of angle α_1 are similar. Their vibration magnitudes at steady state are 37.16 and 38.83 degrees for the linear and the nonlinear systems, respectively. Examining the control torques, u_1 initiates at 0.3223 and 0.3290 N-m respectively for the linear and the nonlinear systems, u_2 is -0.0402 and -0.0381 N-m for both systems, and u_3 are -0.0320 and -0.0314 N-m for both systems. All of them take around 0.002 orbits to reach the steady state. At steady state, the time responses of u_1 for the linear and the nonlinear systems oscillate within $\pm 7.393 \times 10^{-7}$ and $\pm 2.024 \times 10^{-7}$, respectively. The control torques u_2 for the two systems are $\pm 1.019 \times 10^{-6}$ and $\pm 1.116 \times 10^{-6}$. The control torques u_3 for the two systems are $\pm 2.796 \times 10^{-7}$ and $\pm 3.408 \times 10^{-7}$.

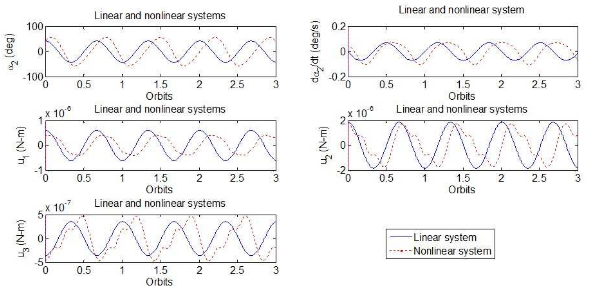

Fig. 6: Time responses of angle α_2 and control torques by a three-link manipulator based on $(\alpha_2+\alpha_3)_{ss}=0$

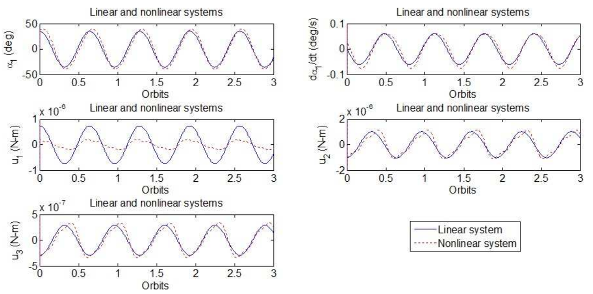

Fig. 7: Time responses of angle α_1 and control torques by a three-link manipulator based on $(\alpha_1+\alpha_2)_{ss}=0$

4 Conclusions

This paper presents the study of the attitude dynamics and control of spacecraft hinged by a serial manipulator under the effect of the earths gravitational field. In literature, most of them study the manipulator with one- or two links. This paper proposes a general form of equations of motion for the spacecraft-manipulator system, where the number of links can be arbitrarily assigned. The attitude motion of spacecraft is regulated by applying external torques to each link based on a simple controller design, which uses the combination of the attitude dynamics and the desired responses of systems. The attitude controls of the system individually with one-, two- and three-link manipulators are demonstrated to show the validity of the proposed approach. The results show that the rotating angle of the spacecraft attitude can reach the steady state within 0.002 orbits based on the systems dynamics and the desired system responses.

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References

[1] L. C. G. De Souza, Mechanics Based Design of Structures and Machines, **34**, 351-364 (2006).
[2] D. Prieto and B. Bona, Proceedings of the IEEE Networking, Sensing and Control, 728-733 (2005).
[3] M. Aliabbasi, H. A. Talebi and M. Karrari, Proceedings of the 41st IEEE Conference on Decision and Control, 4078-4083 (2002).
[4] P. Guan, X. J. Liu, F. Lara-Rosano, and B.J. Chen, Proceedings of the American Control Conference, **2**, 1091-1096 (2004).
[5] M. Belanger and J. De Lafontaine, Proceedings of the AAS/AIAA Astrodynamics Conference, 2701-2711 (2006).
[6] H. Bolandi, F. Bayat, and M. Nasirian, the first International Symposium on Systems and Control in Aerospace and Astronautics, 1413-1419 (2006).
[7] C. H. Won, Proceedings of the American Control Conference, 3538-3543 (2004).
[8] J. Rodden, L. McGovern, J. Higham and X. Price, Advances in the Astronautical Sciences, **111**, 35-44 (2002).
[9] S. N. Singh and W. Yim, IEEE Transactions on Aerospace and Electronic Systems, **41**, 770-779 (2005).
[10] Z. Vafa and S. Dubowsky, Proceedings of the IEEE International Conference on Robotics and Automation, 579-585 (1987).
[11] Z. Vafa and S. Dubowsky, International Journal of Robotics Researches, **9**, 3-21 (1990).
[12] E. Papadopoulos and S. Dubowsky, ASME Journal of Dynamic Systems, Measurement and Control, **115**, 44-52 (1993).
[13] E. Papadopoulos and S. Dubowsky, IEEE Transactions on Robotics and Automation, **7**, 750-758 (1991).
[14] Y. Umetani and K. Yoshida, IEEE Transactions on Robotics and Automation, **5**, 303-314 (1989).
[15] J. Franch, S. Agrawal and A. Fattah, Proceedings of 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems, 3053-3058 (2003).
[16] H. Shui, S. Peng, X. Li, and H. Ma, Proceedings of the 2009 IEEE International Conference on Robotics and Biomimetics, (2009).
[17] K. Nanos and E. Papadopoulos, Intelligent Service Robotics, **4**, 3-15 (2011).
[18] S. Ali, Robotica, **25**, 537-547 (2007).

Yong-Lin Kuo received his Ph.D. in mechanical engineering from University of Toronto in 2005. He received his M.S. from State University of New York at Buffalo in 1999. He received his B.S. and M.S. in aeronautics and astronautics from National Cheng Kung University, Taiwan, in 1992 and 1994, respectively. He is currently an Assistant Professor with the Graduate Institute of Automation and Control at National Taiwan University of Science and Technology, Taiwan. His research interests are system dynamics and control, computational solid mechanics, flexible multibody dynamic systems, etc.

Tsung-Liang Wu earned his B.S. and M.S. degrees in mechanical engineering at National Cheng Kung University, 1998, and National Taiwan University, 2000, in Taiwan, respectively. He focused on system dynamics field in mechanical engineering in his Ph.D. study and earned the degree at University of Washington, 2009, in USA. He is interesting in rotor dynamic study, vibration analysis, and robotic dynamic modeling and control. Currently, he serves as a Research and Development Manager at Industrial Technology Research Institute, Taiwan.