# Eigen Value and It's Comparison with Gaussian RBF, Multi-Quadratic RBF and Inverse Multi-Quadratic RBF Methods 

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#### Abstract

For any system Eigen value is the characteristic value and It is the characteristic roots for an equation which modeled that system. In this paper we have introduce Radial Basis Function (RBF) to evaluate Eigen value approximation for any given system. We have made some approximation to get best possible output with the help of pseudo inverse technique in Radial Basis Functions. We have experimented and compared the Eigen values of matrices with respect to Gaussian RBF, Multi-Quadratic RBF and Inverse Multi-Quadratic RBF methods. In this paper we worked on several matrices and calculated their Eigen value by using different RBF methods.


Keywords: RBF, Eigen value, Gaussian RBF, Multi-Quadratic RBF, Inverse Multi-Quadratic RBF

## 1 Introduction

Eigen vectors and Eigen values are the characteristic roots, characteristic values and numbers for a given system of equations when the system is operated by a set of matrix equation. Eigen values are modeled by scalar quantities which are related to a square matrix. Eigen vectors are the vectors which are related to the same matrix. Therefore, a system can be characterized by the Eigen values and eigen vector. The Eigen values are used in different fields of, science and technology such as engineering system design, pattern recognition, mechanics, automatic machine, statistics, computational intelligence, geometry, economics, astronomy, image processing, computer, robotics, etc. [4, 20, 25].
$T$ be a matrix whose dimension is $\mathrm{m} \times \mathrm{m}$. $\lambda$ is a number which is an Eigen value of matrix $T$ when a nonzero vector $v$ is exists such that.

$$
\begin{equation*}
T v=\lambda v \tag{1}
\end{equation*}
$$

Vector $v$ is Eigen vector of matrix $T$ while corresponding to $\lambda$. We can rewrite the above equation as,

$$
\begin{equation*}
(T-\lambda I) v=0 \tag{2}
\end{equation*}
$$



Fig. 1: RBF Neural Network.

Here $I$ is an identity matrix whose dimension is $\mathrm{m} \times \mathrm{m}$. For a nonzero vector $v,(T-\lambda I)$ cannot be invertible. So, the determinant of $(T-\lambda I)$ must be zero. Thus, $q(\lambda)=$ $\operatorname{det}(T-\lambda I)$ is the characteristic polynomial of $T$. So, the Eigen value of $T$ is the characteristic polynomial of $T[4]$.

[^0]The RBF neural network architecture is given here. It has three layers. First layer is input layer. The second layer is hidden layer which is different from the computational unit structure of the multilayer network. The computational units are known as radial centers which are being represented by $c_{1}, c_{2}, c_{3} \ldots . . c_{h}$ vectors. Dimension of each center for am input network is $m \times 1$. The centers have the same vector dimension as that of the input. The input space is the space where all possible data are located. The centers represent the clusters in the input space. The output of each center which is $\phi_{i}$ is the function of the Euclidian distance between $c_{i}$ and $x$. Output is obtain by proper choice of $w_{j}$, which is the weight of $j^{\text {th }}$ center. The output is simply the summation of $\phi_{i} w_{j}$. Transformation from input space to hidden unit space in nonlinear and transformation from hidden unit space to output space is linear. Here $\left\|x-c_{j}\right\|$ is the Euclidian distance $[1,2,7,8,9,12,13,21,22,25]$.

$$
\begin{gather*}
y=\sum_{h=1}^{n} \phi_{h} w_{h}  \tag{3}\\
\phi=\phi\left(\left\|x-c_{h}\right\|\right) \tag{4}
\end{gather*}
$$

The reminder of the paper is organized as follows. Section 2 reviews Radial Basis Function types and basics. Section 3 describes simulation, analysis,results and discussions. Section 4 gives conclusion.

## 2 Radial Basis Function

Radial Basis Function (RBF) gives non-zero response while input is in small localized area. Different Radial functions are available namely Gaussian RBF, Quadratic RBF and Inverse Quadratic RBF etc[14, 15, 16].

Function for Gaussian RBF is given below, where $z$ is the Euclidian distance, $\sigma$ is the maximum distance of center[3,24].

$$
\begin{equation*}
\phi(z)=e^{-z / 2 \sigma^{2}} \tag{5}
\end{equation*}
$$

Function for Quadratic RBF is as follows, where $r$ is any numerical number which is greater than zero and $z$ is the Euclidian distance[17,19].

$$
\begin{equation*}
\phi(z)=\left(z^{2}+r^{2}\right)^{1 / 2} \tag{6}
\end{equation*}
$$

Function for Inverse Quadratic RBF is as follows, where $r$ is any numerical number which is greater than zero and $z$ is the Euclidian distance[6].

$$
\begin{equation*}
\phi(z)=\left(z^{2}+r^{2}\right)^{-1 / 2} \tag{7}
\end{equation*}
$$

### 2.1 RBF network learning process

Training of Radial Basis Function Network (RBFN) requires optimal selection of the parameter vectors $c_{i}$ and $w_{i}, i=1 \ldots h$. Both layers are optimized using different techniques and in different time scales[5,10,11,18].

Following techniques are used to update the weights and centers of a RBFN.
1.Pseudo-Inverse Technique.
2.Gradient Descent Learning.
3.Hybrid Learning.

### 2.1.1 Pseudo-Inverse Technique

This is a least square problem. Assume a fixed radial basis functions e.g. Gaussian functions. The centers are chosen randomly. The function is normalized i.e. for any $x, \sum \phi_{i}=$ 1. The standard deviation (width) of the radial function is determined by an adhoc choice[5].

### 2.1.2 Gradient Descent Learning

One of the most popular approaches to update $c$ and $w$, is supervised training by error correcting term which is achieved by a gradient descent technique[18].

### 2.1.3 Hybrid Learning

In hybrid learning, the radial basis functions relocate their centers in a self-organized manner while the weights are updated using supervised learning. When a pattern is presented to RBFN, either a new center is grown if the pattern is sufficiently novel or the parameters in both layers are updated using gradient descent [11].

$$
\begin{gather*}
\mathbf{A 3}=\left[\begin{array}{llll}
2 & 1 & 3 \\
1 & 2 & 1 \\
3 & 1 & 2
\end{array}\right]  \tag{8}\\
\mathbf{A 4}=\left[\begin{array}{lllll}
2 & 2 & 3 & 3 \\
4 & 2 & 2 & 2 \\
4 & 2 & 2 & 3 \\
2 & 2 & 3 & 2
\end{array}\right]  \tag{9}\\
\mathbf{A 5}=\left[\begin{array}{lllll}
2 & 2 & 3 & 3 & 2 \\
4 & 2 & 2 & 2 & 3 \\
2 & 2 & 2 & 3 & 2 \\
4 & 2 & 3 & 2 & 3 \\
2 & 3 & 2 & 2 & 2
\end{array}\right]  \tag{10}\\
\mathbf{A 6}=\left[\begin{array}{llllll}
3 & 2 & 1 & 1 & 0 & 0 \\
2 & 3 & 2 & 3 & 2 & 2 \\
1 & 2 & 3 & 1 & 2 & 1 \\
1 & 3 & 1 & 3 & 1 & 2 \\
0 & 2 & 2 & 1 & 3 & 1 \\
0 & 2 & 1 & 2 & 1 & 3
\end{array}\right] \tag{11}
\end{gather*}
$$

$$
\begin{align*}
\mathbf{A} 7 & =\left[\begin{array}{ccccccc}
2 & -1 & -1 & 0 & -1 & 1 & 0 \\
-1 & 2 & 0 & 0 & 1 & -1 & 0 \\
-1 & 0 & 2 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 2 & 0 & -1 & -1 \\
-1 & 1 & 0 & 0 & 2 & -1 & -1 \\
1 & -1 & 0 & -1 & -1 & 2 & 0 \\
0 & 0 & -1 & -1 & -1 & 0 & 2
\end{array}\right]  \tag{12}\\
\mathbf{A 8} & =\left[\begin{array}{cccccccc}
2 & -1 & -1 & 0 & -1 & 1 & 0 & 0 \\
-1 & 2 & 0 & 0 & 1 & -1 & 0 & -1 \\
-1 & 0 & 2 & 1 & 0 & 0 & -1 & -1 \\
0 & 0 & 1 & 2 & 0 & -1 & -1 & -1 \\
-1 & 1 & 0 & 0 & 2 & -1 & -1 & 0 \\
1 & -1 & 0 & -1 & -1 & 2 & 0 & 0 \\
0 & 0 & -1 & -1 & -1 & 0 & 2 & 1 \\
0 & -1 & -1 & -1 & 0 & 0 & 1 & 2
\end{array}\right] \tag{13}
\end{align*}
$$

Table 1: Comparison of Eigen Values of Matrix A3 Using Normal Method and Different RBF Methods.

| Theoritical | Gaussian <br> RBF | Multi-Quadratic <br> RBF | Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: | :---: |
| -1.000000000000000 | -1.000000000000001 | -0.980392156862739 | -0.980392156862743 |
| 1.438447187191170 | 1.438447187191170 | 1.410242340383501 | 1.410242340383501 |
| 5.561552812808831 | 5.561552812808832 | 5.452502757655707 | 5.452502757655715 |

Table 2: Comparison of Error of Eigen Values of Matrix A3 for Different RBF Methods.

| Error in <br> Gaussian RBF | Error in <br> Multi-Quadratic RBF | Error in Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: |
| 0.000000000000001 | 0.019607843137261 | 0.019607843137257 |
| 0.000000000000000 | 0.028204846807669 | 0.028204846807669 |
| 0.000000000000001 | 0.109050055153124 | 0.109050055153116 |

Table 3: Comparison of Eigen Values of Matrix A4 Using Normal Method and Different RBF Methods.

| Theoritical | Gaussian <br> RBF | Multi-Quadratic <br> RBF | Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: | :---: |
| 10.227535796937078 | 10.227535796937078 | 10.026995879350071 | 10.026995879350075 |
| -1.790729366741630 | -1.790729366741630 | -1.755617026217285 | -1.755617026217275 |
| 0.00000000000002 | 0.00000000000001 | 0.000000000000011 | 0.000000000000000 |
| -0.436806430195437 | -0.436806430195437 | -0.428241598230823 | -0.428241598230817 |

Table 4: Comparison of Error of Eigen Values of Matrix A4 for Different RBF Methods

| Error in <br> Gaussian RBF | Error in <br> Multi-Quadratic RBF | Error in Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: |
| 0.000000000000000 | 0.200539917587006 | 0.200539917587003 |
| 0.000000000000000 | 0.035112340524345 | 0.035112340524355 |
| 0.000000000000001 | 0.000000000000010 | 0.000000000000002 |
| 0.000000000000000 | 0.008564831964614 | 0.008564831964620 |

Table 5: Comparison of Eigen Values of Matrix A5 Using Normal Method and Different RBF Methods.

| Theoritical | Gaussian <br> RBF | Multi-Quadratic <br> RBF | Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: | :---: |
| 12.183971475785187 | 12.183971475785189 | 11.945070074299124 | 11.945070074299194 |
| 0.369769129398551 | 0.369769129398549 | 0.362518754312240 | 0.362518754312308 |
| 0.000000000000001 | 0.000000000000000 | -0.000000000000056 | -0.000000000000003 |
| -1.553740605183741 | -1.553740605183741 | -1.523275103121381 | -1.523275103121312 |
| -1.000000000000000 | -1.000000000000004 | -0.980392156862742 | -0.980392156862749 |

Table 6: Comparison of Error of Eigen Values of Matrix A5 for Different RBF Methods

| Error in <br> Gaussian RBF | Error in <br> Multi-Quadratic RBF | Error in Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: |
| 0.000000000000002 | 0.238901401486062 | 0.238901401485993 |
| 0.000000000000002 | 0.007250375086311 | 0.007250375086243 |
| 0.000000000000001 | 0.000000000000057 | 0.000000000000004 |
| 0.000000000000001 | 0.030465502062360 | 0.030465502062429 |
| 0.000000000000004 | 0.019607843137258 | 0.019607843137251 |

## 3 Results and discussions

Six matrices are being experimented. The matrices are A3,A4,A5,A6,A7 and A8. Dimension of matrices A3, A4, A5, A6, A7,A8 are $3 \times 3,4 \times 4,5 \times 5,6 \times 6,7 \times 7$ and $8 \times 8$ respectively. Matrices are input. We have compared the Eigen values of these matrices with three different RBF methods namely Gaussian, Quadratic and Inverse Quadratic respectively. Fig 2 shows the comparison of errors of Eigen values for matrix A3 with respect to Gaussian, Quadratic and Inverse Quadratic methods. Here error is less in Gaussian RBF method. figure 3.shows the comparison of errors of Eigen values for matrix A4 with respect to Gaussian, Quadratic and Inverse Quadratic methods. Here error is less in Gaussian RBF method. figure 4 .shows comparison of errors of Eigen values for the matrix A5. Here, error in Quadratic and Inverse Quadratic RBF is quite close but Gaussian RBF shows better result. figure 5 .shows the comparison of errors of Eigen values for matrix A6 with respect to Gaussian, Quadratic and Inverse Quadratic methods. Here error is less in Gaussian RBF method. figure 6.shows comparison of Eigen values of matrix A7. Here, Gaussian RBF gives best result with respect to the other two methods. figure 7.shows comparison of Eigen values of matrix A8. Here, Gaussian RBF gives best result with respect to the other two methods. Table 1, Table 3, Table 5, Table 7, Table 9 and Table 11 show the Eigen values. While Table 2, Table 4 and Table 6, Table 8, Table 10 and Table 12 give demonstrate of the relative errors.

Table 7: Comparison of Eigen Values of Matrix A6 Using Normal Method and Different RBF Methods.

| Theoritical | Gaussian <br> RBF | Multi-Quadratic <br> RBF | Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: | :---: |
|  |  |  | -0.506331222882810 |
| -0.516457847340464 | -0.516457847340464 | -0.506331222882816 | -0.565178296246 |
| 0.563873934620447 | 0.563873934620443 | 0.552817582961224 |  |
| 1.100739385595450 | 1.100739385595446 | 1.079156260387692 | 1.079156260387693 |
| 3.023694707307639 | 3.023694707307634 | 2.964406575791775 | 2.964406575791801 |
| 3.346716161469828 | 3.346716161469827 | 3.281094275950823 | 3.281094275950810 |
| 10.481433658347100 | 10.481433658347100 | 10.275915351320661 | 10.275915351320688 |

Table 8: Comparison of Error of Eigen Values of Matrix A6 for Different RBF Methods.

| Error in <br> Gaussian RBF | Error in <br> Multi-Quadratic RBF | Error in Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: |
| 0.000000000000000 | 0.010126624457648 | 0.010126624457654 |
| 0.000000000000004 | 0.011056351659201 | 0.011056351659223 |
| 0.0000000000000004 | 0.021583125207757 | 0.021583125207756 |
| 0.0000000000000005 | 0.059288131515864 | 0.059288131515838 |
| 0.0000000000000001 | 0.065621885519005 | 0.065621885519018 |
| 0.0000000000000000 | 0.205518307026439 | 0.205518307026413 |

Table 9: Comparison of Eigen Values of Matrix A7 Using Normal Method and Different RBF Methods.

| Theoritical | Gaussian <br> RBF | Multi-Quadratic <br> RBF | Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: | :---: |
| 0.000000000000000 | 0.000000000000000 | -0.000000000000066 | 0.000000000000002 |
| 0.000000000000001 | 0.000000000000000 | -0.000000000000035 | -0.000000000000001 |
| 0.913869802348507 | 0.913869802348507 | 0.895950786616148 | 0.895950786616183 |
| 1.999999999999999 | 2.000000000000000 | 1.960784313725466 | 1.960784313725489 |
| 2.000000000000000 | 1.999999999999999 | 1.960784313725481 | 1.960784313725489 |
| 3.571993268316203 | 3.571993268316202 | 3.501954184623703 | 3.501954184623728 |
| 5.514136929335290 | 5.514136929335289 | 5.406016597387500 | 5.406016597387540 |

Table 10: Comparison of Error of Eigen Values of Matrix A7 for Different RBF Methods

| Error in <br> Gaussian RBF | Error in <br> Multi-Quadratic RBF | Error in Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: |
| 0.000000000000000 | 0.000000000000067 | 0.000000000000001 |
| 0.000000000000001 | 0.000000000000035 | 0.000000000000002 |
| 0.000000000000000 | 0.017919015732358 | 0.017919015732324 |
| 0.000000000000001 | 0.039215686274533 | 0.039215686274510 |
| 0.000000000000001 | 0.039215686274520 | 0.039215686274511 |
| 0.00000000000001 | 0.070039083692500 | 0.070039083692475 |
| 0.000000000000001 | 0.108120331947791 | 0.108120331947751 |



Fig. 2: Comparison of error of Eigen values of Matrix A3 for different RBF methods.


Fig. 3: Comparison of error of Eigen values of Matrix A4 for different RBF methods.


Fig. 4: Comparison of error of Eigen values of Matrix A5 for different RBF methods.

Table 11: Comparison of Eigen Values of Matrix A8 Using Normal Method and Different RBF Methods.

| Theoritical | Gaussian <br> RBF | Multi-Quadratic <br> RBF | Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: | :---: |
| 0.000000000000000 | 0.000000000000000 | 0.000000000000127 | 0.000000000000127 |
| 0.000000000000000 | -0.000000000000001 | 0.000000000000081 | 0.000000000000082 |
| 0.000000000000000 | 0.000000000000000 | 0.000000000000075 | 0.000000000000075 |
| 2.000000000000000 | 2.000000000000001 | 1.960784313725525 | 1.960784313725525 |
| 2.000000000000001 | 2.000000000000001 | 1.960784313725535 | 1.960784313725535 |
| 2.000000000000001 | 2.000000000000002 | 1.960784313725546 | 1.960784313725546 |
| 4.000000000000001 | 4.000000000000003 | 3.921568627451014 | 3.921568627451014 |
| 6.000000000000002 | 6.000000000000001 | 5.882352941176585 | 5.882352941176585 |

Table 12: Comparison of Error of Eigen Values of Matrix A8 for Different RBF Methods

| Error in <br> Gaussian RBF | Error in <br> Multi-Quadratic RBF | Error in Inverse <br> Multi-Quadratic RBF |
| :---: | :---: | :---: |
| 0.000000000000001 | 0.000000000000127 | 0.000000000000001 |
| 0.000000000000001 | 0.000000000000081 | 0.000000000000001 |
| 0.000000000000001 | 0.000000000000075 | 0.000000000000001 |
| 0.000000000000001 | 0.039215686274475 | 0.039215686274512 |
| 0.000000000000001 | 0.039215686274465 | 0.039215686274511 |
| 0.000000000000001 | 0.039215686274455 | 0.039215686274510 |
| 0.000000000000002 | 0.078431372548986 | 0.078431372549022 |
| 0.000000000000002 | 0.117647058823417 | 0.117647058823532 |



Fig. 5: Comparison of error of Eigen values of Matrix A6 for different RBF methods.

## 4 Conclusion

Gaussian RBF gives better result for the calculation of Eigen values of matrices. In every experiment Gaussian RBF shows better result corresponding to Quadratic and Inverse Quadratic RBF methods. The experiments are done using MATLAB 7.6.0 software tools. Choosing the proper weight in RBF neural network experimental results are satisfactory. Thus we can conclude that Gaussian RBF may be used for getting good results in the neural network.


Fig. 6: Comparison of error of Eigen values of Matrix A7 for different RBF methods.


Fig. 7: Comparison of error of Eigen values of Matrix A8 for different RBF methods.

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