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Performance Analysis of SR-ARQ Based on $Geom/G/1/\infty$ Queue over Wireless Link

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Abstract: Selective-Repeat ARQ (Automatic Repeat reQuest) protocol is the most efficient error control technology in wireless because it allows the transmitter to retransmit only negatively acknowledges (NACK) packets. In this paper, by analyzing the transmission mechanism of SR-ARQ, the discrete-time Geom/G/1/ queue model is established sententiously based on equivalent delay under the condition that the numbers of packets entering the transmitter are assumed to be independent and identically distributed random variables. With the method of embedded Markov chain, the expression formulations of the packet mean waiting delay, system mean delay and channel utilization are explicitly obtained. And then, the sliding window length control model is built adaptively according to the change of transmission conditions. Furthermore, the influences of packet length, the successful transmission probability and packet arrival rate on system mean delay are analyzed by numerical simulation. The numerical simulation results show that the expression formulations are valid for SR-ARQ protocol.

Keywords: SR-ARQ, $Geom/G/1/\infty$ Queue Model, System Mean Delay, Channel Utilization, Window Length Control Model

1 Introduction

In wireless data communication systems, error control techniques have been used extensively to enhance the reliability of data transmissions. There are two types of error control techniques: Forward Error Control (FEC) and Automatic Repeat reQuest (ARQ) protocols [1]. In FEC, redundancy is introduced in order to correctly decode a corrupted packet. In ARQ protocols, erroneous data are retransmitted. If an error is detected by the error-detecting code, a negative acknowledgement (NACK) will be sent back to the transmitter via the feedback channel. The transmitter, upon the receipt of the NACK, retransmits the packet until a positive acknowledgment is received. The ARQ protocols have been widely used because of their high reliability. Nowadays, as the demand for a wireless transmission is getting larger, they are even of greater importance than ever. Generally, there are three basic ARQ schemes: stop-and-wait (SW), go-back-N (GBN) and selective-repeat (SR).

In order to improve the performance of ARQ protocols, many important performance measures are analyzed by building its stochastic models. Literature [2] presents some new SW-ARQ protocols that significantly improve the throughput, while retaining the simple implementation of the classical SW-ARQ. In [3], by using the expression for the probability generating function (PGF) of the buffer content working under a SW-ARQ retrans-mission protocol, the queue length characteristics and the mean packet delay are derived. But SW-ARQ is inherently inefficient due to the idle time spent on waiting for an acknowledgment of each transmitted packet. In GBN-ARQ, the transmitter does not wait for an acknowledgment but sends packets to the receiver continuously and receives acknowledgements as well. Literature [4] gives an exact analysis of the transmission delay of a GBN-ARQ protocol which simultaneously transmits multiple frames using multiple parallel channels. Literature [5] researches the performance of an adaptive GBN-ARQ protocol in time-varying channel environments with random or correlated feedback errors. The throughput of the three-mode GBN (TM-GBN) (that includes standard GBN (SGBN), *n*-copy GBN (*n*GBN)

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and continuous GBN (CGBN)) are analyzed in [6] and [7].

As opposed to the other two protocols, in the SR-ARQ protocol the transmitter sends packets continuously and retransmits the NACK packet without re-sending the transmitted packets following it, so the SR-ARQ protocol is the most efficient one. Certainly, SR-ARQ is considered especially interesting. In [8], the performance of the basic SR-ARQ protocol has been analyzed, and the two different methods (by using an exact Markov state model and a substantially simpler stochastic model) for the evaluation of the queue length and the block delay are presented. Literature [9] studies the waiting time and queue length distributions for GBN-ARQ and SR-ARQ protocols, provides a performance analysis of ARQ protocols in connection-oriented transmission, and obtains the PGF of packet waiting time and the mean performance measures. The expressions to calculate the throughput for both ACK-NACK-based SR-ARQ and NACK-based SR-ARQ with multi-copy retransmission are derived in [10]. Another delay performance of the unit of delivery to a higher layer is analyzed in [11], which derives an exact closed-form SDU delay expression and its approximation in a much simpler form. The queuing delay at the transmitter and the resequencing delay at the receiver are presented in [12], which analyzes the transmitter queuing statistics, and obtains the joint distribution of the transmitter and the receiver buffer occupancies. The average packet transmission delay of SR-ARQ using the signal-flow graph is analyzed in [14]. However, the packet arrival process is not considered. Shumji Fujii and Yukuo Hayashida analyze the performance of SR-ARQ protocol with limited retransmissions in [15], and the distribution of the packet transmission delay and the required receiver buffer capacity are deduced. The performance of SR-ARQ is analyzed by using a Markov two-state channel with Bernoulli block arrivals [16]. Literature [17] describes the tail asymptotic of the probability mass function of the resequencing buffer content for SR-ARQ.

In this work, based on the mentioned works, we focus on the analysis of the link utilization, the system mean delay, the window length control model and the channel utilization of SR-ARQ protocol with startup time and stochastic vacation. For this purpose, the packet service delay is the time interval from the beginning of transmission to the receiving packet successfully at the receiver, which can be considered an equivalent service delay. The startup time can be regarded as a period of holiday time triggered by the arriving packets in idle period, which can be considered an equivalent holiday delay. We devise the discrete-time $Geom/G/1/\infty$ queue model sententiously based on equivalent service delay and equivalent holiday delay. The stochastic decomposition of the steady queuing length and steady waiting time are got based on the results of [13], then the PGFs of the additional queuing length and the additional latency are obtained respectively. The first moment and second moment of the equivalent service delay and equivalent holiday delay are also solved respectively. Then the precise formulation of the system mean delay, channel utilization and the exact analysis of delay for the SR-ARQ protocol are obtained too. Furthermore, the sliding window length control model is built adaptively according to the change of the system performance index. The system mean delay and channel utilization are examined by using numerical simulation, and the results show that the delay performance can be improved by changing the packet length, the successful transmission probability and the packet arrival rate.

The outline of the paper is as follows. In section 2, by analyzing the scheme of the SR-ARQ, we calculate the maximum mean link utilization. In section 3, we establish the $Geom/G/1/\infty$ queue model based on equivalent service delay and equivalent holiday delay. The system mean delay of SR-ARQ will be derived based on $Geom/G/1/\infty$ model. In section 4, the best sliding window length control model will be built for SR-ARQ, and the channel utilization will be researched too. In section 5, the influences of the successful transmission probability, packet length and packet arrival rate on system mean delay are comparative analyzed by numerical simulation. Moreover, the influences of packet length and the successful transmission probability on channel utilization and three different channel utilization of ARQs are explained by numerical simulation too. Finally, in section 6, conclusions and future work are given.

2 SR-ARQ scheme

In ARQ protocols, the data are sent in terms of packets, each of which is encoded for error detection by the receiver. The packets that arrive at the transmitter are assigned subsequent numbers that identify them uniquely and are referred to as identifies.

Under the SR-ARQ protocol, on which we focus in this paper, the transmitter continuously sends new packets and the receiver accepts every packet that arrives error-free. Upon receipt of a NACK by the transmitter, only the corresponding packet is retransmitted. Assuming a fixed packet length, let T_D be the transmission delay, T_A be the response information transmission delay, and T_P be the propagation delay, then $d = T_D + 2T_P + T_A$ denotes the time interval from the beginning of transmission to the receiving the feedback from the receiver.

Obviously, for the SR-ARQ transmission protocol, the required time of transmitting n packets continuously is nT_D. When $nT_D > d$, the response information can be fed back to the sender within nT_D. At this moment, if the packet is error free, the transmitter can send the packet continuously, and the maximum link utilization of SR-ARQ up to 1. If a NACK arrives at the transmitter, in which case the transmitter retransmits the negatively acknowledge packet without re-sending the transmitted packets following it. When $nT_D < d$, the response information can not be fed back to the transmitter within nT_D, and the maximum link utilization of transmission is nT_D/d.

We assume a packet which is retransmitted for k times, and let p be the successful transmission probability of a packet in channel. Because the transmitter only retransmits the NACK packets, the required mean number of packets that successfully transmitted a packet can be considered as

$$N_f = \sum_{k=0}^{\infty}(k+1)p(1-p)^k = \frac{1}{p}. \tag{1}$$

Thus, the maximum mean link utilization is

$$\mu = \begin{cases} 1/N_f, & nT_D \geq d \\ \dfrac{nT_D}{N_f d}, & nT_D < d \end{cases} = \begin{cases} p, & n \geq 1+\alpha \\ \dfrac{np}{1+\alpha}, & nT_D < 1+\alpha \end{cases} \tag{2}$$

Where $\alpha = (2T_P + T_A)/T_D$.

Apparently, the maximum mean link utilization relates to the relative transmission delay α, the successful transmission probability p and the length of the window n. Numerical analysis shows that the best window length is approximately equal to $\lceil 1+\alpha \rceil$ in the case of p given [1]. Where $\lceil x \rceil$ denotes the smallest positive integers greater than or equal to x.

3 Queuing model and performance analysis

3.1 Mathematical model

We consider a transmitter and a receiver, which communicate data packets of fixed lengths through a slotted channel, and where packet transmission times equal one slot. The transmitter and receivers are synchronized with the channel and packet transmissions start at slot beginnings. To investigate the delay performance of SR-ARQ protocol farther by modeling, our analysis is based on the following assumptions.

Packets are transmitted on FCFS (First Come First Service) basis, so that transmission of a message starts after all packets in the previous message is successfully transmitted; the packet service delay is generally distributed and not dependent on the arrival process; packets arriving at the transmitter are stored in a buffer of an infinite capacity; the feedback channel for acknowledge is assumed to be error free; packet arrival randomly occurs according to a Bernoulli process with rate λ, $0 < \lambda < 1$; μ denotes the packet service rate, $\rho = \lambda/\mu$. The number of arrival packets in the M period to obey binomial distribution.

$$P\{A(M) = i\} = \binom{M}{i}\lambda^i(1-\lambda)^{M-i} \tag{3}$$

for $i = 0, 1, \ldots, M$.

In this paper, the discrete-time $Geom/G/1/\infty$ queuing model with startup is presented. Once the system without packets, the service facilities are shut down until a new packet arrival. However, after a period of startup time V, the system begins service for the data packets. Called $Geom/G/1(ES, SU)$.

$$P\{V = k\} = V_k, k \geq 1. \tag{4}$$

Startup time can be regarded as a period of holiday time triggered by the arriving packets in idle period, which can be considered an equivalent holiday delay. While the packet service delay S which is the interval from the beginning of transmission to the time of the packet is successfully received at the receiver, which can be considered an equivalent service delay (that includes the first transmission delay T_1 and the retransmission delay T_r), which follows general distribution and can be described as a $Geom/G/1/\infty$ queuing model.

$$b_r = P\{S = k\}, k \geq 1. \tag{5}$$

When $\rho < 1$, L_n is the remaining number of packets after the nth packet leaves, and $\{L_n, n \geq 1\}$ is the embedded Markov chain of the queue length process.

$$L_n = \begin{cases} L_n - 1 + C, & L_n \geq 1 \\ C, & L_n = 0 \end{cases} \tag{6}$$

Where C is the number of packets entering the system in a service interval, we get the probability distribution as

$$\begin{aligned} k_j &= P\{C = j\} \\ &= \sum_{r=j}^{\infty} P\{S = r\} \\ &\cdot P\{j \text{ packets entering the system during slot } r\} \\ &= \sum_{r=j}^{\infty} b_r \binom{r}{j} \lambda^j (1-\lambda)^{r-j}, j \geq 0 \end{aligned} \tag{7}$$

The PGF of C as

$$\begin{aligned} C(z) &= \sum_{j=0}^{\infty} z^j k_j \\ &= \sum_{j=0}^{\infty} z^j \sum_{r=j}^{\infty} b_r \binom{r}{j} \lambda^j (1-\lambda)^{r-j} \\ &= \sum_{r=1}^{\infty} b_r \sum_{j=0}^{\infty} \binom{r}{j} (z\lambda)^j (1-\lambda)^{r-j} \\ &= \sum_{r=1}^{\infty} b_r (1-\lambda+\lambda z)^r. \end{aligned} \tag{8}$$

When $\rho < 1$, $Geom/G/1(ES, SU)$ have stochastic decomposition [13]: the steady queuing length and steady waiting time are respectively as

$$L^+ = L_0^+ + L_d, \quad W = W_0 + W_d. \tag{9}$$

Where L_0^+ is the steady queuing length of classical $Geom/G/1$, L_d is additional queuing length; W_0 denotes the waiting time of classical $Geom/G/1$, W_d denotes additional latency. The PGF of L_d and W_d are respectively as

$$L_d(z) = \frac{1-zV[\gamma(z)]}{[1+\lambda E[V]](1-z)}, \tag{10}$$

$$W_d(s) = \frac{\lambda-(\lambda-(1-s))V(s)}{[1+\lambda E(V)](1-s)}. \tag{11}$$

Eq. (10) and (11) can be rewritten as

$$L_d(z) = \frac{1}{1+\lambda E(V)} + \frac{z\lambda E(V)}{1+\lambda E(V)}\frac{1-V[\gamma(z)]}{\lambda E(V)(1-z)}, \tag{12}$$

$$W_d(s) = \frac{V(s)}{1+\lambda E(V)} + \frac{\lambda E(V)}{1+\lambda E(V)}\frac{1-V(s)}{E(V)(1-s)}. \tag{13}$$

It shows that W_d consists of complete startup time with probability $p^* = (1+\lambda E(V))^{-1}$ and surplus startup time with probability $1-p^*$. L_d consists of one packet within the complete startup time with p^* and the number of packets within surplus startup time with probability$1-p^*$.

The mean value expressions are easily obtained by the stochastic decomposition as follows

$$E(L_d) = \frac{2\lambda E(V)+\lambda^2 E(V(V-1))}{2[1+\lambda E(V)]}, \tag{14}$$

$$E(W_d) = \frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]}, \tag{15}$$

$$E(L^+) = \rho + \frac{\lambda^2 E(S(S-1))}{2(1-\rho)} + \frac{2\lambda E(V)+\lambda^2 E(V(V-1))}{2[1+\lambda E(V)]}, \tag{16}$$

$$E(W) = \frac{\lambda E(S(S-1))}{2(1-\rho)} + \frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]}. \tag{17}$$

Therefore, the mean delay of the system is as follows

$$\begin{aligned} E(T) =& E(W)+E(S) \\ =& \frac{\lambda E(S(S-1))}{2(1-\rho)} \\ &+ \frac{2E(V)+\lambda E(V(V-1))}{2[1+\lambda E(V)]} + E(S). \end{aligned} \tag{18}$$

3.2 Performance analysis

For SR-ARQ protocol, the probability that a retransmission packet is located in the ith sliding window is $1/N$. So the mean service delay in the first transmission at the receiver can be calculated as

$$T_1 = \frac{1}{N} l[N+(N-1)+\cdots+2+1] = \frac{l(N+1)}{2}. \tag{19}$$

Where N denotes the length of sliding windows, and l denotes the packet length.

Under receipt of a NACK by the transmitter of the SR-ARQ, only the corresponding packet is retransmitted. Therefore, when a packet is retransmitted for k times, the equivalent retransmission delay T_r is

$$T_r = lk + (k+1)(T_D + 2T_P + T_A). \tag{20}$$

Hence, the equivalent service delay that a packet is successfully transmitted is $S_k = T_1 + T_r$ with probability $p(1-p)^k$ (one times successful transmission, k times retransmission). That is, the probability distribution of stochastic variable S_k is

$$\begin{aligned} P\{S_k = & \frac{l(N+1)}{2} + lk + (k+1)(T_D+2T_P+T_A)\} \\ &= p(1-p)^k. \end{aligned} \tag{21}$$

Assume that the retransmission times of the arrival packet in idle period is $j-1$, then the equivalent holiday delay is $V_k = lj$ with probability of $p(1-p)^{j-1}$. That is, the probability distribution of stochastic variable V_k is

$$P\{V_k = lj\} = p(1-p)^{j-1}. \tag{22}$$

The first moment of S_k and V_k are respectively derived as

$$\begin{aligned} E(S) =& \sum_{k=0}^{\infty}\Big[\frac{l(N+1)}{2} + lk \\ &+ (k+1)(T_D+2T_P+T_A)\Big]p(1-p)^{j-1} \\ =& \frac{l(N+1)}{2} + \frac{l(1-p)}{p} + \frac{T_D+2T_P+T_A}{p}, \end{aligned} \tag{23}$$

$$E(V) = \sum_{j=1}^{\infty} ljp(1-p)^{j-1} = \frac{l}{p}. \tag{24}$$

Also, we obtain

$$\begin{aligned} & E(S(S-1)) \\ &= \sum_{k=0}^{\infty}\Big\{\Big[\frac{l(N+1)}{2} + lk + (k+1)(T_D+2T_P+T_A)\Big]^2 \\ &- \Big[\frac{l(N+1)}{2} + lk + (k+1)(T_D+2T_P+T_A) - 1\Big]\Big\}p(1-p)^k \\ &= \sum_{k=0}^{\infty}\Big\{\Big[\frac{l(N+1)}{2} + (l+T_D+2T_P+T_A)k + T_D+2T_P+T_A\Big]^2 \\ &- \Big[\frac{l(N+1)}{2} + lk + (k+1)(T_D+2T_P+T_A) - 1\Big]\Big\}p(1-p)^k \\ &= \Big[\frac{l(N+1)}{2} + T_D+2T_P+T_A\Big]^2 \\ &+ \frac{(l+T_D+2T_P+T_A)^2(1-p)(2-p)}{p^2} \\ &- \frac{lp(N+1)+2l(1-p)+2(T_D+2T_P+T_A)}{2p} \end{aligned}$$

$$+\frac{[l(N+1)+2(T_D+2T_P+T_A)]}{p} \cdot \frac{(l+T_D+2T_P+T_A)(1-p)}{p}, \tag{25}$$

$$E(V(V-1)) = \sum_{j=1}^{\infty} l^2 j^2 p(1-p)^{j-1} - \sum_{j=1}^{\infty} l j p(1-p)^{j-1} = \frac{l^2(2-p)}{p^2} - \frac{1}{p}. \tag{26}$$

Let $\rho = \lambda E(S)$, when $\rho < 1$, the system reaches a steady state. Then, substituting formula (23)-(3.2) into (18), the mean delay of the system is given by

$$E(T) = \frac{\lambda[K_1 p^2 + K_2 p(1-p) + K_3(1-p)(2-p)]}{2p^2 - \lambda p[lp(N+1)+2l(1-p)+2d]} + \frac{lp(N+1)+2l(1-p)+2d}{2p} + \frac{2lp+\lambda[l^2(2-p)-lp]}{2(p^2+\lambda lp)}. \tag{27}$$

Where
$d = T_D + 2T_P + T_A$,
$K_1 = \left[\frac{l(N+1)}{2} + T_D + 2T_P + T_A\right]^2$,
$K_2 = [l(N+1) + 2(T_D + 2T_P + T_A)](l + T_D + 2T_P + T_A)$,
$K_3 = (l + T_D + 2T_P + T_A)^2$.

The normalized maximum throughput is

$$\gamma = \lambda_{\max}(T_1 + T_D + 2T_P + T_A) = \frac{l(N+1)+2(T_D+2T_P+T_A)}{2E(T)} \tag{28}$$

Where $E(T)$ is a function of N, l, λ, p, d and ρ. $\lambda_{\max} = 1/E[T]$ denotes the maximum throughput under steady state. So we have

$$E(T) = f(N, l, \lambda, p, d, \rho) \tag{29}$$

4 The best sliding window length and channel utilization

The best sliding window length is very important to design SR-ARQ system. By (29), the sliding window length N should be the function as follows

$$N = g(l, \lambda, p, d, \rho). \tag{30}$$

When $\rho < 1$, we can deduce that the best sliding window length is

$$N_{best} = \min\left|\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right|. \tag{31}$$

Where
$a = \lambda l^2 p^2$,
$b = -2\lambda lp\left\{l(2-3p) - d(2+p) + 4(\lambda-1) + \frac{2p}{\lambda_{\max}} + \frac{2p}{\lambda} - \frac{2lp+\lambda[l^2(2-p)-lp]}{p+\lambda l}\right\}$,
$c = 4\lambda(l+d)(1-p)[2l - 2dp - (l+d)(2-p)] + \lambda p^2(l^2-4d^2) - 8lp(1-p) + 4d(2\lambda d + 2\lambda l - p) - 2\lambda p\left\{l(2-3p) - d(2+p) - 4(1-\lambda) + \frac{2p}{\lambda_{\max}} + \frac{2p}{\lambda} - \frac{2lp+\lambda[l^2(2-p)-lp]}{p+\lambda l}\right\} + 4[p - \lambda(1-p) - \lambda d]\left\{\frac{2p}{\lambda_{\max}} - \frac{2lp+\lambda[l^2(2-p)-lp]}{p+\lambda l}\right\}$.

Then, the best sliding window length control model is built adaptively according to the change of $E(T)$, l, λ, p, d and ρ. We can design the best sliding window for SR-ARQ system in terms of (31).

At the discrete-time $Geom/G/1/\infty$ queuing model with startup, the busy period of model can be obtained as follows by [13]

$$B_v(z) = Q_b[B(z)]. \tag{32}$$

Where Q_b denotes the number of packets in the system at the busy period begins, and $B(z)$ denotes the PGF of the busy period in the classical system. Hence, the mean value of busy period is given by

$$E[B_v] = \frac{[1+\lambda E(V)]E(S)}{1-\rho}. \tag{33}$$

Thereby, the channel utilization is given by

$$\eta = \frac{E(B_v)}{E(V)+E(B_v)} = \frac{[1+\lambda E(V)]E(S)}{E(V)+E(S)} = \left\{\left(1+\frac{\lambda l}{p}\right)\left[\frac{l(N+1)}{2} + \frac{l(1-p)}{p} + \frac{T_D+2T_P+T_A}{p}\right]\right\} \div \left[\frac{l(N+1)}{2} + \frac{l(2-p)}{p}\right] + \frac{T_D+2T_P+T_A}{p} = \frac{(p+\lambda lp)[L+2l(1-p)+2d]}{p[L+2l(2-p)+2d]} \tag{34}$$

Where $d = T_D + 2T_P + T_A$, $L = lp(N+1)$.

5 Numerical results and analysis

This section validate our theoretical analysis through numerical simulation. Here we analyze the results about the system mean delay and channel utilization of SR-ARQ protocol based on Eq.(27)-(34) respectively. Note that there are five parameters which affect the system mean delay and channel utilization. Viz., the successful transmis-sion probability p, the packet length l, the arrival rate λ, the length of sliding windows N, and the time interval from the beginning of transmission to the receiving the feedback from the receiver d. In all plots in this section, we choose $N = 3$ and $d = 1.2l$.

As first results, in Fig.1 and Fig.2, we report the system mean delay statistics relying on the successful transmission probability p. In both graphs, the statistics obtained by simulation are compared against the two performance parameters, i.e., the packet length l ($l = 1$, $l = 2$, $l = 3$) and the arrival rate λ ($\lambda = 0.001$, $\lambda = 0.005$, $\lambda = 0.01$). Fig.1 shows when the system is in stable state and the same arrival rate ($\lambda = 0.01$), the system mean delay decreases as the packet length decreases. Similarly, the system mean delay decreases as the arrival rate decreases at the same packet length ($l = 2$) is shown in Fig.2. But when the successful transmission probability is larger ($p > 0.6$), the arrival rate bring on less effect on the system mean delay.

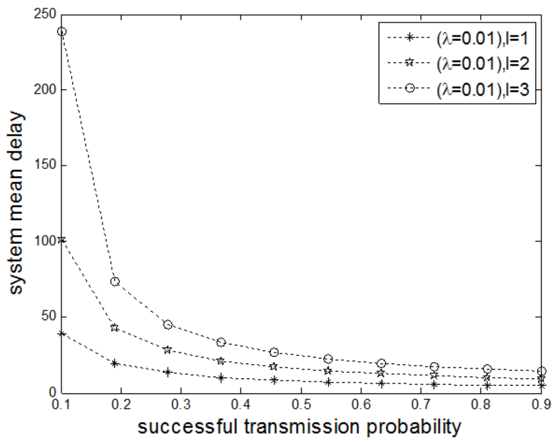

Fig. 1: System mean delay versus successful transmission probability and packet length.

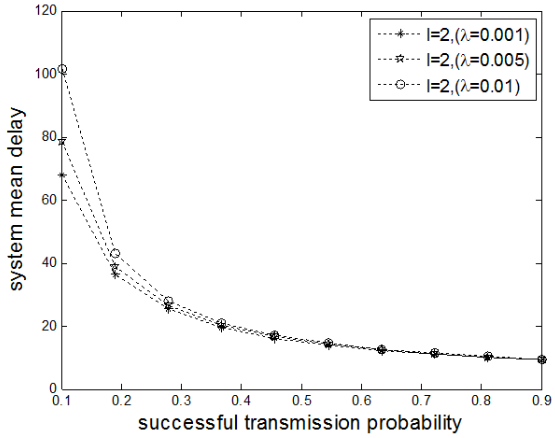

Fig. 2: System mean delay versus successful transmission probability and arrival rate.

In the following Fig.3 and Fig.4, we depict the system mean delay statistics relying on the packet length l. In both graphs, the statistics obtained by simulation are compared against the successful transmission probability p ($p = 0.5$, $p = 0.6$, $p = 0.7$) and the arrival rate λ ($\lambda = 0.001$, $\lambda = 0.005$, $\lambda = 0.01$). Fig.3 shows when the system is in stable state and the same arrival rate ($\lambda = 0.01$), the system mean delay decreases as the successful transmission probability increases. Similarly, the system mean delay decreases as the arrival rate decreases at the same successful transmission probability ($p = 0.5$) is shown in Fig.4. But when the packet length is smaller ($l < 3$), the arrival rate bring on less effect on the system mean delay.

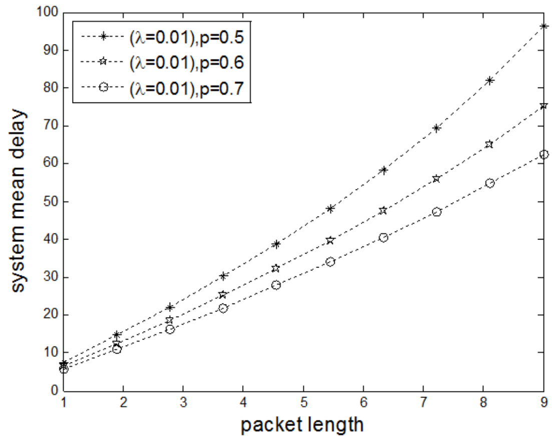

Fig. 3: System mean delay versus packet length and successful transmission probability.

Last, in Fig.5 and Fig.6, we describe the channel utilization by numerical simulation. Fig.5 shows when the system is in the same arrival rate $\lambda = 0.01$, the channel utilization increases as the successful transmission probability and packet length increases. Fig.6 shows when the system is in the same packet length $l = 2$ and the same arrival rate $\lambda = 0.005$, the channel utilization of ARQs versus successful transmission probability. When $p < 0.5$, the GBN-ARQ protocol has the biggest channel

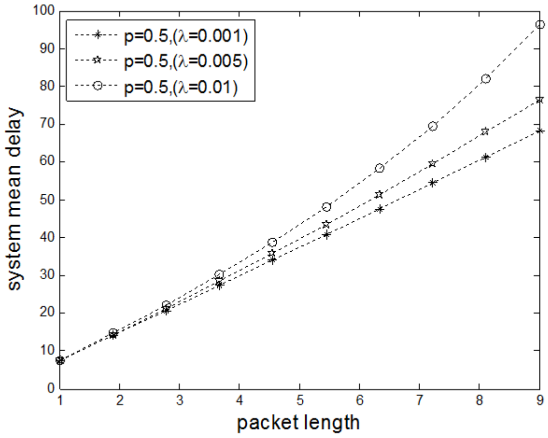

Fig. 4: System mean delay versus packet length and arrival rate.

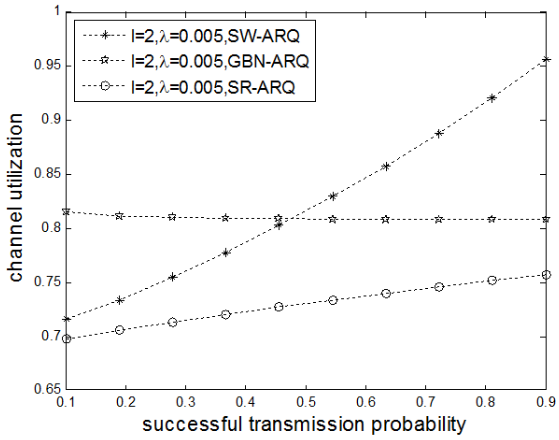

Fig. 6: Channel utilization of ARQs versus successful transmission probability.

utilization. While, the SR-ARQ protocol has the biggest channel utilization when $p > 0.5$.

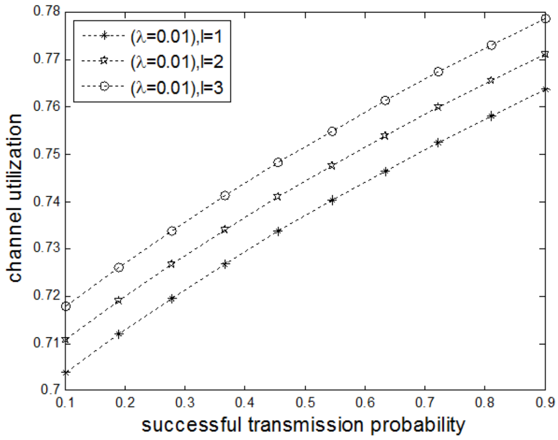

Fig. 5: Channel utilization versus successful transmission probability and packet length.

6 Conclusion

In this paper, we analyze the performance of SR-ARQ protocol. The required mean number of packets that successfully transmitted a packet and the maximum average link utilization of SR-ARQ protocol are derived. Furthermore, the discrete-time $Geom/G/1/\infty$ queue model is established sententiously based on equivalent service delay and equivalent holiday delay, from which the expression formulations of the packet mean waiting delay, system mean delay, the sliding window length control model and channel utilization are explicitly obtained. Last, the influences of packet length, the successful transmission probability and the packet arrival rate on system mean delay, and three different channel utilization of ARQs protocols are comparative analyzed by numerical simulation.

The presented method of calculation could be applied on delay analysis of all ARQ protocol. The system mean delay is influenced by the times of retransmission, which is the subject that remains for future work. Also, the distribution of the packet delay and more general system models will be investigated in the following studies.

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