

A Study of Critical Transmission Range for Connectivity in Ad Hoc Network

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Abstract: In this paper, the critical transmitting range for connectivity in wireless ad hoc networks is analyzed. More specifically, we consider the following problem: Assume n nodes, each capable of communicating with nodes within a radius of r . The nodes are randomly and uniformly distributed in a d -dimensional region with a side of length l . In this paper the critical transmission range in dense and sparse network is studied. The critical transmission range in RWP mobile network and the critical transmission range for k -connectivity are investigated. The results of this paper could be improved the accuracy of ad hoc network which is commonly used to evaluate the performance of ad hoc network protocols.

Keywords: Mobile Ad-Hoc Networks, CTR, dense network, sparse network, RWP, k -connectivity.

1 Introduction

A wireless ad hoc network is a collection of radio devices (transceivers) located in geographical region. Each node is equipped with an omni directional antenna and has limited transmission power. A communication session is established either through a single hop radio transmission if the communication parties are close enough, or through relaying by intermediate devices otherwise. Because of the no need for a fixed infrastructure, wireless ad hoc networks can be flexibly deployed at low cost for varying mission such as decision making in the battlefield, emergency disaster relief and environmental monitoring. In most applications, the ad hoc wireless devices deployed couples with the potential harsh environment often hinder or completely eliminated the possibility of strategic device placement, and consequently, random deployment is often the only viable option. In some other applications, the ad hoc wireless devices may be continuously in motion or be dynamically switched to on or off. For all the applications, it is natural to represent the ad hoc devices by a finite random point process over the (finite) deployment region. Correspondingly, the wireless ad hoc network is represented by a random [1].

In this paper, we make a step forward towards the accurate of ad hoc networks. We consider the critical transmission range (CTR) for connectivity, and we study how this network changes in the presence of node mobility. The critical transmission range (CTR) corresponds to the minimum common values of the nodes transmitting range that produces a connected graph. It is known that setting the nodes transmitting range to the critical value minimizes energy consumption while maximizing network capacity [2, 3].

Due to the possible occurrence of the border effect, the CTR in presence of mobility is in general different from the critical transmitting range in the stationary case with uniformly distributed nodes. This observation discloses another potential source of inaccuracy in the simulation of mobile networks. Suppose we want to evaluate the performance of a routing protocol for mobile network. The main effects of mobility on a routing protocol are (i) frequent route reconfigurations and (ii) occasional network disconnections. In order to fully understand the behavior of the protocol, the relative effects of (i) and (ii) on the routing performance should be carefully evaluated. It is clear that the frequency of network disconnections depends on the choice of the nodes transmitting range the larger the range, the less

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likely it is that the network becomes disconnected. on the other hand, for the reason described above (energy consumption and network capacity) the nodes transmitting range cannot be excessively large. Thus setting the transmitting range to the critical value for connectivity is a reasonable choice. For instance if the CTR in presence of mobility is larger than in case of stationary networks and the CTR is wrongly set as if the network were stationary, then there is a relatively high likelihood of generating a disconnected topology as the nodes move. In turn this causes a relatively low packet delivery rate, which could erroneously be interpreted as a scarce protocol's ability of performing route maintenance.

In this paper we have assumed that all the network nodes have the same transmission range r and the problem is to identify the minimum value of r (critical range). The most studied version of CTR problem in ad hoc network is the characterization of the CTR for connectivity. In section 2 we have been proved that the CTR for connectivity equals the length of the longest edge of the Euclidean Minimum Spanning Tree (EMST). In section 3 we have been studied the theory of geometric random graph (GRG) that has been often used in the derivation of analytical characterization of the CTR. Also in section 4 and section 5 we have presented several characteristics of the CTR for connectivity in the case of dense and sparse network respectively. We conclude the dense and sparse ad hoc network displays the same behavior. In section 6 we characterize the CTR for connectivity in case of Random WayPoint (RWP) mobility. In section 7 we consider characterizations of the critical value of the range for other important network properties, such as k -connectivity. In section 8 the CTR for connectivity with Bernoulli nodes is presented. In section 9 the experimental results are introduced. Our conclusion is presented in section 10.

2 Euclidean Minimum Spanning Tree

The following theorem shows that the CTR for connectivity equals the length of the longest edge of the Euclidean Minimum Spanning Tree (EMST) built on the network nodes.

Definition 2.1. (Euclidean MST) Given a set N of nodes placed in the d -dimensional space (with $1, 2, 3$) and a set of edges E between these nodes, an Euclidean MST (EMST) is a MST of the edge weighted graph $G=(N,E)$, where each edge has weight equal to the Euclidean distance between its endpoints.

Theorem 2.1 [4]. Let N be a set of n nodes placed in $R = [0, l]^d$, with $d = 1, 2, 3$. The CTR for connectivity r_c of the network composed of nodes in N equals the length of the longest edge of the EMST T built on the same set of nodes.

Proof: Let e denote the longest edge in T , and let $l(e)$ denote its length. We first show that r_c cannot be larger than $l(e)$. This follows by observing that the

$l(e)$ -homogeneous range assignment produces a graph that contains T as a subgraph and that T is connected by definition of CTR, we must have $r_c = l(e)$. Let us now prove that it cannot be that $r_c < l(e)$. Consider the sets of nodes corresponding to the two connected components T_1 and T_2 obtained from T by removing edge e (see Figure 1). By definition of EMST, edge e is the shortest edge connecting any pair (u, v) of nodes such that $u \in T_1$ and $v \in T_2$. Thus, any node in T_1 is at distance at least $l(e)$ from any node in T_2 . This implies that setting the transmitting range to a value smaller than $l(e)$ would leave the communication graph disconnected, and the theorem is proved.

According to theorem 2.1, computing the CTR is equivalent to computing the EMST on the network nodes, and finding the longest edge in the EMST. Unfortunately, this way of calculating the CTR is not apt to distributed implementation, since building the EMST requires global knowledge (the exact positions of all the nodes in the network), which can be acquired in a distributed setting only by exchanging a considerable amount of messages.

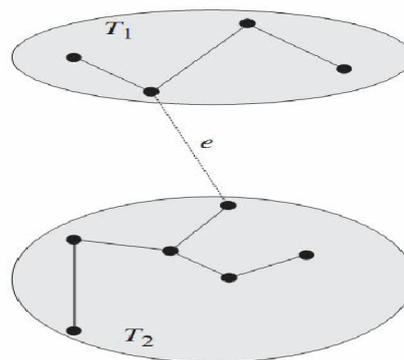


Figure 1: Connected components resulting from removing the longest edge e from the EMST.

For the reasons described above, considerable attention has been devoted to characterizing the CTR in the presence of some form of uncertainty about node positions. If nodes' positions are not known, the minimum value of r ensuring connectivity in all possible cases is $r = l\sqrt{d}$ since nodes could be concentrated at the opposite corners of R . However, this scenario is overly pessimistic in many real-life situations. For this reason, a typical approach is to assume that nodes are distributed in R according to some probability density function f , and to study the conditions for asymptotically almost sure connectivity.

3 Geometric Random Graphs

The theory of geometric random graph (GRG) has been often used in the derivation of analytical characterization of the CTR. In the theory of GRG, a set of points is

distributed according to some probability density function (pdf) in a d -dimension region and some property of the resulting node placement is investigated. For example, the longest nearest neighbor link, the longest edge of the Euclidean Minimum Spanning Tree (MST), and the total cost of the MST have been investigated. For a survey of GRG, the reader is referred to [5].

Some of these results can be applied in the study of connectivity in ad hoc networks. For instance, consider a set S of points distributed in the deployment region R . It is known that the minimum common value of the transmitting range such that the resulting communication graph is connected equals the length of the longest edge of the Euclidean Minimum Spanning Tree built on S [6]. Hence, results concerning the asymptotic distribution of the longest MST edge [7, 6] can be used to characterize the critical transmission range, as it has been done by the authors [8]. Another notable result of the theory of GRG is that, under the assumption of uniformly distributed points, the longest nearest neighbor link and the longest MST edge have the same value (asymptotically). In term of the resulting communication graph, this means that connectivity occurs (asymptotically) when the last isolated node disappears from the graph. This observation can be generalized to the case of k -connected [9]. This result has been showed in [10] to characterize the k -connectivity of ad hoc networks.

We have been used the following result due to Penrose [7], which characterizes the distribution of the longest MST edge for points distributed according to an arbitrary pdf with connected and compact support.

Theorem 3.1 [11]. *Let $X_1, X_2, X_3, \dots, X_n$ be independent random points in R^2 and assume that the points are distributed according to a common pdf f , having connected and compact support Ω with smooth boundary $\partial\Omega$. Further, assume that f is continuous on $\partial\Omega$. let M_n denotes the length of the longest MST edge built on the first n points of this random process. Then:*

$$\lim_{n \rightarrow \infty} \frac{n\pi(M_n)^2}{\ln n} = \frac{1}{\min_{\Omega} f}$$

Almost surely.

We recall that the boundary $\partial\Omega$ is smooth if and only if it is twice differentiable. Theorem 3.1 holds in the hypothesis that $\min_{\Omega} f > 0$. However, Penrose states that given the similarities with the result on the largest nearest neighbor link of [12], the theorem holds also when $\min_{\Omega} f = 0$. In this case the value of the limit must be intended as $+\infty$. in words theorem 3.1 states that the asymptotic behavior of the longest MST edge (and, consequently, of the critical transmitting range) depends only on the minimum value of the pdf used to distributed the nodes in Ω . In the next sections, we use these results to characterize the asymptotic behavior of the CTR in presence of mobility.

4 The CTR in Dense Networks

The CTR in dense networks can be characterized using results taken from a recent applied probability theory, the theory of Geometric Random Graphs (GRGs). Since the CTR equals the longest EMST edge, probabilistic solutions to the CTR problem in dense networks can be derived using results concerning the asymptotic distribution of the longest EMST edge. The following theorem is proven in [13].

Theorem 4.1 [13] *Assume n points are distributed uniformly at random in the unit square $[0, 1]^2$, and let M_n be the random variable denoting the length of the longest MST edge built on the n nodes. Then,*

$$\lim_{n \rightarrow \infty} P[n\pi(M_n)^2 - \log n \leq \beta] = \frac{1}{\exp(e^{-\beta})}$$

for any $\beta \in R$.

Corollary 4.1 *If R is the unit square and n nodes are distributed uniformly at random in R , then the CTR for connectivity is*

$$r_c = \sqrt{\frac{\log n + f(n)}{n\pi}}$$

where $f(n)$ is an arbitrary function such that $\lim_{n \rightarrow \infty} f(n) = +\infty$.

Proof: Let G_r denote the communication graph obtained when the transmitting range is set to r . Given the characterization of the CTR for connectivity of Theorem 2.1 and Theorem 4.1, G_r is asymptotically almost surly (a.a.s) connected if and only if

$$\lim_{n \rightarrow \infty} P \left[r \leq \sqrt{\frac{\log n + \beta}{n\pi}} \right] = 1.$$

It is immediate to see that corollary (??) is satisfied if and only if $\beta = f(n)$ for any function $f(n)$ such that $\lim_{n \rightarrow \infty} f(n) = +\infty$.

The CTR in case of three-dimensional networks can be derived by combining theorem 1.4 of [14] and theorem 1.1 of [15].

Theorem 4.2[4] *If R is the unit cube $[0, 1]^3$ and n nodes are distributed uniformly at random in R , then the CTR for connectivity is*

$$r_c = \sqrt[3]{\frac{\log n - \log \log n}{n\pi} + \frac{3}{2} \cdot \frac{1.41 + g(n)}{\pi n}}$$

where $g(n)$ is an arbitrary function such that $\lim_{n \rightarrow \infty} g(n) = +\infty$.

Note that, with respect to the case of two-dimensional networks and disregarding constants, the expression of the CTR in three-dimensional networks contains an additional $\log \log$ term. It is observed in [14] that this term is due to the boundary effect which is asymptotically negligible in the two-dimensional case, while it is not

negligible for three-dimensional networks. In case of one-dimensional networks (nodes along a line), the CTR can be characterized by combining Theorem 3.1 of [16], Theorem 2 of [13], and Theorem 2 of [12].

Theorem 4.3[4] *If R is the segment of unit length $[0, 1]$ and n nodes are distributed uniformly at random in R , and then the CTR for connectivity is*

$$r_c = \frac{\log n}{n}$$

5 The CTR in Sparse Networks

A common assumption of the GRG model is that the node deployment region R is fixed (typically, it is a d -dimensional cube), and the asymptotic investigation is for increasing number of deployed nodes (i.e. for increasing density). Combining this observation with the fact that the rate of convergence of the actual CTR to the theoretical value of the CTR is quite low, we can conclude that the results presented in the previous section in principle can be applied only to networks with very high node density. To circumvent this problem, some authors suggested adding a further parameter to the model, the side l of the deployment region. In this model, l is the independent variable, and the asymptotic values of r and n (which can be seen as functions of l) yielding connectivity with high probability are investigated for $l \rightarrow \infty$. Differing from the GRG model, node density $\frac{n}{l^d}$ can either converge to 0, or to a constant $c > 0$, or diverge as $l \rightarrow \infty$ depending on the relative magnitude of n and l . Let us first consider one-dimensional networks. The following result, as well as the other results presented in this section, has been proven in [17] by making use of the occupancy theory, which is another applied probability theory used in the analysis of ad hoc network properties.

Theorem 5.1 [17] *Assume n nodes, each with transmitting range r , are placed uniformly at random in $[0, l]$, and assume that $rn = kl \log l$, for some constant $k > 0$. Further, assume that $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $k > 2$, or $k=2$ and $r = r(l) \gg 1$, then the resulting communication graph is a.a.s. connected. If $k \leq (1 - \varepsilon)$ and $r = r(l) \in \Theta(l^\varepsilon)$ for some $0 < \varepsilon < 1$, then the communication graph is a.a.s. disconnected. If $r = r(l)$ is not of the form $\Theta(l^\varepsilon)$ but, $rn \ll l \log l$, then the communication graph is a.a.s. disconnected.*

Corollary 5.1 *If $R = [0, l]$ and n nodes are distributed uniformly at random in R , the CTR for connectivity is*

$$r_c = k \frac{l \log l}{n}$$

where k is a constant with $1 = k = 2$.

As compared to Theorem 4.3, the statement of Theorem 5.1 is more involved, and contains several technical conditions. In particular, there are assumptions on the relative magnitudes of r and n when expressed as functions of the independent variable l , namely,

$r = r(l) \ll l$ and $n = n(l) \gg 1$. Given the more general nature of this model as compared to the GRG model, these assumptions are necessary to investigate the asymptotic behavior of the CTR in a nontrivial setting. In fact, suppose $r \sim l$. In this case, each node has a direct connection to most of the other network nodes, and connectivity is ensured independent of n .

On the other hand, if n would remain constant as l increases, the only way of obtaining a connected network would be to have $r \sim l$, which is also a trivial case. It is interesting to compare Corollary 5.1 with the analogous theorem for dense networks. First of all, we observe that the characterization of the CTR in case of sparse networks is only partial since the exact value of the constant k is not known. The authors of [17] argue that k is probably 1, indicating a clear similarity with Theorem 4.3. Assuming $k = 1$, the only difference between the formulas presented in the two theorems is the 'geometric factor' while in case of fixed deployment region R the product $r_c n$ is proportional to $\log n$, in case of deployment region of side l , the product is proportional to $l \log l$. The l term can be interpreted as the scaling factor, while the $\log l$ term indicates the dependence of the CTR on a geometric parameter.

Theorem 5.2 [17] *Assume n nodes, each with transmitting range r , are placed uniformly at random in $[0, l]^d$, with $d = 2, 3$ and assume that $r^d n = kl^d \log l$, for some constant $k > 0$. Further, assume that $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $k > dk_d$, or $k = dk_d$ and $r = r(l) \gg 1$, then the resulting communication graph is a.a.s. connected, where $dk_d = 2^d d^{d/2}$.*

Theorem 5.3 [17] *Assume n nodes, each with transmitting range r , are placed uniformly at random in $[0, l]^d$, with $d = 2, 3$, and assume that $r = r(l) \ll l$ and $n = n(l) \gg 1$. If $r^d n \in O(l^d)$, then the resulting communication graph is not a.a.s. connected.*

Note the asymptotic gap between the necessary and sufficient condition for a.a.s. connectivity: it is known that $r^d n \in \Theta(l^d \log l)$ is sufficient for a.a.s. connectivity [Theorem 5.2] and that $r^d n \gg l^d$ is necessary for a.a.s. connectivity [Theorem 5.3]. Thus, the CTR for connectivity r_c might be any function of the following type:

$$\frac{l^d f(l)}{n}$$

where $f(l)$ is a function such that $f(l) = O(\log l)$ and $f(l) \geq 1$.

The authors of [17] argue that $f(l) = \log l$ is also a necessary condition for a.a.s. connectivity. We then claim the following result, which is only partially proven.

Proposition 5.1 *If $R = [0, l]^d$, with $d = 2, 3$, and n nodes are distributed uniformly at random in R , the CTR for connectivity is*

$$r_c = k \frac{l^d \log l}{n}$$

where k is a constant with $0 \leq k \leq 2^d d^{d/2+1}$.

6 The CTR in Mobile Networks

Our first result is the characterization of the CTR in the presence of bounded and obstacle free mobility, which we known define.

Definition 6.1 Let R be a bounded region, and let ∂R be its boundary. Let M be an arbitrary mobility model, and let f_M be the pdf that resembles the long-term node spatial distribution generated by M -like mobility. M is bounded within R if the support of f_M is contained in R . M is obstacle free if the support of f_M contains $R - \partial R$.

In words, a mobility model is bounded within R if the nodes are allowed to move only within that region, while it is obstacle free if the probability of finding a mobile node in any subregion of R (excluding the border) is greater than 0. For instance, the random waypoint model, the random direction model, and the Brownian-like model are bounded and obstacle free.

For simplicity, in the rest of this paper, we assume that $R = [0, 1]^2$, i.e., it is the unit square.

Theorem 6.1 [18]. *Let M be an arbitrary mobility model which is bounded within $R = [0, 1]^2$ and obstacle free. Furthermore, assume that f_M is continuous on ∂R and $\min_R f_M > 0$. The critical transmitting range for connectivity of an ad hoc network with M -like mobility is*

$$r_M = c \sqrt{\frac{\ln n}{\pi n}}$$

with high probability, for some constant

Proof: We observe that, if the hypotheses of Penrose's theorem hold, our result follows immediately since $\min_R f_M > 0$ by hypothesis. Thus, we only have to show that the hypotheses of Penrose's theorem are satisfied. First, we observe that, since M is bounded within R and obstacle free, the support of f_M is contained in R and contains $R - \partial R$. Since R is connected and compact, it follows that the support of f_M is connected and compact also. Furthermore, f_M is continuous on ∂R by hypothesis. The only hypothesis left to prove is that ∂R is smooth. Unfortunately, this is not true due to the presence of the corners. This problem can be circumvented by considering the region R_ϵ , obtained by "rounding" the corners of ∂R with a portion of the circle of radius ϵ (see [18]). The boundary of R_ϵ is smooth for any value of $0 < \epsilon < 1/2$ and $\lim_{\epsilon \rightarrow 0} R_\epsilon = R$.

6.1 The CTR in RWP Mobile Networks

In this section, we characterize the CTR for connectivity in case of RWP mobility, which is by far the most popular mobility model used in the simulation of ad hoc networks. It is known that the asymptotic node spatial distribution generated by RWP mobility is not uniform but is somewhat concentrated in the center of the deployment region [19, 20]. This phenomenon, which is called the *border effect*, is due to the fact that the waypoints (i.e. the destinations of a movement) in the RWP model are selected uniformly at random in a bounded deployment region R . To better understand this

point, consider a RWP mobile node u , and assume that node u is currently resting at a waypoint that is close to the border of R (see Figure 2). Since the next waypoint is chosen uniformly at random in R , it is very likely that the trajectory connecting node u with its next waypoint will cross the center of R . So, the probability of finding a mobile node close to the center of R is higher than the probability of finding the node on the boundary. This means that mobile nodes contribute a nonuniform component to the asymptotic node spatial distribution generated by RWP mobility, which we denote by f_m (m stands for 'mobile'). On the other hand, a node resting at a waypoint contributes a uniform component f_u to the asymptotic RWP distribution, since the waypoints are chosen uniformly at random in R . Then, the asymptotic node spatial distribution generated by RWP mobility, denoted by f_{RWP} is given by $f_{RWP} = f_m + f_u$, which is nonuniform. The amount of this nonuniformity (and, hence, the intensity of the border effect) depends on the relative strength of the two components of f_{RWP} . It is easy to see that a longer pause time strengthens f_u , since the nodes remain stationary for a longer time. Conversely, f_m is maximal when the pause time is 1 because, in this case, nodes are constantly moving.

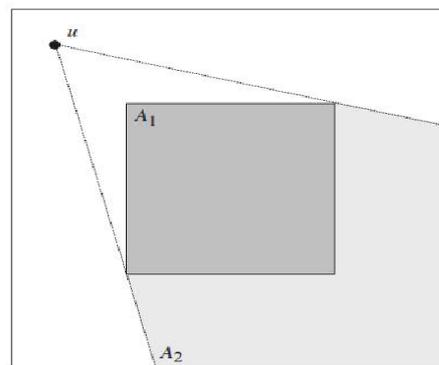


Figure 2: The border effect in RWP mobile networks

When a node is resting close to the border, it is likely that the trajectory to the next waypoint crosses the center of the deployment region (dark shaded area). In the figure 2, the probability that the trajectory of node u to the next waypoint intersects A_1 equals the sum of the areas of A_1 and A_2 (we are assuming $R = [0, 1]^2$). The informal argument above is theoretically supported by the following theorem proven in [21], which derives a very good approximation of f_{RWP} when nodes move in $R = [0, 1]^2$.

Theorem 6.1.1 [21] *the asymptotic spatial density function of a node moving in $R = [0, 1]^2$ according to the RWP model with pause time t_p and velocity v is closely approximated by*

$$f_{RWP}(x,y) = \begin{cases} P_{\text{pause}} + (1 - P_{\text{pause}})f_m(x,y) & \text{if } (x,y) \in [0, 1]^2 \\ 0 & \text{otherwise} \end{cases}$$

where

$$P_{Pause} = \frac{t_p}{t_p + \frac{0.521405}{v}}$$

$$f_{RWP}(x,y) = \begin{cases} 0 & \text{if } (x=0) \text{ or } (y=0) \\ f_R(x,y) & \text{otherwise} \end{cases}$$

The expression of $f_R(x,y)$ could be written as:

$$f_R(x,y) = 6y + \frac{3}{4}(1 - 2x + 2x^2)(\frac{y}{y-1} + \frac{y^2}{(x-1)x}) + \frac{3}{2}((2x - 1)y(1 + y) \log(\frac{1-x}{x}) + y(1 - 2x + 2x^2 + y) \log(\frac{1-y}{y}))$$

We remark that the expression of $f_m(x,y)$ above is valid only for $(x,y) \in R = \{(x,y) \in [0, 1]^2 \mid (x \geq y) \wedge (x \leq 1/2)\}$ the expression of $f_m(x,y)$ on the remainder of $[0, 1]^2$ can be easily obtained observing that by symmetry we have

$$f_m(x,y) = f_m(y,x) = f_m(1-x,y) = f_m(x,1-y).$$

The CTR in presence of RWP mobility can be characterized by using the following result of the GRG theory, which is due to Penrose [22] [23].

Theorem 6.1.2 [22] Assume n nodes are distributed independently at random in R^2 according to a common probability density function f , having connected and compact support Ω with smooth boundary $\partial\Omega$. Further, assume that f is continuous on $\partial\Omega$. Let M_n denote the length of the longest MST edge built on the n points. Then,

$$\lim_{n \rightarrow \infty} \frac{n\pi(M_n)^2}{\log n} = \frac{1}{\min_{\Omega} f}$$

Almost surely

We recall that the support Ω of a probability density function is the set of points in which it has nonzero value, and that the boundary $\partial\Omega$ is smooth if and only if it is twice differentiable. Informally speaking, Theorem 6.1.2 states that the asymptotic behavior of the CTR for connectivity with arbitrary density f depends only on the minimum value of f in its support. In case $\min_{\Omega} f = 0$, the limit of equation in theorem 6.1.2 must be intended as $+\infty$. In order to apply Theorem 6.1.2 to f_{RWP} , we have to check that all the conditions of the theorem are satisfied. It is immediate to see that $R = [0, 1]^2$, the support of f_{RWP} , is connected and compacted. However, the boundary of R is not smooth because of the presence of the corners. This problem can be circumvented by using the ‘corner-rounding’ technique described in [24]. Thus, we are in the hypotheses of Theorem 6.1.2, and the only thing left to do to characterize the CTR is to determine the minimum value of f_{RWP} in R . This can be easily done, given the expression of f_{RWP} introduced in Theorem 6.1.1.

Corollary 6.1.1 Let $f_{RWP}^{t_p}$ denote the asymptotic node spatial density generated by RWP mobile networks with pause time t_p and velocity v . The minimum value of $f_{RWP}^{t_p}$ is achieved on R , and it equals $P_{Pause} = \frac{t_p}{t_p + \frac{0.521405}{v}}$. When $t_p \rightarrow \infty$, $f_{RWP}^{t_p}$ becomes the uniform distribution on

$[0, 1]^2$, and $\min_R f_{RWP}^{\infty} = 1$.

We are now ready to characterize the CTR in presence of RWP mobility.

Theorem 6.1.3 [24] If $R = [0, 1]^2$ and n nodes move in R according to the RWP mobility model with pause time t_p and velocity v , then the CTR for connectivity is

$$r_{RWP}^{t_p} = \frac{1}{P_{Pause}} \sqrt{\frac{\log n}{\pi n}} = \frac{t_p + \frac{0.521405}{v}}{t_p} \sqrt{\frac{\log n}{\pi n}}$$

if $t_p > 0$. If $t_p = 0$, we have

$$r_{RWP}^0 \gg \sqrt{\frac{\log n}{n}}$$

a.a.s.

Note that the CTR in presence of RWP mobility is always larger than the CTR in case of uniform node distribution since $1/P_{Pause}$ is larger than 1 for any value of t_p . For instance, with $t_p = 75$ and $v = 0.01$, we have $1/P_{Pause} = 1.69485$. Clearly, a longer pause time results in a more uniform node distribution and, consequently, in a smaller value of the CTR. For instance, with $t_p = 150$, we have $1/P_{Pause} = 1.34743$.

Note also the asymptotic gap of the CTR in the most extreme case of RWP mobility, that is, when $t_p = 0$: in this case, for any constant $c > 0$, setting the transmitting range to $c\sqrt{\frac{\log n}{n}}$ is not sufficient for achieving a.a.s. connectivity. The exact value of the CTR with RWP mobility when $t_p = 0$ is not known to date. In [24], it is conjectured that $r_{RWP}^0 \approx \frac{1}{4} \log n \sqrt{\frac{\log n}{\pi n}}$.

Theorem 6.1.4 [4] A network with RWP mobility is ergodic with respect to the CTR for connectivity.

Proof: In order to prove the theorem, we have to show that the RWP mobility model is stable and c -independent, for some constant $c > 0$. The first property is an immediate consequence of Theorem 6.1.1. As for the second, consider an arbitrary time instant i . We have to determine a certain value $c > 0$ such that the positions of all the nodes at time $i+c$ are independent of node positions at time i . Let us define a movement epoch as the time needed for a node just arrived at a waypoint to reach the next waypoint. In other words, a movement epoch is composed of the pause time plus the travel time between two consecutive waypoints. Since the length of the trajectory and node velocity is in general random variables, the duration of a movement epoch is also a random variable. Indeed, we have a sequence of random variables representing the duration of the various epochs that constitute the movement trace of a node. We denote these variables with $E_{u,j}$, where u is the node to which the variable is referred and j denotes the j^{th} epoch of node u . By definition of RWP mobility, node u 's position at time $i+c$ is independent of its position at time i if and only if c is larger than $E_{u,j} + E_{u,j+1}$, where j is the index of the epoch occurring at time i . In words, the node must conclude the current and the next epoch before its

position is independent of the position at time i . Note that it is not enough for the node to terminate the current epoch, since a node which is traveling at time i is on its trajectory to a certain waypoint $W_{u,j}$, which is also the starting point of the next trajectory. However, after the node has reached the next waypoint, the conditions for independence are satisfied. So, proving the theorem reduces to proving that there exists constant $c > 0$ such that $E_{u,j} + E_{u,j+1} = c$, for any $j = 0$ and for any node u . This is accomplished by setting $c = 2 \frac{\sqrt{n}}{v_{min}}$. In fact, the maximum length of a linear trajectory in $R = [0, 1]^2$ is $\sqrt{2}$, and node velocity in the RWP model is at least $v_{min} > 0$. Note that, by setting $c = 2 \frac{\sqrt{n}}{v_{min}}$, we ensure that the positions of all the nodes at time $i + c$ are independent of their positions at time i . This follows from the fact that inequality $E_{u,j} + E_{u,j+1} = c$ is satisfied for any epoch and for any node.

7 The CTR for k -connectivity

The k -connectivity graph property is an immediate extension of the concept of graph connectivity. Formally, k -connectivity is defined as follows:

Definition 7.1 [Connectivity] A graph G is said to be k -connected, where $1 \leq k < n$, if for any pair of nodes u, v there exist at least k node disjoint paths connecting them. The connectivity of G , denoted as $k(G)$, is the maximum value of k such that G is k -connected. A 1-connected graph is also called simply connected.

A similar definition of connectivity can be given by considering edge, instead of node, disjoint paths between nodes. Denoting with $\xi(G)$ the edge-connectivity of G , it is seen immediately that $k(G) \leq \xi(G)$. Figure 3 illustrates the concepts of k -connectivity and k -edge connectivity.

The interest in studying the CTR for k -connectivity is motivated by the fact that, when a network is k -connected, at most $k - 1$ node or link faults can be tolerated without disconnecting the network. So, a k -connected network is more resilient to faults than a simply connected network, where a single node or link failure might partition the network. A network satisfying k -connectivity in general achieves also a better load balancing with respect to a simply connected network in fact, messages between any two nodes u and v can be routed along at least k different paths, instead of along at least one single.

On the other hand, a connectivity value that is too high is detrimental for network capacity since any transmission would interfere with a large number of nodes. For instance, if $k(G) = \frac{n}{2}$, it is seen immediately that any node in the communication graph has at least $\frac{n}{2}$ neighbors. In turn, this implies that when any node transmits, it interferes with at least $\frac{n}{2}$ nodes, and the network traffic carrying capacity is compromised. Thus, from a practical point of view, only networks with relatively low connectivity are of some interest. The first study of k -connectivity that can be applied to ad hoc

networks is due to Penrose. In [15], Penrose shows that the giant component phenomenon occurs in case of k -connectivity also, for any constant $1 \leq k < n$. More formally, Penrose proved the following theorem.

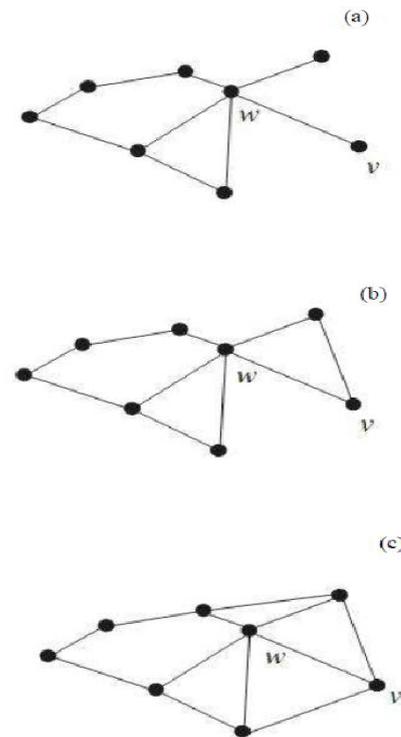


Figure 3: Simple and 2-connectivity.

(a)-The graph is simply connected (removing node w , or edge (w, v) , is sufficient to disconnect the network).

(b)-The graph is 2-edge-connected, but not 2-(node) connected. In fact, removing any edge does not disconnect the graph, but removing node w does disconnect the graph.

(c)-The graph 2-connected: removing any node or edge does not disconnect the graph path. In turn, better load balancing means a more evenly distributed energy consumption in the network, which potentially results in a longer network lifetime.

Theorem 7.1 [15] Assume n nodes are distributed uniformly at random in $R = [0, 1]^d$, with $d = 2, 3$. Let r_n (respectively, σ_n) denote the minimum value of the transmitting range at which the communication graph becomes k -connected (respectively, has minimum degree k), where $1 \leq k < n$ is an arbitrary constant. Then,

$$\lim_{n \rightarrow \infty} P[r_n = \sigma_n] = 1$$

In words, Theorem 7.1 states that, with high probability, the network becomes k -connected when the minimum node degree in the communication graph becomes k . Theorem 7.1 proved useful in the characterization of the CTR for k -connectivity, which can be derived by analyzing the probability of the relatively simpler event that every node in the network has degree at least k . The value of the CTR for k -connectivity, which was partially characterized in [15], has been derived in [15] in case of two-dimensional networks.

Theorem 7.2 [25] Assume n nodes are distributed uniformly at random in the unit square $R = [0, 1]^2$. The CTR for k -connectivity, for any constant k , with $1 \leq k < n$, is

$$r_k = \sqrt{\frac{\log n + (2k - 3) \log \log n + f(n)}{\pi n}}$$

where $f(n)$ is a function such that $\lim_{n \rightarrow \infty} f(n) = +\infty$.

Wan and Yi [26, 27] proved that a similar expression holds when nodes are uniformly distributed in the disk of unit area.

Comparing the expression of the CTR for k -connectivity with that of the CTR for simple connectivity (Corollary 4.1), we see that the difference between the two values is only in the second-order term $(2k - 3) \log \log n$ (we recall that k is a constant). This means that, asymptotically, k -connectivity with $k > 1$ is achieved by slightly increasing the transmitting range with respect to the critical value for simple connectivity. The CTR for k -connectivity has also been studied under the assumption that n nodes are distributed in a two-dimensional region A with very large area [26]. With this assumption, the number of nodes per units of area is $\rho = \frac{n}{a}$ with high probability, where a is the area of A .

Theorem 7.3 [26] Assume n nodes, each with transmitting range r_0 , are distributed uniformly at random in A , where A has a very large area. The probability that the minimum node degree in the communication graph is at least k , for some $1 \leq k < n$, is closely approximated by

$$P(\text{deg}_{\min} \geq k) \approx \left(1 - \sum_{i=0}^{k-1} \frac{(\rho \pi r_0^2)^i}{i!} \cdot e^{-\rho \pi r_0^2} \right)^n$$

a.a.s., where $\rho = \frac{n}{a}$.

Given Theorem 7.1, the expression reported in Theorem 7.3 is also a close approximation of the probability of having a k -connected network [4].

8 The CTR for Connectivity with Bernoulli Nodes

Wireless ad hoc networks with Bernoulli nodes provide a unified model of various important problems including fault-tolerances, randomized construction of virtual

backbone, randomized broadcast routing and randomized wake/sleep management. We assume that the wireless ad hoc network consists of n nodes which are distributed independently and uniformly in a unit-area disk and are active independently with some constant probability p . Let r_n denote the random variable which is the smallest transmission range at which the active nodes form a connected network, and r'_n denote the random variable which is the smallest transmission range at which the active nodes from a connected network and each inactive nodes is adjacent to at least one active node. r_n is referred to as the critical transmission range for connectivity of active nodes, and r'_n is referred to as the critical transmission range for connectivity of all nodes. The precise asymptotic distribution of r_n and r'_n . The Asymptotic Critical Transmission Range for Connectivity in Wireless Ad Hoc Network with Bernoulli nodes paper has been studied by Peng-Jun Wan and Chih-Wei Yi [1].

9 Experimental results

Figure 4 depicts the rate convergence of the CTR to the asymptotic value in case of two dimensional networks, where the asymptotic value of the CTR is obtained by setting the formula in corollary 4.1 with n nodes are distributed uniformly at random $[0,1]^2$. As seen from figure 4 the CTR in dense network is decreased with the increasing of the node density (number of network nodes).

Table 1 CTR in dense network

n	CTR	n	CTR
10	0.323348	500	0.073219
20	0.261181	1000	0.054284
50	0.187547	1500	0.045468
100	0.142973	2000	0.040061
250	0.098184	2500	0.036297

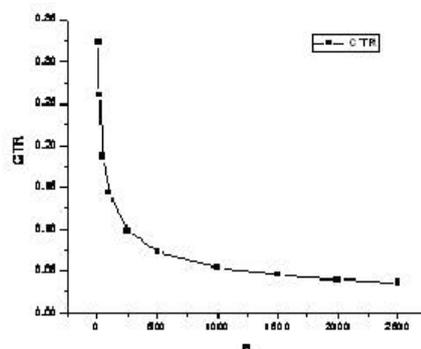


Figure 4: CTR for connectivity according to Corollary 4.1 in dense network.

Figure 5 shows the rate of convergence of the actual CTR in one-dimensional networks to the asymptotic value as predicted by Corollary 5.1, where k is set to 1. as in the case of dense networks. In the experiments, the number n of nodes to distribute for a given value of l is set to \sqrt{l} .

As seen from the figure, in this case. The asymptotic CTR formula of Corollary 5.1 is a very good approximation of the actual CTR for moderate to high values of l . Note that these values of l correspond to values of n . Thus, contrary to the case of dense networks, the formula of Corollary 5.1 is very accurate even for networks composed of few nodes. In case of two- and three-dimensional networks, the characterization of the CTR proven in [17] is weaker.

Table2 CTR in sparse network

l	$CTR_{k=0.5}$	$CTR_{k=0.7}$	$CTR_{k=1.0}$
10	0.00768	0.01075	0.01535
20	0.01997	0.02796	0.03994
50	0.0652	0.09128	0.1304
100	0.15351	0.21491	0.30701
250	0.46012	0.64417	0.92024
500	1.03577	1.45008	2.07154
1000	2.30259	3.22362	4.60517
1500	3.65661	5.11925	7.31322
2000	5.06727	7.09418	10.1345
2500	6.52004	9.12805	13.0401

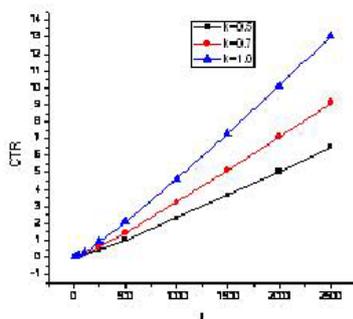


Figure 5: CTR for connectivity according to Corollary 5.1 in sparse network

Figure 6 shows the rate of convergence of the actual CTR for connectivity to the asymptotic value stated in theorem 6.1.3 in case of RWP mobility with $t_p = 1, 50,$ and $t_p = 150$. The actual CTR value is computed as follows: Initially, n nodes are distributed uniformly randomly in $R = [0, 1]^2$. Then, they start moving according to the RWP mobility model. From figure 6, we show that the formula of theorem 6.1.3 is quite accurate only for large values of n . The experimental value of the CTR for RWP mobile networks with different values of the pause time is reported in table 3.

Table 3 CTR in RWP in mobile network

n	$Tp=1$	$Tp=50$	$Tp=150$
10	1.68274	0.29904	0.28021
20	1.35721	0.24119	0.226
50	0.9809	0.17431	0.16334
100	0.75255	0.13373	0.12531
250	0.52116	0.09261	0.08678
500	0.39096	0.06948	0.0651
1000	0.29146	0.05179	0.04853
1500	0.24486	0.04351	0.04077
2000	0.21619	0.03842	0.036
2500	0.19618	0.03486	0.03267

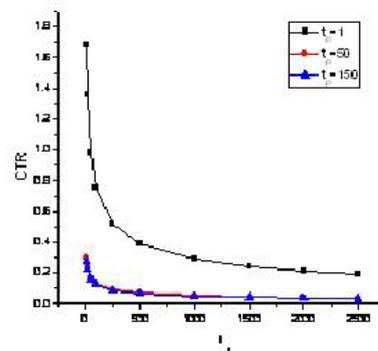


Figure 6: CTR for connectivity in RWP mobility with $t_p=1, 50, 150$

Figure 7 depicts comparing the expression of the CTR for k -connectivity with that of the CTR for simple connectivity (Corollary 4.1), we see that the difference between the two values is only in the second-order term $(2k - 3) \log \log n$ (we recall that k is a constant). This means that, asymptotically, k -connectivity with $k > 1$ is achieved by slightly increasing the transmitting range with respect to the critical value for simple connectivity. The CTR is decreased with the increasing of the node density of network.

Table 4 CTR for k-connectivity

n	$CTR_{k=2}$	$CTR_{k=3}$
10	0.363806	0.43354
20	0.294112	0.35082
50	0.210399	0.24991
100	0.159787	0.188979
500	0.081111	0.094946
750	0.067988	0.079388
1000	0.059925	0.069853
1500	0.050095	0.058256
2000	0.044078	0.051176
2500	0.039897	0.046264

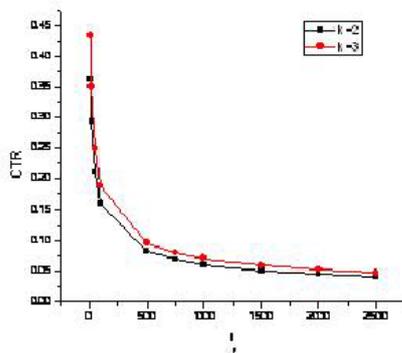


Figure 7: the CTR for k-connectivity.

10 Conclusion

In this paper, we have analyzed the critical transmitting range for connectivity in dense, sparse network and mobile wireless ad hoc networks. For dense and sparse networks, we have provided both analytical and experimental results. The most notable aspect of our analysis is that, contrary to the case of existing theoretical results, it can be applied to both dense and sparse ad hoc networks. We have also investigated the critical transmitting range in two-dimensional mobile networks. We have considered a mobility random waypoint.

For the case of RWP mobility, we have proven a more accurate characterization of the CTR and shown that, if the pause time is 1, there is an asymptotic gap between the mobile and uniform scenario. We have verified the quality of our experimental results. We have also presented a formula that, given the value of the CTR in the uniform case, provides a good approximation of the CTR in the most extreme case of RWP mobility, i.e., when the pause time is set to 1. We remark that the approach presented in this paper could be easily extended to other mobility models. If the expression of the pdf f_m that resembles the long-term node distribution is known and satisfies certain properties, it is sufficient to compute the minimum value of f_m on R to determine the value of the critical range for connectivity. In this paper we consider characterization of the CTR for other important network prosperities such as k-connectivity and connectivity with Bernoulli nodes. We believe that the results of this paper can be improving the accuracy of ad hoc network which is commonly used to evaluate the performance of ad hoc network protocols.

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