Applied Mathematics & Information Sciences
An International Journal

Mathematical Analysis of Super Low Frequency Ground Loop Receiving Antennas

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Received: 20 Sep. 2012, Revised: 30 Dec. 2012, Accepted: 14 Jan. 2013
Published online: 1 May 2013

Abstract: A compact super low frequency receiving antenna in the form of a loop can be built for which the induced atmospheric noise voltage dominates both the antennas thermal noise voltage and preamplifier noise. Air-core and ferromagnetic-core ground loop receiving antennas were introduced, and mathematical calculation methods to analyse the ENF spectrum of the atmospheric noise and thermal noise were derived. The research identified the roles played by the various contributing factors such as antenna dimensions, wire conductivity, ferromagnetic-core permeability, etc. One air-core and three ferromagnetic-core antennas were designed, and a compromise core design was recommend, which takes advantage of the improvement that ferromagnetic cores can provide while largely avoiding the disadvantages.

Keywords: super low frequency, loop receiving antenna, air-core loop, ferromagnetic core loop

1. Introduction

The two most conventional super low frequency (SLF) receiving antennas are the whip and the loop. The extremely high input impedance of the whip, however, causes the problems of sensitivity variation and noise due to static electricity [1,2]. These, coupled with the fact that it is suitable only for above-surface reception make it unattractive compared with the loop.

The loop receiving antenna is the receiving counterpart of the transmitting loop antenna. The received signal voltage across the terminals of the loop is given simply by the time rate of change of the signal flux density linking the turns of the loop [3]. If the stray capacitance of the loop is large enough, however, or if the self-inductance of the loop is large enough, the self-resonant frequency of the loop may be low enough to be commensurate with the operating frequency. Then the simple flux-linkage formula is inaccurate. But since appreciable stray capacitance lead to a non-uniform current distribution along the wire of the loop winding, the signal induced in one turn is not perfectly in phase with that induced in another. The result is a reduced figure of merit. Thus even though the voltage amplification factor of self-resonance may increase the output signal voltage, this is more than offset by a concomitant increase in the resistance of the antenna [4]. Usually, therefore, at SLF, it is better to arrange for the self-resonant frequency of the loop to be large compared with the operating frequency. Then the simple flux linkage relationship is accurate. This may mean that the output signal voltage is not high enough compared with the noise voltage of the preamplifier. In that case, a signal transformer can be used to raise the voltage level. If the transformer were perfect, it would not change the figure of merit of the antenna. Since it is not perfect, some increase in thermal ENF occurs which must be balanced off against the relative decrease in amplifier ENF. For geophysical applications, the frequency of interest is usually so low that the dominant noise is the amplifier noise. Then, increase the number of turns to bring the antenna to self-resonance at the frequency of interest results in a net decrease in the total ENF, even though the thermal ENF is increased [5]. This effect is less likely in SLF. It also has less appeal because a large front-end bandwidth may be required to take full advantage of the impulsive nature of atmospheric noise.

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2. Air-core loop antenna

The simplest loop antenna is the air-core loop. The resistance Ra of the antenna is given by

$$R_a = \frac{2\pi a n^2}{\sigma_c C_w A_c} \tag{1}$$

where a is the average turn radius, n is the total number of turns, σ_c is the conductivity of the wire conductor, A_c is the cross-sectional area available for the winding and C_w is the filling factor. (C_w is the factor, less than unity, by which the actual total conductor cross section is smaller than the winding area. Thus $C_w = \frac{A_c}{nA_w}$, where A_w is the cross-sectional area of a single wire.) If the winding area A_c is square of side length s, the inductance L_a of the loop is given by [6]

$$L_a = \frac{\mu_0 a n^2 p}{2} \tag{2}$$

where p depends on the former factor m_f of the loop, defined as $m_f = \frac{s}{2a}$, for $m_f < 0.2$, the formula for p is

$$p = (1 + \frac{m_f^2}{6})\ln(\frac{8}{m_f^2}) + 0.41 m_f^2 - 1.70 \tag{3}$$

This square cross-section coil is compact, and the computation of its inductance is simple. However, it has the disadvantage that its self-inductance and stray capacitance are both larger than they need be, making the self-resonant frequency lower than it would be were the winding spread out further. The spreading could be done axially, to obtain a solenoid winding, or radially, in which case a pancake type of coil would result. For these, and other, more general shapes, the inductance may be calculated from Grovers [7] formulas and Tables.

The radiation resistance of the loop in air is so small compared with the resistance of the winding that it is completely negligible. On the other hand, since the wavelength is so long, the loop antenna is unlikely ever to be so far from the earth or other conducting media for the radiation resistance to be larger than the mutual resistance between the loop and the closest conducting medium. However, even when it is completely buried in natural Earth media, for which the skin depth is large compared with the loop radius, the total external resistance of the air-core loop would still usually be small compared with its winding resistance.

For example, if the external resistance R_e of the loop is given by [8]

$$R_e = \frac{8n^2}{3a\sigma}\frac{a^4}{\delta^4} \tag{4}$$

Where σ and δ are the conductivity and skin depth of the surrounding medium. Thus the ratio $\frac{R_e}{R_a}$ can be written, from (1) and (4), as

$$\frac{R_e}{R_a} = \frac{4A_c a^2 C_w}{3\pi\delta_c^2\delta^2} \tag{5}$$

To obtain this expression, the ratio $\frac{\sigma_c}{\sigma}$ has been identified with the ratio $\frac{\delta^2}{\delta_c^2}$ by multiplying both numerator and denominator by $2\omega\mu_0$ and denoting by δ_c the skin depth of the conductor used to wind the loop antenna. Substituting into formula (5), the dimensions $A_c = 0.01\mathrm{m}^2$, $a = 0.5\mathrm{m}$, the maximum possible filling factor of unity and the frequency 100Hz, one finds $\frac{R_e}{R_a} = 0.038$ if the winding is with copper wire ($\sigma_c = 0.580 \times 10^8$ S/m) and the antenna is immersed in ocean water ($\sigma = 4$ S/m).Since ocean water is a typical worst case and since, also, the antenna example chosen is, at a weight of 280 kg, atypically large for this type of receiving antenna, the external resistance would usually be even less than the already small 3.8% of the winding resistance.

The absolute magnitude V of the voltage induced in the loop by an external uniform magnetic field strength H is, by Faradays law,

$$V = A\omega\mu_0 H n \tag{6}$$

Assuming the loop axis is parallel to the magnetic field lines and where A is the average turns area. Thus, since the open-circuit thermal noise voltage spectral density $S_v(\omega)$ is

$$S_v(\omega) = 4k_b T_k R_a \tag{7}$$

Where k_b is Boltzmanns constant (1.38×10^{-23} J/K) and T_k is the absolute temperature in Kelvins, the spectral density $S_h(\omega)$ of the magnetic equivalent noise field is given by $\frac{4k_b T_k R_a}{(A\omega\mu_0 n)^2}$. By virtue of formula (1), this can be rewritten as

$$S_h(\omega) = \frac{4\pi k_b T_k \delta_c^2 a}{\omega\mu_0 C_w A_c A^2} \tag{8}$$

in $\mathrm{A}^2/\mathrm{m}^2/\mathrm{Hz}$, provided R_e is negligible compared with R_a. For the dimensions ($A_c = 10^{-2}\ \mathrm{m}^2, a = 0.5\mathrm{m}$), wire conductivity ($\sigma_c = 0.58 \times 10^8$ S/m), filling factor (1) and frequency (100 Hz) used in the earlier example, and with $T_k = 300\mathrm{K}$, this works out to be $S_h(\omega) = 2.34 \times 10^{19}\ \mathrm{A}^2/\mathrm{m}^2/\mathrm{Hz}$. This is several orders of magnitude smaller than atmospheric noise and justifies the statement made earlier that this loop antenna is atypically large.

If the loop is being used to receive the magnetic field component of a plane wave in a conducting medium, the electric ENF component of a plane wave in a conducting medium, the electric ENF spectrum $S_e(\omega)$ corresponding to formula (8) is given by multiplying formula (8) by the squared magnitude $\frac{\omega\mu_0}{\sigma}$ of the wave impedance in the medium. Thus

$$S_e(\omega) = \frac{2\pi\omega\mu_0 k_b T_k \delta^2 \delta_c^2 a}{C_w A_c A^2} \tag{9}$$

in $\mathrm{V}^2/\mathrm{m}^2/\mathrm{Hz}$. On the other hand, if it is being used at, or above, the surface for the reception of long-range SLF transmissions, the electric ENF spectrum can be expressed either in the manner of formula (9) (if the horizontal electric field is taken as the reference) or by multiplying formula (8) by the squared magnitude of the wave impedance of the Earth-ionosphere waveguide (if the vertical electric field is taken as the reference). For SLF communications, the vertical whip antenna is unlikely to be used in preference to the loop or horizontal insulated electric dipole. Thus, in practice, there would be no practical reason to use the vertical electric field as the reference.

A significant feature of both (8) and (9) is that the number of turns on the loop does not appear. Thus the intrinsic quality of the number of antenna, as far as thermal noise is concerned, is independent of the number of turns. This means that the number of turns is a parameter which is free for the designer to use for other purposes. One of these might be to avoid operating the antenna at a frequency near its self-resonant frequency. Another would be to ensure that the received signal voltage is large compared with the electronic noise of the preamplifier, or to establish a self-impedance for the antenna which achieves the optimum noise match with the preamplifier. Yet a third might be to rise to the impedance level of the antenna to make its resistance large compared with the resistance of the transmission line connecting the antenna to the preamplifier.

It is usually advantageous to surround the antenna with an electrostatic shield. It can be applied as a flexible tape after the antenna has been wound on its non-conducting coil form, or the coil form itself could be made of metal. In either case, it is usual to provide an electrical break in the shield at some point around the circumference to avoid the short-circuited turn that the shield would otherwise be. By operating the antenna in an electrically balanced fashion with the shield grounded, electrostatic pick-up is minimized. (See Figure 1) It may be necessary to space the shield away from the winding, if the winding has a great many turns, to avoid increasing the stray capacitance of the coil-shield combination. An undesirably low self-resonant frequency may otherwise be the result.

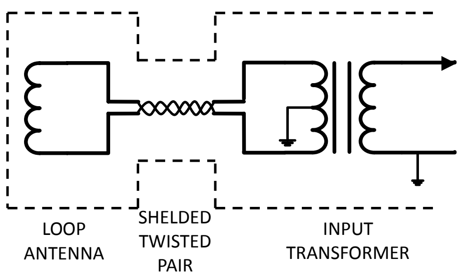

Figure 1 Shielded loop antenna in an electrically balanced connection mode.

The SLF loop antenna is versatile, in that it operates equally well above or below the surface. The fact that it requires no electrical connection to be made with the ground makes it more suitable than the horizontal insulated-dipole antenna for mobile or portable use over land. Its signal sensitivity can usually be calculated accurately from its mechanical dimensions, and the stability of this sensitivity with time is excellent. Calibration of the sensitivity can be readily carried out by using a single-turn circular loop of wire carrying an accurately known current as a source to provide a known incident field.

The one serious disadvantage of the loop antenna is that it has a high sensitivity to vibration. The SLF radio-wave band coincides in frequency with that in the acoustic spectrum over which appreciable vibrational energy of mechanically structure is found. And when the loop vibrates mechanically, some of the vibration modes are such as to change the linkage of the geomagnetic field with the loop. Since the geomagnetic flux density is of the order of 0.5×10^{-4} T, whereas the atmospheric noise spectrum is of the order of $1\mu\mathrm{A/m}/\sqrt{\mathrm{Hz}}$ or less, an angular vibration spectrum of only 0.25×10^{-7} radians$/\sqrt{\mathrm{Hz}}$ can more than double the total noise voltage at the antenna terminals. This makes the antenna very difficult to use in a mobile application, and even for stationary use on land, it is necessary to provide protection for the antenna from the wind and to select a site away from sources of ground vibration, the proximity of a tree can produce a measurable vibration-induced noise voltage. The effect of the wind is to distort the ground around the tree and thereby vibrate the antenna.

3. Ferromagnetic-core loop antenna

The addition of a ferromagnetic core to the loop antenna has a profound effect on its performance. Its function is to increase the signal flux threading the winding without changing the resistance of the antenna. At higher frequencies, to reduce the losses in the core, the core material is usually ferrite. However, at SLF, laminated signal-transformer metal of the permalloy type can be used. Very high small-signal magnetic permeability can be attained with this type of material as much as 5×10^4 or 10×10^4 times that of free space. Building up the core cross-section from thin insulated laminations can reduce core losses to the point where they are negligible compared with the winding losses. This means that by winding the same number of turns around a ferromagnetic core, one can achieve the same signal sensitivity as that of an air-core loop having an enclosed area as much as 100,000 times bigger. At the same time, the length of wire needed is less than one percent of that of the air-core loop, and so the winding resistance is much less. Thus the figure of merit is much higher, that is, the thermal ENF is much lower, and the weight of the antenna is much less.

In order to take full advantage of the high permeability offered by currently available signal-transformer material, the core should be very long and slender. Paper [9] and paper [10] display a family of curves showing the effective permeability of long cylindrical cores as a function of the ratio of length to diameter with the intrinsic permeability as a parameter. When the small-signal permeability is 10^5 μ_0, for example, the length to diameter ratio has to be as much as 2,000 for the flux density at the mid-section of the core to be within a few percent of 10^5 times the incident flux density.

For the reasons discussed above, therefore, the general shape of the loop antenna with a ferromagnetic core is that of a long thin cylinder, whereas the air-core loop has the shape of a torus. Figure 2 is a sketch of the ferromagnetic-core loop antenna.

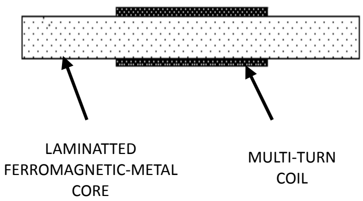

Figure 2 Shielded loop antenna in an electrically balanced connection mode.

The coil would be shielded in practice, to minimize electrostatic pick-up, and a balanced electrical connection to the preamplifier might also be used as shown in Fig.2 for the air-core loop. Another advantage of the ferromagnetic core loop is that the smaller size of the coil leads to a smaller effect of the stray capacitance. Thus the designer has more freedom to choose the number of turns on the coil to achieve a better noise match than he has for the air-core loop.

There are three disadvantages to the ferromagnetic-core antenna. One is that the permeability of the magnetically soft materials used for the core cannot be determined accurately in advance. This means that a specific voltage sensitivity cannot be achieved by design. Rather, the design must be based on a conservatively low value for the permeability which would then normally be exceeded by the material procured for job. The actual sensitivity achieved would then be evaluated by a calibration measurement.

The second disadvantage is that the magnetic material is also mechanically soft and very sensitive in its magnetic properties to mechanical strain, both within and exceeding the elastic limit, and to temperature. Thus then calibration of the antenna can be changed irreversibly by slight physical damage to the core and reversibly by the elastic strains to which the core is subjected when it is mounted or emplaced and by changing temperature.

Finally there is the problem that making the core long and slender to achieve good coupling with the signal field also subjects it to good coupling with the geomagnetic field. This means that depending on the orientation of the core axis with respect to the geomagnetic field direction, there will typically be a strong bias flux density in the core. And the more sensitive magnetic materials would be fully magnetically saturated by the geomagnetic field which would drastically reduce the small-signal permeability. Thus the voltage sensitivity of the loop antenna with a very long slender ferromagnetic core will vary widely as a function of the orientation of the antenna.

Fortunately, there is a compromise design that takes advantage of the improvement that ferromagnetic cores can provide while largely avoiding the disadvantages. In this design, the length to diameter ratio of the core is chosen to be much smaller than it would have to be to make its effective permeability close to its intrinsic permeability. Then the demagnetization factor is so large compared with μ_0/μ that the effective permeability μ' is defined, for practical purposes, solely by the length to diameter ratio of the core. Since this ratio can be obtained with precision by simple mechanical means, the effective permeability (and therefore also the voltage sensitivity) can be accurately predetermined. Also, the effects of minor damage, of strain and of temperature change will be very much less [11], provided the relative intrinsic permeability is not reduced so far that its reciprocal becomes commensurate with the demagnetization factor.

It is more difficult to counteract in the same way the effect of geomagnetic bias. This is because the geomagnetic field is so large that the effective permeability would have to be depressed, by reducing the length to diameter ratio, to perhaps $1000\mu_0$ or less to make the voltage sensitivity of the loop largely independent of orientation. This problem can, of course, be dealt with by providing a degaussing winding around the core to cancel the axial component of the geomagnetic field. If the winding is supplied by a very low noise d.c. source, it is possible to remove completely the detrimental biasing effects of the geomagnetic field.

From the foregoing discussion one perceives that the ferromagnetic core loop antenna can be of two basic types. The first, which makes use of the full intrinsic permeability of the core material, is long, slender and light in weight but its voltage sensitivity is uncertain and varies with core strain and it also requires a degaussing system to counteract the effects of geomagnetic bias. The second, which uses the core in a geometry-limited configuration, is shorter, fatter and considerably heavier, for the same thermal ENF. However, its voltage sensitivity is calculable and stable and can be essentially independent of orientation.

The design equations for the antenna are readily derived. The antenna resistance is again given by equation

(1), since the core losses can be made negligible by using thin core laminations and the external resistance, if the antenna is above ground, is also typically negligible. The signal induced voltage υ is given, according to Faradays law, by

$$\upsilon = A_f \omega \mu' H n \tag{10}$$

assuming that the flux in the annular region between the core and each turn is negligible and that the core axis is parallel to the magnetic signal field lines, where A_f and μ' are the cross-sectional area and effective permeability of the core and n is the total number of turns. Following the derivation carried out earlier for the air-core loop, therefore, one may express the thermal ENF of the ferromagnetic-core loop as the magnetic field spectrum $4k_bT_kR_a/(A_f\omega\mu' n)^2$ or from equation (1),

$$S_h(\omega) = \frac{4\pi k_b T_k \delta_c^2 a}{\omega\mu_0(\mu'/\mu_0)^2 C_w A_c A_f^2} \tag{11}$$

where the units are $\mathrm{V}^2/\mathrm{m}^2/\mathrm{Hz}$. Here, it is to be recalled, a is the average turns radius, A_c is the cross-sectional area of the winding region and C_w is the filling factor of the winding.

Some care is necessary in interpreting μ' in formula (11) because the signal flux induced in the core is not uniform. Thus if the signal coil around the core is short enough in axial length, the flux linking it is essentially uniform over the length of the coil and has the value implied by the demagnetizing relationship. If the signal coil is long, however, the flux linking it is non-uniform and the average flux linkage is somewhat less. If the geometry-limited type of core construction is adopted, an estimate of the factor by which the average flux linking the coil is less than its peak value can be obtained by assuming the flux distribution is parabolic. The permeability-limited type of core construction, on the other hand, can have a flat-top flux distribution of a shape more difficult to estimate.

As an example of the three types of loop antenna, Figure 3 has been prepared showing the particular sizes and weights that would result in a thermal ENF of $1.71\times10^{-18}\ \mathrm{A}^2/\mathrm{m}^2/\mathrm{Hz}$ or -178dB re. $1\mathrm{A/m}/\sqrt{\mathrm{Hz}}$. In the calculations, no allowance was made for the thickness or weight of insulation, the winding factor C_w was assumed to be equal to unity, the intrinsic core permeability to be $10^5\mu_0$ and since the winding is assumed to lie in the central region of the core, the flux distribution over the winding length was taken to be uniform and equal to its maximum value. The ENF was calculated for the frequency of 100Hz, with copper as the conductor material, using equation (8) for the air-core antenna and equation (11) for the ferromagnetic-core antennas. The differences between the three examples are dramatic. The weights cover a range of three orders of magnitude and the physical proportions range from the solid torus of the air-core antenna to a thin cylinder, in the case of the geometry-limited ferromagnetic-core antenna, and a long

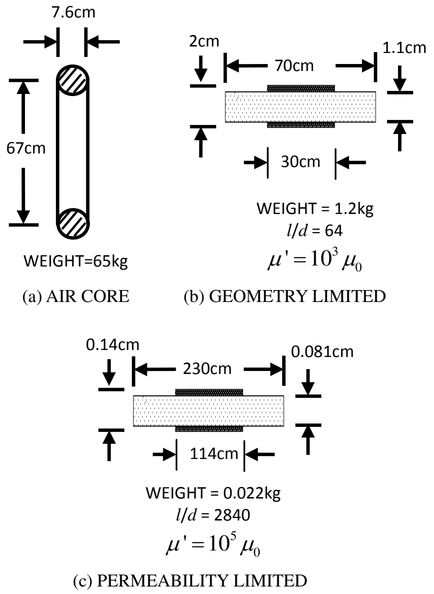

Figure 3 Sample weights and sizes of three loop receiving antennas having the same thermal ENF at a frequency of 100 Hz.

slender wire in the case of the permeability-limited ferromagnetic-core type. The contrast between the types is exaggerated by the large intrinsic small-signal permeability ($10^5\mu_0$) assumed for the core material, by the low thermal ENF chosen to base the comparison on, and by the fact that both the insulating material and the very essential support structure has been neglected. Nevertheless, the trend exhibited by these results is so strong that it persists even when these conditions are relaxed.

For the ferromagnetic-core loop, the effect of burial or submergence on the antenna resistance in more difficult to analyze than it is in the case of the air-core loop. It is more probable, in view of the field-concentrating effect of the core, that the external resistance will be significant. On the other hand, the total resistance of a low-resistance air-core loop was still attributable almost wholly to the winding resistance even in the sea water. Thus only in the extremely cases (highly conducting media, very compact but low-loss antenna) is it likely that the external resistance would need to be considered.

In any case, the simple technique of placing the antenna in a radome, to exclude the conducting medium from the immediate vicinity of the antenna, will suppress the external resistance effectively [12]. For a small air-core loop laced at the center of a spherical cavity in the conducting medium, the external resistance R_e is given by

$$R_e = \frac{2n^2A^2}{3\pi\sigma b\delta^4} \tag{12}$$

where b is the cavity radius and $b \ll \delta$ [13]. The corresponding expression for the ferromagnetic-core case is obtained, by virtue of (6) and (10), by replacing A in this expression by $(\mu'/\mu)A_f$. Thus

$$R_e = \frac{2n^2(\mu'/\mu)^2A_f^2}{3\pi\sigma b\delta^4} \tag{13}$$

The question of submerging a ferromagnetic-core antenna directs attention to a particular design intended for towing behind a submarine. Its characteristics are sufficiently unique and were described in paper [14].

4. Conclusion

There are a number of different antennas available for SLF reception. The goal of the designer is to obtain a total antenna ENF that is low compared with ambient noise while, at the same time, ensuring that the antennas effective length is large enough to make the ambient-noise antenna voltage much larger than the preamplifier noise.

The loop antenna is stable, can be used buried or above ground, is relatively light in weight especially when it has a laminated ferromagnetic core is portable, and passive. The ferromagnetic core, while reducing the size and weight of the antenna, introduces special problems of its own. If it is too long and slender, the geomagnetic field can severely reduce its otherwise very high small-signal permeability. Such a core is also sensitive to damage and strain. A compromise core design, which avoids these problems, is preferable unless the need for minimum weight is crucial.

Acknowledgement

The authors acknowledge the financial support of National Natural Science Foundation of China, project No. 41074047. The author is grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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