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Stochastic Model Of Crude Oil Spot Price Process As A Jump-Diffusion Process

Chisara Peace Ogbogbo

Department of Mathematics, University of Ghana, Legon Accra, Ghana

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Abstract: Although crude oil is a major source of energy throughout the world, unexpected jumps in its price cause remarkable economic events globally. Countries such as Nigeria and Venezuela, whose major foreign exchange earner is crude oil. have experienced economic recession because of the recent huge drop in its price. Thus, understanding the dynamics of its price has become necessary. Moreover, obtaining a model that properly describes crude oil spot price movement is useful for strategic investment planning, managing market risks and even for short time forecasting. Crude oil price exhibits jumps, so Gaussian models will fail to capture the real market events. In this paper, a Lévy model of the jump-diffusion type is used to model crude oil price and its fit is confirmed using empirical spot price data.

Keywords: Crude Oil Spot Price, Jumps, Stochastic Differentials Equations, Le ?vy Market, Jump-Diffusion Model, Yuima Package.

1 Introduction

To develop viable strategies for managing risk and pricing assets, one needs to know the stochastic process describing asset dynamics. Crude oil is a very important commodity which has been described as a consumption and an investment (asset) commodity [1].

The investor will be interested in identifying the process that drives oil price movement to plan the strategic investment, maximize profit, minimize risk and financial loss. Identifying oil price process is also beneficial for predicting the future circumstances of the market.

Modeling risky assets using stochastic processes with continuous paths, based on the Brownian motion, involves several defects. This is because the path continuity assumption is inconsistent with the real situation of the market. Price movements show small jumps in short periods in traditional diffusion models. However, prices may show big jumps in short time periods in real markets. As a result, diffusion models used in finance are insufficient. A good model should permit discontinuities and jumps in price process [2].

Crude oil prices exhibit significant volatility over time. The distribution of returns on crude oil price shows fat tails, skewness and barely follows a normal distribution [3]. Accordingly, more efficient models that could effectively describe crude oil prices.

To address these limitations and defects, stochastic models with jumps do well to account for sudden variation in prices. The Lévy model used in this paper is based on the Poisson distribution which has slower tail decay than the Gaussian distribution. Higher probability is assigned to the unexpected events and a more realistic model is created.

Most studies addressing crude oil prices have used Gaussian models [4,5,6,7,8]. Gonzalo and Noranjo [4] used n-factor Gaussian models for futures prices. Massino, Barcellona and D'Ecclesia [5] investigated energy commodity prices using neural networks. Mina Hosienni [7] proposed a stochastic model for the future prices of oil. Schwartz [9] also handled the stochastic behaviour of commodity prices.
Noureddine Krichene [1] modeled the future prices of crude oil as a Lévy process specifically as a Variance Gamma process. He [10] modeled them by using a Lévy driven process, which is the generalized hyperbolic type. In particular, he obtained parameters for the normal

* Corresponding author e-mail: chisaraogbogbo@yahoo.com

inverse Gaussian (NIG) process. He applied the model for option pricing.

Hossein Askari and Noureddine Krichene [11] addressed the dynamics of daily prices of oil for 2002-2006, extracted price process parameters which consistent with underlying market fundamentals.

In the present paper, we consider a Lévy market and use a Lévy process to model crude oil spot prices.

The dynamics of the crude oil spot price was formulated as a Stochastic Differential Equation (SDE) driven by a jump-diffusion Lévy process which was solved analytically using Itô's formula for jump processes. Empirical data were used to validate the model. Parameters of the model were estimated and used to compute the theoretical crude oil spot price values. The paper is outlined as follows;

Section 1 comprises the introduction. **Section 2** discusses Lévy processes and motivation for Lévy market, and presents the crude oil price data used for validation of results. **Section 3** is devoted to the jump-diffusion model and the estimation of the model parameters. Fit of the jump-diffusion model to empirical crude oil price data is illustrated in **section 4**. **Section 5** involves analysis of the results and conclusion.

2 Lévy process and Motivation for Lévy Market

2.1 Motivation for the Lévy Markets

Despite its huge success (simplicity and popularity) Black-Scholes model (which starts off with a GBM) had some limitations. It presents unrealistic assumptions that lead to drawbacks. For example, returns follow a Brownian motion, return volatilities are constant, interest rate is risk-free, and the returns as well as their volatilities are uncorrelated.

Simple statistical tests prove that one cannot assume constant volatility and normality of stock returns in real markets. Observation of empirical data have shown that commodity returns have distributions with some (stylized) features, such as fat or semi-heavy tails, skewness, jumps, and no-normality [12]. Therefore, the normal distribution is not a viable tool for modeling this. Researchers, such as Carr et al [12] as well as Carr and Wu [13] have suggested using Lévy processes, characterized by stationary and independent increments, for modeling asset prices.

Lévy model is used in this paper, because Lévy processes are more versatile than the diffusion processes. Moreover they can model skewness, excess kurtosis, and jumps.

2.2 Empirical Crude Oil Price Data

Monthly data on crude oil spot price for four Niger-Delta crude types, during the period from January 2005-December 2009, were collected from the Department of Petroleum resources, Lagos. The price data used in the paper is in line with oil price data from yahoo finance. The crude types are Benny Light (BL), Brass Blend (BB), Pennington light (PL) and Antan (ANTAN). The data were tested for properties of Lévy process using Augmented Dickey-Fuller, Kolmogorov-Smirnov and Durbin-Watson tests for stationarity, normality and autocorrelation, respectively.
The data were found to be consistent with the existence of an underlying Lévy process [14].

3 The Jump - Diffusion Model for crude oil Spot Price Process

In 1976, Merton [15] also identified the presence of jumps in asset prices. Crude oil has been investigated in the recent pieces of literature as a consumption commodity as well as an asset. [16]

Through adding a jump part to the dynamics of the process, the presence of the peculiar features of crude oil price series are justified. Skewness and kurtosis are observed in the distribution of oil price returns [10,1], so a process that includes jumps is proposed. Jump processes are generally based on the Poisson distribution which has slower tail decay than the Gaussian distribution [2].
We consider a probability space $(\Omega,\mathscr{F},P)$, and all random variables discussed in the paper are measurable on $(\Omega,\mathscr{F},P)$.

An important implication of the Lévy-Itô decomposition theorem is that every Lévy process is made up of a Brownian motion with drift and possibly an infinite sum of independent compound Poisson processes. [17]
The stochastic continuity property of a Lévy process implies that discontinuity occurs at random times, i.e. probability of a jump at some fixed time t, is zero.

$$\lim_{h\to 0} P\left[|X_{t+h}-X_t|\geq \in\right]=0$$

A stochastic differential equation (SDE) with Poissonian jumps for an asset price S_t , is of the form:

$$\frac{dS_t}{S_{t^-}}=\mu(t,S_t)dt+\sigma(t,S_t)dW_t+c(t^-,S_{t^-})dN_t, \quad S(0)=S_0$$

$W=\{W_t,t\geq 0\}$ is a standard Brownian motion, $N=\{N_t,t\geq 0\}$ is a Poisson process with intensity $\lambda>0,t\geq 0$, with $X_0>0,c(.,.)$ is the jump coefficient which defines the jump size at an event, i.e is an impulse function which determines the jump from S_{t^-} to S_t. μ is linear drift and σ is volatility.

Events driven by uncertainty, such as arrival of abnormal information, are incorporated through a Poisson process with intensity λ. The model describes the movement of the crude oil price justifying the jumps which periodically arise. In the model, event driven uncertainty is expressed by jumps.

3.1 Model Assumptions

In formulating the model, the following assumptions were made:
Let S_t be crude oil price at time t, S_t is a set of random variables on $(\Omega, \mathscr{F}, P)$

(i)*The crude oil price process has a continuous component and some discontinuities.*
(ii)*The discontinuities are in the form of jumps.*
(iii)*The number of jumps is finite*
(iv)*There are jumps of various sizes.*
(v)*Standard Brownian motion is associated with the continuous component of the process and distributed as $W_t \sim N(0,1)$. μ is the instantaneous expected return (drift) and σ, the volatility of the process.*
(vi)*Jumps are special events which occur with arrival of new (unusual) information.*
(vii)*The discontinuities of the price process which are jumps are described by a Poisson counter N_t.*
(viii)*The intensity of the Poisson process is given as λ*
(ix)*Jump size is denoted J_t*
(x)*λ is the mean number of arrival of abnormal information; mean number of jumps occurring per unit time.*

3.2 Construction of the dynamics of crude oil spot price

(a)*Let S_{t^-} denote the left hand limit of S_t at time t, S_{t^-} describes the value just before a potential jump at time t.*
(b)*From time t to $t+dt$, a Poisson event occurs if some unusual but important information about crude oil arrives.*
(c)*Let ε denote the random variable description coming from a distribution which determines the impact of the information on crude oil price.*
(d)*Let $S_t = \varepsilon S_{t^-}$ denote the new price value, as arrival of information causes price to go from S_{t^-} to S_t at time $t+dt$*
(e)*Let $\varepsilon = e^{J_t}$, then $S_t = e^{J_t} S_{t^-}$*
(f)*Since in the interval dt, the price of oil goes from S_{t^-} to S_t and $S_t = e^{J_t} S_{t^-}$ then the percentage change in the oil price caused by the jump i.e percentage change in oil price if Poisson event occurs is*

$$\frac{dS_t}{S_{t^-}} = \frac{e^{J_t} S_{t^-} - S_{t^-}}{S_{t^-}} = e^{J_t} - 1.$$

(g)*Since J_t is normally distributed, e^{J_t} is log normally distributed.*
(h)*We assume $J_t \sim N(0,1)$.*
(i)*$\Delta N_t = N_{t+\Delta} - N_t$ is the number of jumps which occurs in the interval $(t, t+\Delta)$.*

The crude oil price process is now described with the following SDE

$$\frac{dS_t}{S_{t^-}} = \mu dt + \sigma dW_t + \left(e^{J_t} - 1\right) dW_t \qquad (1)$$

Using Itô's lemma for jump processes, we obtain a closed form solution to equation (1) as given in equation (1.1) below.

Crude oil price process has jumps and is shown to be a Lévy process [14]. As a model for the crude oil spot price, we use the following jump-diffusion process given in equation 1.1 for the price of crude oil at time t.

$$S_t = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t + \sum_{t=1}^{N_t} J_t\right\}. \qquad (1.1)$$

In the following section, we estimate the parameters of the jump-diffusion process given in equation 1.1. Using the parameter estimates, in an R-computational algorithm, we compute the crude oil price at time t.

3.3 Parameter estimates for empirical data

Parameters of the model are estimated using the YUIMA package. The YUIMA package is an R-program which uses quasi maximum likelihood estimation method, in its scheme. The YUIMA package is used for parameter estimation because it surmounts the challenges of inefficiency of other methods of estimation, including MLE, CF and method of cumulants. Using these other methods, one parameter has to be fixed to obtain others and sometimes a parameter is assumed to be zero to estimate others. [18,19]. In addition, QMLE method, which overcomes the limitation of the MLE for estimating parameters of the jump-diffusion model, is used in the package. [20] The package is programmed to do the following:

–*Estimate the empirical data, the parameters of the model, drift of the diffusion part μ, volatility of the diffusion σ, intensity of the compound Poisson (jump) process λ, (mean of jump size β , variance of size δ) .*
–*Estimate other statistics, such as number of jumps, average jump size and jump threshold.*
–*Randomly generate sample paths for the model, and plot the graphs.*
–*Plot the empirical data.*

The empirical data set is setup as YUIMA object using the "setYUIMA( )function", and the model is specified as jump-diffusion through the "setModel( )function", giving the desired flexibility including jumps. [21] Then, the parameters are estimated, and the estimates are presented in table 1 below, with other important statistics. Codes are attached in the appendix.

4 Fit of the Jump-Diffusion model to Empirical crude oil price oil Data

The jump-diffusion process consists of two independent components; the diffusion component and the jump component. Thus, the jump component is incorporated into the diffusion simulation process. The parameters, obtained for the jump-diffusion, are substituted into equation 1.1 to achieve the following equations for the four crude types.
For Bonny Light (BL), the model equation for the crude oil spot price is:

$$\begin{aligned} S_t &= 44.1667\exp\left\{\left(0.0133-\frac{0.1154^2}{2}\right)t+0.1154W_t+\sum_{t=1}^{N_t}J_t\right\} \\ &= 44.1667\exp\left\{0.0066t+0.1154W_t+\sum_{t=1}^{N_t}J_t\right\}. \end{aligned}$$

For Brass Blend (BB), the model equation for the crude oil spot price is:

$$\begin{aligned} S_t &= 44.2267\exp\left\{\left(0.0497-\frac{0.1297^2}{2}\right)t+0.1297W_t+\sum_{t=1}^{N_t}J_t\right\} \\ &= 44.2267\exp\left\{0.0413t+0.1297W_t+\sum_{t=1}^{N_t}J_t\right\} \end{aligned}$$

For Pennington Light (PL), the model equation for the crude oil spot price is:

$$\begin{aligned} S_t &= 44.2967\exp\left\{\left(0.0120-\frac{0.1136^2}{2}\right)t+0.1136W_t+\sum_{t=1}^{N_t}J_t\right\} \\ &= 44.2967\exp\left\{0.0055t+0.1136W_t+\sum_{t=1}^{N_t}J_t\right\} \end{aligned}$$

For Antan (ANTAN), the model equation for the crude oil spot price is:

$$\begin{aligned} S_t &= 42.7717\exp\left\{\left(0.0578-\frac{0.1272^2}{2}\right)t+0.1272W_t+\sum_{t=1}^{N_t}J_t\right\} \\ &= 42.7717\exp\left\{0.0497t+0.1272W_t+\sum_{t=1}^{N_t}J_t\right\}. \end{aligned}$$

4.1 Simulation of the Jump-Diffusion Model

From the price process i.e equation 1.1, S_t values were computed using an R-program (code is attached in the appendix). The program runs as an iterative process. It uses crude oil price for January 2005 as initial price S_0 for all the crude types BL, BB, PL, and ANTAN. To explain, $44.1667, 44.2267, 44.2967, 42.7717$, respectively.
The estimated parameters μ (drift) and σ (volatility) were used for simulation of diffusion process. The program randomly generates W_t, from normal distribution, $N(0,1)$. Time was defined in months. Since the process is a jump-diffusion process, the program generated the jump component, which is incorporated into the diffusion part. For the jump component, we defined a price change greater than or equal to $\pm 3\%$ of, current price suggesting a jump. Krichene [10] has defined a price change of $\pm 3\%$ indicating a jump.
Since the jump size is considered a standard normal random variable, i.e. $J_t \sim N(0,1)$, the program randomly generated jump sizes J_t , from $N(0,1)$. The generated numbers J_t, are summed up by the program $\left(\sum_{t=1}^{N_t} J_t \text{ is computed}\right)$ and introduced (added directly) to the actual simulated diffusion component. The sum could be positive, negative or zero. When the sum is zero, the process coincides with the diffusion model. S_t values for the diffusion process were obtained by the described simulation procedure of the program.

When jump size is considered a normal random variable with parameters β and δ different from $(0,1)$, good results are not obtained. This is consistent with the approach of Ball and Torous [18].

4.2 Computation of Crude Oil Price using the model, In-Sample Forecast

Table [2] displays the computed values for in-sample forecast, obtained from the Jump-diffusion process with the empirical values.
These computed price values are also denoted the theoretical price values.

4.3 Graphs of Empirical and Computed Crude Oil Price Values

The following graphs (Figures 1,2,3,4,5,6,7 and 8) show the paths for empirical and theoretical oil price for various crude types. They illustrate jumps and green dotted lines indicating the jump size.

5 Analysis of Results and Conclusion

A jump-diffusion process was used to model crude oil price process. The theoretical values for crude oil price S_t for $2005-2009$ were obtained from this equation. The theoretical values are well-compared with the empirical price data obtained from DPR [22]. This validates the model due to providing a good description of the movement of crude oil spot price.

Table 1: Empirical and computed $\mathbf{S_t}$ values for the crude types (2005-2009): in-sample values.

	BL		**BB**		**PL**		**ANTAN**	
Month	**Empirical**	**Computed**	**Empirical**	**Computed**	**Empirical**	**Computed**	**Empirical**	**Computed**
1	44.1667	52.29821	44.2267	48.58325	44.2967	45.01970	42.7717	42.33898
2	45.2401	53.31987	45.3001	49.84527	45.3701	45.99206	45.874	45.50809
3	52.9998	60.72929	53.0598	58.99780	53.1298	53.04391	51.6048	51.36583
4	51.8679	59.65034	51.9279	57.66502	51.9979	52.01703	50.4729	50.21064
5	48.936	56.85271	48.996	54.20923	49.066	49.35441	47.541	47.21532
6	54.898	62.54345	54.958	61.23876	55.028	54.77053	53.503	53.30819
7	58.0465	65.55115	58.1065	64.95402	58.1765	57.63307	56.6515	56.52841
8	65.945	73.09425	66.005	74.27168	66.075	64.81215	64.55	64.60453
9	65.1395	72.32084	65.1995	73.31633	65.2695	64.07606	63.7445	63.77647
10	60.1407	67.54673	60.2007	67.41908	60.2707	59.53234	58.7457	58.67523
11	55.8968	63.49828	55.9568	62.41821	56.0268	55.67927	54.5018	54.33048
12	57.154	64.69181	57.214	63.89252	57.284	56.81520	55.759	55.61857
13	63.3527	70.61171	63.4127	71.20511	63.4827	62.44941	61.9567	61.95679
14	60.74	68.11962	60.8	68.12675	60.87	60.07759	59.344	59.27838
15	63.2524	70.51622	63.3124	71.08717	63.3824	62.35854	61.8564	61.85456
16	72.1166	78.98550	72.1766	81.54889	72.2466	70.41909	70.3206	70.50316
17	71.164	78.06887	71.224	80.41662	71.294	69.54670	69.571	69.73644
18	69.5066	76.49341	69.5666	78.47053	69.6366	68.04727	69.989	70.16581
19	75.2569	81.98364	75.3169	85.25237	75.3869	73.27255	73.8609	74.12208
20	74.4721	81.22933	74.5321	84.32060	74.6021	72.55464	73.0761	73.32469
21	61.94	69.26541	62	69.54209	62.07	61.16808	60.544	60.50513
22	58.757	66.22907	58.817	65.79143	58.887	58.27828	57.361	57.25424
23	60.3197	67.71859	60.3797	67.63138	60.4497	59.69592	58.9237	58.84902
24	64.2745	71.49014	64.3345	72.29021	64.4045	63.28546	62.8785	62.89730
25	55.5622	63.17364	55.6222	62.01719	55.6922	55.37030	54.12	53.94201
26	59.395	66.84016	59.455	66.54628	59.525	58.85987	58.005	57.91873
27	64.058	71.28963	64.118	72.04252	64.188	63.09462	58.331	58.24586
28	70.332	77.27637	70.392	79.43768	70.462	68.79244	64.73	64.78854
29	70.104	77.05676	70.164	79.16640	70.234	68.58343	66.584	66.67978
30	73.801	80.58960	73.861	83.53037	73.931	71.94578	68.401	68.54036

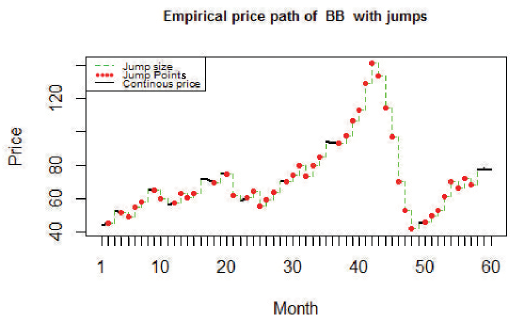

Fig. 1: Empirical Price Path of BB showing Jumps

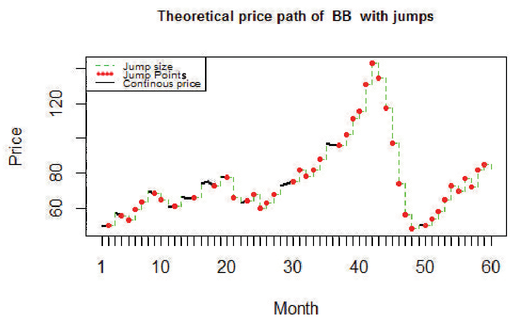

Fig. 2: Theoretical Price Path of BB showing Jumps.

The values displayed in Table [2] reveal that the computed values for the Jump-diffusion model, are close to the empirical values. The price difference is less than or equal to eight dollars ($\leq \pm\$8$) for all crude types and less than or equal to five dollars ($\leq \pm\$5$) for PL and Antan crude types in particular. Thus, Jump-diffusion model, provides a good fit for the empirical oil price data. The theoretical results obtained from the jump-diffusion model showed that computed price values are close to the empirical data. The model is validated based on these results.

Table 2: Empirical and Computed S_t values for the crude types (2005- 2009): in-sample values.

	BL		BB		PL		ANTAN	
Month	Empirical	Computed	Empirical	Computed	Empirical	Computed	Empirical	Computed
31	79.456	85.99390	79.516	90.20606	79.586	77.08927	73.421	73.67227
32	73.344	80.15038	73.404	82.98782	73.474	71.52776	73.344	73.59049
33	79.465	86.00345	79.525	90.21785	79.595	77.09836	79.465	79.85715
34	84.584	90.88259	84.644	96.24484	84.714	81.74204	84.584	85.08107
35	94.461	100.31624	94.521	107.89781	94.591	90.72043	94.461	95.18133
36	92.8548	98.77898	92.9148	105.99890	92.9848	89.25735	92.8548	93.53543
37	94.0321	99.90567	93.4394	106.62401	92.8512	89.13922	87.75	88.32174
38	99.2161	104.86120	97.5285	111.44796	98.0903	93.90104	96.252	97.01123
39	95.5473	101.35700	106.8165	122.40505	105.256	100.41673	101.39	102.26582
40	114.0519	119.02122	113.0931	129.80021	114.3591	108.68630	113.178	114.31866
41	129.1207	133.41039	128.5945	148.08169	129.574	122.50829	126.2017	127.62892
42	130.4259	134.66121	140.727	162.40021	137.959	130.13265	140.249	141.99214
43	132.6695	136.80001	133.4284	153.79023	137.0709	129.32387	130.885	132.42348
44	115.1328	120.05242	113.9404	130.80274	117.885	111.89416	109.7788	110.84286
45	98.9226	104.57475	96.8924	110.69311	98.673	94.42811	89.6836	90.29476
46	66.7688	73.87720	70.3236	79.35511	72.584	70.71898	59.3712	59.30905
47	52.1634	59.92724	52.8436	58.73832	54.212	54.02536	45.404	45.02761
48	43.1876	51.36248	41.944	45.88231	44.569	54.02536	37.2078	36.65503
49	45.54142	53.60631	45.38057	49.93962	46.158	46.70997	41.3517	40.88732
50	45.02294	53.10981	45.93185	50.58832	48.348	48.70012	39.85829	39.36410
51	51.85602	59.64079	49.87174	55.23535	48.444	48.78190	46.55918	46.21347
52	52.3576	60.11821	53.2512	59.22190	48.444	48.78190	49.7834	49.50525
53	62.7257	70.01972	61.1513	68.53955	62.6772	61.72242	59.2899	59.22727
54	69.0588	76.06374	69.9299	78.89513	70.87	69.16503	66.4178	66.51622
55	68.23453	75.27124	66.4206	74.75526	69.728	68.12906	66.04679	66.13797
56	71.19531	78.10706	72.16594	81.53710	74.086	72.09118	71.53944	71.75036
574	68.79465	75.80594	68.46146	77.16134	70.483	68.81062	66.84437	66.94558
58	75.66168	82.36557	77.17534	87.44614	72.966	71.07339	74.91835	75.20571
59	78.03899	84.63805	78.06139	88.48406	79.492	76.99840	76.48787	76.81071
60	76.87649	83.53046	77.06593	87.31640	79.9419	77.40733	76.41673	76.73915

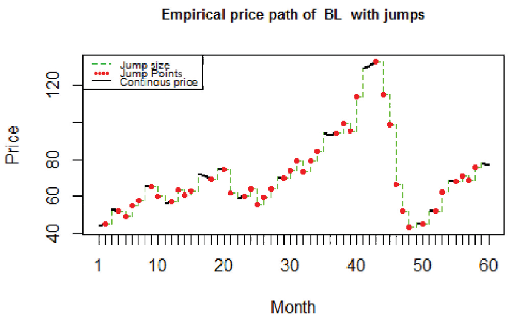

Fig. 3: Empirical Price Path of BL showing Jumps

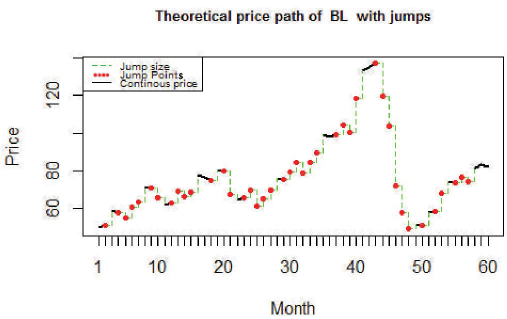

Fig. 4: Theoretical Price Path of BL showing Jumps

The empirical data are monthly data obtained from DPR Lagos. Theoretical data generated is also monthly. Consequently, the continuous parts of the graphs are not clusters of several values as would be observed for daily or weekly data.

For almost all the crude types, the 25th month to 48th month were dominated by jumps. This is between January 2007 and December 2008.

A jump threshold of $\pm 3\%$ of current price led to several jumps in the price series. Increments in price which is

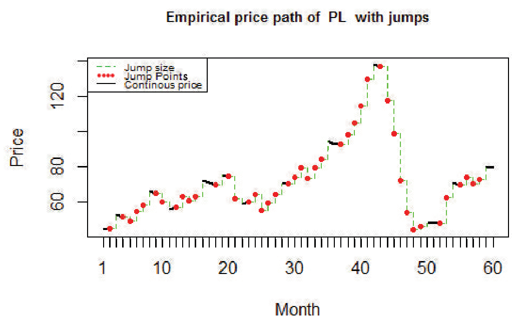

Fig. 5: Empirical Price Path of PL showing Jumps.

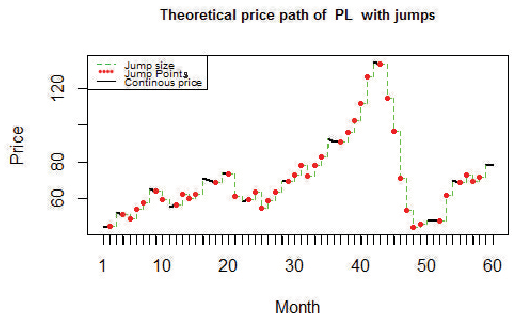

Fig. 6: Theoretical Price Path of PL showing Jumps

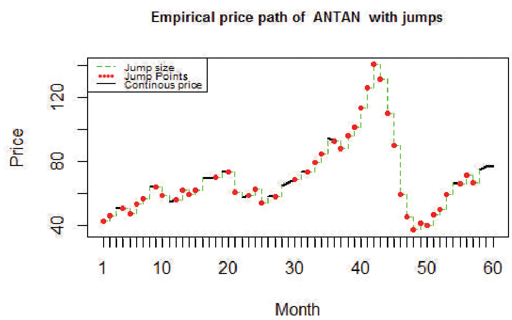

Fig. 7: Empirical Price Path of Antan showing Jumps

greater than or equal to $\pm 3\%$ were observed for several months for both empirical and theoretical price series.

Highest jump sizes were recorded between May 2008 (41^{st} month) and December 2008 (48th month) for both empirical and theoretical price values.

For almost all crude types, the months, when jumps occurred for the empirical data, are also the months jumps occurred for the theoretical data. When the jumps occurred, for both the empirical and theoretical graphs, the jump sizes were about the same for almost all the

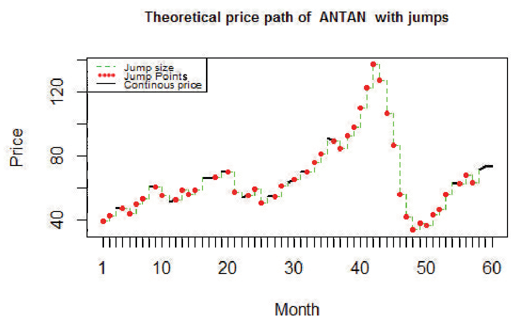

Fig. 8: Theoretical Price Path of Antan showing Jumps

crude types. These are observed throughout the 60 months for each crude type, making the theoretical graphs as mirror images of the empirical graphs.

This presents a good fit of data to the model, thereby validating the jump-diffusion model for describing crude oil spot price process.

Crude oil price responds to global supply and global, inventory, as well as speculative demand.

Jumps occur in crude oil price as a result of special (unexpected) events and the arrival of abnormal information into the market. Significant events can lead to increase or decrease in global demand and supply. Activities of speculators also cause strong fluctuations that result in jump occurrence.

Between 2005 and 2008, there was a rise in commodity index trading. This coupled with the boom in commodity prices speculated by the financial traders, i.e. the main driver of oil price. As a result, a large percentage of price jumps emerge.

From 2005 to the first half of 2008, price jumps are attributed to speculation [23,24]. The period from 2005-2006 was dominated by small jumps as price changed gradually. Jumps in oil price in mid 2006 are attributed to the tension from the event of North-Korea's missile launch [23]. Jumps observed in October 2007 occurred as a result of tension in Turkey at that time [23].

Price jumps experienced during the downward price trend of the second half of 2008, are attributed to the sharp decline in global demand as a result of the global financial crisis of that period. Increased global supply caused the initial downward jumps. Other downward price jumps occurred in the second half of 2008 were the result of the global economic recession [24]. Jump sizes were the largest in this period compared to other price jumps in the whole study period (2005 - 2009).

There was an upward price reversal from December 2008, and the price steadily picked up in 2009. Jumps of small sizes, which are observed in 2009, occurred because of reduction in OPEC production, and expectation of growth in US GDP.

6 Perspective

Crude oil is the main source of energy throughout the world. Understanding the movement of crude oil spot price has been a concern for investors, traders, speculators and end-users of the product. This paper has provided a good model that explains crude oil price movement through describing the dynamics of the price process.
Empirical data for crude oil spot price for some Niger-Delta crude types, for January 2005- December 2009, were collected and used for validating the results. The data were tested for stationarity and independence of price increments. The tests confirmed that the crude oil spot price process is a Lévy process. Thus, a Lévy model of the jump-diffusion type was used in modeling crude oil price process. The parameters of the jump-diffusion process were estimated using the YUIMA package. A computational program written in R-programming language was developed and used to compute in-sample data forecasts for the jump-diffusion process. An out-of-sample forecast was attempted for 2010 oil price data. Values of the first four months were close empirical data, indicating that the model needs to be modified for long-term forecasting. Providing a good fit for the oil price data (2005-2009), the S-D model is presented as a good model describing the dynamics of crude oil price process.

Despite the challenges, the model showed its excellence in modeling the crude oil price process through fitting the empirical data. Accordingly, it will serve the oil industry through defining the price trends in the market.

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Chisara P. Ogbogbo received PhD. Degree in Mathematics (Major: Financial Mathematics) from the University of Ibadan, Nigeria. Her research interests are Financial Mathematics, Applied Mathematics and Mathematical Modelling. She is a lecturer in the Department of Mathematics, University of Ghana. She is a visiting research scholar to the Department of Mathematical Sciences, Stellenbosch University, South Africa. Dr. Ogbogbo has received various a number of grants and academic fellowships, including awards from MacArthur Foundation in the USA, Mathematical Research Institute (MRI), The African Mathematics Millennium Science Initiative(AMMSI) and Association of African Universities (AAU). She has written research papers published in competitive peer-reviewed International Journals of Mathematics and Science.