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Performance Analysis of an M/M/1 Queue with Close Down Periods, Server under Maintenance Subject to Catastrophe

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Abstract: Mathematical analysis of Queueing systems shows a significant part in wireless communication network such as channel control, energy saving schemes etc. Here, we consider a M/M/1 Queue with the server operating in three modes -Active mode, maintenance mode , sleep mode/close down period (with transitions from the maintenance mode to the sleep mode/close down period and active mode), subject to catastrophes. Catastrophes occur only when there are customers in the system and they wipe out the entire system resulting in the system being rendered inactive for a random period of time. Explicit expressions have been obtained for the transient probabilities and steady state probabilities of the close down period, maintenance state and system size along with the many performance measures. Influence of the different parameters on the steady state probabilities, system performance measures are studied using numerical examples.

Keywords: Catastrophe, close down state, maintenance state, transient probabilities, steady state probabilities

1 Introduction

The development of queueing theory has its origin in the study of congestion of telephone systems. Over the years, the study of queueing systems has grown tremendously, primarily due to the fact that queueing systems have enormous applications in wireless networks, telecommunication networks etc. A comprehensive over view of the fundamental techniques and standard results in queueing theory are given in the monographs of [1], [2], [3], [4], [5], [6], and [7].Wireless communications is one of the fastest growing segments of the communication industry. The rapid growth of wireless systems in areas such as Wireless sensor networks, automated factories, remote telemedicine etc. promise a bright future for wireless networks. WiMAX evolved to satisfy the requirement of having a wireless internet access and other broadband services which work well in those areas where it is difficult to provide wired infrastructure and economically not viable.
The IEEE 802.16 standard allows for subscriber stations to move around. On account of the movement of subscriber stations, the idea of power saving is a very substantial issue for the battery powered mobile stations [MSs]. The IEEE 802.16 e standard outlines sleep mode and idle mode operations on MAC layer to save the energy of the MSs. The idea behind the power saving mechanism is to employ sleep mode operation to minimize MS power usage due to minimum power consumption in sleep mode.
Various researchers have proposed different analytical models and acquired the performance of the close down period operations in the IEEE 802.16e system. See [8], [9], [10], [11], [12], [13], [14], [15],[16], [17], [18]. [19], [20], [21], [22], [15]. Several models analysed related to this area in [22]. In [23], analyzed the transient behaviour of a M/M/1 queue with the server operating in three modes- Active mode, maintenance mode , sleep mode. Transient analysis of M/M/1 queue with the server operating in three modes- Active mode, maintenance mode analyzed in [24] , sleep mode wherein at the the end of the maintenance period, a server either enters into

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active mode or it goes to sleep mode.
An important aspect in the dynamics of communication networks is the study of dramatic but relatively rare events which result in abrupt and often catastrophic changes in the network state. Such catastrophic events are commonly regarded to as phase transition. [25] gives an extensive survey of research phase transition in communication networks. When a catastrophe occurs, it invariably paralyzes/ destroys the systems and the system needs to start fresh. See [26], [27], [28]. Events that cause site failure, which propagate through a network in [29], [30], [31], [32], [33], [34], [35], [36], [37]. The Performance analysis measures for M/M/1 queue with the server operating in three modes- Active mode, maintenance mode , sleep mode subject to Catostrophe derived by [38].
This has motivated us to study and analyze the behavior of a M/M/1 queue where the server operates in three modes-active mode, maintenance mode , sleep mode/close down period, with transitions from the maintenance mode to the sleep mode/close down period and active mode, subject to catastrophes. The organization of the paper is as follows: Section 2 describes the mathematical model and closed-form expressions for the transient probabilities of the system are derived through the approach of integral equations; Section 3 provides an expression for the total power saved up to any time t. Steady state probabilities of the system are derived in section 4. Some fundamental performance measures under steady state conditions are deduced in section 5. Numerical examples are explained to illustrate The effect of system parameters on the various performance measures are illustrated by numerical examples in section 6. Section 7, concludes the paper.

2 Model description and Analysis

Consider a M/M/1 queue with infinite capacity. The arrivals follow a Poisson process with arrival rate λ while the service times are exponentially distributed with mean $\frac{1}{\mu}$.The system enters the close down state (sleep state) D when the server completes the services of all customers in the system. The close down period is assumed to follow an exponential distribution with mean $\frac{1}{\xi}$. If a new customer arrives into the system during the close down period, the close down period is interrupted and the server resumes service. If no customer arrives into the system for the entire duration of the close down period, then the system enters the preventive maintenance state M. The server remains in the maintenance mode for a random time is exponentially distributed with mean $\frac{1}{\eta}$. Any customer arriving to the system during the preventive maintenance state will not be permitted to enter the system and will be lost ever. Once the maintenance of the server is completed, the server returns to the sleep mode with probability $(1-p)$ and enters its functioning state (active state) with probability p ready to serve new customers. Customers arrive into the system as a Poisson process with arrival rate λ, during the working state, which comprises the active period (idle and busy) and close down period. Catastrophes are assumed to arrive as a Poisson process with rate γ. Once a catastrophe strikes the system, all the customers are wiped out from the system and the system enters the maintenance state.
We adopt the following notations: $X(t)$ - number of customers in the system at time t when the server is in active state
$J(t)$ - the state of the server at time t.
Then $X(t) \in \{0,1,2...\}$ and $J(t)$ = A if the server is in active state, D if the server is in sleep state, M if the server is in maintenance state
The process $X(t),J(t),t \geq 0$ is Markovian. The state space of the system is
$\Omega = \{(0,D),(0,M)\} \bigcup \{(0,A),(1,A),(2,A),...\}$
For brevity, the states $(0,D)$ and $(0,M)$ are denoted by D and M respectively; and the states (n,A), $n = 0,1,2,...$ are simply denoted by 0,1,2,...

The state transition diagram is as follows,

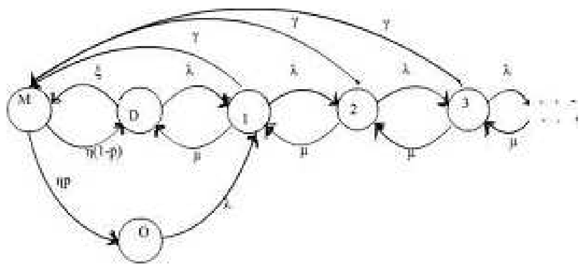

Fig. 1 Transition Diagram

Let $p(n,t) = p[X(t)=n], n = 0,1,2...$ be the probability that, n customers in the system at time t when the server is in active state, $p(M,t) = p[X(t)=M]$ be the probability that the server is in maintenance state (and that no customer arrives to the state) at time t, and $p(D,t) = p[X(t)=D]$ be the probability that the server is in sleep mode at time t.

Using Probability laws, we derive the following integral equations,

$$p(M,t) = \int_0^t p(D,u)\,\xi e^{-\eta(t-u)}du + \sum_{n=1}^{\infty}\int_0^t p(n,u)\,\gamma e^{-\eta(t-u)}du \quad (1)$$

$$p(D,t) = \int_0^t p(1,u)\,\mu e^{-(\lambda+\xi)(t-u)}du + \int_0^t p(M,u)\,\eta(1-p)e^{-(\lambda+\xi)(t-u)}du \quad (2)$$

$$p(0,t) = e^{-\lambda t} + \int_0^t p(M,u)\,\eta p e^{-\eta(t-u)}du \quad (3)$$

$$p(1,t) = \int_0^t p(D,u)\,\lambda e^{-(\lambda+\mu+\gamma)(t-u)}du + \int_0^t p(0,u)\,\lambda e^{-(\lambda+\mu+\gamma)(t-u)}du + \int_0^t p(2,u)\,\mu e^{-(\lambda+\mu+\gamma)(t-u)}du \quad (4)$$

$$p(n,t) = \int_0^t p(n-1,u)\,\lambda e^{-(\lambda+\mu+\gamma)(t-u)}du + \int_0^t p(n+1,u)\,\mu e^{-(\lambda+\mu+\gamma)(t-u)}du \quad (5)$$

Taking Laplace Transform of (1) to (5),

$$p^*(M,s) = \frac{\xi}{s+\eta}p^*(D,s) + \sum_{n=1}^{\infty}\frac{\gamma}{s+\eta}p^*(n,s) \quad (6)$$

$$p^*(D,s) = \frac{\mu}{s+\lambda+\xi}p^*(1,s) + \frac{\eta(1-p)}{s+\lambda+\xi}p^*(M,s) \quad (7)$$

$$p^*(0,s) = \frac{1}{s+\lambda} + \frac{\eta p}{s+\lambda}p^*(M,s) \quad (8)$$

$$p^*(1,s) = \frac{\lambda}{s+\lambda+\mu+\gamma}[p^*(D,s)+p^*(0,s)] + \frac{\mu}{s+\lambda+\mu+\gamma}p^*(2,s) \quad (9)$$

$$p^*(n,s) = \frac{\lambda}{s+\lambda+\mu+\gamma}p^*(n-1,s) + \frac{\mu}{s+\lambda+\mu+\gamma}p^*(n+1,s), n \geq 2 \quad (10)$$

Laplace Transform of the generating function,

$$G^*(u,s) = p^*(D,s) + p^*(M,s) + \sum_{n=0}^{\infty}p^*(n,s)u^n \quad (11)$$

From Equations (6) to (10),

$$G^*(u,s) = p^*(D,s) + p^*(M,s) + p^*(0,s) + \frac{\lambda u^2[p^*(D,s)+P^*(0,s)] - \mu u p^*(1,s)}{-\lambda u^2 + (s+\lambda+\mu+\gamma)u - \mu} \quad (12)$$

The zeros of $\lambda u^2 + (s+\lambda+\mu+\gamma)u - \mu = 0$ are

$\theta_1 = \frac{(s+\lambda+\mu+\gamma)-\sqrt{(s+\lambda+\mu+\gamma)^2-4\lambda\mu}}{2\lambda}$ and

$\theta_2 = \frac{(s+\lambda+\mu+\gamma)+\sqrt{(s+\lambda+\mu+\gamma)^2-4\lambda\mu}}{2\lambda}$

The zeros satisfy the following conditions where $0 < \theta_1 < 1 < \theta_2$ Invoking the analyticity of $G^*(u,s)$, we have
$\lambda u^2[p^*(D,s)+p^*(0,s)] - \mu u p^*(1,s) = 0$

$$p^*(D,s) + p^*(0,s) = \frac{\mu p^*(1,s)}{\lambda\theta_1} \quad (13)$$

On substituting (13) in (12),

$$G^*(u,s) = p^*(D,s) + p^*(M,s) + p^*(0,s) + \sum_{n=1}^{\infty}\left(\frac{\lambda\theta_1}{\mu}\right)^{n-1}u^n p^*(1,s) \quad (14)$$

Comparing Equations (11) and (14),

$$p^*(n,s) = \left(\frac{\lambda\theta_1}{\mu}\right)^{n-1}p^*(1,s); n = 2,3,\ldots \quad (15)$$

Using Equations (15) in (9),
$p^*(1,s) = \frac{\lambda\theta_1}{\mu}[p^*(D,s)+p^*(0,s)]$

$$p^*(1,s) = \frac{(s+\lambda+\mu+\gamma)-\sqrt{(s+\lambda+\mu+\gamma)^2-4\lambda\mu}}{2\mu}[p^*(D,s)+p^*(0,s)] \quad (16)$$

Inverting equation (16),

$$p(1,t) = \beta e^{\{-(\lambda+\mu+\gamma)t\}}I_1\frac{\left(2\sqrt{\lambda\mu}t\right)}{t}©[p(D,t)+p(0,t)] \quad (17)$$

where $\beta = \sqrt{\frac{\lambda}{\mu}}$ and ©denotes convolution and $I_n(.)$ is the modified Bessel Function of order n defined in [39],
where we have used the formula in [40].
$L^{-1}\left[\left(\frac{(s+\lambda+\mu+\gamma)-\sqrt{(s+\lambda+\mu+\gamma)^2-4\lambda\mu}}{2\lambda}\right)^n\right] = \left(\frac{\lambda}{\mu}\right)^{\frac{n}{2}}\frac{I_n\left(2t\sqrt{\lambda\mu}\right)}{t}$
Rewriting equations (7), (6) and (8) we have

$$p^*(D,s) = \frac{\mu(s+\eta)+\eta(1-p)\gamma\left(1-\frac{\lambda\theta_1}{\mu}\right)^{-1}}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi}p^*(1,s) \quad (18)$$

$$p^{*}\,(M,s)=\frac{\xi}{s+\eta}\left[\frac{\mu(s+\eta)+\eta(1-p)\gamma\left(1-\frac{\lambda\theta_1}{\mu}\right)^{-1}}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi}\right]p^{*}(1,s)+\frac{\gamma}{s+\eta}\left(1-\frac{\lambda\theta_1}{\mu}\right)^{-1}p^{*}(1,s) \tag{19}$$

$$p^{*}\,(0,s)=\frac{\eta p}{s+\lambda}\left\{\frac{\xi}{s+\eta}\left[\frac{\mu(s+\eta)+\eta(1-p)\gamma\left(1-\frac{\lambda\theta_1}{\mu}\right)^{-1}}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi}\right]p^{*}(1,s)+\frac{\gamma}{s+\eta}\left(1-\frac{\lambda\theta_1}{\mu}\right)^{-1}p^{*}(1,s)\right\}+\frac{1}{s+\lambda} \tag{20}$$

By sustituting (18) and (20) in (16) we get,

$p^{*}\,(1,s)=\frac{1}{s+\lambda}\left(\frac{\lambda\theta_1}{\mu}\right)\sum_{n=0}^{\infty}(H^{*}\,(s))^{n}$

$$p^{*}\,(1,s)=\frac{1}{s+\lambda}\left(\frac{\lambda\theta_1}{\mu}\right)\frac{1}{1-H^{*}\,(s)} \tag{21}$$

where

$$H^{*}(s)=\frac{\lambda\theta_1}{\mu}\left[H_1(s)+H_2(s)+H_3(s)\right] \tag{22}$$

where

$$H_1(s)=\frac{\mu\,(s+\eta)-\eta(1-p)\gamma\sum_{n=0}^{\infty}\left(\frac{\lambda\theta_1}{\mu}\right)^{n}}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi};$$

$$H_2(s)=\frac{\eta p\xi}{(s+\lambda)(s+\eta)}\left(\frac{\mu\,(s+\eta)-\eta(1-p)\gamma\sum_{n=0}^{\infty}\left(\frac{\lambda\theta_1}{\mu}\right)^{n}}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi}\right);$$

$$H_3(s)=\frac{\eta p\gamma}{(s+\lambda)(s+\eta)}\sum_{n=0}^{\infty}\left(\frac{\lambda\theta_1}{\mu}\right)^{n}$$

We have the inverse Laplace transform,

$$L^{-1}\,(\theta_1)=e^{-(\lambda+\mu+\gamma)t}\left(\frac{\mu}{\lambda}\right)^{1/2}\frac{I_1\left(2\sqrt{\lambda\mu t}\right)}{t}$$

$$L^{-1}\,(\theta_1^{n})=e^{-(\lambda+\mu+\gamma)t}\left(\frac{\mu}{\lambda}\right)^{n/2}\frac{nI_n\left(2\sqrt{\lambda\mu t}\right)}{t}$$

By using the notation

$$\Phi_{1;n}\,(t)=L^{-1}\,(\theta_1^{n})$$

By inverting (22), we get

$$H(t)=\frac{\lambda}{\mu}\Phi_{1;1}(t)\copyright\left[H_1(t)+H_2(t)+H_3(t)\right] \tag{23}$$

where

$$H_1(t)=\mu\sum_{l=1}^{\infty}\frac{\eta^{l}(l-p)^{l}\xi^{l}}{l!(l-1)!}e^{-(\lambda+\xi)t}t^{l}\copyright e^{-\eta t}t^{l-1}+\sum_{l=1}^{\infty}\sum_{n=0}^{\infty}\frac{\eta^{l+1}(l-p)^{l+1}\xi^{l}}{l!l!}e^{-(\lambda+\xi)t}t^{l}\copyright e^{-\eta t}t^{l}\copyright\left(\frac{\lambda}{\mu}\right)^{n}\Phi_{1;n}(t);$$

$$H_2(t)=\eta p\xi e^{-\lambda t}\copyright\left[\mu\sum_{m=0}^{\infty}\frac{\eta^{m}(1-p)^{m}\xi^{m}}{m!m!}e^{-(\lambda+\xi)t}t^{m}\copyright e^{-\eta t}t^{m}+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\gamma\eta^{m+1}(1-p)^{m+1}\xi^{m}}{m!(m+1)!}e^{-(\lambda+\xi)t}t^{m}\copyright e^{-\eta t}t^{m+1}\copyright\left(\frac{\lambda}{\mu}\right)^{n}\Phi_{1;n}(t)\right];$$

$$H_3(t)=\eta p\gamma e^{-\lambda t}\copyright e^{-\eta t}\copyright\sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^{n}\Phi_{1;n+1}(t)$$

Inverse Laplace Transform of (21),

$$p(1,t)=e^{-\lambda t}\copyright\Phi_{1;1}(t)\copyright\sum_{n=0}^{\infty}\left(H^{\copyright(n)}(t)\right)^{n} \tag{24}$$

where $H^{\copyright(n)}$ is the n-fold convolution of $H(t)$. Taking Inverse Laplace transform of (18), we get

$$p(D,t)=\mu\left[e^{-(\lambda+\xi)t}+\sum_{l=1}^{\infty}\frac{\eta^{l}(l-p)^{l}\xi^{l}}{l!(l-1)!}e^{-(\lambda+\xi)t}t^{l}\copyright e^{-\eta t}t^{l-1}+\frac{\gamma}{\mu}\copyright\left\{\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{\eta^{l+1}(1-p)^{l+1}\xi^{l}}{l!l!}e^{-(\lambda+\xi)t}t^{l}\copyright e^{-\eta t}t^{l}\copyright\left(\frac{\lambda}{\mu}\right)^{n}\left(\frac{\mu}{\lambda}\right)^{n}\Phi_{1;n}(t)\right\}\right]\copyright p(1,t) \tag{25}$$

Taking Inverse Laplace transform of (19), we get

$$p(M,t)=\xi\mu\left[e^{-(\lambda+\xi)t}\copyright e^{\eta t}+\sum_{l=1}^{\infty}\frac{\eta^{l}(l-p)^{l}\xi^{l}}{l!(l-1)!}e^{-(\lambda+\xi)t}t^{l}\copyright e^{-\eta t}t^{l-1}+\frac{\gamma}{\mu}\copyright\left\{\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{\eta^{l+1}(1-p)^{l+1}\xi^{l}}{l!l!}e^{-(\lambda+\xi)t}t^{l}\copyright e^{-\eta t}t^{l}\copyright\left(\frac{\lambda}{\mu}\right)^{n}\Phi_{1;n}(t)\right\}\right]\copyright p(1,t) \tag{26}$$

Taking Inverse Laplace transform of (20), we get

$$p(0,t)=e^{-\lambda t}+\xi\mu\left[e^{-(\lambda+\xi)t}©e^{\eta t}+\sum_{l=1}^{\infty}\frac{\eta^l(l-p)^l\xi^l}{l!(l-1)!}e^{-(\lambda+\xi)t}t^l©e^{-\eta t}t^{l-1}+\frac{\gamma}{\mu}©\left\{\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{\eta^{l+1}(1-p)^{l+1}\xi^l}{l!l!}e^{-(\lambda+\xi)t}t^l©e^{-\eta t}t^l©\left(\frac{\lambda}{\mu}\right)^n\Phi_{1;n}(t)\right\}\right]©p(1,t) \quad (27)$$

Taking Inverse Laplace transform of (15), we get

$$p(n,t)=\Phi_{1;n-1}(t)©p(1,t),n=2,3,... \quad (28)$$

3 TOTAL POWER SAVED UP TO TIME t

Power is saved by switching the server to sleep or maintenance mode.During the sleep or maintenance modes, less power is consumed. The rate of power saving in these modes are different. We calculate them separetely.

Mean time spent in sleep in the interval$[0,t]$:

Let Y(t) be the total sleep in [0,t]. Using J(t), we obtain the stochastic integral representation of Y(t) as follows:

$$Y(t)=\int_0^t\delta_{J(u),D}du, \quad (29)$$

where $\delta_{j,k}$ is the kronecker delta function defined by,

$$\delta_{j,k}=\begin{cases}1 \text{ if } j=k\\ 0 \text{ otherwise}\end{cases}$$

We obtain,

$$E(Y(t))=\int_0^t E(\delta_{J(u),D})du=\int_0^t P\{J(u)=D\}du \quad (30)$$

Taking laplace transform on both sides of equation(30) and using equations (18) and (20), we get

$$y^*(s)=\left(\frac{1}{s}\right)\left[\frac{\mu(s+\eta)+\eta(1-p)\gamma\sum_{n=0}^{\infty}\left(\frac{\lambda\theta_1}{\mu}\right)^n}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi}\left\{\frac{\lambda\theta_1}{(s+\lambda)\mu}\sum_{n=0}^{\infty}(H^*(s))^n\right\}\right]$$

$$y^*(s)=\frac{\lambda\theta_1}{s(s+\lambda)}\left\{\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\frac{\eta^l(1-p)^l\xi^l}{(s+\lambda+\xi)^{l+1}(s+\eta)^l}(H^*(s))^n+\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{\eta^{l+1}(1-p)^{l+1}\xi^l\gamma}{\mu(s+\lambda+\xi)^{l+1}(s+\eta)^l}\left(\frac{\lambda\theta_1}{\mu}\right)^k(H^*(s))^n\right\} \quad (31)$$

where $y^*(s)=L(E(Y(t)))$,
Inverting $y*(s)$, we get

$$E(Y(t))=\lambda\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\eta^l(1-p)^l\xi^l©e^{-\lambda t}©\Phi_{1;1}(t)©e^{-\eta t}\frac{t^{l-1}}{(l-1)!}©e^{-(\lambda+\xi)t}\frac{t^l}{l!}©H^{©(n)}t+\lambda\gamma\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\eta^{l+1}(1-p)^{l+1}\xi^l©e^{\lambda}t©\Phi_{1;1}(t)©e^{-\eta t}\frac{t^l}{l!}©e^{-(\lambda+\xi)t}\frac{t^l}{l!}©\left(\frac{\lambda}{\mu}\right)\Phi_{1;1}(t)©H^{©(n)}t \quad (32)$$

Mean time spent in Maintenance mode in the interval$[0,t]$:

Let Z(t) be the total maintenance time of the server in [0,t].

$$Z(t)=\int_0^t\delta_{J(u),M}du, \quad (33)$$

$$E(Z(t))=\int_0^t E(\delta_{J(u),M})du=\int_0^t P\{J(u)=M\}du \quad (34)$$

Taking laplace transform on both sides of equation(34) and using equations (19) and (20), we get $z^*(s)$

where $z^*(s) = L(E(Z(t)))$,
Inverting $z*(s)$, we get

$$E(Z(t)) = \sqrt{\lambda\mu}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\eta^l(1-p)^l\xi^{l+1}©e^{-\lambda t}© \Phi_{1;1}(t)©e^{-\eta t}\frac{t^l}{l!}©e^{-(\lambda+\xi)t}\frac{t^l}{l!}©H^{©(n)}t + \sqrt{\frac{\lambda}{\mu}}\gamma\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\eta^{l+1}(1-p)^{l+1}\xi^{l+1}©e^{\lambda}t© \Phi_{1;1}(t)©e^{-\eta t}\frac{t^{l+1}}{l+1!}©e^{-(\lambda+\xi)t}\frac{t^l}{l!} ©\left(\frac{\lambda}{\mu}\right)\Phi_{1;k}(t)©H^{©(n)}t \frac{\gamma}{\mu}\sqrt{\lambda\mu}\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}e^{-\lambda t}©\Phi_{1;1}(t)e^{-\eta t} ©\left(\frac{\lambda}{\mu}\right)\Phi_{1;k}(t)©H^{©(n)}t \quad (35)$$

4 STEADY STATE DISTRIBUTION

We define the steady state probabilities of the queueing system by the equations:

$$p(D) = \lim_{t\to\infty} p(D,t)$$
$$p(M) = \lim_{t\to\infty} p(M,t)$$
$$p(n) = \lim_{t\to\infty} p(n,t), n = 0,1,2...$$

Then,by using the final value theorem for Laplace transform, we get

$$p(D) = \lim_{s\to 0} sp^*(D,s) \quad (36)$$
$$p(M) = \lim_{s\to 0} sp^*(M,s) \quad (37)$$
$$p(n) = \lim_{s\to 0} sp^*(n,s), n = 0,1,2... \quad (38)$$

$$p(D) = \lim_{s\to 0} s\left\{\frac{\mu(s+\eta)+\eta(1-p)\gamma\left(1-\frac{\lambda\theta_1}{\mu}\right)^{-1}}{(s+\lambda+\xi)(s+\eta)-\eta(1-p)\xi}\right\}p^*(1,s)$$
$$p(D) = \frac{\mu\eta+\frac{\eta(1-p)\gamma}{1-\rho}}{(\lambda+\xi)\eta-\eta(1-p)\xi}p_1$$

$$p(D) = \frac{\mu+\frac{(1-p)\gamma}{1-\rho}}{\lambda+p\xi}p_1 \quad (39)$$

$$p(M) = \left[\frac{\xi}{\eta}\left(\frac{\mu+\frac{(1-p)\gamma}{1-\rho}}{\lambda+p\xi}\right)+\frac{\gamma}{\eta(1-\rho)}\right]p_1 \quad (40)$$

$$p(0) = \frac{\eta p}{\lambda}\left[\frac{\xi}{\eta}\left(\frac{\mu+\frac{(1-p)\gamma}{1-\rho}}{\lambda+p\xi}\right)+\frac{\gamma}{\eta(1-\rho)}\right]p_1 \quad (41)$$

$$p(n) = \rho^{n-1}p_1 \quad (42)$$

By total probability
$p(D)+p(M)+p(0)+\sum_{n=1}^{\infty}p(n) = 1$

$$p(1) = \frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1} \quad (43)$$

$p(M)$ may be written as

$$p(M) = \left[\frac{\mu}{\lambda+p\xi}\frac{\xi}{\eta}\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)+\frac{\gamma}{\eta}\frac{1}{1-\rho}\right]\left[\frac{\mu}{\lambda+p\xi}\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(1+\frac{\xi}{\eta}+\frac{p\xi}{\lambda}\right)+\frac{1}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{p\gamma}{\lambda}\right)\right]^{-1}$$
$$= \frac{1}{\eta}\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)+\frac{\gamma}{\eta}\frac{1}{1-\rho}\left(\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1} \quad (44)$$

$p(0)$ may be written as

$$p(0) = \left[\frac{\mu}{\lambda+p\xi}\frac{p\xi}{\lambda}\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)+\frac{p\gamma}{\lambda}\frac{1}{1-\rho}\right]\left[\frac{\mu}{\lambda+p\xi}\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(1+\frac{\xi}{\eta}+\frac{p\xi}{\lambda}\right)+\frac{1}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{p\gamma}{\lambda}\right)\right]^{-1}$$
$$= \left[\frac{p}{\lambda}\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)+\frac{\gamma}{\mu}\frac{p}{1-\rho}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1} \quad (45)$$

$p(D)$ may be written as

$$p(D) = \frac{1}{\xi}\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1} \quad (46)$$

$p(n)$ may be written as

$$p(n) = \rho^{n-1}\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1} \quad (47)$$

where

$$\rho = \frac{(\lambda+\mu+\gamma)-\sqrt{(\mu-\lambda+\gamma)^2+4\lambda\gamma}}{2\mu}$$

5 Steady-state Performance Measures

Mean and Variance of the number of customers in the system:
Let X be the number of customers in the steady-state condition and $\Pi(z)$ be its probability generating function. Then we obtain

$$\Pi(z) = E\left(z^X\right) = p(D)+p(M)+p(0)+\textstyle\sum_{n=1}^{\infty} p(n)$$

$$\Pi(z) = \left(\frac{1-\rho}{1-\rho z}\right)\left[\left\{\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{\left[(1-\rho)+\frac{(1-\rho)\gamma}{\mu}\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)\right]}\frac{\left[z+\left(\frac{1-\rho z}{1-\rho}\right)\left(\frac{\gamma}{\eta}+\frac{p\gamma}{\lambda}\right)\right]}{\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left(1+\frac{\gamma}{\eta}+\frac{p\gamma}{\lambda}\right)}\right\}\left\{\frac{\left(\frac{1-\rho z}{1-\rho}\right)}{\left[(1-\rho)+\frac{(1-\rho)z}{\mu}\right]}\frac{\left[(1-\rho)+\frac{(1-\rho)\gamma}{\mu}\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)\right]}{\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left(1+\frac{\gamma}{\eta}+\frac{p\gamma}{\lambda}\right)}\right\}\right]$$

Differentiating (41) with respect to z and substituting $z = 1$, we get

$$E(X) = \frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

To find $E(X^2) = E[X(X-1)] - E(X)$, Diff $\Pi(z)$ twice with respect to z,

$$E\left(X^2\right) = \left[\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left(\frac{1+\rho}{(1-\rho)^2}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

Variance of X as $E\left[X^2\right]-[E(X)]^2$

$$V(X) = \frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left(\frac{1+\rho}{(1-\rho)^2}\right)}{\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)} - \left\{\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)}\right\}^2$$

6 System throughput

The system throughput U, the rate at which customers exit the queue, there are one or more customers in the system, with the exit rate μ

$U = [1-p(0)-p(D)-p(M)]\mu$
Using equations $(37),(38)$ and (39), we get

$$U = \frac{\lambda}{1-\rho}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

The effective arrival rate λ_{eff} (the total arrival rate when the server is available) is defined as,
$\lambda_{eff} = [1-p(M)]\lambda$

$$\lambda_{eff} = \lambda\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{p\gamma}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

There are several general descriptions of our queuing system,
Under Steady state,

$$P(serverisbusy) = \sum_{n=1}^{\infty} p(n) = p(1)+\sum_{n=2}^{\infty} p(n) = \frac{p(1)}{1-\rho}$$

$$= \left[\frac{1}{1-\rho}\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

$P(serverisavailable) = 1 - p(M)$

$$= \left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma p}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

P(customer is served immediately upon arrival)=$p(0)+p(D)$

$$= \left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\frac{\gamma p}{\lambda}\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

P_W=P[an arriving customers has to wait for service]=$1-[p(0)+p(D)]$

$$= \left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\frac{1}{\eta}+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

The probability of atleast k or more customers in the system is given as,

$$P(X \geq k) = \sum_{n=1}^{\infty} p(n) = \sum_{n=1}^{\infty} \rho^{n-1} p(1) = \left(\frac{\rho^{k-1}}{1-\rho}\right) p(1)$$

$$P(X \geq k) = \left[\left(\frac{\rho^{k-1}}{1-\rho}\right)\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\eta}+\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\eta}+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

P(server is busy/server is available)=$\frac{\sum_{n=1}^{\infty} p(n)}{1-p(M)}$

$$= \frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

P(server is idle/server is available)=$\frac{p(0)+p(D)}{1-p(M)}$

$$= \left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\frac{1}{\eta}+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma}{\lambda}\right)\right]\left[\left(1+\frac{(1-p)\gamma}{(1-\rho)\mu}\right)\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)+\frac{\frac{\lambda}{\mu}\left(\frac{1}{\xi}+\frac{p}{\lambda}\right)}{1-\rho}\left(1+\frac{\gamma p}{\lambda}\right)\right]^{-1}$$

7 Numerical Illustrations

We study the effects of the various parameters on the following steady state probabilities: p(0) -the server in idle mode, p(M)-maintenance mode and p(D)-sleep mode; the system throughput U, and the average number of customers E(X) in the system. The parameters are so chosen that they satisfy the condition that $\lambda < \mu, \xi > 0, \eta > 0$ and $\gamma > 0$

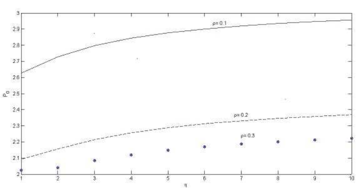

Fig. 2 $p(0)$ -for fixed value of $\xi = 2$

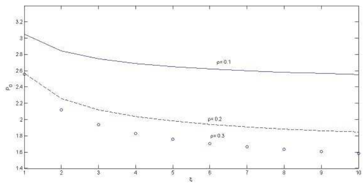

Fig. 3 $p(0)$ -for fixed value of $\eta = 4$

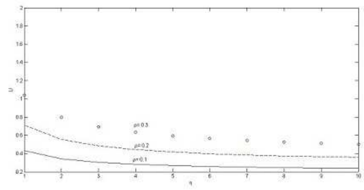

Fig. 4 U-for fixed value of$\xi = 2$

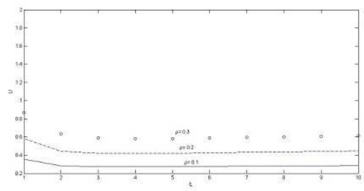

Fig. 5 U-for fixed value of$\eta = 4$

Figure 2 shows that $p(0)$ is a decreasing function of η, while Figure 3 shows that $p(0)$ is an increasing function of ξ and in both the cases p(0) is a decreasing function of ρ.

Figures 4 and 5 reveal that the system throughput U decreases slowly for increasing values of η and ξ while decreasing for increasing values of ρ.

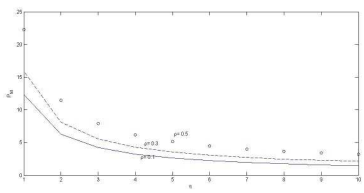

Fig. 6 $p(M)$-for fixed value of$\xi = 2$

From figures 6 and 7 we see that $p(M)$ is a decreasing function of η and ξ and is an increasing function of ρ.

In figures 8 and 9, we see that the close down probability $p(D)$ is an increasing function of η, is a

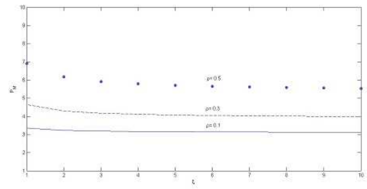

Fig. 7 $p(M)$-for fixed value of $\eta = 4$

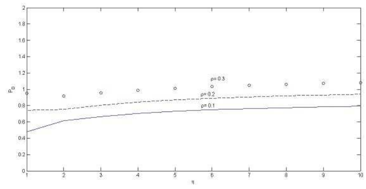

Fig. 8 $p(D)$ -for fixed value of $\xi = 2$

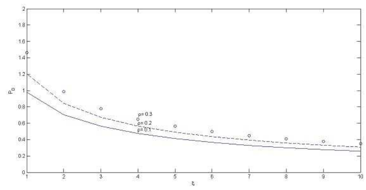

Fig. 9 $p(D)$-for fixed value of $\eta = 4$

gradually decreasing function of ρ; $p(D)$ is a sharply decreasing function of ξ. Figures 10 and 11 show that ,

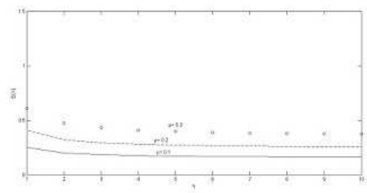

Fig. 10 $E(X)$-for fixed value of $\xi = 2$

$E(X)$ is a decreasing function of η and ξ for fixed values of ρ.

8 Conclusion

We have studied a M/M/1 system with server operating in three modes subject to catastrophes. Explicit

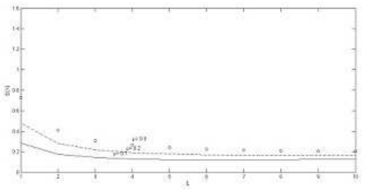

Fig. 11 $E(X)$-for fixed value of $\eta = 4$

expressions for the transient probabilities of the system in the three different modes are obtained. Further Mean time spent in sleep and maintenance mode have been derived. The steady state probabilities and performance measures such as mean of the system size, availability of the server, system throughput and mean waiting time of an arbitrary customer in the system have been obtained. Finally, graphical illustrations have been presented and the effects of various parameters on the system performance measures are studied.

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