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Software Analysis on Free-Edge Stresses in the composite laminatesusing various Approaches and its Applications

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Abstract: A novel model is presented to determine the inter-laminar stresses near the free edges of composite laminate plates subjected to mechanical loading. The proposed method is based on an admissible function representing inter-laminar stresses which account for the effects of both the global mismatches and the local mismatches in two elastic properties such as poisson's ratio and coefficient of mutual influence. For this purpose, new mechanical mismatch terms are defined to reflect an effective deformation on a $0/90/90/0$ cross ply composite laminated structure. It is found that the present approximate method yields inter laminar stress results in an efficient, fast and reliable way. It is observed that unlike some previous approximation methods, the proposed method is numerically robust and stable. For validating the results of the proposed approach, experimental and numerical analysis were performed and presented.

Keywords: inter-laminar stress; free edge stress; cross ply composite structure; lamination theory

1 Introduction

Various research works are going presently towards the applications and usage of selective composite materials because of their varieties of desirable advantages. Many works on composite materials have been focused in the past decades such as characterization, synthesis, mechanical property study, study on property improvement, etc., by adding suitable reinforcement particles and some of the works were with replacing synthetic fibers by natural fibers with reasonable strength. Concentration on the analytics of free edge stress part is less in the research side. Consideration of free edge stress evaluations in the analytics of composite materials for suitable application is most important since it plays a major role. Free edge stress is the stress acting near the free edge of the composite material while subjected to mechanical loading; and which will initiate the failure of the structure after crossing the safe loading condition. The measurement of free edge stress is somewhat difficult; and few researchers have worked on it and given some approximate approaches to calculate it.

N.J Pagano et al (1977) invented a new approximate theory to define the complete stress fields within an arbitrary composite laminate. Weakness points in the previously proposed laminate theories were discussed and they were demonstrated how those could be overcome in the present formulation [1]. Gerry Flanagan et al (1993) found a method that was developed for determining the free edge stresses in composite laminates. A relatively simple and efficient method was demonstrated for determining the stresses near the free edge of general composite laminates. Based on an average stress convergence criterion, a 16-ply laminate was analyzed using 16 terms in the series [2]. MasoudTahani et al (2002) explained a layer wise theory is used to investigate analytically the inter laminar stresses near the free edges of general cross – ply composite laminates under uniform axial extension. In their work, composite laminates of finite dimensions were considered and three-dimensional stresses in the interior and the boundary- layer regions were calculated. The results obtained from this theory are compared with those available in the literature [3]. ZongshuTian et al (2004) suggested a new three

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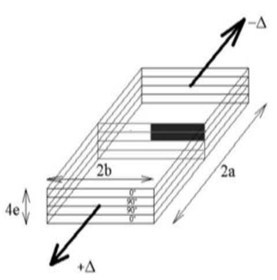
Table 1: Loading details of the composite structure

Material properties		Geometric details	Loading details
Material Name	E glass-Epoxy	e = 1mm	
Youngs Modulus	E1 = 39 GPa; E2 = E3 = 8.6 GPa	h = 4e	
Shear Modulus	G12 = G13 = G23 = 3.8 GPa	b = 10e	$\Delta = 100KN/m$
Poissons Ratio	$\nu 12 = \nu 13 = \nu 23 = 0.28$	a = 8.25b	

dimensional hybrid stress element with a traction-free planar surface has been developed for efficient analysis of the inter-laminar stresses near traction-free straight boundaries. The use of such special element in the finite element solution has been shown to be highly accurate when only a very coarse element mesh is used near the free-edge [4]. Viet-Tung Nguyen et al (2007) investigated free-edge laminates subjected to uni axial extension and uniform temperature variation by using multi-particle finite element. The two dimensional finite element approach which they followed provides very accurate three dimensional free edge stress field [5].
From the literature review as detailed above, estimation of inter-laminar stresses is important because they have a marked effect on the failure strengths of composite laminates. Accurate determination of inter laminar stresses near the free edge is therefore crucial to correctly describe the laminate behavior, since this causes the de-lamination failure of the composite laminate. This work explained the determination of inter-laminar stress values with the help a novel analysis termed as lamination theory, numerical analysis and experimental analysis.

2 Experimental Analysis of Laminate Subjected to Uniform Axial Load



4cm

Fig. 1: Laminate subjected to uniform axial load

The four layer finite width bidirectional composite laminate of length 2a, width 2b and height 4e was considered for this work has an uniform axial load at the two ends as shown in fig 1. The properties of the composite laminate are shown in table 1. The test specimen was prepared in ASTM standard size (ASTM D3039) for tensile testing. This test method is designed to produce tensile property data for material specifications, research and development, quality assurance, and structural design and analysis. Several factors influence the tensile response of the material or structure such as material, methods of material preparation, specimen preparation, specimen stacking sequence, environment of testing, specimen conditioning, speed of testing, specimen alignment and gripping, time at temperature, volume percent reinforcement and void content. The prepared test specimen with ASTM D3039 is shown in figure 2. In the proposed work, the inter laminar stress

Fig. 2: Test specimen with ASTM D 3039

values in free edges are to be predicted to near the fixed edges. But, it is not possible to measure the stress induced exactly at a particular location with the help of Universal Testing Machine. For that, we fit the strain gauges in the test specimen to predict the strain values in some required particular points near the free edges by the use of strain indicator. The strain values are calculated from the strain indicator results by using the following formula.

$$GF = \frac{(mVN)1000}{D \times B}$$

Here, GF = Gauge factor setting on the instrument
mVN = Rated output of the transducer/ sensitivity
D = Desired display of full scale output
B = Bridge selection
Full Bridge = 1 Half Bridge =$\frac{1}{2}$ Quarter Bridge =$\frac{1}{4}$.

The strain value of the desired points are measured then the free edge stress values were calculated by using the relation between stress and strain. The test specimen fixed with strain gauge is shown in the figure 3.

3 Numerical Analysis

By using the ANSYS 11 software, the four layer finite width bidirectional composite laminate of length 165 mm, width 20 mm and thickness 4 mm was modeled by the

Fig. 3: Test Specimen with strain Gauge

use of 4 node shell 181 element. The model was meshed with the size of element is 5 mm. The input axial loading condition was given as $100KN/m$ at the two ends.

4 Lamination Theory

Lamination theory is generally used [16, 17] to calculate the stresses and strains in each lamina of a thin laminated structure Stiffness matrix:

$$[\underline{Q}] = [\underline{Q}_{11}\ \underline{Q}_{12}\ \underline{Q}_{16}\ \underline{Q}_{12}\ \underline{Q}_{22}\ \underline{Q}_{26}\ \underline{Q}_{16}\ \underline{Q}_{26}\ \underline{Q}_{66}]$$

Here,

$$\underline{Q}_{11} = U_1 + U_2 \cos\cos 2\theta + U_3 \cos 4\theta$$
$$\underline{Q}_{12} = \underline{Q}_{12} = U_4 - U_3 \cos 4\theta$$
$$\underline{Q}_{22} = U_1 - U_2 \cos\cos 2\theta + U_3 \cos 4\theta$$
$$\underline{Q}_{16} = \frac{1}{2} U_2 \sin\sin 2\theta + U_3 \sin 4\theta$$
$$(\underline{Q})_{26} = \frac{1}{2} U_2 \sin\sin 2\theta - U_3 \sin 4\theta$$
$$\underline{Q}_{66} = U_5 - U_3 \cos 4\theta$$
$$U_1 = \frac{1}{8}(3Q_{11} + 3\underline{Q}_{22} + 2\underline{Q}_{12} + 4\underline{Q}_{66})$$
$$U_2 = \frac{1}{2}(Q_{11} - Q_{22})$$
$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})$$
$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})$$
$$U_5 = \frac{1}{2}(U_1 - U_4)$$
$$Q_{11} = \frac{E_{11}}{1 - \circ_{12}v_{21}}$$
$$Q_{22} = \frac{E_{22}}{1 - \circ_{12}v_{21}}$$
$$Q_{12} = Q_{21} = \frac{v_{21}E_{22}}{1 - \circ_{12}V_{21}}$$
$$Q_{66} = G_{12}$$
$$[\sigma_{xx}\sigma_{yy}\tau_{xy}] = [\underline{Q}_1 1\underline{Q}_{12}\underline{Q}_{16}\underline{Q}_{21}\underline{Q}_{22}\underline{Q}_{26}\underline{Q}_{16}\underline{Q}_{26}\underline{Q}_{66}]$$
$$[\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{xy}]$$

The midplane strains are,

$$\odot_{xx} = \frac{A_{22}}{(A_{11}A_{22} - A_{12}^2)} N_{XX},$$
$$\odot_{yy} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} N_{XX},$$
$$\odot_{xy} = 0$$

Here, N_{XX} is the tensile load in X direction.

$$A_{mn} = \sum_{j=1}^{N} (Q_{mn})_j (h_j - h_{j-1})$$
$$B_{mn} = \frac{1}{2}\sum_{j=1}^{N} \odot(Q_{mn})_(j)(h_j^2 - h_{j-1}^2)$$
$$D_{mn} = \frac{1}{3}\sum_{j=1}^{N} \odot(\underline{Q}_{mn})_j (h_j^3 - h_{j-1}^3)$$

4.1 Prediction of Inter-laminar Stress by Lamination Theory

$$Q_{11} = \frac{E_{11}}{1 - v_{12}v_{21}} = \frac{44.8}{1 - (0.28 * 0.28} = 48.611GPa$$

$$Q_{22} = \frac{E_{22}}{1 - v_{12}v_{21}} = \frac{7.27}{1 - (0.28 * 0.28)} = 7.88GPa$$

$$Q_{12} = Q_{21} = \frac{v_{12}E_{22}}{1 - v_{12}v_{21}} = \frac{0.28 * 7.27}{1 - (0.28 * 0.28)} = 2.201GPa$$

$$Q_{66} = G_{12} = 4.86GPa$$

$$U_1 = \frac{1}{8}(3 \times 48.611 + 3 \times 7.88 + 2 \times 2.208 + 4 \times 4.86)$$
$$= 24.166GPa$$

$$U_2 = \frac{1}{2}(48.611 - 7.88)$$
$$= 20.365GPa$$
$$U_3 = \frac{1}{8}(48.611 + 7.88 - 2 \times 2.208 - 4 \times 4.86)$$
$$= 4.079GPa$$
$$U_4 = \frac{1}{8}(48.611 + 7.88 + 6 \times 2.208 - 4 \times 4.86)$$
$$= 6.28GPa$$
$$U_5 = \frac{1}{2}(48.611 - 7.88)$$
$$= 8.939GPa$$

Stiffness Matrix $[\underline{Q}]$ for θ° Lamina:

$$\underline{Q}_{11} = 24.166 + 20.365\,\mathrm{coscos}(2*0) + 4.079\,\mathrm{coscos}(4*0)$$
$$= 48.61\,GPa$$
$$\underline{Q}_{12} = \underline{Q}_1 2 = 19.38 - 16.2?coscos(4*0)$$
$$= 2.201\,GPa$$
$$\underline{Q}_{22} = 63.5 - 64.56\,\mathrm{coscos}(2*0) + 16.2?coscos(4*0)$$
$$= 7.88\,GPa$$
$$\underline{Q}_{16} = \frac{1}{2}(64.56\,\mathrm{sinsin}(2*0) + 16.2\,\mathrm{sinsin}(4*0))$$
$$= 0$$
$$\underline{Q}_{26} = \frac{1}{2}(64.56\,\mathrm{sinsin}(2*0) - 16.2\,\mathrm{sinsin}(4*0))$$
$$= 0$$
$$\underline{Q}_{66} = 22.06 - 16.2\,\mathrm{coscos}(4*0)$$
$$= 4.86\,GPa$$
$$[\underline{Q}]_{\theta^{\circ}} = [48.612.20102.2017.880004.86]\,GPa$$

Similarly, $[\underline{Q}]_{90^{\circ}} = [7.882.20102.20148.610004.86]$ GPa
Extensional stiffness matrix

$$[A] = 2\times[10]^{-3}[48.612.20102.2017.880004.86] +$$
$$2\times 10^{-3}[7.882.20102.20148.610004.86]$$
$$[A] = [112.988.80408.804112.9800019.44]\times 10^{6}\,N/m$$

Coupling stiffness matrix

$$[B] = \frac{1}{2}\sum_{j=1}^{N}(\sqcap \underline{Q}_{mn})_j(\sqcap_j^2 - \sqcap_{j-1}^2)$$
$$[B] = 0$$

Bending stiffness matrix

$$[D] = \frac{1}{3}\sum_{j=1}^{N}(\underline{Q}_{mn})_j(\sqcap_j^3 - \sqcap_{j-1}^3)$$
$$= 37.33\times 10^{-9}[\underline{Q}_{mn}]_{\theta^{\circ}} + 5.33\times 10^{-9}[\underline{Q}_{mn}]_{90^{\circ}}$$
$$[D] = 37.33\times 10^{-9}[48.612.20102.2017.880004.86] +$$
$$5.33\times 10^{-9}[7.882.20102.20148.610004.86]$$
$$[D] = [5465.93135.660135.661334.09000249.98]\frac{N}{m}$$

Here, $N_{XX} = 100\frac{KN}{m}$

$$\odot_{XX}? = \frac{112.98}{(112.98\times 112.98 - 8.804^2}\times 100\times 10^{6}$$
$$= 8.905\times 10^{-6}$$
$$\odot_{YY} = \frac{-8.804}{112.98\times 112.98 - 8.804^2}\times 100\times 10^{6}$$
$$= -0.964\times 10^{-6}$$
$$\odot_{XY} = 0$$
$$[\odot] = [8.905\times 10^{-6} - 0.964\times 10^{-6}0]$$

Table 2: Comparison of results

Method	A	B	C
Numerical analysis	43.13	1.625	$0.443\times\ 10^{-12}$
Proposed analytical approach	41.13	1.63	0
Experimentation analysis	41.14	1.62	0

Stress for each lamina:

$$[\odot_{xx}\,\odot_{yy}\,\odot_{xy}] = [48.612.20102.2017.880004.86]$$
$$[8.905\times 10^{-6} - 0.964\times 10^{-6}0]$$
$$[\odot_{xx}\,\odot_{yy}\,\odot_{xy}] = [43.241.380]\frac{N}{mm^2}$$

5 Results and Discussion

Experimental Result:

From the experimental results, strain components in each direction were noted and with the help of hook's law stress components have been calculated as 41.14 N/mm^2 in pure x direction and 1.62 N/mm^2 in pure y direction.

Results of Numerical Analysis:

After finishing the preprocessor procedure in ANSYS the results are obtained in the general post processor. The inter laminar stress in X direction is 43.13 N/mm^2 , the inter laminar stress value in the Y direction is 1.625 N/mm^2, the inter laminar shear stress in XY plane due to the tensile load is 0.443 $e^{(}-12)N/mm^2$. For the applied tensile load, the stress value in the X direction was high when compared to the other directions. **Lamination Theory Results:** The flow of calculating the inter-laminar stress values using the proposed lamination theory has been clearly depicted and the results obtained from which are being tabulated below along with the results obtained from experimental and numerical methods.

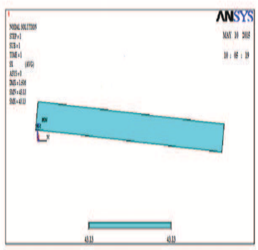


Fig. 4: Stress in X direction (σ_{xx}) for [0/90]s laminate

Where A=Stress in X direction $(\sigma_{xx})\frac{N}{mm^2}$
B=Stress in Y direction $(\sigma_{yy})\frac{N}{mm^2}$
C=Shear Stress in XY Plane $(\tau_{xy})\frac{N}{mm^2}$

From the comparison table, we can observe that the proposed analytical approach provides the result which was very nearest to the experimental results. This confirms the suitability of the higher order lamination theory in comparison with the numerical analysis.

6 Conclusion

More researchers are currently giving more attention on the prediction of inter-laminar and free edge stresses by knowing their significant effect on the structural application areas. In line with the same objective, various numbers of different approaches have been practiced so far in the field of composites. Similar to those, in this attempt a new form of lamination theory was taken and considered for checking its suitability to use it as better alternative approach to the previous approaches. The inter-laminar stress value induced in the chosen composite laminate [00/900/900/00] under the loading condition of $N_{xx} = 100kN/m$ with the help of the proposed lamination theory. The objective of this research work was fulfilled by proving the better suitability of following the proposed lamination theory to predict the inter-laminar stress values induced in the composite structures. Numerical results were in the form of validating the proposed approach.

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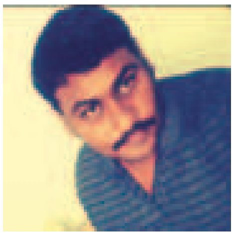

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