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On Face Magic Labeling of Duplication of a Tree

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Abstract: This paper deals with the problem of labeling the vertices, edges and faces of a plane graph. Let $(a,b,c) \in \{0,1\}$. A labeling of type (a,b,c) assigns labels from the set $\{1,2,\ldots,a|V(G)|+b|E(G)|+c|F(G)|\}$ to the vertices, edges and faces of G in such a way that each vertex receives a labels, each edge receives b labels and each face receives c labels and each number is used exactly once without repetition as a label. The weight of the face $w(f)$ under a labeling is the sum of the labels of the face itself together with the labels of vertices and edges surrounding that face. A labeling of type (a,b,c) is said to be face magic, if for every positive integer k all k-sided faces have the same weight. Here we study the existence of face magic labeling of duplication and double duplication of trees.

Keywords: duplication, double duplication, strong face graph, face magic labeling.

1 Introduction

Graph labeling was first introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The notion of face magic labeling of plane graphs was introduced by Ko-Wei Lih in 1983 [3]. Lih described face magic labeling of type (1,1,0) for the wheels, the friendship graphs, the prisms and some of the platonic polyhedra.

Let $G(V,E,F)$ be a finite plane graph where V,E,F are its vertex set, edge set and set of interior faces with $|V|=p$, $|E|=q$ and $|F|=f$. A labeling of type (a,b,c) of a graph G assigns labels from the set $\{1,2,3,\ldots,ap+bq+cf\}$ to the vertices, edges and faces of a graph G such that each vertex gets a-label, each edge gets b-label and each face gets c-label and each label is used exactly once. The values of a,b and c are restricted to 0 and 1. Labeling of type (1,0,0), (0,1,0) and (0,0,1) are called vertex labeling, edge labeling and face labeling respectively. The weight of a face $w(f)$ under a labeling is the sum of labels of face together with labels of vertices and edges forming that face. A labeling is said to be magic, if for every positive integer s, all s-sided faces have the same weight.

Baca had investigated about face magic labeling in [5]. He gave magic labeling of type (1,1,1) for fans, ladders, planar bipyramids, grids, hexagonal lattices and mobius ladders. In [1] Amarajothi et al. has proved the existence of face magic labeling of vertex and edge duplication of graphs. We studied the existence of face magic labeling of double duplication of all edges by vertices of type (0,1,1), duplication of an edge by a vertex followed by duplication of a vertex by an edge of type (0,1,1) of a tree T_n, $n \geq 2$. Also the existence of the strong face of the duplication of all edges by vertices of type (0,1,1) and the strong face of duplication of a vertex by edge of types (1,0,1), (1,1,0), and (0,1,1) in a tree T_n, $n \geq 2$ are proved.

The definitions which are used in our study are as follows:

Definition 1.*Duplication of a vertex v_k by a new edge $e=v'v''$ in a graph G produces a new graph G' such that $N(v')=\{v_k,v''\}$ and $N(v'')=\{v_k,v'\}$.*

Definition 2.*Duplication of an edge $e=v_iv_{i+1}$ by a vertex v' in a graph G produces a new graph G' such that $N(v')=\{v_iv_{i+1}\}$.*

Definition 3.*[8] The double duplication of edge by vertex followed by edge by vertex of a graph is defined as, a duplication of an edge $e_k=\{v_kv_{k+1}\}$ by a vertex v_k' in a graph G produces a graph G' in which $N(v_k')=\{v_k,v_{k+1}\}$. Again duplication of the edges $\{v_kv_k'\}$ and $\{v_{k+1}v_k'\}$ by the vertices v_k'' and v_k''' respectively in G'*

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produces a new graph G'' such that $N(v_k''') = \{v_{k+1}, v_k'\}$ and $N(v_k'') = \{v_k, v_k'\}$. It is denoted by $DD_{EE}(G)$.

Definition 4. *The double duplication of edge by vertex followed by vertex by edge of a graph is defined as, a duplication of an edge $e_k = \{v_k v_{k+1}\}$ by a vertex v_k' in a graph G produces a graph G' in which $N(v_k') = \{v_k, v_{k+1}\}$. Again duplication of a vertex v_k' by an edge $e' = \{u_1 u_2\}$ in G' produces a new graph G'' such that $N(u_1) = \{u_2, v_k'\}$ and $N(u_2) = \{u_1, v_k'\}$. It is denoted by $DD_{EV}(G)$.*

Definition 5. *[4] Let G be a simple, connected, plane graph. A strong face graph G^* is obtained from G by adding a new vertex to every face of G except the external face and joining this vertex with all vertices surrounding that face, so that all faces of the graph G^* are isomorphic to the cycle C_3.*

2 Main Results

In this section, we identify the face magic labeling of duplication and double duplication of trees.

Theorem 1. *The graph $DD_{EE}(T_n)$, $n \geq 2$ of type (0, 1, 1) is face magic.*

Proof. Let G be an arbitrary tree of order n with vertex set $V = \{u_t : 1 \leq t \leq n\}$ and edge set $E = \{e_t : 1 \leq t \leq n-1\}$. Let $DD_{EE}(G)$ be the graph obtained by double duplication of all edges by vertices of G with edge set $E' = \{e_t', e_t'' : 1 \leq t \leq n-1\} \cup \{p_t, q_t, p_t', q_t', p_t'', q_t'' : 1 \leq t \leq n-1\} \cup E$, where $\{e_t', e_t'', p_t, q_t\}$ are adjacent to e_t, $\{p_t', q_t', e_t, e_t''\}$ are adjacent to e_t', $\{p_t'', q_t'', e_t, e_t'\}$ are adjacent to e_t'' respectively. The face set is given by $F' = \{g_1 : e_t e_t' e_t'' : 1 \leq t \leq n-1\} \cup \{f_t : e_t p_t q_t : 1 \leq t \leq n-1\} \cup \{f_t' : e_t' p_t' q_t' : 1 \leq t \leq n-1\} \cup \{f_t'' : e_t'' p_t'' q_t'' : 1 \leq t \leq n-1\}$

The following pattern gives the face magic labeling of type (0,1,1)

Define a mapping $\varphi : E \rightarrow \{1, 2, 3, \ldots (13n-13)\}$ as follows:
For $1 \leq t \leq n-1$

$\varphi(e_t) = t$	$\varphi(e_t') = 4n-3-t$
$\varphi(e_t'') = 8n-7-t$	$\varphi(g_t) = 12n-12+t$
$\varphi(p_t) = 6n-6+t$	$\varphi(p_t') = 8n-8+t$
$\varphi(p_t'') = 4n-4+t$	$\varphi(f_t) = 12n-11-t$
$\varphi(q_t) = 6n-5-t$	$\varphi(q_t') = 2n-2+t$
$\varphi(q_t'') = 2n-1-t$	$\varphi(f_t') = 10n-9-t$
$\varphi(f_t'') = 10n-10+t$	

From the above labeling pattern we conclude that the weights of all 3-sided faces as:
$w(g_t) = \varphi(g_t) + \varphi(e_t) + \varphi(e_t') + \varphi(e_t'') = 12n-12+t+t+4n-3-t+8n-7-t = 24n-22$.
Similarly, $w(f_t) = w(f_t') = w(f_t'') = 24n-22$.
Hence the graph $DD_{EE}(G)$ admits face magic labeling.

Theorem 2. *The graph $DD_{EV}(T_n)$, $n \geq 2$ of type (0,1,1) is face magic.*

Proof. Let G be an arbitrary tree of order n with vertex set $V = \{u_t : 1 \leq t \leq n\}$ and edge set $E = \{e_t : 1 \leq t \leq n-1\}$. Let $DD_{EV}(G)$ be the graph obtained by duplication of edge by vertex followed by vertex by edge in G with edge set $E' = \{e_t', e_t'' : 1 \leq t \leq n-1\} \cup \{p_t, p_t', p_t'' : 1 \leq t \leq 2n-1\} \cup E$, where e_t' and e_t'' are adjacent to e_t, p_t' and p_t'' are adjacent to p_t respectively, and face set is defined by $F' = \{f_t : e_t e_t' e_t'' : 1 \leq t \leq n-1\} \cup \{g_t : p_t p_t' p_t'' : 1 \leq t \leq 2n-1\}$.
The following pattern gives the face magic labeling of type (0,1,1).
Define a mapping $\varphi : E' \cup F' \rightarrow \{1, 2, 3, \ldots (12n-8)\}$ as follows:
For $1 \leq t \leq n-1$,

$\varphi(e_t) = t$	$\varphi(e_t') = 6n-3-t$
$\varphi(e_t'') = 6n-4-t$	$\varphi(g_t) = 12n-7+t$

For $1 \leq t \leq 2n-1$

$\varphi(p_t) = 3n-1-t$	$\varphi(p_t') = 3n-2+t$
$\varphi(p_t'') = 9n-5-t$	$\varphi(g_t) = 9n-6+t$

From the above labeling pattern we infer that the weight of all 3-sided faces are given by,
$w(g_t) = \varphi(g_t) + \varphi(p_t) + \varphi(p_t') + \varphi(p_t'') = 9n-6+t+3n-1-t+3n-2+t+9n-5-t = 24n-14$.
Similarly, $w(f_t) = 24n-14$.
Hence the graph $DD_{EV}(G)$ admits face magic labeling.

Theorem 3. *For a tree of order $n \geq 2$, the strong face of the duplication of all edges by vertices simultaneously admits face magic labeling of type (0,1,1).*

Proof. Let G be an arbitrary tree of order n with vertex set $V = \{u_t : 1 \leq t \leq n\}$ and edge set $E = \{e_t : 1 \leq t \leq n-1\}$. Let G^* be the graph obtained by duplicating all the edges by vertices in G and applying the concept of strong face on it with edge set $E' = \{e_t', e_t'' : 1 \leq t \leq n-1\} \cup \{p_t, p_t', p_t'' : 1 \leq t \leq n-1\} \cup E$, where p_t' and p_t'' are adjacent to e_t, p_t' and p_t are adjacent to e_t', p_t and p_t'' are adjacent to e_t'' respectively, and the face set $F' = \{f_t : e_t p_t' p_t'' : 1 \leq t \leq n-1\} \cup \{f_t' : p_t p_t' e_t' : 1 \leq t \leq n-1\} \cup \{f_t'' : e_t'' p_t p_t'' : 1 \leq t \leq n-1\}$.

Define a mapping $\varphi : E' \cup F' \rightarrow \{1, 2, 3, \ldots 9(n-1)\}$ as follows:

For $1 \leq t \leq n-1$,

$\varphi(e_t) = 2n-2+t$	$\varphi(e_t') = t$
$\varphi(e_t'') = 2n-1-t$	$\varphi(p_t) = 6n-5-t$
$\varphi(p_t') = 4n-3-t$	$\varphi(p_t'') = 4n-4-t$
$\varphi(f_t) = 8n-7-t$	$\varphi(f_t') = 8n-8+t$
$\varphi(f_t'') = 6n-6-t$	

By the above labeling pattern, the weights of all 3-sided faces are given by,
$w(f_t) = \varphi(e_t) + \varphi(p_t') + \varphi(p_t'') + \varphi(f_t) = 18n-16$

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Similarly, $w(f_t') = w(f_t'') = 18n - 16$.
Hence the graph obtained by the strong face of the duplication of all edges by vertices of G^* admits face magic labeling.

Theorem 4. *The strong face of duplication of a vertex by edge of a tree T_n, $n \geq 2$ of types (1,0,1), (1,1,0), and (0,1,1) are face magic.*

Proof. Let G be an arbitrary tree of order n with the vertex set $V = \{u_t : 1 \leq t \leq n\}$ and edge set $E = \{e_t : 1 \leq t \leq n-1\}$. Let G^* be the graph obtained by duplicating all the vertices by the edges in G and applying the concept of strong face to it. Let $V' = V \cup \{v_t, w_t, x_t : 1 \leq t \leq n\}$ where v_t, w_t are adjacent to u_t and x_t is adjacent to u_t, v_t, w_t; $E' = E \cup \{u_tv_t, u_tw_t, v_tw_t, u_tx_t, v_tx_t, w_tx_t : 1 \leq t \leq n\}$ and the face set $F' = \{u_tv_tx_t, u_tw_tx_t, v_tw_tx_t : 1 \leq t \leq n\}$. The three types of labeling are discussed below

Type i: (1, 0, 1)-Face magic
Define a mapping $\varphi_1 : V' \cup F' \to \{1, 2, \ldots 7n\}$ as follows:
For $1 \leq t \leq n$

$\varphi_1(u_t) = t$ $\quad$ $\varphi_1(v_t) = 2n + 1 - t$
$\varphi_1(w_t) = 2n + t$ $\quad$ $\varphi_1(x_t) = 7n + 1 - t$
$\varphi_1(u_tv_tx_t) = 5n + t$ $\quad$ $\varphi_1(v_tw_tx_t) = 3n + t$
$\varphi_1(u_tw_tx_t) = 5n + 1 - t$

By the above labeling technique the weights of all 3-sided faces are given by $w(u_tv_tx_t) = \varphi_1(u_t) + \varphi_1(v_t) + \varphi_1(x_t) + \varphi_1(u_tv_tx_t) = 14n + 2$
Similarly $w(v_tw_tx_t) = w(u_tw_tx_t) = 14n + 2$

Type ii: (1, 1, 0)-Face magic
Define a mapping $\varphi_2 : V' \cup E' \to \{1, 2, \ldots (11n - 1)\}$ as follows:
For $1 \leq t \leq n$

$\varphi_2(u_t) = t$ $\quad$ $\varphi_2(v_t) = 2n + 1 - t$
$\varphi_2(w_t) = 2n + t$ $\quad$ $\varphi_2(x_t) = 10n + 1 - t$
$\varphi_2(u_tx_t) = 3n + t$ $\quad$ $\varphi_2(u_tx_t) = 5n + 1 - t$
$\varphi_2(v_tx_t) = 5n + t$ $\quad$ $\varphi_2(u_tv_t) = 7n + t$
$\varphi_2(v_tw_t) = 7n + 1 - t$ $\quad$ $\varphi_2(u_tw_t) = 9n + 1 - t$

For $1 \leq t \leq n - 1$ $\varphi_2(e_t) = 10n + t$
With the above labeling pattern, the weight of all 3-sided faces $(u_tv_tx_t)$ are given by
$w(u_tv_tx_t) = \varphi_2(u_t) + \varphi_2(v_t) + \varphi_2(x_t) + \varphi_2(u_tv_t) + \varphi_2(u_tx_t) + \varphi_2(v_tx_t) = 29n + 3$
Similarly $w(v_tw_tx_t) = w(u_tw_tx_t) = 29n + 3$

Type iii: (0, 1, 1)-Face magic
Define a mapping $\varphi_3 : E' \cup F' \to \{1, 2, \ldots (10n - 1)\}$ as follows:
For $1 \leq t \leq n$

$\varphi_3(u_tv_t) = t$ $\quad$ $\varphi_3(u_tw_t) = n + t$
$\varphi_3(v_tw_t) = 2n + t$ $\quad$ $\varphi_3(w_tx_t) = 3n + t$
$\varphi_3(u_tx_t) = 5n + 1 - t$ $\quad$ $\varphi_3(v_tx_t) = 6n + 1 - t$
$\varphi_3(v_tw_tx_t) = 7n + 1 - t$ $\quad$ $\varphi_3(u_tv_tx_t) = 7n + t$
$\varphi_3(u_tw_tx_t) = 9n + 1 - i$

For $1 \leq t \leq n - 1$ $\varphi_3(e_t) = 9n + t$
The weight of all 3-sided faces $(u_tv_tx_t)$ from the above labeling pattern is given by
$w(u_tv_tx_t) = \varphi_3(u_tv_t) + \varphi_3(u_tx_t) + \varphi_3(v_tx_t) + \varphi_3(u_tv_tx_t) = 18n + 2$
Similarly $w(v_tw_tx_t) = w(u_tw_tx_t) = 18n + 2$.
Therefore the graph obtained by the strong face of the duplication of a vertex by edge admits face magic labeling.

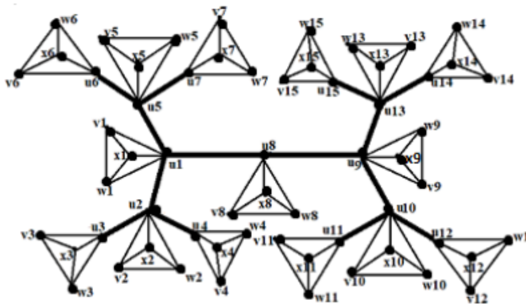

Fig. 1: Strong face of duplication of a vertex by an edge of tree T_{15}

3 Conclusion

In this paper, we have studied face magic labeling of $DD_{EE}(T_n)$, $n \geq 2$ and $DD_{EV}(T_n)$, $n \geq 2$ of type (0,1,1). Also the strong face of the duplication of all edges by vertices and the strong face of duplication of all vertices by edge of a tree T_n, $n \geq 2$ are investigated. Studying the face labeling of strong face of a double duplication of some special type of graphs are the future work.

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