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A Linguistic Approach of IFSM with Euclidean Distance in Decision Making of Compatible Waste Treatment Methods

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Abstract: Decision making of suitable waste treatment method for promoting environmental sustainability is indeed a challenging task for the industrial sectors. There are various approaches and methods of decision making ranging from simple to complex, but in recent times the concept of soft sets represented as soft matrix are used in making appropriate decisions by associating several parameters. To overcome the uncertainty and impreciseness the notion of intutionistic fuzzy soft matrix is used (IFSM). In this paper a new approach of linguistic IFSM is employed in which the membership and non-membership values are represented in terms of linguistic variables instead of numerical values. This approach presents the realistic opinion of experts and it creates a new paradigm of decision making. This paper primarily aims in introducing LAIFSM and validates the proposed approach with the real life application.

Keywords: Linguistic variable, intutionistic, fuzzy, soft matrix, waste management, environmental conservation

1 Introduction

Industrial sectors are the prime boon for the economic growth of our nation. The extent of urbanization and industrialization depends on the progress of these industries and its interrelation with the society in which it exists. The sustenance of these industries hinges on the benefits it renders and receives, but in recent times, the rate of production has started to increase with the implementation of advanced technology to meet the demands of the growing population. This has resulted in the degradation of the environment to great degree by the ejection of waste to the surroundings without treatment. As the effects of these wastes are highly dreadful, the government has enforced strict environmental regulations which have obliged the industrial sectors to adopt suitable waste management techniques and the problem of decision making with regard to selection of suitable waste treatment method begins from this point.
There are ample methods of selecting the best method for waste mitigation such as AHP, DEMATEL,TOPSIS and so on, but these methods are not too efficient to handle the situations of uncertainty and impreciseness. To overcome such ambiguity, the concept of fuzzy came into existence which was first introduced by Lofti.A.Zadeh. Besides these decisions making tools, researchers have also begun to use the notion of soft sets. The theory of soft set was first proposed by Molodstov[1] and it was applied by Maji[2] to deal the risks of impreciseness. Yang and Ji[6] introduced the aspect of matrix representation which was extended to intutionistic fuzzy soft matrix theory by Chetia and Das [9]. Shanmugasundaram.et.al has applied max-min average composition in decision making of suitable jobs to the individuals according to their skills[17]. The same method was applied by Nivetha.et.al[12] in determining the suitable waste treatment methods for the industrial sectors. This method is modified into max-min Euclidean Distance composition with the incorporation of linguistic approach in this paper. The representation of membership and non-membership values are made in terms of linguistic variables to be more realistic as it will duly assist in making apposite decisions. Efforts are taken in this paper to make an attempt of introducing novel approaches in the

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existing decision making tools so as to have a comparative analysis of all the proposed methods to select the most feasible method of selecting the suitable waste treatment method.
The paper is organized as follows: section 2 consists of basic definitions; section 3 presents the methodology of the new approach; section 4 comprises the adaptation of the method to the decision making problem considered for analysis; section 5 presents the results and discussion and section 6 concludes the paper.

2 Basic definitions

This section consists of the preliminaries pertaining to LIFSM.
Definition 2.1
A soft set F is a mapping from E to P(U), where E is the of parameters and P(U) is the power set of the universe set U.

Definition 2.2.
A fuzzy soft set F is a mapping from A to I^U, where A is the subset of E, the set of parameters and I^U is the collection of all fuzzy subsets over of the universe set U.

Definition 2.3
An intutionistic fuzzy soft set F is a mapping from A to I^U, where A is the subset of E, the set of parameters and I^U is the collection of all intutionistic fuzzy subsets over of the universe set U.

Definition 2.4
An intutionistic fuzzy soft matrix $A=[a_{ij}]$, i = 1,2,...m and j =1, 2,...n is represented as

$$a_{ij} = \begin{cases} (\mu_j(c_i), v_j(c_i)) & if e_j \in A \\ (0,1) & if e_j \notin A \end{cases}$$

Where $\mu_j(c_i)$ denotes the membership value of c_i in the intutionistic fuzzy set, $v_j(c_i)$ denotes the non-membership value of c_i in the intutionistic fuzzy set and e_j is the element of the set E.

Definition 2.5
The complement of an intutionistic fuzzy soft matrix $A=[a_{ij}]$, where $a_{ij}=(\mu_j(c_i), v_j(c_i))$ is AC = $[b_{ij}]$, where b_{ij} = $(v_j(c_i), \mu_j(c_i))$ for all i,j.

Definition 2.6
A variable becomes linguistic variable if its value is linguistic rather than numeric.

Definition 2.7
A linguistic intutionistic fuzzy soft matrix $A=[a_{ij}]$, i = 1,2,...m and j =1, 2,...n is represented as a_{ij} = $(L1, L2)$.

Definition 2.8
The complement of a linguistic intutionistic fuzzy soft matrix $A=[a_{ij}]$, where a_{ij} = $(L2, L1)$.

Definition 2.9
The Euclidean distance $E(A,B)$ between two intutionistic fuzzy sets is

$$\sqrt{\frac{1}{2}\sum_{i=1}^{n}[\mu_{\widetilde{A}}(x_i)-\mu_{\widetilde{B}}(x_i)]^2+[v_{\widetilde{A}}(x_i)-v_{\widetilde{B}}(x_i)]^2+[\pi_{\widetilde{A}}(x_i)-\pi_{\widetilde{B}}(x_i)]^2}$$

Where $\pi_{\widetilde{A}}(x)=1-\mu_{\widetilde{A}}(x)-v_{\widetilde{A}}(x)$

Definition 2.10

A Triangular fuzzy number $\widetilde{B}=(a,b,c), a<b<c$, defined on R has the membership function

$$\mu_{\widetilde{B}}(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \le x \le b \\ \frac{c-x}{c-b}, & \text{if } b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.11
Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two intutionistic fuzzy soft matrixes with linguistic variables represented in terms of triangular alpha -cuts, the max-min Euclidean Distance composition of fuzzy soft matrix relation is defined as $A\Phi_E B$ = Max $\{\frac{1}{2}\sum[A^L_\mu - B^L_\mu]^2 + [A^U_\mu - B^U_\mu]^2]\}$, Min $\{\frac{1}{2}\sum[A^L_v - B^L_v]^2 + [A^U_v - B^U_v]^2\}$
Definition 2.12
$A=[a_{ij}]$ and $B=[b_{ij}]$ be two intutionistic fuzzy soft matrixes, then the score of the matrixes A and B is defined as $S(A,B)=\frac{1}{2}(V+W)$, where V and W are matrixes defined as
$V=[\mu_{A\Phi_E B}-V_{A^C\Phi_E B^C}]$ and $W=[V_{A\Phi_E B - \mu_{A^C\Phi_E B^C}}]$

3 Methodology

1. The linguistic intutionistic fuzzy soft matrices A and B corresponding to the intutionistic fuzzy soft sets (F,R) and (G,D) along with their complements are obtained in terms of linguistic variables.
2. The LIFSM are represented in terms of triangular fuzzy numbers and the alpha cuts are determined.
3.Find $A\Phi_E B$ and $A^C\Phi_E B^C$.
4.The score of the matrixes A and B is computed for making suitable decisions.

4 Adaptation to the Proposed Problem of Decision Making

The decision making problem solved by Nivetha.et,al [12] using max-min average composition approach is once again taken in this paper for applying the new approach of max-min Euclidean distance composition with modifications in data presentation.
The four major pollution causing industries in India are Fertilizer industries (F), Tannery industries (T), Pesticide industries (P) and Chemical industries (C). The major

types of waste emanated from these industries are solid (S), liquid (L), hazardous (H) and organic (O). The available waste treatment methods are incineration (I), Gasification (G), Adsorption (A). Let P ={F,T,P,C} be the universal set, R ={ S,L,H,O}and D = {I,G,A}.

The matrix A

	S	L	H	O
F	(H,L)	(H,L)	(H,L)	LH
T	(M,M)	(H,L)	(H,L)	(H,L)
P	(M,M)	(H,L)	(H,L)	(M,M)
C	(H,L)	(H,L)	$(H.L)$	(L,H)

The matrix B

	I	G	A
S	(H,L)	(H,L)	(H,L)
L	(H,L)	(H,L)	(H,L)
H	(M,M)	(H,L)	(H,L)
O	(H,L)	(H,L)	(H,L)

The matrix A^C

	S	L	H	O
F	(L,H)	(L,H)	(L,H)	(H,L)
T	(M,M)	(L,H)	(L,H)	(L,H)
P	(M,M)	(L,H)	(L,H)	(M,M)
C	(L,H)	(L,H)	(L,H)	(H,L)

The matrix B^C

	I	G	A
S	(L,H)	(L,H)	(L,H)
L	(L,H)	(L,H)	(L,H)
H	(M,M)	(L,H)	(L,H)
O	(L,H)	(L,H)	(L,H)

The Triangular representation of the linguistic variable is tabulated as follows

Linguistic Variable	Triangular Membership Value
Low	(0.1,0.3,0.4)
Moderate	(0.4,0.5,0.7)
High	(0.7,0.9,1)

Linguistic Variable	Alpha-cut ($\alpha = 0.5$)
Low	[0.2,0.35]
Moderate	[0.45,0.6]
High	[0.8,0.95]

where [(b-a) α+ a, c- (c-b) α]

The modified matrix A

	S	L
F	$([0.8,0.95],[0.2,0.35])$	$([0.8,0.95],[0.2,0.35])$
T	$([0.45,0.6],[0.45,0.6])$	$([0.8,0.95],[0.2,0.35])$
P	$([0.45,0.6],[0.45,0.6])$	$([0.8,0.95],[0.2,0.35])$
C	$([0.8,0.95],[0.2,0.35])$	$([0.8,0.95],[0.2,0.35])$

	H	O
F	$([0.8,0.95],[0.2,0.35])$	$([0.2,0.35],[0.8,0.95])$
T	$([0.8,0.95],[0.2,0.35])$	$([0.8,0.95],[0.2,0.35])$
P	$([0.8,0.95],[0.2,0.35])$	$([0.45,0.6],[0.45,0.6])$
C	$([0.8,0.95],[0.2,0.35])$	$([0.2,0.35],[0.8,0.95])$

The modified matrix B

	I	G
S	$([0.8,0.95],[0.2,0.35])$	$([0.8,0.95],[0.2,0.35])$
L	$([0.8,0.95],[0.2,0.35])$	$([0.8,0.95],[0.2,0.35])$
H	$([0.45,0.6],[0.45,0.6])$	$([0.8,0.95],[0.2,0.35])$
O	$([0.8,0.95],[0.2,0.35])$	$([0.8,0.95],[0.2,0.35])$

	A
S	$([0.8,0.95],[0.2,0.35])$
L	$([0.8,0.95],[0.2,0.35])$
H	$([0.8,0.95],[0.2,0.35])$
O	$([0.8,0.95],[0.2,0.35])$

The modified matrix A^C

	S	L
F	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$
T	$([0.45,0.6],[0.45,0.6])$	$([0.2,0.35],[0.8,0.95])$
P	$([0.45,0.6],[0.45,0.6])$	$([0.2,0.35],[0.8,0.95])$
C	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$

	H	O
F	$([0.2,0.35],[0.8,0.95])$	$([0.8,0.95],[0.2,0.35])$
T	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$
P	$([0.2,0.35],[0.8,0.95])$	$([0.45,0.6],[0.45,0.6])$
C	$([0.2,0.35],[0.8,0.95])$	$([0.8,0.95],[0.2,0.35])$

The modified matrix B^C

	I	G	A
S	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$
L	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$
H	$([0.45,0.6],[0.45,0.6])$	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$
O	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$	$([0.2,0.35],[0.8,0.95])$

The matrix $A\Phi_E B$ is

	I	G	A
F	$(0.43,0)$	$(0.43,0)$	$(0.43,0)$
T	$(0.247,0)$	$(0.247,0)$	$(0.247,0)$
P	$(0.247,0)$	$(0.247,0)$	$(0.247,0)$
C	$(0.43,0)$	$(0.43,0)$	$(0.43,0)$

The matrix $A^C\Phi_E B^C$ is

	I	G	A
F	$(0.434,0)$	$(0.43,0)$	$(0.43,0)$
T	$(0.17,0)$	$(0.176,0)$	$(0.17,0)$
P	$(0.17,0)$	$(0.17,0)$	$(0.172,0)$
C	$(0.43,0)$	$(0.424,0)$	$(0.43,0)$

The matrix V is

	I	G	A
F	0.43	0.43	0.43
T	0.247	0.247	0.247
P	0.247	0.247	0.247
C	0.43	0.43	0.43

The matrix W is

	I	G	A
F	0.434	0.43	0.43
T	0.17	0.176	0.17
P	0.17	0.17	0.172
C	0.43	0.424	0.43

The score matrix S(A,B)is as follows

	I	G	A
F	0.432	0.43	0.43
T	0.2085	0.2115	0.2085
P	0.2085	0.2085	0.2095
C	0.43	0.432	0.43

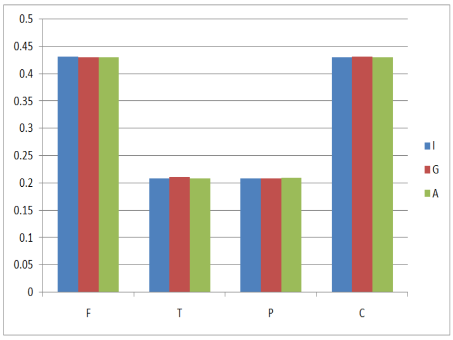

Fig. 1: Graphical representation of the Score Matrix

5 Results and Discussion

The values in the score matrix clearly show that the incineration method can be used by the Fertilizer, Gasification by Tannery and Chemical industries and Adsorption by Pesticide industry. The Fig.1 is the graphical representation of the obtained results. The approach of max-min Euclidean Distance composition method is more realistic than max-min average composition method in determining the suitable waste treatment method for treating the pollutants and waste emanated from the major waste producing industrial sectors. This research work presents a new linguistic approach integrating the notion of Euclidean distance with the composition.This work can be extended with different types of intutionistic fuzzy sets such as trapezoidal, hexagonal and so on.
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