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Computational and Mathematical Analysis of Fuzzy Quota Harvesting Model in Fuzzy Environment using Homotopy Perturbation Method

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Abstract: The mathematical model of fuzzy quota harvesting in fuzzy environment under non-steady state conditions is discussed. Analytical expressions for population density for all values of harvesting constant rate and growth rate are presented by using Homotopy perturbation method. Further, the population densities are compared with simulation results and fuzzy solution in which a good agreement was noted.

Keywords: Fuzzy environment, Harvest model, Simulation, Fuzzy differential equations, Fuzzy quota harvesting model, Homotopy Perturbation Method.

1 Introduction

Harvesting intensity performs a prime representation on exploited and a dynamic property of a population. It is beneficial in numerous cases to avoid the unwanted population compactness (Ludwig, Jones and Holling 1978) [1]. In specific atmosphere, harvesting models are considered by majority of the researchers and scientists. It is predictable that the constraint and preliminary setting or populations are specific in nature. Nevertheless, the actual problems are drenched with impreciseness. To recognize the part of the impreciseness is crucial to investigate the biological limitations in the population communities behavior. In real-world ecosystems, concurrently, many parameters may fluctuate with the from time to time in changeable environments. Owing together to environment and human culture, like as climate warming, earthquake, financial crisis, fire, etc., the parameters also vary. Consequently, the interface procedure linking the types and dynamics are powerfully prejudiced by these conservation dissimilarities. The information assortment, element of investigation, the process of measurement, as well as shaping the original circumstances is also taken in place for impreciseness.

In situations, where the parameters and the initial conditions are described in inaccurate forms, fuzzy theory [2] is applied to represent the inaccuracy. Fuzzy Differential Equations (FDEs) are increasingly secondhand for representative problems in science and engineering. Chang and Zadeh [3] first presented the notion of a fuzzy derivative tracked by Dubois and Prade [4], which are well-defined and the extension value is used in their method. In a fuzzy atmosphere, the perception of differential equations was primarily conveyed by Kaleva [5]. All derivatives of the fuzzy differential equations was broadly considered by Hukuhara or generalized derivatives. A large set of boundary value problems is displayed by Bede [6]; however, it has no result if the Hukuhara derivative is applied. Adopting the generalized derivative, an outsized category of frontier value problems are established by Nieto and Khastan [7]. Accordingly, H-differentiability is in relationship with shortcoming of robustly comprehensive differentiability function. It is also a fuzzy

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differential equation, which eventually has no distinctive explanation [8]. The perception of differentiability has associations with inadequately and strongly generalized differentiability which was compered by Stefanini [9].

In science and engineering problems, Fuzzy differential equations (FDEs) provides the better solutions. In order to attain the thorough solution of FDEs is too complicated in real life applications. Therefore, many researchers are interested to find solutions of FDEs. In order to solve FDE, there are numerous methods. Over the years, many authors such have attempted to solve fuzzy differential equations using numerical methods. Allahviranloo et al. [10], Chen et al. [11] and OŔegan et al. [12] altered the FDE interested in equivalent fuzzy integral equation and consequently, resolved the problem. The next approach was Zadeh extension principle method used by Buckley and Feuring [13]. For the approach, the fuzzy is altered to a crispy problem. Tapaswini and Chakraverty in [14] applied Homotopy Perturbation Method (HPM), ([15], [16]) to handle the numerical solution of fuzzy fractional predator-prey system with fuzzy initial conditions.

Bio-mathematical models are based on impreciseness. Bassanezi et al. [17] laid a base stone to learn the constancy of a dynamical scheme utilizing fuzzy differential equations. Barros et al. [18] depicted ecological fuzziness of a existence anticipation model by bearing in mind the limitations are fuzzy in character. Guo et al. [19] intended logistic prototypical and Gompertz model under fuzziness. Peixoto et al. [20] considered predator-prey model under fuzziness and they use fuzzy rule-based method to solve the model. Mizukoshi et al.[21] reported the fuzzy primary responsibility with limitations and preliminary circumstances under fuzziness. Ahmad and Baets [22] predictable the predator-prey model with fuzzy principal populations by Runge-Kutta format. Utilizing the fuzzy dynamical systems Najariyan et al. [23] conferred the optimal control of HIV infection. Omar and Hasan [24] measured the interface of predator prey with unknown preliminary inhabitant's dimensions. Pal et al. [25] considered comparative harvesting model by means of fuzzy inherent growth rate and fuzzy reaping quantity. The arrangement of fuzzy differential equations in the biological model was solved by Ahmad and Hasan [26]using Euler's method. Mann et al. [27] utilized interrupted differential equation in predator-prey interface in addition to firmness of steady state. Pal and Mahapatra [28] encompassed a bio-economic mould of binary quarry and one-predator fishery mould through finest harvesting policy by hybridization approach.

In a solitary classes population, the model equations are previously simulated using stability analysis by Pal et al. [29]. Recently, Paul et al. [30] established the considerable mathematical effect for the dynamical performance of harvest quota mould by means of fuzzy constraint using numerical simulation and graphical representation. According to the previous evidence, there is no demanding logical solutions for non-steady state of population density have been reported. The rate of intrinsic growth in fuzzy environment for all conceivable standards of the parameters is also not available. The determination of this exertion is to originate investigative appearance of population density in fuzzy environment.

In this work, it is considered that the same basic definition related to fuzzy differential equations which are discussed in preliminary section of Paul et al. [30].

2 Crisp Quota Harvesting Model Formulation

The logistic proportional harvesting model is given by Pastor [31] as follows:

$$\frac{dN}{dt} = rN\left(1-\frac{N}{k}\right) - f(N)n_2, \tag{1}$$

where r, N and k be the inherent growth rate, population compactness and troposphere transport competence of the quarry inhabitants. n_2 is the population density of the harvester/consumer. Here, we neglect the population dynamics of the harvester and treat n_2 as parameter. $f(N)$ is the per capita harvest rate of the N by individuals of n_2. A invariable quantity of N is yielded by all persons of n_2 per unit of period, that is, $f(N)n_2 = h = constant$.

Therefore, the representation (1) turn out to be crisp quota yielding mould as follows:

$$\frac{dN}{dt} = rN\left(1-\frac{N}{k}\right) - h. \tag{2}$$

2.1 Fuzzy Quota Harvesting Model and Mathematical Analysis of the Problem

The quota harvesting model Eq. (1), are identified by all the limitations and have convinced the values with no improbability. It is not so accurate, nevertheless, in authentic conditions. A fuzzy quota harvesting model is currently formulated. Owing to indistinct information, the intrinsic escalation speed is deliberated. It is a fuzzy in character. In addition, it is also regarded as the fuzziness harvested character by all those per unit of time, that is, the harvesting crisp share model is altered to a fuzzy inaccurate nature of information. At this juncture, it is regarded as the fundamental expansion rate $\tilde{r}$. It is also a fuzzy in character as well as believed that the uncertainty temperament of N is harvested by all individuals of n_2 per unit of time, that is, $f(N)n_2 = \tilde{h}$. The harvesting crisp quota mould is transformed to a fuzzy harvesting quota model in fuzzy environment as given below [30]:

$$\frac{d\tilde{N}}{dt} = \tilde{r}\tilde{N}\left(1-\frac{\tilde{N}}{k}\right) - \tilde{h}. \tag{3}$$

Therefore, three potential cases might arise:

Case 1 At first and foremost altitude, the population density is of fuzzy number.
Case 2 Fuzzy numbers are intrinsic growth rate and quota harvesting .
Case 3 Population density, intrinsic growth rate and quota harvesting are all fuzzy numbers.

The corresponding initial conditions are as follows:

$$N(t_0) = \tilde{N_0} = (1,3,5), \tag{4}$$

$$N(t_0))_\alpha = [1+2\alpha, 5-3\alpha]. \tag{5}$$

In this research work, Homotopy perturbation technique is utilised to clarify the solution non-linear fuzzy differential equations. The population density of the analytical expression is set by equation (6),

$$N(t) = 5e^{rt} + \left(\frac{25}{k} - \frac{h}{r}\right)e^{rt} + \frac{h}{r} - \frac{25e^{2rt}}{k}. \tag{6}$$

Case(ia). Take $\tilde{N_0}$ is a fuzzy number. Currently, we apply $(i)-gH$ differentiable to equation (3). The dimensionless form of harvesting model can be given as follows [30]:

$$\left.\begin{aligned} \frac{dN_l(t,\alpha)}{dt} &= rN_l(t,\alpha)\left(1-\frac{N_r(t,\alpha)}{k}\right)-h, \\ \frac{dN_r(t,\alpha)}{dt} &= rN_r(t,\alpha)\left(1-\frac{N_l(t,\alpha)}{k}\right)-h. \end{aligned}\right\} \tag{7}$$

By solving the equation (7) applying Homotopy perturbation mechanism, the population density is attained when α-cut is zero as follows:

$$N_l(t,\alpha) = e^{rt} + \left(\frac{h}{r}+\frac{3}{k}\right)e^{rt} - \frac{h}{r} - \frac{3}{k}e^{2rt}, \tag{8}$$

$$N_r(t,\alpha) = 3e^{rt} + \left(\frac{h}{r}+\frac{3}{k}\right)e^{rt} - \frac{h}{r} - \frac{3}{k}e^{2rt}. \tag{9}$$

Similarly using the initial condition (5), we have found the solution:

$$\begin{aligned} N_l(t,\alpha) = (1+2\alpha)e^{rt} &+ \left[\frac{(1+2\alpha)(5-2\alpha)}{k} - \frac{h}{r}\right]e^{rt} \\ &+\frac{h}{r} - \frac{(1+2\alpha)(5-2\alpha)}{k}e^{2rt} \end{aligned} \tag{10}$$

$$\begin{aligned} N_r(t,\alpha) = (5-2\alpha)e^{rt} &+ \left[\frac{(1+2\alpha)(5-2\alpha)}{k} - \frac{h}{r}\right]e^{rt} \\ &+\frac{h}{r} - \frac{(1+2\alpha)(5-2\alpha)}{k}e^{2rt} \end{aligned} \tag{11}$$

The equations (8)-(11) satisfy the initial conditions (4) and (5). These equations characterize the novel analytical appearance of the population density for all possible standards of the parameters α, r and h.

Case (ib). We apply $(ii)-gH$ differentiable to the model equation (3) when population density is fuzzy number, then model reduce to [30]:

$$\left.\begin{aligned} \frac{dN_r(t,\alpha)}{dt} &= rN_l(t,\alpha)\left(1-\frac{N_r(t,\alpha)}{k}\right)-h, \\ \frac{dN_l(t,\alpha)}{dt} &= rN_r(t,\alpha)\left(1-\frac{N_l(t,\alpha)}{k}\right)-h. \end{aligned}\right\} \tag{12}$$

Adopting the initial conditions (5) ,in (12) we obtain:

$$\begin{aligned} N_r(t,\alpha) = (1+2\alpha) &+ \Big[r(5-2\alpha) \\ &-\frac{r(1+2\alpha)(5-2\alpha}{k} - h\Big]t, \end{aligned} \tag{13}$$

$$\begin{aligned} N_l(t,\alpha) = (5-2\alpha) &+ \Big[r(1+2\alpha) \\ &-\frac{r(1+2\alpha)(5-2\alpha}{k} - h\Big]t \end{aligned} \tag{14}$$

Case(iia). The solution of the model when $\tilde{r}$ and $\tilde{h}$ are fuzzy numbers and $\tilde{N(t)}$ $(i)-gH$ differentiable. The dimensionless form of harvesting model reduces to the following forms [30]:

$$\left.\begin{aligned} \frac{dN_l(t,\alpha)}{dt} &= r_l(\alpha)N_l(t,\alpha)\left(1-\frac{N_r(t,\alpha)}{k}\right)-h_r(\alpha), \\ \frac{dN_r(t,\alpha)}{dt} &= r_r(\alpha)N_r(t,\alpha)\left(1-\frac{N_l(t,\alpha)}{k}\right)-h_l(\alpha). \end{aligned}\right\} \tag{15}$$

Using HPM

$$\begin{aligned} N_l(t,\alpha) = (1+2\alpha)e^{r_lt} &+ \left[\frac{(1+2\alpha)(5-2\alpha}{k} - \frac{h_r}{r_l}\right]e^{r_lt} \\ &-\frac{h_r}{r_l} - \frac{(1+2\alpha)(5-2\alpha)}{k}e^{2r_lt} \end{aligned} \tag{16}$$

$$\begin{aligned} N_r(t,\alpha) = (5-2\alpha)e^{r_rt} &+ \left[\frac{(1+2\alpha)(5-2\alpha)}{k} - \frac{h_l}{r_r}\right]e^{r_rt} \\ &-\frac{h_l}{r_r} - \frac{(1+2\alpha)(5-2\alpha)}{k}e^{2r_rt} \end{aligned} \tag{17}$$

Case(iib). The solution of the model when $\tilde{N(t)}$ (ii)-gH differentiable, $\tilde{r}$ and $\tilde{h}$ are fuzzy numbers. The (3) can effectively as displayed:

$$\left.\begin{aligned} \frac{dN_r(t,\alpha)}{dt} &= r_l(\alpha)N_l(t,\alpha)\left(1-\frac{N_r(t,\alpha)}{k}\right)-h_r(\alpha), \\ \frac{dN_l(t,\alpha)}{dt} &= r_r(\alpha)N_r(t,\alpha)\left(1-\frac{N_l(t,\alpha)}{k}\right)-h_l(\alpha). \end{aligned}\right\} \tag{18}$$

we obtain the solution

$$\begin{aligned} N_r(t,\alpha) = (1+2\alpha) &+ \Big[r_l(\alpha)(5-2\alpha) \\ &-\frac{r_l(\alpha)(1+2\alpha)(5-2\alpha)}{k} - h_r(\alpha)\Big]t \end{aligned} \tag{19}$$

$$\begin{aligned} N_l(t,\alpha) = (5-2\alpha) &+ \Big[r_r(\alpha)(1+2\alpha) \\ &-\frac{r_r(\alpha)(1+2\alpha)(5-2\alpha)}{k} - h_l(\alpha)\Big]t \end{aligned} \tag{20}$$

Case(iiia). In this circumstance, the elucidation of the intrinsic growth rate, the proportion harvesting rate in addition to population density at initially are fuzzy numbers$\tilde{N(t)}$ (i)-gH differentiable, at that moment, the model is reduced to [30] :

$$\left.\begin{aligned} \frac{dN_l(t,\alpha)}{dt} &= r_l(\alpha)N_l(t,\alpha)\left(1-\frac{N_r(t,\alpha)}{k}\right)-h_r(\alpha), \\ \frac{dN_r(t,\alpha)}{dt} &= r_r(\alpha)N_r(t,\alpha)\left(1-\frac{N_l(t,\alpha)}{k}\right)-h_l(\alpha). \end{aligned}\right\} \tag{21}$$

The approximate solutions of the (21) are expressed as follows:

$$N_l(t,\alpha) = (1+2\alpha)e^{r_l t} + \left[\frac{(1+2\alpha)(5-2\alpha}{k} - \frac{h_r}{r_l}\right]e^{r_l t} - \frac{h_r}{r_l} - \frac{(1+2\alpha)(5-2\alpha)}{k}e^{2r_l t} \quad (22)$$

$$N_r(t,\alpha) = (5-2\alpha)e^{r_r t} + \left[\frac{(1+2\alpha)(5-2\alpha}{k} - \frac{h_l}{r_r}\right]e^{r_r t} - \frac{h_l}{r_r} - \frac{(1+2\alpha)(5-2\alpha)}{k}e^{2r_r t} \quad (23)$$

Case(iiib). The solution of the model when $\tilde{N(t)}$ (ii)-gH differentiable and$\tilde{N(0)}$, $\tilde{r}$and $\tilde{h}$ are fuzzy numbers. The (3) can effectively as presented below:

$$\left.\begin{aligned} \frac{dN_r(t,\alpha)}{dt} &= r_l(\alpha)N_l(t,\alpha)\left(1-\frac{N_r(t,\alpha)}{k}\right) - h_r(\alpha), \\ \frac{dN_l(t,\alpha)}{dt} &= r_r(\alpha)N_r(t,\alpha)\left(1-\frac{N_l(t,\alpha)}{k}\right) - h_l(\alpha) \end{aligned}\right\} \quad (24)$$

Using the corresponding initial conditions, we've

$$N_r(t,\alpha) = (1+2\alpha)\Big[r_l(\alpha)(5-2\alpha) - \frac{r_l(\alpha)(1+2\alpha)(5-2\alpha}{k} - h_r(\alpha)\Big]t \quad (25)$$

$$N_l(t,\alpha) = (5-2\alpha) + \Big[r_r(\alpha)(1+2\alpha) - \frac{r_r(\alpha)(1+2\alpha)(5-2\alpha}{k} - h_l(\alpha)\Big]t \quad (26)$$

2.2 Comparison with Paul et al. [30]work :

Paul et al. were obtained the graphical representation of population density with different α-cut values and intrinsic growth rate coefficient at quota harvesting model under non-steady state conditions by using the concept of stability analysis. However, these population densities are fundamentally based on the simulation approach. Further, it has no analytical solution. In this article, we have presented an analytical expression for the non-steady state population density using HPM. The investigative results are associated with numerical and fuzzy solution [30] in Figure (1-6). Upon comparison, it is observed that the results are identical when α-cut is large and for some possible values of other parameters.

3 Analytical solutions of the population density using the Homotopy perturbation method

The current model is positioned on non-steady state system of diffusion equations incorporating a non-linear reaction term related to fuzzy environment. By applying standard analytical techniques, it is not possible to solve the equations. Over the years, many researchers primarily had attempted to examine a solution of non-linear equations by distinct techniques, such as Backlund transformation [32], Darboux transformation [33], Inverse scattering method [34], Bilinear method [35], The tanh method [36], Variational iteration method [37] and Homotopy perturbation method [38] among others.In this article we have used HPM to find the analytical expression of the fuzzy quota harvesting model

4 Numerical simulation

To represent the effectiveness of the HPM method, our non-steady state outcome is correlated with numerical solution. The SCILAB/MATLAB 6.1,The Math Works Inc,Natick,MA,2000 [39] to find numerical solution of the non-linear equations. The numerical values of parameters used were similar to those employed by Paul et al.[30]. The statistical solution is correlated with the analytical result in Figs. 1-6. Upon comparison, it gives a satisfactory agreement for all possible values of parameters r , h and α.

5 Discussion

Fig. 1(a-d) to Fig. 6(a-d) illustrates the population density as a function of time t for numerous values of parameter . However, the population density increases rapidly when the value of α-cut decreases. (8)-(11) represent the population density for all values of parameters r , h and k. When $\alpha = 0$, the equation (10)-(11) is identical to Fig (1-6) of reference [Paul et al.[30]].Fig. (1) to Fig. (2) represent the population density of $N_l(t)$ and $N_r(t)$ respectively for numerous values of α (α=0, 0.5, 1) and fixed values of r , h and k.From these graph it is inferred that the population density of $N_l(t)$ is always greater than $N_r(t)$ for all values of α. Fig. (3) to Fig. (4) Show the population density for fixed values of$r$, h and k. From these Figs., it is inferred that the species of $N_r(t)$ reaches the steady state value when $t \geq 0.01$ and $N(t) \geq 0.1$ .In Fig. (5) to Fig. (6) Observe that, when α increases, the difference between $N_l(t)$ and $N_r(t)$also decreases except $\alpha = 1$, it is overlap with one and another for the fixed biological parameters $r = 0.41$, $h = 0.01$ and $k = 100$. Also, there is no significant difference in the population density$N_l(t)$ and $N_r(t)$.It is concluded that, the population density $N_l(t)$ increases when $N_r(t)$ decreases with increasing values of α and fixed value of possible parameters.

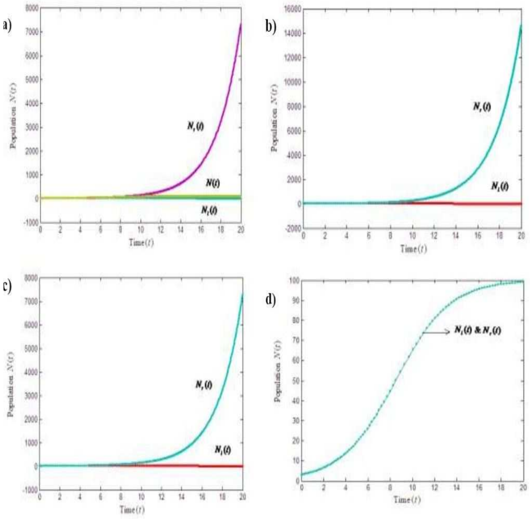

Fig. 1: Fig. 1(a-d) Estimation of our population density $N(t)$, (8)-(11) with the numerical simulation and fuzzy solution of (Paul et al.[30] for possible values of parameters r, h and k. The population densities were computed using for values of $\alpha = 0, 0.5, 1$ and some hypothetical values of other parameter $r = 0.41$, $h = 0.01$ and $k = 100$.

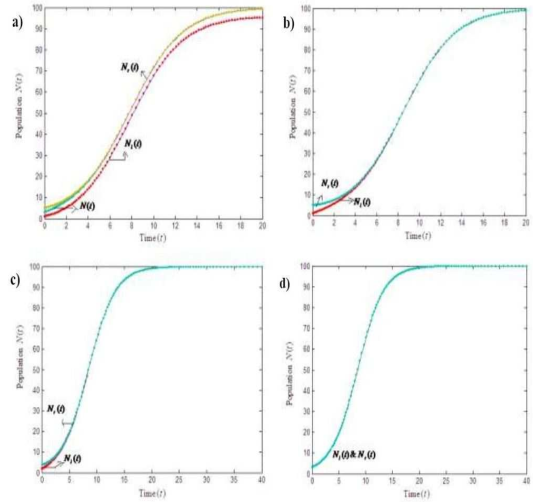

Fig. 2: Fig. 2(a-d) Estimation of our population density $N(t)$, (13)-(14) with the numerical simulation and fuzzy solution of (Paul et al.[30] for possible values of parameters r, h and k. The population densities were computed using for values of $\alpha = 0, 0.5, 1$ and some hypothetical values of other parameter $r = 0.41$, $h = 0.01$ and $k = 100$.

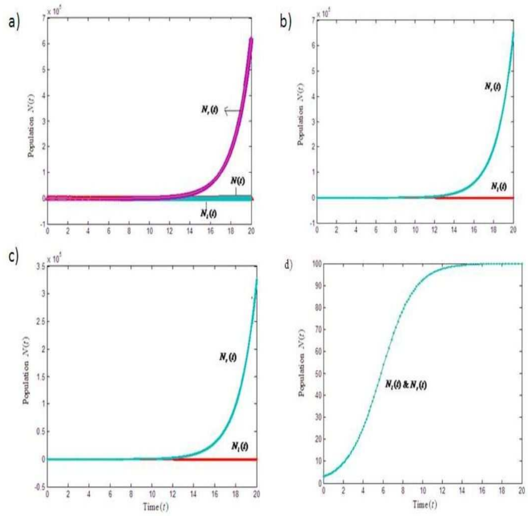

Fig. 3: Fig. 3(a-d) Estimation of our population density $N(t)$, (16)-(17) with the numerical simulation and fuzzy solution of (Paul et al.[30] for possible values of parameters r, h and k. The population densities were computed using for values of $\alpha = 0, 0.5, 1$ and some hypothetical values of other parameter $r = 0.41$, $h = 0.01$ and $k = 100$.

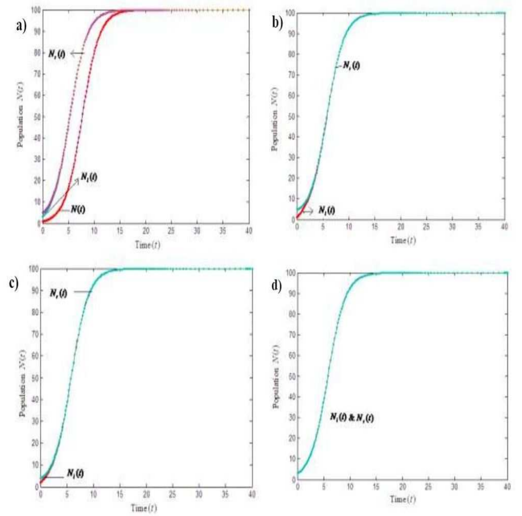

Fig. 4: Fig. 4(a-d) Estimation of our population density $N(t)$, (19)-(20) with the numerical simulation and fuzzy solution of (Paul et al.[30] for possible values of parameters r, h and k. The population densities were computed using for values of $\alpha = 0, 0.5, 1$ and some hypothetical values of other parameter $r = 0.41$, $h = 0.01$ and $k = 100$.

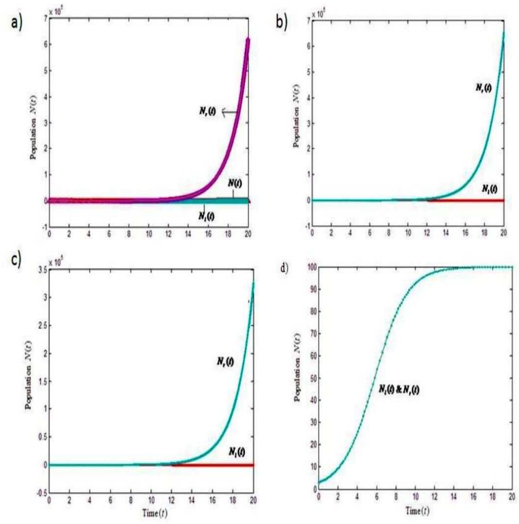

Fig. 5: Fig. 5(a-d) Estimation of our population density $N(t)$, (22)-(23) with the numerical simulation and fuzzy solution of (Paul et al.[30] for possible values of parameters r, h and k. The population densities were computed using for values of $\alpha = 0, 0.5, 1$ and some hypothetical values of other parameter $r = 0.41$, $h = 0.01$ and $k = 100$.

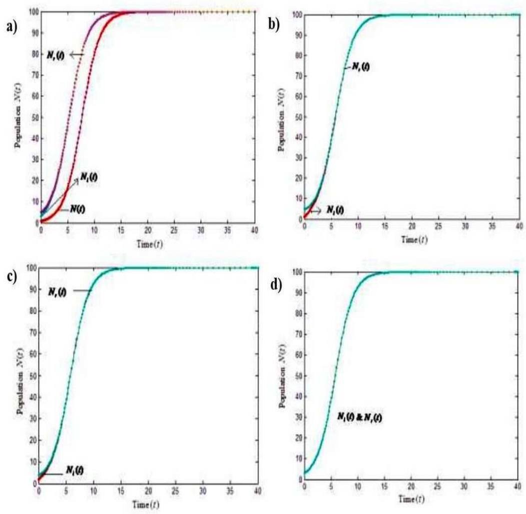

Fig. 6: Fig. 6(a-d) Estimation of our population density $N(t)$, (25)-(26) with the numerical simulation and fuzzy solution of (Paul et al.[30] for possible values of parameters r, h and k. The population densities were computed using for values of $\alpha = 0, 0.5, 1$ and some hypothetical values of other parameter $r = 0.41$, $h = 0.01$ and $k = 100$.

6 Conclusions

Comparing to the day-to-day circumstance, it is difficult to identify the value of parameters of the biological model precisely. To overcome these difficulties one may take the fuzzy model in consideration to get better results.

Biological model of quota harvesting process for a single species population density in fuzzy environment is conferred. In this article, an effort is made to find the inspiration of diversified operating parameters, i.e., population density, intrinsic growth rate and prey population in fuzzy environmental containing non-linear contributions are reported using Homotopy perturbation method. Basically the outcome of this work is to appraise the approximate values of the population density for all possible values of the parameters. This theoretical analysis is useful for fuzzy environment process. The derived population densities are correlated with simulation results which are in expectional accord.

Hence, it is concluded that this approach (Homotopy perturbation method) for quota harvest model in fuzzy environment is extraordinarily encouraging the researchers. However, they also concerned the modeling through inconclusiveness in some linear and non-linear inequality equation problem in rich field of engineering sciences.

References

[1] D. Ludwig, D. D. Jones, and C. S. Holling, Journal of Animal Ecology, **47**, 315-352, (1978) .
[2] L. A. Zadeh, Information and Control, **8**, pp. 338-353, (1965).
[3] S. L. Chang and L. A. Zadeh, IEEE Trans. Syst. Man. Cybern., **2**, 30-34, (1972).
[4] D. Dubois and H. Prade, Differ Fuzzy Sets Syst **8**:225-233, (1982).
[5] O. Kaleva, Fuzzy Sets and Systems, **24**, pp. 301- 317, (1987).
[6] B. Bede, Fuzzy Sets Syst 157:986-989, (2006).
[7] A. Khastan and J. J. Nieto, Nonlinear Analysis: Theory, Methods and Applications, 2010.
[8] B. Bede and S. G. Gal, Fuzzy Sets Syst. 151, 581-599 (2005).
[9] L. Stefanini, D. Dubois, M.A. Lubiano, H. Prade, M. A. Gil, P. Grzegorzewski, O. Hryniewicz (Eds.), In Series on Advances in Soft Computing, vol. 48, Springer, 2008.
[10] T. Allahviranloo , S. Abbasbandy,S. Salahshour ,H. Hakimzadeh, Computing 92:181-197, (2011).
[11] M. Chen ,C. Wu ,X. Xue , G. Liu , Inf Sci 178:1877-1892, (2008).
[12] D. O'Regan ,V. Lakshmikantham ,J. Nieto ,Non-linear Anal 54:405-415 (2003).
[13] J. J. Buckley , T. Feuring , Fuzzy Sets Syst 110:43-54, (2000).
[14] S. Tapaswini, S. Chakraverty, Appl Math 8(2):647-672, (2013).
[15] J. H. He, Computer Methods in Applied Mechanics and Engineering, Vol. 178, pp. 257-262, (1999).

[16] J. H. He, International Journal of Non-Linear Mechanics, Vol. 35, pp. 37-43, (2000) .

[17] R. C. Bassanezi, L. C. Barros and P. A. Tonelli , Fuzzy Sets Syst. 113 :473-483, (2000) .

[18] L. C. Barros, R. C. Bassanezi, P. A. Tonelli, Ecol. Model. 128,27-33 (2000).

[19] M. Gou, X. Xu, R. Li, Fuzzy Sets Syst. 138 : 601-615, (2003) .

[20] M. Peixoto,L. Barros and R. Bassanezi , Ecological Modelling, 214, 39-44, (2008).

[21] M. T. Mizukoshi, L. C. Barros,R. C. Bassanezi , Int. J. Uncertainty Fuzziness Knowledge Based Syst. 17, 69-84, (2009).

[22] M. Z. Ahmad ,B. D. Baets , IFSA-EUSFLAT, Lisbon, Portugal, p 1311-1314, (2009).

[23] M. Najariyan , M. H. Farahi , M. Alavian ,J Math ComputSci 2(4):639-649, (2011).

[24] A. H. A. Omar ,Y. A. Hasan , J QualMeas Anal JQMA 7(2):75-83, (2011).

[25] D. Pal, G. S. Mahapatra, G. P. Samanta., Pacific Asian J. Math. 6 ,199-213,(2012).

[26] M. Z. Ahmad and M. K. Hasan , International Journal of Modern Physics: Conference Series vol 9, pp 354-363, (2012).

[27] Maan, Normah, Barzinji, Khabat, Aris, Nor'aini, In: Proceedings of the World Congress on Engineering 2013, vol I, WCE 2013, July 3-5, London, UK,(2013).

[28] D. Pal and G. S. Mahapatra , Appl Math Comput 242:748-763, (2014).

[29] D. Pal,G. S. Mahaptra and G. P. Samanta ,IJMS 12(1-2):33-46, (2013b).

[30] Susmita Paul, Sankar Prasad Mondal, Paritosh Bhattacharya ,Model. Earth Syst. Environ, 2:70, (2016).

[31] J. Pastor , Mathematical Ecology of Populations and Ecosystems, Wiiley-blackwell,(2008) .

[32] Rogers C and Shadwick W F, Backlund Transformations and Their Applications, Vol.161, in Mathematics in Science and Engineering, Academic Press, New York, 1982.

[33] Bueno.M.I, Marcellan.F (2004) ,Darboux transformation and perturbation of linear functionals ,Linear Algebra and its Applications 384 :215-242

[34] Gel'fand, I. M. and Levitan, B. M.,(1995) , American Mathematical Society Translations, (2)1:253-304.

[35] Hietarinta, Phys. AUC, 15(part 1):31- 37,(2005) .

[36] Huibin.L, Kelin.W, I. J. Phys. A: Math. Gen. 23 :3923-3928(1990).

[37] He.J.H, Internat. J. Non-Linear Mech., vol. 34, no. 4, pp. 699-708(1999).

[38] He.J.H, Comput. Methods Appl. Mech. Engrg. Vol. 167(1-2) p. 57,(1998).

[39] MATLAB 6.1, The MathWorks Inc., Natick, MA, 2000; www.scilabenterprises.com.

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