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N Two-Transmon-Qubit Quantum Logic Gates Realized in a Circuit QED System

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Abstract: We theoretically present an effective method to realize NiSWAP, N(sqrt(iSWAP)), NSWAP, N(sqrt(SWAP)) and NTQ-NOT gates based on the qubit-qubit interaction in a circuit QED using N+1 transmon qubits driven by a strong microwave field. In this system, the interaction between the qubits and the resonator field can be achieved by turning the gate voltage and the external flux. The operation time is independent of the number of qubits involved in the scheme, and the gates operations are insensitive to the initial state of the resonator. These quantum logic gates can be realized in a time (nanosecond-scale) much smaller than decoherence time (microsecond-scale), and it is more immune to the 1/f charge noise and has longer dephasing time due to the favorable properties of the transmon qubits in the circuit QED. Numerical simulation under the influence of the gates operations shows that the scheme can be implemented with high fidelity. We also propose a detailed procedure and experimentally analyze its feasibility. Moreover, the scheme might be experimentally achieved efficiently within current state-of-the-art technology.

Keywords: Transmon qubit, *Ni*SWAP gate, *N*SWAP gate, *N*TQ-NOT gate, $N\sqrt{i\text{SWAP}}$gate, $N\sqrt{\text{SWAP}}$ gate, circuit QED.

1 Introduction

Quantum computation has attracted much attention since a quantum computer has an ability to solve hard computational problems with high efficiency compared to a classical computer [1,2,3]. Also, it is well known that it is difficult to simulate the behaviour of a quantum mechanical system with a classical computer. The difficulty arises because quantum systems are not confined to their eigenstates but can in general exist in any superposition of them, thus the vector space needed to describe the system is extremely large. Quantum logic gates are the basic building blocks of a quantum computer [4,5]. A universal quantum computer can be built on a series of two-qubit logic gates [6,7]. Up to now, a large number of theoretical proposals have been introduced to implement various gates, such as two-qubit Controlled-NOT (CNOT) gate [8], SWAP gate [9], *i*SWAP gate [10], $\sqrt{i\text{SWAP}}$ gate [8] and $\sqrt{\text{SWAP}}$ gate [11] in different systems, such as cavity QED system [13] and circuit QED system [14]. The circuit QED is currently the promising candidate for future quantum computer.

Recently, Yang et al. [15] proposed a novel scheme for implementing multi-qubit quantum controlled-phase gate with one superconducting qubit simultaneously controlling n qubits selected from N qubits ($1 < n < N$). In ref. [16] the authors proposed a scheme to realize *N*TCP gate in a circuit QED by using the system of $(N+1)$ transmon qubits. The method of implementing n qubits SWAP gates simultaneously between one superconducting charge qubit with n qubits coupled to a cavity was firstly proposed by Song et al. [17].

In recent works, We have proposed another type of multiqubit controlled gate that is a *N*TCP gate by using qubit-qubit interaction in a circuit QED [18]. Also, we have presented a method for realizing *N*TCP-NOT and *N*TQ-NOT gates with one control superconducting qubit simultaneously controlling N target qubits in a cavity QED [19]. In the present study, we present and demonstrate a method for realizing N two-transmon-qubit quantum logic gates (*Ni*SWAP, $N\sqrt{i\text{SWAP}}$, *N*SWAP,

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$N\sqrt{\text{SWAP}}$, and NTQ-NOT gates) of one transmon qubit simultaneously controlling N target transmon qubits in circuit QED with nearest qubit–qubit interaction by adding a strong microwave field. These type of quantum logic gates are useful in quantum computation and quantum information processing. On the other hand, the qubit-qubit interaction could influence the evolution of the system used in ref. [17]. Moreover, the presence of both qubit-qubit interaction and Jaynes-Cummings make our system powerful.

The paper is organized as follows: In Sec.2, we concretely illustrate the way NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates via circuit QED by using the system of $N+1$ transmon qubits coupled to a resonator with nearest qubit–qubit interaction, we have calculated the evolution operator a three-step, we use the overall evolution operator for obtaining these quantum logic gates, we also give a brief discussion about our proposal. In Sec.3, we study the fidelity and then discuss its feasibility based on current experiments in circuit QED. A concluding summary is given in Sec.4.

2 Basic theory

The method presented for implementing N two-transmon-qubit quantum logic gates (NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates) via circuit QED is based on the qubit-qubit interaction. In this subsection, we show how to apply the first method to implement these quantum logic gates which the transmon qubits are capacitively coupled to a superconducting transmission line resonator (TLR) driven by a strong microwave field. The physical setup considered in this study is schematically illustrated in Fig.1. The operation time of these gate is shorten because the coupling constant between the transmon qubit and the quantum date bus (QDB) is large enough in the case of TLR has been chosen as QDB. The similar architectures have been used in refs. [20].

Let us now consider an $(N+1)$ transmon qubits which are capacitively coupled to a TLR driven by a strong microwave field [20] as depicted in Fig.1. A microwave field of frequency ω_d is applied to the input wire of the TLR. The $(N+1)$ transmon qubits each having two-level subspaces driven by a conventional field added, these transmon qubits are capacitively coupled to it. Moreover, the qubit-qubit interaction should be included in the circuit QED [21], where the direct interaction between the qubits, in circuit QED, is virtually realized in the dispersive regime [22]. The Hamiltonian of the whole system (assuming $\hbar=1$) given by [18]

$$H=\omega_q\sum_{j=1}^{N+1}\sigma_{z,j}+\omega_r a^+a+\Omega\sum_{j=1}^{N+1}(\sigma_j^+e^{-i\omega_d t}+\sigma_j^-e^{i\omega_d t})+\sum_{j=1}^{N+1}g_j(a^+\sigma_j^-+a\sigma_j^+)+\sum_{j,k=1,\,j\neq k}^{N+1}\Gamma_{jk}\sigma_j^+\sigma_k^-, \quad (1)$$

we get $\Omega=g_j\varepsilon/\delta$ where $\delta=\omega_r-\omega_d$ is the detuning between the resonance frequency of the TLR ω_r and the frequency of the external drive applied to the TLR ω_d, $\Gamma_{jk}=g_jg_k(\frac{1}{\Delta_j}+\frac{1}{\Delta_k})$ [22] which describes the effective interaction between qubits j and k, and $\Delta_j(\Delta_k)$ is large detuning as compared to the qubit-TLR coupling $g_j(g_k)$ [23]. $\sigma_{z,j}$, σ_j^-, and σ_j^- are the collective operators for the $(1,2,...,N+1)$ qubits, which are given by $\sigma_{z,j}=\frac{1}{2}(|0_j\rangle\langle 0_j|-|1_j\rangle\langle 1_j|)$, $\sigma_j^+=|1_j\rangle\langle 0_j|$, $\sigma_j^-=|0_j\rangle\langle 1_j|$ where $|1_j\rangle(|0_j\rangle)$ is the excited state (ground state) of the transmon qubit, $\omega_r=1/\sqrt{LC}$ is the resonance frequency of the TLR where the transmission line resonator can be modeled as a simple harmonic oscillator composed of the parallel combination of an inductor L and a capacitor C, ω_q is the transition frequency of the transmon qubit with $\omega_{q_1}=\omega_{q_2}=...=\omega_{q_{N+1}}$, a^+,a are the creation and annihilation of the resonator mode. In the high E_J/E_C limit the transition frequency between the ground state $|0_j\rangle$ and excited state $|1_j\rangle$ is given by $\omega_q=\sqrt{8E_JE_C}/\hbar$, where $E_C=e^2/2C_\Sigma$ and $E_J(\Phi)=E_{J0}|\cos(\pi\Phi/\Phi_0)|$, $C_\Sigma=C_S+(C_g^{-1}+C_g'^{-1})^{-1}$. Here, C_g, C_g' are the gate capacitance, C_S is the additional capacitor, C_Σ is the effective total capacitance, Φ is the external magnetic flux applied to the SQUID loop, $\Phi_0=h/2e$ is the flux quantum, $E_J(\Phi)$ is the effective Josephson coupling energy, E_C is the charging energy, and E_{J0} is the Josephson coupling energy, where the qubit work at the regime $E_J/E_C\gg 1$. It is obvious that the frequency of the transmon qubit ω_q can be tuned by external magnetic flux Φ. When we work with a large amplitude driving field, quantum fluctuations in the drive are very small with respect to the drive amplitude and the drive can be considered, for all practical purposes, as a classical field.

For convince, we assume that we can tune the coupling constant g_j and the coupling strength Γ_{jk} at the same time to have $g_j=g_k=g$ and $\Gamma_{jk}=\Gamma=\frac{2g^2}{\Delta}$ (in the case $\Delta_j=\Delta_k=\Delta$). By assuming that $\omega_d=\omega_q$, we have the following Hamiltonian H_I in the interaction picture [18]

$$H_I=\Omega\sum_{j=1}^{N+1}\left(\sigma_j^++\sigma_j^-\right)+g\sum_{j=1}^{N+1}\left(e^{i\delta t}a\sigma_j^++e^{-i\delta t}a^+\sigma_j^-\right)+\Gamma\sum_{j,k=1,j\neq k}^{N+1}\sigma_j^+\sigma_k^-, \quad (2)$$

where $\delta=\omega_r-\omega_q$ is the detuning between the frequency of the qubit ω_q and the frequency of the resonator ω_r. When $\Omega\gg\delta,g,\Gamma$ and $\delta\gg g,\Gamma$ we can get the Hamiltonian H_I in the interaction picture [15,24,25,26]

$$H_I=H_0+H_{eff}, \quad (3)$$

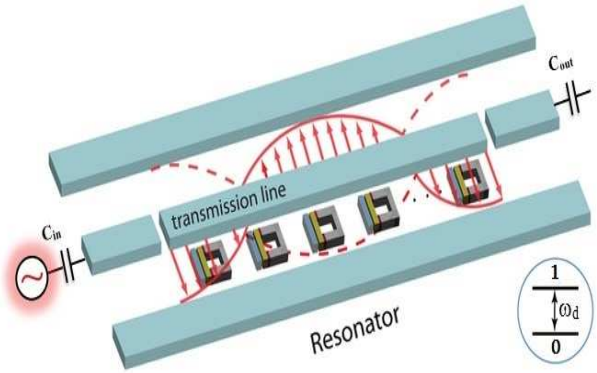

Fig. 1: (Color online) Schematic diagram of a TLR and several $(N+1)$ transmon qubits are coupled in a circuit QED (blue) driven by a strong microwave field of frequency ω_d (this field $\sim$ is applied to the input wire of the TLR). The TLR is connected to the input wiring with a capacitor C_{in}, and output wiring with a capacitor C_{out}. In addition, the coupling between qubits i and j is via inductances coupling, where this coupling can be interpreted as an energy coupling between qubits.

with

$$H_0 = 2\Omega \sum_{j=1}^{N+1} \sigma_{x,j} \tag{4}$$

$$H_{eff} = \frac{g^2}{\delta}\left[\sum_{j=1}^{N+1}(\sigma_j^+\sigma_j^- + a^+a\sigma_{z,j}) + \sum_{j,k=1,\, j\neq k}^{N+1}\sigma_j^+\sigma_k^-\right] + \Gamma \sum_{j,k=1, j\neq k}^{N+1}\left(\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+\sigma_k^- + \sigma_j^-\sigma_k^+\right)\right), \tag{5}$$

where $\sigma_{x,j} = \frac{1}{2}\left(\sigma_j^+ + \sigma_j^-\right)$. From *Eq.*3, we can easily obtain the corresponding evolution operator $U(t)$ as follows [15,25,27]

$$U(t) = \exp\left\{-i2\Omega t\sum_{j=1}^{N+1}\sigma_{x,j} - i\frac{g^2}{\delta}t\left[\sum_{j=1}^{N+1}\left(\sigma_j^+\sigma_j^- + a^+a\sigma_{z,j}\right) + \sum_{j,k=1,\, j\neq k}^{N+1}\sigma_j^+\sigma_k^-\right] - i\Gamma t\sum_{j,k=1,\, j\neq k}^{N+1}\left[\frac{1}{2}\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+\sigma_k^- + \sigma_j^-\sigma_k^+\right)\right]\right\}. \tag{6}$$

For a charge qubit 1 coupled to the resonator, assumed $\delta \gg |g|$. Then, the corresponding evolution operator can be expressed as [17,28]

$$U_{I1}(t) = \exp\left[-2i\Omega t\sigma_{x,1} - i\frac{g^2}{\delta}t(\sigma_1^+\sigma_1^- + a^+a\sigma_{z,1})\right]. \tag{7}$$

We now show how to utilize the above model (*Eq.*6) to implement an NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates. We note that: (*i*) for each one of qubits $1,2,...,N+1$ of each step of the operations, the dc gate voltage $V_g^{dc} = e/C_g$, such that $E_z = 0$ for each qubit, and (*ii*) the resonator mode frequency ω_r is fixed during the entire operation. We now consider two special cases: $\delta > 0$ as well as $\delta < 0$. The results from the unitary evolution, obtained for these two special cases, are employed below for the gate implementation. The detailed procedure is given as follows:

Step 1. Let us begin with the case $\delta > 0$, set $V_g^{ac} = V_0\cos(\omega t)$ for each qubit, adjust the transition frequencies of $(N+1)$ qubits by external flux Φ_j, then $\omega_q = \omega_d$ is satisfied, with the pulse Rabi frequency $\Omega = g\varepsilon/\delta$ of the amplitude V_0 where the detuning $\delta = \omega_r - \omega_q > 0$. So, after a period of evolution time $\tau_1 = 2\pi/\delta$, the evolution operator for the qubit system corresponding to this step would be the $U_I(t)$ of *Eq.*6.

$$U(\tau_1) = \exp\left\{-i2\Omega\tau\sum_{j=1}^{N+1}\sigma_{x,j} - i\frac{g^2}{\delta}\tau\left[\sum_{j=1}^{N+1}(\sigma_j^+\sigma_j^- + a^+a\sigma_{z,j}) + \sum_{j,k=1,\, j\neq k}^{N+1}\sigma_j^+\sigma_k^-\right] - i\Gamma\tau\sum_{j,k=1,\, j\neq k}^{N+1}\left[\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+\sigma_k^- + \sigma_j^-\sigma_k^+\right)\right]\right\}, \tag{8}$$

Step 2. Let us now consider the case $\delta < 0$, set $V_g^{ac} = 0$ for qubit 1, adjust the level spacings of qubit 1 such that the resonator mode is coupled to qubits $(2,3,...,N+1)$, so $\omega_q = \omega_d$ is satisfied, with the pulse Rabi frequency is Ω' of the amplitude V_0', where the detuning $\delta' = \omega_r - \omega_q' = -\delta > 0$. Then, after a period of time $\tau_2 = 2\pi/\delta'$, the evolution operator of the system corresponding to this step can be described by

$$U(\tau_2) = \exp\left\{-i2\Omega\tau\sum_{j=2}^{N+1}\sigma_{x,j} - i\frac{g^2}{\delta}\tau\left[\sum_{j=2}^{N+1}(\sigma_j^+\sigma_j^- + a^+a\sigma_{z,j}) + \sum_{j,k=2,\, j\neq k}^{N+1}\sigma_j^+\sigma_k^-\right] - i\Gamma\tau\sum_{j,k=2,\, j\neq k}^{N+1}\left[\sigma_{x,j}\sigma_{x,k} + \frac{1}{4}\left(\sigma_j^+\sigma_k^- + \sigma_j^-\sigma_k^+\right)\right]\right\}, \tag{9}$$

The combined time evolution operator of the whole system, after these two steps, is arrived at

$$U(\tau_1+\tau_2) = \exp\left\{-i2\Omega\tau\sigma_{x,1} - i\frac{g^2}{\delta}\tau(\sigma_1^+\sigma_1^- + a^+a\sigma_{z,1}) - i\frac{g^2}{\delta}\tau\sum_{j=2}^{N+1}(\sigma_1^+\sigma_j^- + \sigma_1^-\sigma_j^+) - i\Gamma\tau\sum_{j=2}^{N+1}\left[\sigma_{x,1}\sigma_{x,j} \frac{1}{2}\left(\sigma_1^+\sigma_j^- + \sigma_1^-\sigma_j^+\right)\right]\right\}. \tag{10}$$

Step 3. We adjust the transition frequency of qubits $2,3,...,N+1$ such that the resonator mode is largely

decoupled from each qubit, but we leave the transition frequency unchanged for qubit 1 such that $g^2/|\delta'| \ll |g|$ (i.e. $|\delta'| \gg |g|$ where $\delta' < 0$) and $\Omega' = g\varepsilon/\delta'$. When the condition $\delta' = -\delta$ is satisfied, the time evolution operato after a period of time τ is [17,28]

$$U_{I1}(\tau_3) = \exp\left[2i\Omega\tau\sigma_{x,1} + i\frac{g^2}{\delta}\tau(\sigma_1^+\sigma_1^- + a^+a\sigma_{z,1})\right]. \tag{11}$$

After the three-step process, a phase flip (i.e., $|1\rangle \to -|1\rangle$) on the state $|1\rangle$ of each target transmon qubit is achieved when the control transmon qubit 1 is initially in the state $|1\rangle$, but nothing happens to the states $|0\rangle$ and $|1\rangle$ of each target transmon qubit when the control transmon qubit 1 is initially in the state $|0\rangle$. When the conditions $\frac{\Gamma}{2} + \frac{g^2}{\delta} = \gamma$ and $\Gamma = 4\chi$, the evolution operator of the system, after the above three steps operation, is given by

$$U_s(\tau) = \prod_{j=2}^{N+1} U_{gate}(1,j), \tag{12}$$

where
$U_{gate}(1,j) = \exp[-i\chi\tau 4\sigma_{x,1}\sigma_{x,j} - i\gamma\tau(\sigma_1^+\sigma_j^- + \sigma_1^-\sigma_j^+)]$, $j = 2,3,...,N+1$. In the next section, we will use this evolution operator of the whole system (*Eq.*12) for demonstrate how *Ni*SWAP, $N\sqrt{i\text{SWAP}}$, *N*SWAP, $N\sqrt{\text{SWAP}}$ and *N*TQ-NOT gates can be realized.

3 Preparation of quantum logic gates and discussion

3.1 Implementation of NiSWAP and N$\sqrt{iSWAP}$gates

It is demonstrated that more complex quantum gates can be created by the *i*SWAP and $\sqrt{i\text{SWAP}}$gates [29]. Furthermore, the *i*SWAP and $\sqrt{i\text{SWAP}}$ gates forms a universal gate set, a CNOT gate for example can be obtained from two *i*SWAP or two $\sqrt{i\text{SWAP}}$ gates and single-qubit rotations [4]. Recently, it has been shown that the *i*SWAP and $\sqrt{i\text{SWAP}}$ gates can be very useful for applications in quantum information process QIP and quantum computing [30]. The purely quantum iSWAP and iSWAP gates are an appropriate elementary two-qubit gates, these quantum logic gates can be represented by the the following input–output operators:

$$iSWAP \equiv |00\rangle\langle 00| + i|01\rangle\langle 10| + i|10\rangle\langle 01| + |11\rangle\langle 11|. \tag{13}$$

$$\sqrt{iSWAP} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

The goal of this subsection is to demonstrate how the *Ni*SWAP and $N\sqrt{i\text{SWAP}}$ gates can be realized based on the evolution operator of the whole system (*Eq.*12). According to the evolution operator $U_{gate}(1,j)$ above (from *Eq.*12) on the basis $(|0_1\rangle, |1_1\rangle)$ for qubit 1, so the basis $(|0_j\rangle, |1_j\rangle)$ for qubit $(2,3,....N+1)$, we can obtain following evolutions

$$\begin{aligned}
U_{gate}(1,j)|0_1\rangle|0_j\rangle &= \cos\chi\tau|0_1\rangle|0_j\rangle - i\sin\chi\tau|1_1\rangle|1_j\rangle \\
U_{gate}(1,j)|0_1\rangle|1_j\rangle &= e^{i\eta\pi}\left[[\cos(\gamma+\chi)\tau|0_1\rangle|1_j\rangle\right] \\
&\quad -i\sin(\gamma+\chi)\tau|1_1\rangle|0_j\rangle] \\
U_{gate}(1,j)|1_1\rangle|0_j\rangle &= e^{i\eta\pi}\left[[\cos(\gamma+\chi)\tau|1_1\rangle|0_j\rangle\right] \\
&\quad -i\sin(\gamma+\chi)\tau|0_1\rangle|1_j\rangle] \\
U_{gate}(1,j)|1_1\rangle|1_j\rangle &= \cos\chi\tau|1_1\rangle|1_j\rangle - i\sin\chi\tau|0_1\rangle|0_j\rangle,
\end{aligned} \tag{15}$$

A phase factor $\eta\pi$ in the previous evolutions can be produced by several different proposals [31]. By setting $\chi\tau = 2k\pi$ (with *k* being an integer), $\eta = 2m$ (with *m* being an integer) and $\gamma\tau = \left(2n + \frac{3}{2}\right)\pi$ (with *n* being an integer), we obtain *N*-two-qubit *i*SWAP operations as (from *Eq.*15)

$$\begin{aligned}
U_{iSWAP}(1,j)|0_1\rangle|0_j\rangle &= |0_1\rangle|0_j\rangle \\
U_{iSWAP}(1,j)|0_1\rangle|1_j\rangle &= i|1_1\rangle|0_j\rangle \\
U_{iSWAP}(1,j)|1_1\rangle|0_j\rangle &= i|0_1\rangle|1_j\rangle \\
U_{iSWAP}(1,j)|1_1\rangle|1_j\rangle &= |1_1\rangle|1_j\rangle.
\end{aligned} \tag{16}$$

By setting $\chi\tau = 2k\pi$ (with *k* being an integer), $\eta = 2m$ (with *m* being an integer) and $\gamma\tau = \left(2n + \frac{5}{4}\right)\pi$ (with *n* being an integer), we obtain *N*-two-qubit $\sqrt{i\text{SWAP}}$ operations as

$$\begin{aligned}
U_{\sqrt{iSWAP}}(1,j)|0_1\rangle|0_j\rangle &= |0_1\rangle|0_j\rangle \\
U_{\sqrt{iSWAP}}(1,j)|0_1\rangle|1_j\rangle &= \frac{1}{\sqrt{2}}[|0_1\rangle|1_j\rangle + i|1_1\rangle|0_j\rangle] \\
U_{\sqrt{iSWAP}}(1,j)|1_1\rangle|0_j\rangle &= \frac{1}{\sqrt{2}}[|1_1\rangle|0_j\rangle + i|0_1\rangle|1_j\rangle] \\
U_{\sqrt{iSWAP}}(1,j)|1_1\rangle|1_j\rangle &= |1_1\rangle|1_j\rangle.
\end{aligned} \tag{17}$$

Then, the *N* two qubit *i*SWAP and $\sqrt{i\text{SWAP}}$ operations are simultaneously performed on *N* qubit pairs $(1,2)$, $(1,3)$, ..., $(1,N+1)$. Each gate operation (*i*SWAP and $\sqrt{i\text{SWAP}}$ gate) includes the same control qubit and a different target qubit. Thus, the *Ni*SWAP and $N\sqrt{i\text{SWAP}}$ gates are implemented simultaneously between the first qubit and the *N* qubits. Finally, we give a brief discussion about our proposal. We notice that δ satisfies the equation $(g/\delta_1)^2 = n + \frac{3}{4}$ of the *Ni*SWAP gate and the equation $(g/\delta_2)^2 = n + \frac{5}{8}$ of the $\sqrt{i\text{SWAP}}$ gate, where *n* is an integer. So, when $n = 0$, δ takes maximum $\delta_1 = \frac{2\sqrt{3}}{3}g$ and $\delta_2 = \frac{2\sqrt{10}}{5}g$. Then, the operation time $t_{op_1} = 3\tau = 2\pi \times \frac{3\sqrt{3}}{2g}$ (*Ni*SWAP gate) and $t_{op2} = 3\tau = 2\pi \times \frac{3\sqrt{10}}{4g}$ ($\sqrt{i\text{SWAP}}$ gate) are independent of the number of target qubits *N*. Then, the direct

calculation shows that the operations times required to implement the NiSWAP and $\sqrt{i\text{SWAP}}$ gates with transmon qubits are $t_{op_1} = 13ns$ and $t_{op_2} = 11.86ns$. In the next, we use the above evolutions ($Eq.15$) for implementing NSWAP, NTQ-NOT and $N\sqrt{\text{SWAP}}$ gates.

3.2 *Implementation of NSWAP and N$\sqrt{SWAP}$ gates*

The SWAP and $\sqrt{\text{SWAP}}$ gates are an appropriate elementary two-qubit gates, where they are useful in quantum computation and quantum information processing [4], such as establishing the universality of two-qubit gates [32], programmable gate arrays [33], and constructing quantum circuits [34]. On the other hand, the SWAP gate is equivalent to three CNOT gates and the $\sqrt{\text{SWAP}}$ gate constitutes a universal set of quantum gates together with single qubit rotations around an arbitrary axis [11]. these type of quantum logic gates can be represented by the operators:

$$SWAP \equiv |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 11| \quad (18)$$

$$\sqrt{SWAP} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (19)$$

The main objective of this subsection is to realize NSWAP and $\sqrt{\text{SWAP}}$ gates using the evolutions ($Eq.15$). By selecting $\chi\tau = 2k\pi$ (with k being an integer), $\eta = 2m + \frac{1}{2}$ (with m being an integer) and $\gamma\tau = \left(2n + \frac{1}{2}\right)\pi$ (with n being an integer), an N two qubit SWAP operations are simultaneously performed on N qubit pairs $(1,2)$, $(1,3)$, ..., $(1,N+1)$ as (from $Eq.15$)

$$\begin{aligned} U_{SWAP}(1,j)|0_1\rangle|0_j\rangle &= |0_1\rangle|0_j\rangle \\ U_{SWAP}(1,j)|0_1\rangle|1_j\rangle &= |1_1\rangle|0_j\rangle \\ U_{SWAP}(1,j)|1_1\rangle|0_j\rangle &= |0_1\rangle|1_j\rangle \\ U_{SWAP}(1,j)|1_1\rangle|1_j\rangle &= |1_1\rangle|1_j\rangle. \end{aligned} \quad (20)$$

By setting $\chi\tau = 2k\pi$ (with k being an integer), $\eta = 2m + \frac{1}{4}$ (with m being an integer) and $\gamma\tau = \left(2n + \frac{1}{4}\right)\pi$ (see $Eq.15$), we obtain N-two-qubit $\sqrt{\text{SWAP}}$ operations as

$$\begin{aligned} U_{\sqrt{SWAP}}(1,j)|0_1\rangle|0_j\rangle &= |0_1\rangle|0_j\rangle \\ U_{\sqrt{SWAP}}(1,j)|0_1\rangle|1_j\rangle &= \frac{1+i}{2}|0_1\rangle|1_j\rangle + \frac{1-i}{2}|1_1\rangle|0_j\rangle \\ U_{\sqrt{SWAP}}(1,j)|1_1\rangle|0_j\rangle &= \frac{1+i}{2}|1_1\rangle|0_j\rangle + \frac{1-i}{2}|0_1\rangle|1_j\rangle \\ U_{\sqrt{SWAP}}(1,j)|1_1\rangle|1_j\rangle &= |1_1\rangle|1_j\rangle. \end{aligned} \quad (21)$$

By this way, one can see that NSWAP and N $\sqrt{\text{SWAP}}$ gates are simultaneously performed on the qubit pairs $(1,2)$, $(1,3)$, ..., $(1,N+1)$, respectively. So, the NSWAP and N $\sqrt{\text{SWAP}}$ gates are implemented simultaneously between one transmon qubit and N transmon qubits. Hence, it is clear that the NSWAP and N $\sqrt{\text{SWAP}}$ gates can be realized after the three-step process. Now, we give a brief discussion about quantum NSWAP and N $\sqrt{\text{SWAP}}$ gates, we notice that δ satisfies the equation $(g/\delta_3)^2 = n + \frac{1}{4}$ of the NSWAP gate and the equation $(g/\delta)^2 = n + \frac{1}{8}$ of the $N\sqrt{\text{SWAP}}$, where n is an integer, when $n = 0$, δ takes maximum $\delta_3 = 2g$ and $\delta_4 = 2\sqrt{2}g$. Then, the operations times required to implement the NSWAP and $N\sqrt{\text{SWAP}}$ gates are $t_{op_3} = 2\pi \times \frac{3}{2g}$ and $t_{op_4} = 2\pi \times \frac{3}{2g}$ respectively. So, the direct calculation shows that the operations times are $t_{op3} = 7.5ns$ and $t_{op_4} = 15ns$.

3.3 *Implementation of NTQ-NOT gate*

In this subsection, we focus on how to realize NTQ-NOT gate (N two-transman qubit-NOT gate) with one transmon qubit simultaneously controlling N target qubits by introducting the qubit-qubit interaction. The quantum NTQ-NOT operation on two-qubit computational basis is defined through the following input–output relations:

$$\begin{aligned} &U_{NOT}|0_i\rangle|0_j\rangle = |1_i\rangle|1_j\rangle; U_{NOT}|0_i\rangle|1_j\rangle = |1_i\rangle|0_j\rangle, \\ &U_{NOT}|1_i\rangle|0_j\rangle = |0_i\rangle|1_j\rangle; U_{NOT}|1_i\rangle|1_j\rangle = |0_i\rangle|0_j\rangle. \end{aligned} \quad (22)$$

The method presented so far for implementing the NTQ-NOT gate is based on the evolutions of the subsubsection 1 (see $Eq.15$) where an overall phase factor $e^{-i\eta\pi}$ is omitted. By setting $\chi\tau = (2k + \frac{1}{2})\pi$ (with k being an integer), $\eta = 2m + \frac{3}{2}$ (with m being an integer) and $\gamma\tau = (2n+1)\pi$ (with n being an integer), we can easily verify that

$$\begin{aligned} U_{NOT}(1,j)|0_1\rangle|0_j\rangle &= |1_1\rangle|1_j\rangle \\ U_{NOT}(1,j)|0_1\rangle|1_j\rangle &= |1_1\rangle|0_j\rangle \\ U_{NOT}(1,j)|1_1\rangle|0_j\rangle &= |0_1\rangle|1_j\rangle \\ U_{NOT}(1,j)|1_1\rangle|1_j\rangle &= |0_1\rangle|0_j\rangle. \end{aligned} \quad (23)$$

It's worth to point out that N two-transmon qubit-NOT operations are simultaneously performed on N qubit pairs $(1,2),(1,3),\ldots,(1,N+1)$, Each SWAP operation includes the same control qubit and a different target qubit. Then, the N two-transmon qubit-NOT gates are implemented simultaneously between one transmon qubit and N qubits. Hence, it is clear that the NTQ-NOT gate can be realised in circuit QED. Now, we give a brief discussion about quantum NTQ-NOT gate. We notice that δ satisfies the equation $(g/\delta_5)^2 = n + \frac{1}{2}$, where n is an integer, when $n = 0$, δ takes maximum $\delta_5 = \sqrt{2}g$. Then, the operation time is $t_{op_5} = 2\pi \times \frac{3\sqrt{2}}{2g}$. So, the direct calculation shows that the operation time required to implement the NTQ-NOT gate will be $t_{op_5} = 10.61ns$.

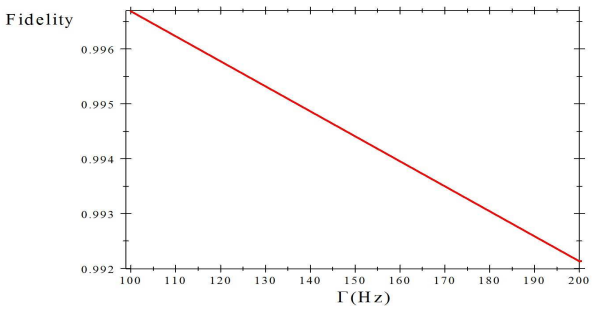

Fig. 2: Numerical results for fidelity of the gates operations versus the force qubit-qubit coupling Γ.

4 Possible experimental implementation

Let us now study the fidelity of the gate operation. In order to check the validity of our proposal, we define the following fidelity to characterize the deviation of how much the output states $|\Psi(t)\rangle$ deviate in amplitude and phase from the ideal logical gate transformation for the different input states [35]

$$F = |\langle\Psi(t)|U(t)|\Psi(0)\rangle|^2, \tag{24}$$

where $|\Psi(t)\rangle$ is the final state of the whole system after the gate operations, $|\Psi(0)\rangle$ is the initial state followed by an ideal phase operation and $U(t)$ is the overall evolution operator of the system are performed in a real situation. We numerically simulate the relationship between the fidelity of the system and the force qubit-qubit coupling Γ. Our numerical calculation shows that a high fidelity can be achieved when $\Gamma = 100$ Hz (see Fig. 2).

For this method to work, we discuss some issues which are relevant for future experimental implementation of our proposal. Compared with the usual charge qubit, the transmon qubit is immune to the $1/f$ charge noise and it has much longer dephasing time as a result of the transmon qubits chosen in the system. For the sake of definiteness, let us consider the experimental parameters of the transmon qubits as $C_g = 1aF$, $C_{J0} = 300aF$, $E_c = 2\pi \times 2GHz$ and $E_J \simeq 2\pi \times 100GHz$ [15]. The transmon qubits with these parameters are available at present. Thus, the coupling three qubits with a transmission line resonator has been experimentally demonstrated [36]. Our scheme may have potential applications in multipartite entanglement. The charge fluctuations are principal only in low-frequency region and can be reduced by the echo technique [37] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations and keep the state coherent for a longer time is highly desired. So our proposal is realizable with presently available circuit QED techniques.

In a recent experiments, it was showen that the decoherence times T_1 and dephasing time T_2 can be made to be on the order of $20-100$ μs for transmon qubits (when $E_J/E_C = 50$) [38,39]. In addition, the coupling strength is $g = 2\pi \times 200MHz$ [22,16], which is experimentally available. Furthermore, the operation time required to implement the N*i*SWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates with transmon qubits is independent of the number of target qubits N. It is clear from the above calculation that the total operation time (t_{op_1}, t_{op_2}, t_{op_3}, t_{op_4} and t_{op_5}) of the quantum logic gates (N*i*SWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates) which is much shorter than the decoherence times T_1 and dephasing time T_2, which satisfies our experimental requirement. Also, we note that the decoherence time of field state inside a resonator depends on the initial field state. However, the gate operation is independent of the initial state of the resonator because of the operator U_{gate} does not incluse the photon operator a and a^+ of the circuit QED.

5 Conclusion

In Summary, We have used the system in which the transmon qubits are capacitively coupled to a TLR driven by a strong microwave field. As shown above, we have presented and demonstrated a method for realizing an N*i*SWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates in circuit QED by introducing the qubit-qubit interaction. The operation time is independent of the number of qubits involved in the scheme, and the gates operations are insensitive to the initial state of the resonator. The system requires no disagreement between the qubits and the resonator. In addition, the operation time is only dependent of the detuning and the time can be controlled by adjusting the frequency between the $|0_j\rangle$ and $|1_j\rangle$. However, we have calculated an evolution operator in the case of the qubit-qubit interaction. Finally, we have applied the overall evolution operator to working basis of the qubit 1 and the qubits j ($j = 2,...,N+1$) for find the N*i*SWAP, $N\sqrt{i\text{SWAP}}$, NSWAP, $N\sqrt{\text{SWAP}}$ and NTQ-NOT gates. In the our system, when we choose suitable detuning, the quantum logic gates can be realized in a time much shorter than decoherence time, and it is more immune to the $1/f$ charge noise and have longer dephasing time as a result of the transmon qubits chosen in the system. In addition, numerical simulation of the gate operations shows that the scheme could be achieved with high fidelity under current state-of-the-art technology. Therefore, the present scheme might be realizable using the presently available techniques, and the experimental implementation of the present scheme would be a important step toward more complex quantum logic gates, serving to show the power of the transmon-qubit system for quantum information processing.

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